Series 11 – Solutions

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Chapter 23: Arterial Input Impedance

Exercise 23.1 – Solution

1) The experimental measurements are:

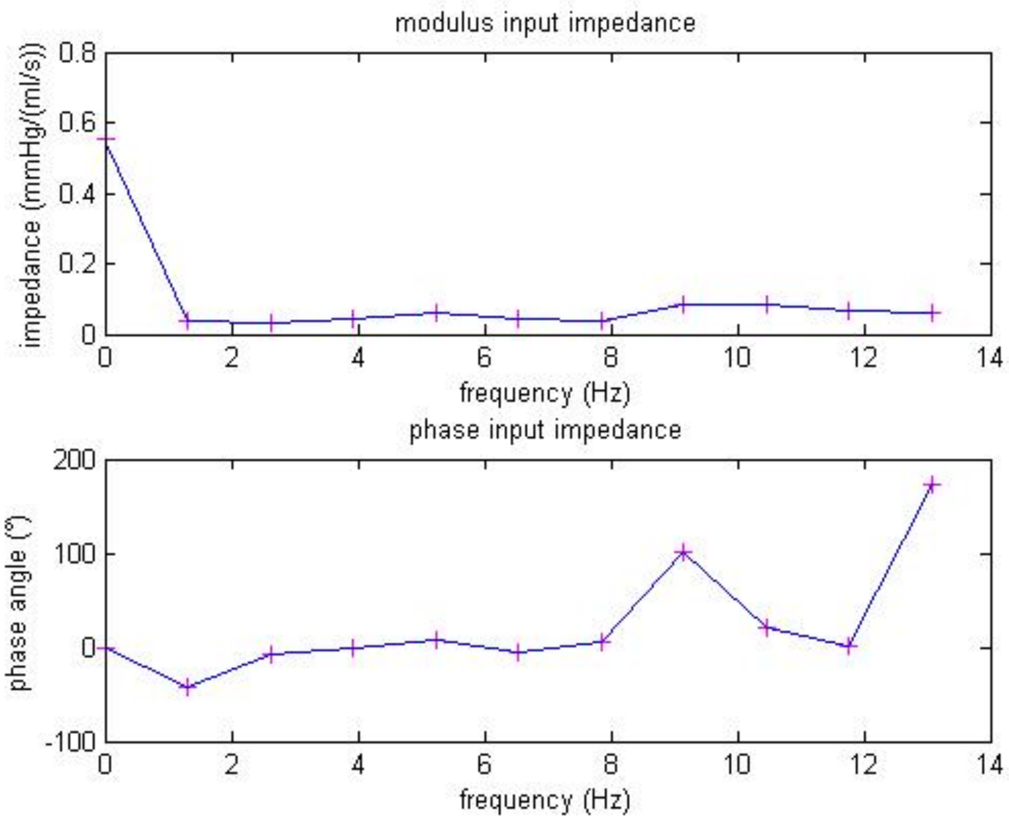
Time [s]	Pressure [mmHg]	Flow [ml/s]
0	65.537	68.649
0.00765	69.934	130.96
0.0153	79.611	226.94
0.02295	83.892	310.35
0.0306	86.697	392.74
0.03825	88.123	475.43
0.0459	89.165	538.57
0.05355	90.793	580.71
0.0612	92.098	617.24
0.06885	92.729	636.64
0.0765	93.378	641.32
0.08415	94.328	642.38
0.0918	95.24	642.58
0.09945	95.934	644.79
0.1071	96.029	634.82
0.11475	95.593	606.2
0.1224	95.516	600.31
0.13005	95.534	597.47
0.1377	95.152	583.05
0.14535	94.582	548.87
0.153	93.963	507.95
0.16065	93.335	506.59
0.1683	92.677	502.14
0.17595	92.06	470.31
0.1836	91.703	431.85
0.19125	91.395	393.3
0.1989	90.668	359.47
0.20655	90.073	332.55
0.2142	90.274	305.46
0.22185	90.656	279.27
0.2295	90.841	245.41
0.23715	90.448	218.21
0.2448	89.234	190.7
0.25245	87.474	100.99
0.2601	84.323	29.497
0.26775	77.737	-34.776

0.2754	79.139	7.2367
0.28305	85.705	37.539
0.2907	85.977	56.693
0.29835	85.443	68.467
0.306	85.145	61.324
0.31365	85.173	45.25
0.3213	85.41	34.065
0.32895	85.743	34.899
0.3366	85.801	38.409
0.34425	85.539	47.685
0.3519	85.207	60.368
0.35955	84.966	54.695
0.3672	84.689	42.525
0.37485	84.223	33.965
0.3825	83.749	25.874
0.39015	83.438	18.712
0.3978	83.463	11.887
0.40545	83.365	3.9519
0.4131	82.891	-2.9418
0.42075	82.301	-4.7806
0.4284	81.937	-1.9367
0.43605	81.591	1.5466
0.4437	81.144	2.3029
0.45135	80.403	-0.71659
0.459	79.714	-6.4193
0.46665	79.432	-15.921
0.4743	79.303	-21.537
0.48195	79.163	-18.029
0.4896	78.748	-12.542
0.49725	78.286	-8.9652
0.5049	77.805	-5.5526
0.51255	77.336	-1.3187
0.5202	76.91	2.0489
0.52785	76.604	2.4272
0.5355	76.382	-0.19447
0.54315	76.192	-4.5725
0.5508	76.002	-11.404
0.55845	75.782	-17.923
0.5661	75.476	-21.341
0.57375	75.027	-18.003
0.5814	74.571	-14.033
0.58905	74.231	-15.165
0.5967	74.02	-18.562
0.60435	73.854	-21.855
0.612	73.704	-23.082
0.61965	73.542	-22.198
0.6273	73.338	-15.919

0.63495	73.025	-8.3889
0.6426	72.572	-6.4657
0.65025	72.118	-12.034
0.6579	71.831	-19.579
0.66555	71.64	-24.75
0.6732	71.384	-27.962
0.68085	70.76	-29.825
0.6885	70.059	-29.712
0.69615	69.47	-31.472
0.7038	69.151	-35.588
0.71145	68.831	-37.807
0.7191	68.379	-34.519
0.72675	67.925	-28.306
0.7344	67.558	-19.694
0.74205	67.303	-12.938
0.7497	67.14	-8
0.75735	66.732	-5
0.765	66.988	0

Using MatLab or any mathematical software, calculate and plot the input impedance Z_{in} in the frequency domain $\omega = 2\pi f$:

$$Z_{in}(\omega) = \frac{p(\omega)}{Q(\omega)} \tag{1}$$



2i) To obtain the characteristic impedance Z_c in the frequency domain, we calculate the average of input impedance modulus over the range of frequencies 5 to 10 Hz, which gives:

$$Z_c = 0.062 \ \frac{\text{mmHg}}{\text{ml/s}} \tag{2}$$

2ii) To calculate the characteristic impedance Z_c in the time domain, we use the following relationship in the early systole (reflectionless):

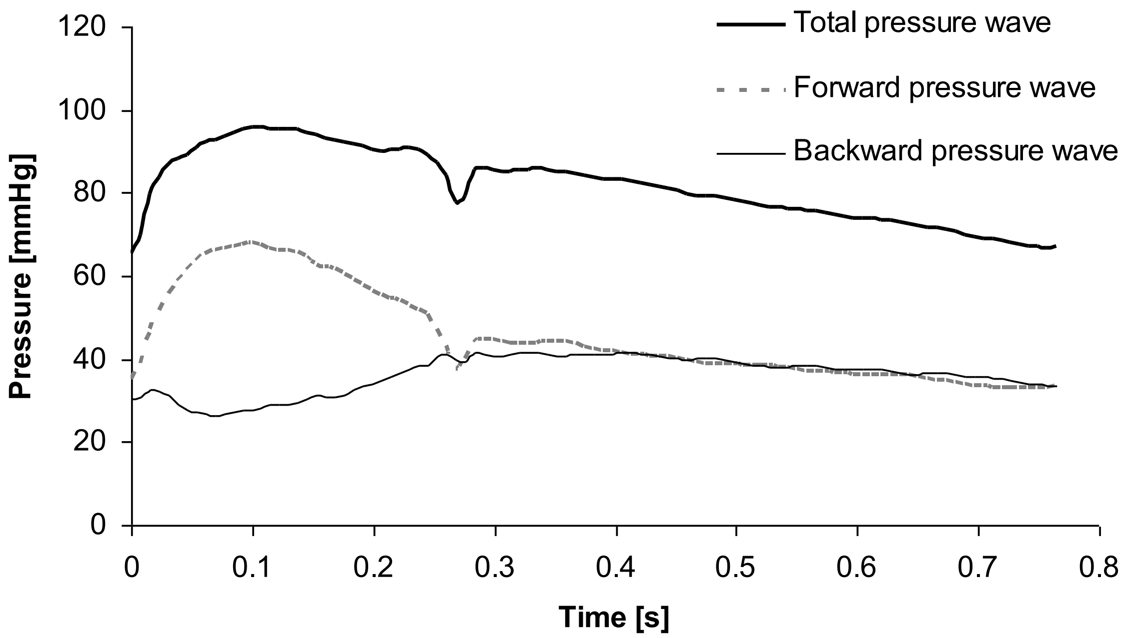
$$Z_c = \frac{\Delta p}{\Delta Q} \simeq 0.077 \ \frac{\text{mmHg}}{\text{ml/s}} \tag{3}$$

3) Waves are separated into their forward and backward running components in order to detect any early reflection. Using the below equations given in the class for the forward and backward waves of pressure

$$p_f = (p + Q \cdot Z_c) / 2 \tag{4}$$

$$p_b = (p - Q \cdot Z_c) / 2 \tag{5}$$

we obtain the following graphs:



From the graph, during the early part of the ejection phase, the reflection is small from the backward wave allowing us an approximate calculation of the characteristic impedance Z_c in the time domain from the total pressure wave. Note that characteristic impedance is a pressure-flow relation in the absence of reflections.