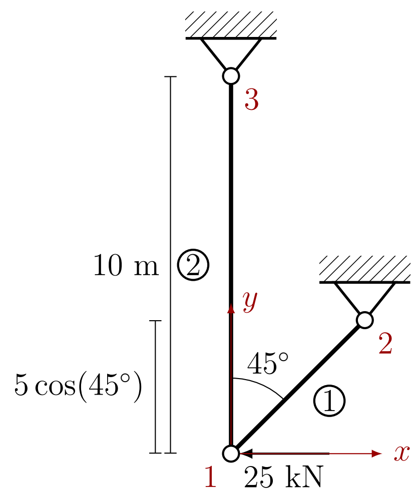
Problem set 5

Problem 1

Consider the two-dimensional truss structure depicted in the figure below. The system consists of two linear elastic truss elements, each characterized by a modulus of elasticity E and a constant cross-sectional area A. The structure is modeled using three nodes and two finite elements within a global Cartesian coordinate system whose origin is located at node 1.



The truss is subjected to an external horizontal load of 25 kN applied at node 1.

a) Degree of freedom classification: using the finite element discretization, the system has a total of six degrees of freedom, two per node. Identify which of these degrees of freedom are constrained due to boundary conditions, and which are unconstrained.

b) Reduced system formulation: by eliminating the constrained degrees of freedom, compute the reduced stiffness matrix and the reduced applied loads vector corresponding to the unconstrained degrees of freedom of the structure.

c) Displacement calculation: solve the reduced system to obtain the displacements at the unconstrained degrees of freedom.

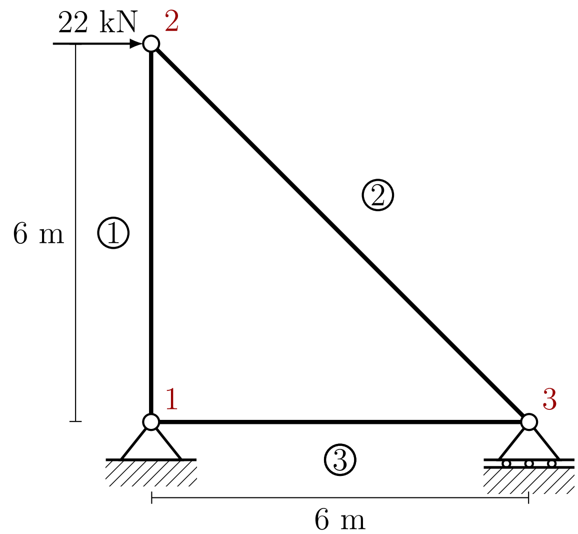
d) Stress evaluation: using the computed displacements, determine the approximate axial stress in each truss element. Then, provide a physical interpretation of the results: which member is in tension or compression, and why?

Problem 2

Consider the two-dimensional truss structure illustrated below, composed of three inclined bars made of structural steel. The material and geometric properties of each member are as follows:

- Young's modulus: $E = 210\,\text{GPa}$
- Cross-sectional area: $A = 6.45 \times 10^{-4}\,\text{m}^2$
- Density $\rho = 7800\,\text{kg/m}^3$

Node 1 is fully constrained, while node 3 is constrained in the vertical (y) direction only. These boundary conditions result in a system with three unconstrained degrees of freedom.



Develop a `MATLAB` script that performs the following tasks:

1. Assemble the global stiffness matrix $\mathbf{K}$ and consistent mass matrix $\mathbf{M}$ of the truss structure.
2. Compute the natural frequencies ω_i and the corresponding mode shapes $\boldsymbol{\phi}_i$.
3. Determine the time response of the structure when subjected to a constant external force $f = 22\,\text{kN}$ applied at node 1 along the horizontal (x) direction.

   a) Use modal decomposition to transform the coupled second-order differential system

   $$\mathbf{M}\ddot{\mathbf{q}}(t) + \mathbf{K}\mathbf{q}(t) = \mathbf{f}(t)$$

   into a system of uncoupled scalar equations by expressing the displacement vector as $\mathbf{q}(t) = \boldsymbol{\Phi}\,\mathbf{z}(t)$, where $\boldsymbol{\Phi}$ is the modal matrix formed by mass-normalized mode shapes.

   b) More precisely, apply the orthogonality conditions

   $$\boldsymbol{\Phi}^T\mathbf{M}\boldsymbol{\Phi} = \mathbf{I}, \qquad \boldsymbol{\Phi}^T\mathbf{K}\boldsymbol{\Phi} = \boldsymbol{\Lambda}$$

   with $\boldsymbol{\Lambda} = \text{diag}(\lambda_1, \ldots, \lambda_n)$, to derive and solve the uncoupled system governing the modal coordinates $\mathbf{z}(t)$.

   c) Solve the uncoupled system and compute the time response vector $\mathbf{q}$ at the unconstrained degrees of freedom.