r-direction

$$\rho\left(\frac{\partial V_r}{\partial t}+V_r\frac{\partial V_r}{\partial r}+\frac{V_\theta}{r}\frac{\partial V_r}{\partial \theta}-\frac{{V_\theta}^2}{r}+V_x\frac{\partial V_r}{\partial x}\right)=\rho g_r-\frac{\partial p}{\partial r}+\mu\left(\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial V_r}{\partial r}\right)+\frac{1}{r^2}\frac{\partial^2 V_r}{\partial \theta^2}+\frac{\partial^2 V_r}{\partial x^2}-\frac{V_r}{r^2}-\frac{2}{r^2}\frac{\partial V_\theta}{\partial \theta}\right)$$

θ-direction

$$\rho\left(\frac{\partial V_\theta}{\partial t}+V_r\frac{\partial V_\theta}{\partial r}+\frac{V_\theta}{r}\frac{\partial V_\theta}{\partial \theta}+\frac{V_\theta V_r}{r}+V_x\frac{\partial V_\theta}{\partial x}\right)=\rho g_\theta-\frac{1}{r}\frac{\partial p}{\partial \theta}+\mu\left(\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial (rV_\theta)}{\partial r}\right)+\frac{1}{r^2}\frac{\partial^2 V_\theta}{\partial \theta^2}+\frac{\partial^2 V_\theta}{\partial x^2}-\frac{2}{r^2}\frac{\partial V_r}{\partial \theta}\right)$$

x-direction

$$\rho\left(\frac{\partial V_x}{\partial t}+V_r\frac{\partial V_x}{\partial r}+\frac{V_\theta}{r}\frac{\partial V_x}{\partial \theta}+V_x\frac{\partial V_x}{\partial x}\right)=\rho g_x-\frac{\partial p}{\partial x}+\mu\left(\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial V_x}{\partial r}\right)+\frac{1}{r^2}\frac{\partial^2 V_x}{\partial \theta^2}+\frac{\partial^2 V_x}{\partial x^2}\right)$$

Assume:

- Steady flow: $\frac{\partial}{\partial t}=0$
- Velocity components V_r and V_θ are zero
- Velocity component V_x depends only on r.
- Gravitational terms can be neglected.

So you have:

r-direction

$$0=\frac{\partial p}{\partial r}$$

θ-direction

$$0=\frac{\partial p}{\partial \theta}$$

x-direction

$$\rho\left(\frac{\partial V_x}{\partial t}+V_r\frac{\partial V_x}{\partial r}+\frac{V_\theta}{r}\frac{\partial V_x}{\partial \theta}+V_x\frac{\partial V_x}{\partial x}\right)=\rho g_x-\frac{\partial p}{\partial x}+\mu\left(\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial V_x}{\partial r}\right)+\frac{1}{r^2}\frac{\partial^2 V_x}{\partial \theta^2}+\frac{\partial^2 V_x}{\partial x^2}\right)$$

with :

$$\frac{\partial V_x}{\partial t}=0\ \ (steady)$$
$$V_r\frac{\partial V_x}{\partial r}=0\ \ (V_r=0)$$
$$\frac{V_\theta}{r}\frac{\partial V_x}{\partial \theta}=0\ \ (V_\theta=0)$$

$$\mathrm{V}_x \frac{\partial \mathrm{V}_x}{\partial x} = 0 \; (\mathrm{V}_x \; \textit{depends only on } r)$$

$$\rho g_x \; \textit{neglected}$$

$$\frac{1}{\mathrm{r}^2} \frac{\partial^2 \mathrm{V}_x}{\partial \theta^2} = 0 \; (\mathrm{V}_x \; \textit{depends only on } r)$$

$$\frac{\partial^2 \mathrm{V_x}}{\partial \mathrm{x}^2} = 0 \; (\mathrm{V}_x \; \textit{depends only on } r)$$

So we are left with:

$$0 = -\frac{\partial \mathrm{p}}{\partial \mathrm{x}} + \mu \left(\frac{1}{\mathrm{r}} \frac{\partial}{\partial \mathrm{r}} \left(\mathrm{r} \frac{\partial \mathrm{V}_x}{\partial \mathrm{r}} \right) \right)$$