Exercise 1.1

Phonon mean free path and relaxation time. Given the thermal conductivity of Si at room temperature as $145\,\mathrm{Wm^{-1}K^{-1}}$, the speed of sound as $6400\,\mathrm{ms^{-1}}$, the volumetric specific heat as $1.66 \times 10^{6}\,\mathrm{Jm^{-3}K^{-1}}$,

a) Estimate the phonon mean free path in Si at room temperature from the kinetic theory. In reality, this estimation usually leads to a much shorter mean free path (about a factor of 10 shorter) than with more sophisticated modelling.

b) Estimate the relaxation time of phonons in silicon.

Solution

(a) According to the kinetic theory for a phonon gas, the thermal conductivity κ equals to $\frac{1}{3}Cv\Lambda$, so the mean free path Λ of phonons in Si at room temperature is

$$\Lambda = \frac{3\kappa}{Cv} = \frac{3 \times 145}{6400 \times 1.66 \times 10^{6}} \approx 4.1 \times 10^{-8}\,\mathrm{m} = 41\,\mathrm{nm}.$$

(b) The corresponding relaxation time of phonons in silicon at room temperature is

$$\tau = \frac{\Lambda}{v} = \frac{4.1 \times 10^{-8}}{6400} \approx 6.4 \times 10^{-12}\,\mathrm{s} = 6.4\,\mathrm{ps}.$$

Exercise 1.2

$k_B T$ energy. One unit for energy is the electron-volt (eV). It is the energy difference of one electron under a potential difference of 1 V. Convert 1 $k_B T$ at 300 K into milli-eV (meV).

Note: The Boltzmann constant $k_B = 1.38064852 \times 10^{-23}\,\mathrm{J/K}$.

Solution

When $T = 300\,\mathrm{K}$ and $1\,\mathrm{eV} = 1.602177 \times 10^{-19}\,\mathrm{J}$,

$$k_B T = \frac{1.38064852 \times 10^{-23} \times 300 \times 1000}{1.602177 \times 10^{-19}} = 25.85\,\mathrm{meV}.$$

Roughly, 1 $k_B T$ at 300 K is about 26 meV. This is an important energy-scale that you are encouraged to remember, and the kinetic energy of an atom at room temperature is $\frac{3}{2} k_B T = 39\,\mathrm{meV}$.

Exercise 1.3

Fick's Law. Using a simple kinetic argument that is similar to the derivation of the Fourier law, derive the Fick's law of diffusion, which gives the mass flux for species i under a concentration gradient as:

$$J_i = -\rho D \frac{dm_i}{dz}$$

where D is the mass diffusivity, p is the density of the mixture, and m_i the local mass fraction of species i. Also, state the assumptions made during this analysis.

Solution

Let us determine the diffusive flux of molecules from a region of high concentration to a region of low concentration (z-direction) through the imaginary surface with arbitrary area A and position z. We follow a similar derivation of $L1$ for the heat flux.

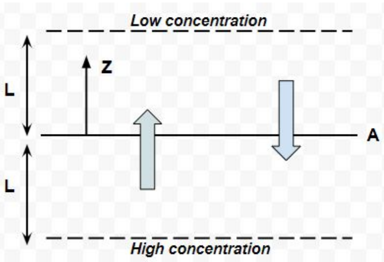

Figure 1: Illustration of mass diffusion due to molecules's random motion across the imaginary surface

Let c_i be the concentration and m_i be the mass fraction of the i^{th} species. Then it can be shown that:

$$c_i = \frac{\rho}{M_i} * m_i$$

where ρ is the density of the mixture and M_i is the Molecular mass of the i^{th} species.

If L is the mean free path of the molecules, we can write the mass flux of the i^{th} specie as:

$$j_i(z) = \frac{1}{2} v_z M_i [c_i(z-L) - c_i(z+L)]$$

Using Taylor expansion, $c_i(z-L) = c_i + \frac{dc_i}{dz}(z_L)$, we get:

$$j_i(z) = -v_x L M_i \frac{dc_i}{dz}$$

We then assume that* $v_z = 1/3v$ and therefore combining with the expression of the concentration we get:

$$j_i(z) = -\frac{1}{3} v L \rho \frac{dm_i}{dz}$$

If $D = -\frac{1}{3}vL$ is defined as the diffusion coefficient, we get:

$$j_i(z) = -D\rho \frac{dm_i}{dz}$$

*What if we don't make this assumption and start with the fact that molecules can move randomly in all direction. In this case molecules can move any angle θ not just along the 3 mutually perpendicular axis. Can we do such an analysis? If yes, then will the results be the same?

Exercise 1.4

Planck-Einstein relations.

(1) An argon laser emits light at 514 nm and at a power of 1 W. Calculate:

a) The frequency of the photons in Hz

b) Their wavelength, expressed as a wavenumber

c) The energy of each photon

d) The momentum of each photon

e) The number of photons generated per second

(2) If the photons are completely absorbed by a $1mm^2$ surface, calculate

a) The pressure exerted on the surface by the photons

b) The heat flux generated by the photon absorption

Solution

1.a) For an Electromagnetic wave, frequency and wavelength are related as: $\lambda * \nu = c$, where c is the speed of light.

$$\nu = \frac{c}{\lambda} = \frac{3 \times 10^8}{514 \times 10^{-9}} = 5.8366 \times 10^{14} Hz$$

1.b)

$$wavenumber = k = \frac{1}{\lambda} = \frac{1}{514 \times 10^{-9}} = 1.9455 \times 10^6 m^{-1} = 19455 cm^{-1}$$

1.c)

$$E = h\nu = (6.626 \times 10^{-34}) \times (5.8366 \times 10^{14}) = 3.8673 \times 10^{-19} J$$

$$Energy\ in\ eV = \frac{Energy\ in\ Joule}{electronic\ charge} = \frac{3.8673 \times 10^{-19}}{1.6 \times 10^{-19}} = 2.417 eV$$

1.d)

$$momentum = p = \frac{h}{\lambda} = \frac{6.626 \times 10^{-34}}{514 \times 10^{-9}} = 1.289 \times 10^{-23} Ns$$

1.e) The number of photons generated, $n \left(\frac{photons}{sec}\right)$

$$Power \left(\frac{J}{sec}\right) = Energy\ of\ each\ Photon \left(\frac{J}{photon}\right) \times n \left(\frac{photons}{sec}\right)$$

$$n = \frac{Power}{Energy\ of\ each\ photon} = \frac{1}{3.8673 \times 10^{-19}} = 2.586 \times 10^{18} \left(\frac{photons}{sec}\right)$$

2.a) Force exerted on the surface due to complete absorption of the photons = rate of change of momentum due to photon absorption

$$F = \frac{\Delta p}{\Delta t} = n \left(\frac{photons}{sec}\right) \times p \left(\frac{Ns}{photon}\right)$$

$$F = \left(2.586 \times 10^{18}\right) \times \left(1.289 \times 10^{-23}\right) = 3.33 \times 10^{-5} N$$

$$Pressure\ exerted = \frac{Force}{Area} = \frac{3.33 \times 10^{-5} N}{10^{-6} m^2} = 33.33 \frac{N}{m^2}$$

2.b) Assuming complete absorption of photons and 100 percent conversion efficiency of light-to-heat, we have that the heat flux q" is equal to the energy absorbed per unit area and per unit time, i.e.

$$Heat\ flux\ = \frac{Power}{Area\ of\ the\ surface} = \frac{1W}{10^{-6}\ m^2} = 10^6 \frac{W}{m^2}$$

Note: In reality the flux generated will be much smaller than this because, photon absorption is never 100 percent due reflection and transmission. Even after the absorption of certain percentage of incident photons, the light-to-heat conversion is not 100 percent effective.