Problem set 4

Problem 1

We consider the transient heat conduction problem on a circular aluminum plate, denoted by Ω, with a radius $a = 10\,\text{cm}$. The governing equation for the temperature distribution $T(x, y, t)$ is given by

$$\rho c \frac{\partial T}{\partial t} = \text{div}(k\nabla T) + f_0, \quad \text{in } \Omega, \quad t > 0,$$

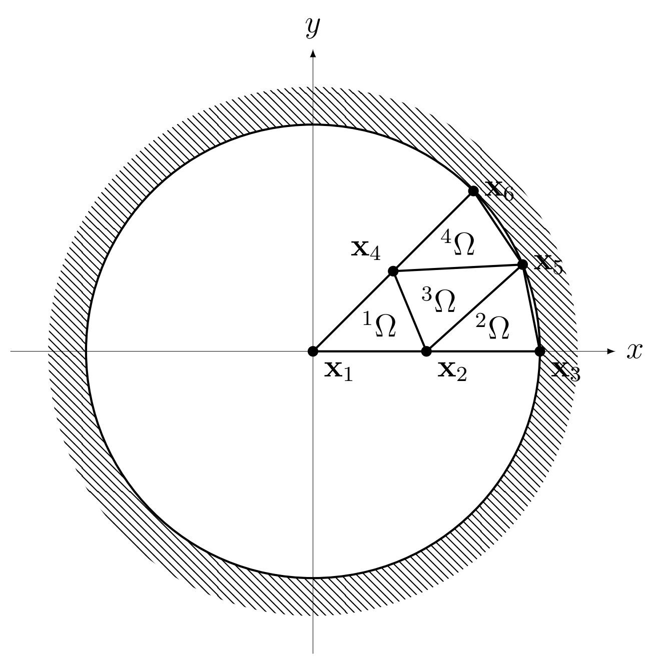
where:

- $\rho = 2700\,\text{kg/m}^3$ is the density of aluminum,
- $c = 900\,\text{J/kg} \cdot \text{K}$ is the specific heat capacity of aluminum,
- $k = 205\,\text{W/m} \cdot \text{K}$ is the thermal conductivity of aluminum,
- $f_0 = 10^6\,\text{W/m}^2$ is the uniform internal heat source.

The temperature satisfies the following conditions:

- Boundary condition: $T = 0$ on $\partial\Omega$, the boundary of Ω.
- Initial condition: $T(x, y, 0) = 0$, for all $(x, y) \in \Omega$.

Due to the symmetry of the problem, the temperature distribution is approximated using four bilinear triangular elements on an angular sector of 45°, as illustrated in the following diagram:



The spatial discretization of the weak form leads to the following semi-discrete system:

$$\mathbf{M}\dot{\mathbf{q}} + \mathbf{K}\mathbf{q} = \mathbf{r},$$

where:

- $\mathbf{q}$ is the vector of nodal temperatures,
- $\mathbf{M}$ is the heat capacity matrix,
- $\mathbf{K}$ is the thermal conductivity matrix,
- $\mathbf{r}$ is the heat source vector.

Write a `MATLAB` script to:

1. compute the global thermal conductivity and heat capacity matrices,
2. compute the heat source vector $\mathbf{r}$ corresponding to the internal heat source f_0,
3. solve the time-dependent system for a time span of 20 s,
4. plot the temperature distribution over time.