Series 12 – Solutions

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Chapter 26: Transfer of Pressure

Exercise 26.1 – Solution

1) Pressure wave propagation of a single harmonic can be described as:

$$p(z,t) = p_0\, \mathrm{e}^{\mathrm{i}\omega t - \gamma z} \tag{1}$$

To include attenuation we assume that γ is a complex number:

$$\gamma = a \cdot f + \mathrm{i}k \tag{2}$$

where $a \cdot f$ is the function we apply to describe the attenuation in dependency of frequency f, and $k = 2\pi f/c$ is the wave number with phase velocity c. The transfer function T_{12} between two points z_1 and z_2 is defined as the ratio (output / input):

$$T_{12} = \frac{p(z_2,t)}{p(z_1,t)} \tag{3}$$

If we take into consideration the first wave reflection, the pressure at point $z_1 = 0$ is:

$$p(z_1,t) = p_0 \mathrm{e}^{\mathrm{i}\omega t} + p_0 \Gamma \mathrm{e}^{\mathrm{i}\omega t - \gamma(2L)} = p_0 \mathrm{e}^{\mathrm{i}\omega t}\left(1 + \Gamma \mathrm{e}^{-af2L - \mathrm{i}\frac{2\pi f}{c}2L}\right) \tag{4}$$

Similarly, the pressure at point $z_2 = L$ is:

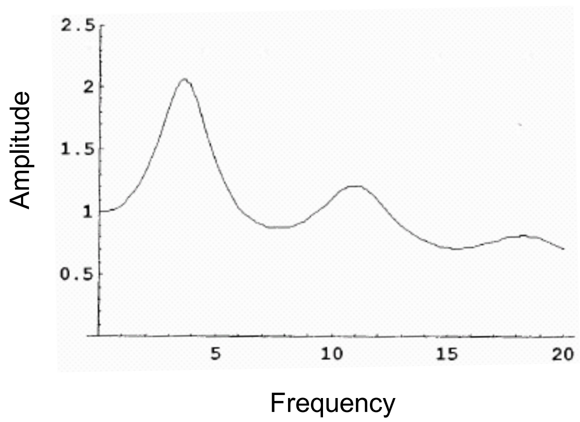
$$p(z_2,t) = p_0 \mathrm{e}^{\mathrm{i}\omega t - \gamma L} + p_0 \Gamma \mathrm{e}^{\mathrm{i}\omega t - \gamma L} = p_0 \mathrm{e}^{\mathrm{i}\omega t}(1+\Gamma)\mathrm{e}^{-afL - \mathrm{i}\frac{2\pi f}{c}L} \tag{5}$$

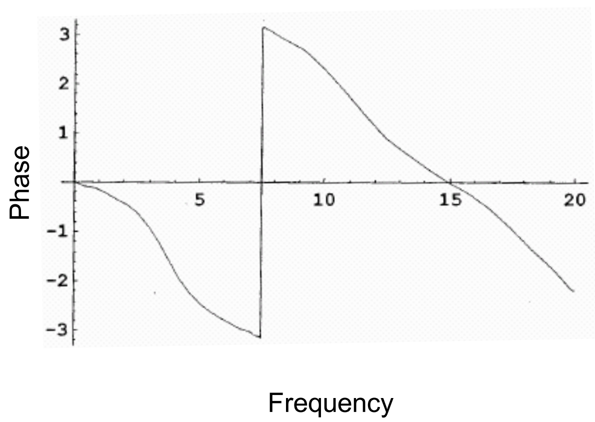
Thus Eq. (3) now becomes:

$$T_{12} = \frac{p_0 \mathrm{e}^{\mathrm{i}\omega t}(1+\Gamma)\mathrm{e}^{-afL - \mathrm{i}\frac{2\pi f}{c}L}}{p_0 \mathrm{e}^{\mathrm{i}\omega t}\left(1 + \Gamma \mathrm{e}^{-af2L - \mathrm{i}\frac{2\pi f}{c}2L}\right)} = \frac{(1+\Gamma)\mathrm{e}^{-afL - \mathrm{i}\frac{2\pi f}{c}L}}{1 + \Gamma \mathrm{e}^{-af2L - \mathrm{i}\frac{2\pi f}{c}2L}} \tag{6}$$

We notice that T_{12} is independent of both time t and pressure amplitude p_0. However, T_{12} depends on the frequency f.

2) We plot the amplitude $\left|T_{12}(f)\right|$ and phase of T_{12} over range of 0 … 20 Hz:





3) We find (numerically) the first maximum of $\left|T_{12}\left(f\right)\right|$ at 3.62 Hz. This is the 'resonance' frequency of this aortic model which we can estimate directly.

4) For the lowest 'resonance' mode, the open node (entry at z_1) and closed node (z_2) are ¼ of a wavelength apart:

$$\frac{\pi}{2} = \frac{2\pi f}{c} L \tag{7}$$

Solving Eq. (7) for frequency *f* we get:

$$f = \frac{c}{4L} = 3.75 \text{ Hz} \tag{8}$$

This is slightly more than the 3.62 found in T_{12}, since our estimate does not account for the attenuation.