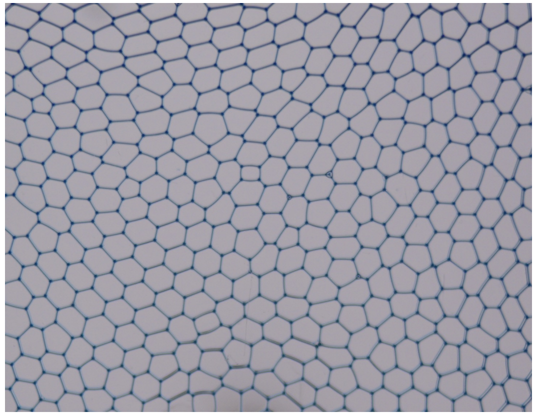
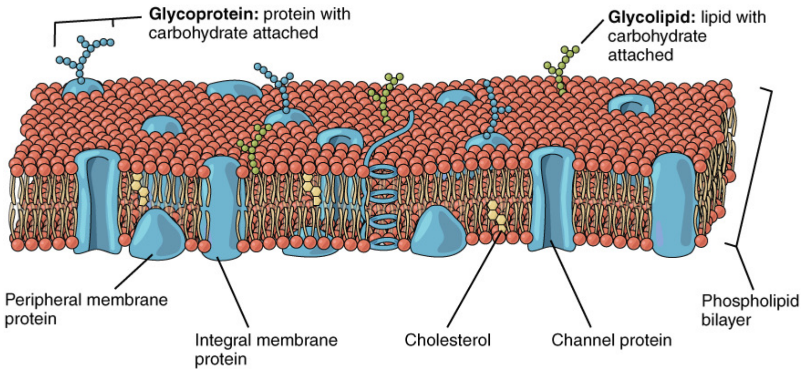
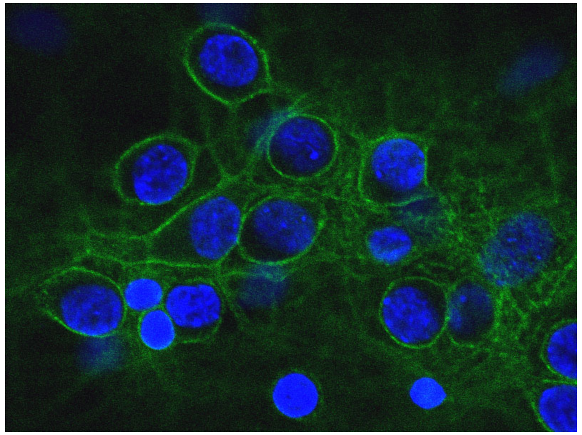
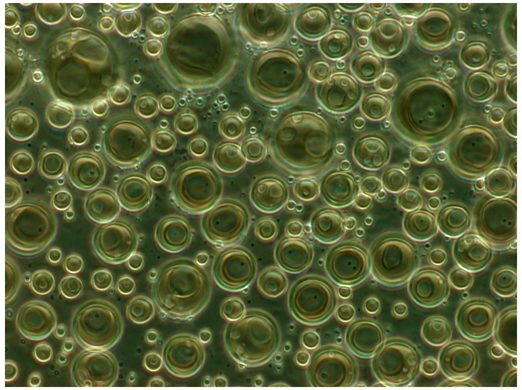
ME470: Mechanics of Soft and Biological Matter
Lecture12: Cell Mechanics









Red Blood Cells

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MESOBIO – IGM – STI – EPFL

Adhesion of Membranes

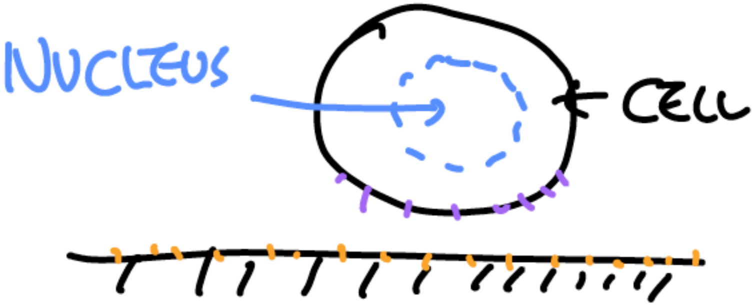
How to cell collectives maintain their structural integrity ? ⇨ Adhesion!

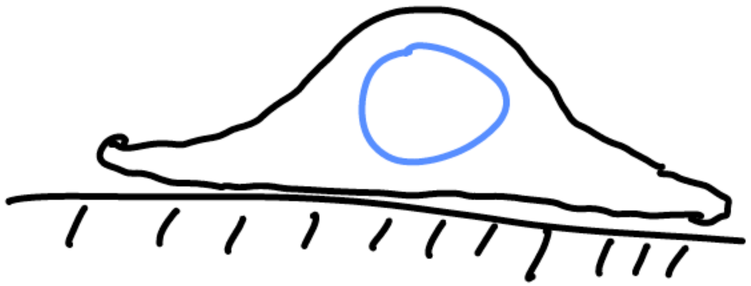
- Cell – substrate: e.g. integrins, organized in focal adhesion
- Cell – cell: cadherins, organized in desmosomes

⇨ Described as $\frac{Adhesion\ energy}{area} \equiv W$

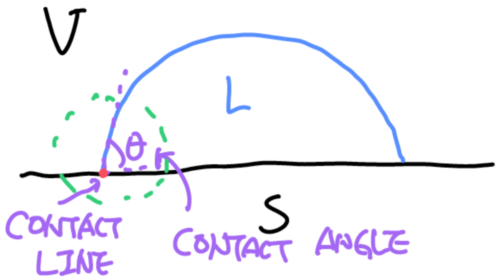
- Typical bond energies of adhesion molecules: $1 - 35k_BT$
- Typical adhesion energy: $\mathrm{W}{\sim}10^{-5}J/m$
  (molecule density: $n{\sim}5{\times}10^{16}1/m^2$

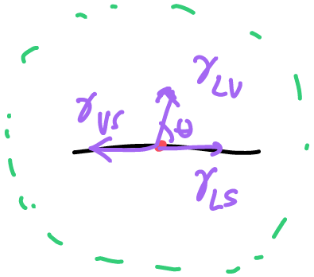
Contact Line & Adhesion





Analogous to droplet spreading?





$$\gamma_{VS} = \gamma_{LS} + \gamma_{LV} \cos\theta$$

$$\cos\theta = \frac{\gamma_{VS} - \gamma_{LS}}{\gamma_{LV}}$$

Alternatively, minimize energy

$$E_{tot} = \gamma_{LV} \cdot A_{curved} + \gamma_{LS} \cdot A_{flat} + \gamma_{VS}\left(A_0 - A_{flat}\right)$$

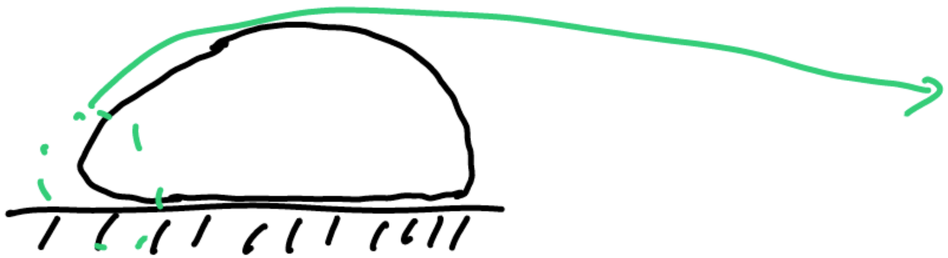
$$\mathrm{d}E_{tot} = \gamma_{LV} \cdot dA_{curved} + (\gamma_{LS} - \gamma_{VS})dA_{flat}$$

$$\frac{dA_{curved}}{dA_{flat}} = \cos\theta$$

$$\mathrm{d}E_{tot} = 0 \Rightarrow \gamma_{VS} - \gamma_{LS} = \gamma_{LV} \cos\theta$$

Membrane-Substrate adhesion

Droplet picture has divergent H @ contact line ⇨ U_{bend} diverges!



Simple energy balance: $U_{bend} + U_{adh} = U_{tot}$

$$\Rightarrow \quad \frac{1}{2}k_b\left(\frac{1}{R_c}\right)^2 R_c(\pi - \theta_{eff}) - W\left[L_0 - R_c(\pi - \theta_{eff})\right] = U_{tot}$$

$$\delta U_{tot} = 0 \quad \Rightarrow \quad W = \frac{k_b}{2R_c{}^2}$$

Estimate, assuming only local change of geometry (near contact line), but generally valid in 2D and 3D objects

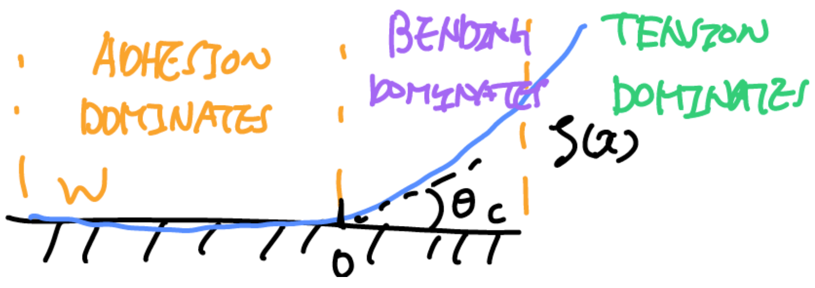
Seifert & Lipowsky, PRA, 1990, 1991

EPFL

Membrane-Substrate adhesion

What if there is also a tension τ in membrane?

$$U_{tot} = \frac{1}{2}k_b \iint (2H)^2 dA + \tau \int dA - W \int_{flat} dA$$



$$U_{tot} = \frac{1}{2}k_b \int_0^{\infty} \varsigma''(x)^2\, dx - W\int_{-\infty}^{0} dx + \tau \int_{-\infty}^{0} dx + \tau\int_0^{\infty} \sqrt{1+\varsigma'(x)^2} dA$$

$$\approx \int_0^{\infty} \left(\frac{1}{2}k_b \varsigma''(x)^2 + \frac{1}{2}\tau\varsigma'(x)^2\right) dx - W\int_{-\infty}^{0} dx + \tau\int_{-\infty}^{\infty} dx$$

constant

Variational calculus: $\varsigma(x) = Ae^{x/l} + Be^{-x/l} + Cx + D \qquad l \equiv k_b/\tau$

Boundary conditions:

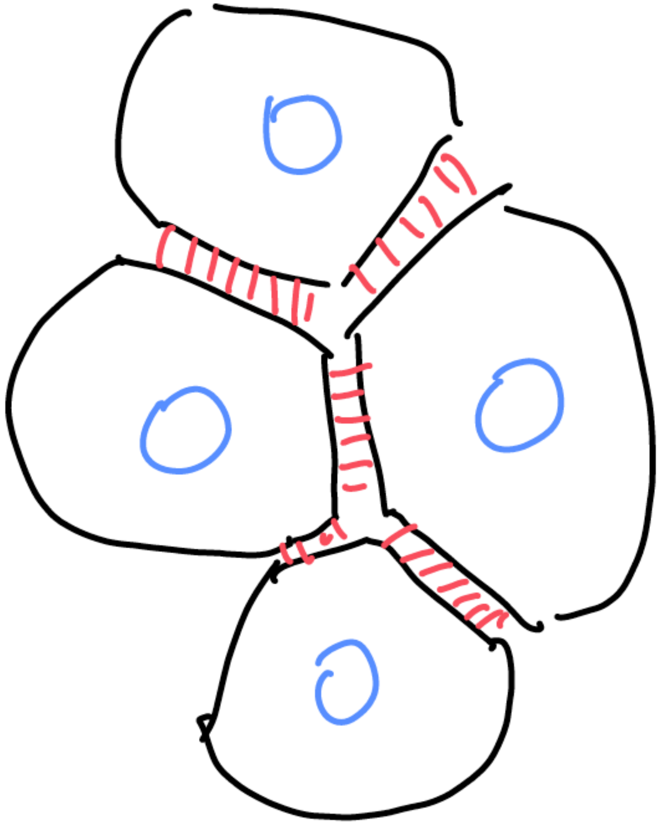
- Regularity, $A = 0$
- $\varsigma(x = 0) = 0$
- $\varsigma(x \to \infty) = (\tan\theta_c)x$
- $\varsigma''(x = 0) = 1/R_c$
- $\varsigma'(x = 0) = 0$

$$\Rightarrow \quad \varsigma(x) = \sqrt{\frac{2W}{\tau}}x + \sqrt{\frac{2Wk_b}{\tau^2}}\left(e^{-\sqrt{\tau/k_b}x} - 1\right)$$

School of Engineering

Simson et. at. Biophysical Journal, 1998

Cell-cell adhesion



Differential adhesion hypothesis (also generalized as differential interfacial tension hypothesis)

Consider cell types A, B: model as droplets with uniform surface tension, γ, and specific surface tensions

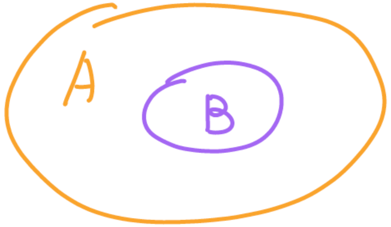
$$\gamma_{AA} = \gamma - W_{AA}$$

$$\gamma_{BB} = \gamma - W_{BB}$$

$$\gamma_{AB} = \gamma - W_{AB}$$

If adhesion strength is such that $W_{AA} > W_{AB} > W_{BB}$

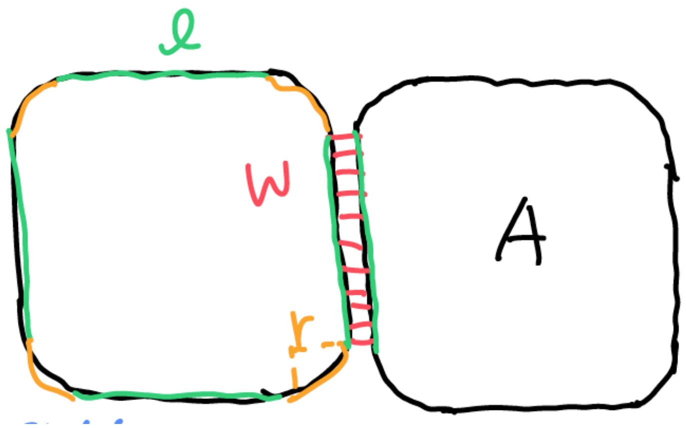
⇨ Tissue segregation



Steinberg, Exp. Zool. 1970

Competition between Adhesion & Bending

Simple model:
(2D, uniform in $\hat{z}$)



Assume no volume change: $A = constant = l^2 + 4rl + \pi r^2$

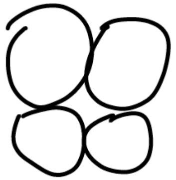
$$r = \frac{1}{\pi}\left[-2l + \sqrt{(4-\pi)l^2 + A\pi}\right]$$

$$U_{tot} = \frac{1}{2}k_b \cdot \frac{2\pi}{r} - 2Wl$$

$$\widetilde{U} = \frac{U_{tot}}{k_b} = \frac{\pi}{r} - 2\widetilde{W}l \qquad \widetilde{W} \equiv \frac{W}{k_b}$$

Competition between Adhesion & Bending

$\widetilde{W} = 0$, expect $l = 0$



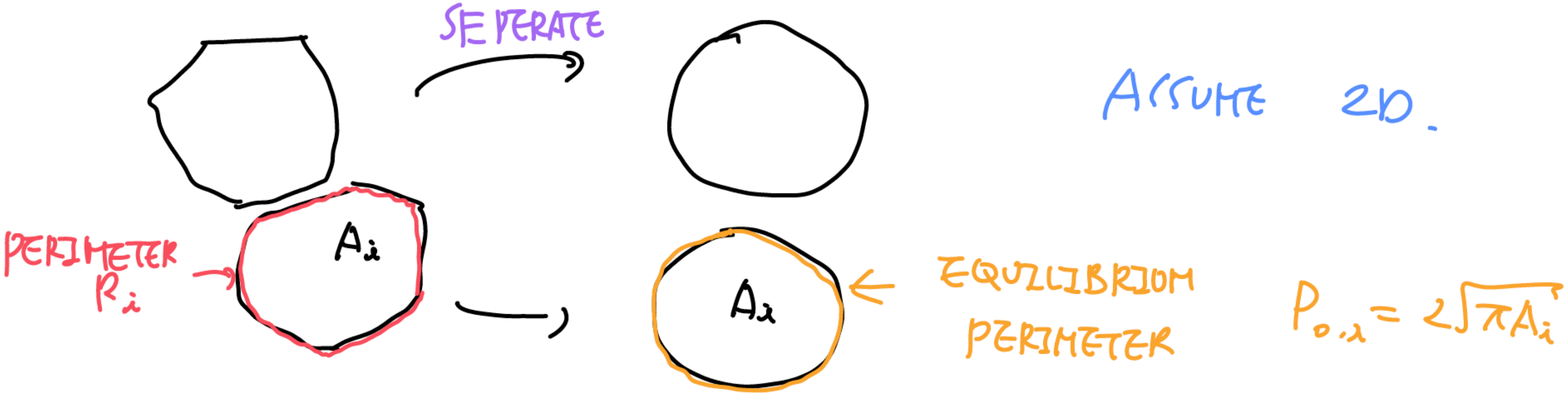
Q: is there a critical adhesion for finite attachment?

⇨ To have finite l at equilibrium, we need a condition such that

$$\left.\frac{d\widetilde{U}}{dl}\right|_{l=0} < 0 \qquad\qquad \frac{d\widetilde{U}}{dl} = -\frac{\pi}{r(l)^2}\frac{dr}{dl} - 2\widetilde{W}$$

⇨ $$\boxed{\widetilde{W} = \frac{\pi}{A}}$$

Competition between Adhesion & Stretching



$$U_{stretch,i} = \frac{1}{2} K_A \frac{\left(P_i - P_{i,0}\right)^2}{P_{i,0}} \qquad U_{adh,i} = -\frac{1}{2} W P_i$$

- Minimize energy to get tissue equilibrium structures
- Can add a bending energy term

$$U_{b,ij} = \frac{1}{2} k_b {c_{ij}}^2 L_{ij} \qquad k_b \approx \frac{1}{12} K_A h^2$$

Between cell i and j

Emergent Properties of Biological Tissues

Cells are main constituent units of biological tissues

⇨ Interactions between cells determines emergent properties of tissues

⇨ Large number of cells = huge # of degrees of freedom

Need numerical models (discrete models) to study tissue mechanics!!