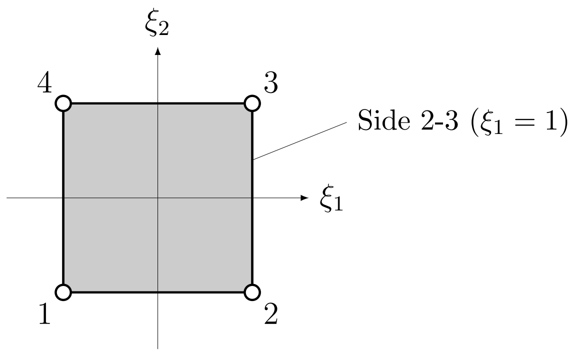
Problem set 7

Problem 1

Consider the shape functions for the AMC element:

$$
{}^{a}\mathbf{h}_i(\boldsymbol{\xi}) = \begin{bmatrix} (1+\xi_1^i\xi_1)(1+\xi_2^i\xi_2)(2+\xi_1^i\xi_1+\xi_2^i\xi_2-\xi_1^2-\xi_2^2)/8 \\ b(1+\xi_1^i\xi_1)(\xi_2^i+\xi_2)(\xi_2^2-1)/8 \\ -a(\xi_1^i+\xi_1)(\xi_1^2-1)(1+\xi_2^i\xi_2)/8 \end{bmatrix}^T
$$

where $\boldsymbol{\xi}^i = \{\xi_1^i, \xi_2^i\}^T$ are the local coordinates of node $i = 1, \ldots, 4$.



a) Show that, for $i, j = 1, \ldots 4$, the Kronecker delta property holds:

$$
{}^{a}\mathbf{h}_i(\boldsymbol{\xi}^j) = \begin{bmatrix} \delta_{ij} \\ 0 \\ 0 \end{bmatrix}^T
$$

where δ_{ij} is the Kronecker symbol.

b) Show that the generalized displacement function defined by

$$
{}^{e}u_3^h(\boldsymbol{\xi}, t) = \sum_{i=1}^{4} {}^{a}\mathbf{h}_i(\boldsymbol{\xi}) \begin{bmatrix} {}^{e}d^i(t) \\ {}^{e}\theta_1^i(t) \\ {}^{e}\theta_2^i(t) \end{bmatrix} = {}^{a}\mathbf{H}(\boldsymbol{\xi})^{e}\mathbf{q}(t)
$$

is a non-conforming one by evaluating u_3^h, θ_1^h and θ_2^h on the side 2-3 ($\xi_1 = 1$) and by showing that θ_2^h could be discontinuous along this edge.

Problem 2

Demonstrate that, in the AMC plate bending element, selecting a displacement function that includes the quartic terms x^4 and y^4 instead of the mixed cubic terms x^3y and xy^3 results in displacement discontinuities along the element boundaries.

Problem 3

Consider the following shape functions:

$$
{}^{a}\mathbf{h}_i(\boldsymbol{\xi}) = \begin{bmatrix} f_i(\xi_1) f_i(\xi_2) \\ b f_i(\xi_1) g_i(\xi_2) \\ -a g_i(\xi_1) f_i(\xi_2) \end{bmatrix}^T
$$

where $\boldsymbol{\xi}^i = \{\xi_1^i, \xi_2^i\}^T$ are the local coordinates of node $i = 1, \ldots, 4$ and f_i and g_i are the Hermite cubic shape functions:

$$
f_i(\xi) = (-\xi^i \xi^3 + 3\xi^i \xi + 2)/4, \qquad g_i(\xi) = (\xi^3 + \xi^i \xi^2 - \xi - \xi^i)/4.
$$

Show that the generalized displacement function, defined by

$$
{}^{e}u_3^h(\boldsymbol{\xi}, t) = \sum_{i=1}^{4} {}^{a}\mathbf{h}_i(\boldsymbol{\xi}) \begin{bmatrix} {}^{e}d^i(t) \\ {}^{e}\theta_1^i(t) \\ {}^{e}\theta_2^i(t) \end{bmatrix} = {}^{a}\mathbf{H}(\boldsymbol{\xi})\, {}^{e}\mathbf{q}(t),
$$

has twist, $\frac{\partial^2}{\partial \xi_1 \partial \xi_2} {}^{e}u_3^h(\boldsymbol{\xi}, t)$, zero at the four nodal points.