Exercise 5.1

Thermal boundary resistance. Estimate the thermal boundary resistance between two materials with the following properties on the basis of the diffuse interface scattering model: material 1: $v_1 = 3900\,\mathrm{ms^{-1}}$, $C_1 = 1.67 \times 10^6\,\mathrm{Jm^{-3}K^{-1}}$; material 2: $v_2 = 6400\,\mathrm{ms^{-1}}$, $C_2 = 1.66 \times 10^6\,\mathrm{Jm^{-3}K^{-1}}$. For a heat flux of $10^8\,\mathrm{Wm^{-2}}$, estimate the temperature drop occurring at the interface.

Exercise 5.2

Thermal boundary resistance at low temperature. Thermal boundary resistance is a phenomenon that is important at low temperatures even for bulk materials and becomes important even at room temperature in nanostructures, determined by

$$\frac{1}{R_e} = \frac{1}{2}\int_0^1 \mu_1 d\mu_1 \int_0^{\omega_D} v_1 C_1(\omega)\tau_{12}(\omega, \mu_1) d\omega.$$

In the above equation, v_1 is sound velocity in medium 1, $C_1(\omega)$ is spectral heat capacity of medium 1, τ_{12} is phonon transmissivity from medium 1 to medium 2, μ_1 is diectional cosine, and ω_D is Debye frequency. At low temperatures, it can be shown that thermal boundary resistance obeys the relation $R_e^{-1} \propto T^3$. Treating the transmissivity τ_{12} as independent of angle and frequency, derive an expression for its proportionality coefficient between R_e^{-1} and T^3 at low temperatures.

Exercise 5.3

Phonon thermal conductivity at intermediate temperature The phonon-phonon scattering relaxation time in the intermediate range of temperature (when $T < \theta_D$) can be approximated as

$$\frac{1}{\tau} = Aexp[-\frac{\Theta}{aT}]T^3\omega^2$$

On the basis of the Debye model(linear dispersion), derive an expression for the thermal conductivity and discuss its dependence on temperature.

Exercise 5.4

Heat generation distribution due to absorption. A plane wave with an intensity of $10^4\ Wm^{-2}$ at $\lambda_0 = 0.517\ \mu m$ meets a gold surface at 30^o of incidence. Determine the heat generation distribution inside the gold specimen. The refractive index at $0.517\ \mu m$ is $N = n + i\kappa = 0.608 + 2.12i$.