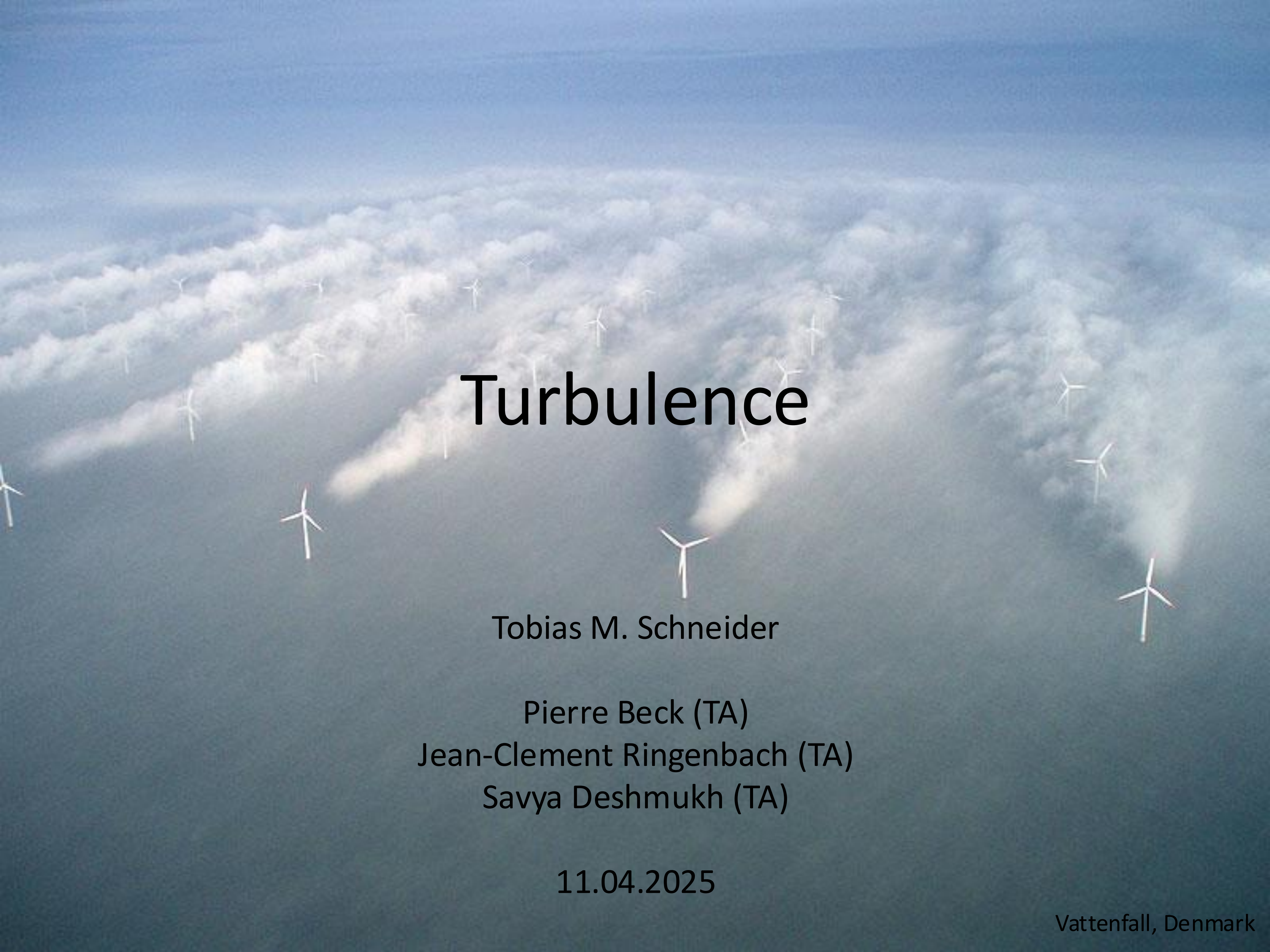


Turbulence

Tobias M. Schneider

Pierre Beck (TA)
Jean-Clement Ringenbach (TA)
Savya Deshmukh (TA)

11.04.2025

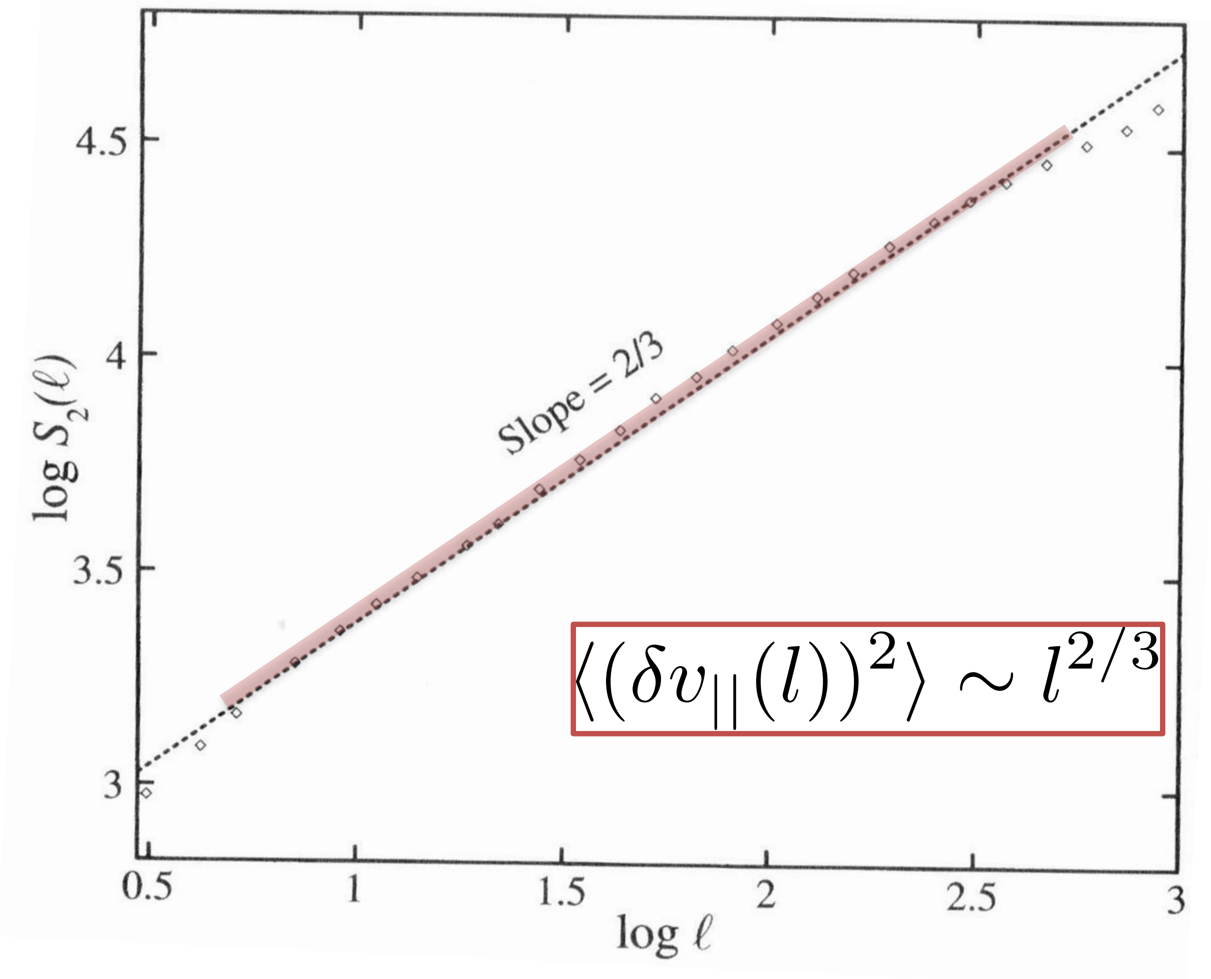
Vattenfall, Denmark

Two experimental laws of fully developed turbulence

REMINDER

5.1. The two-third law of the second order structure function

ONERA S1 windtunnel

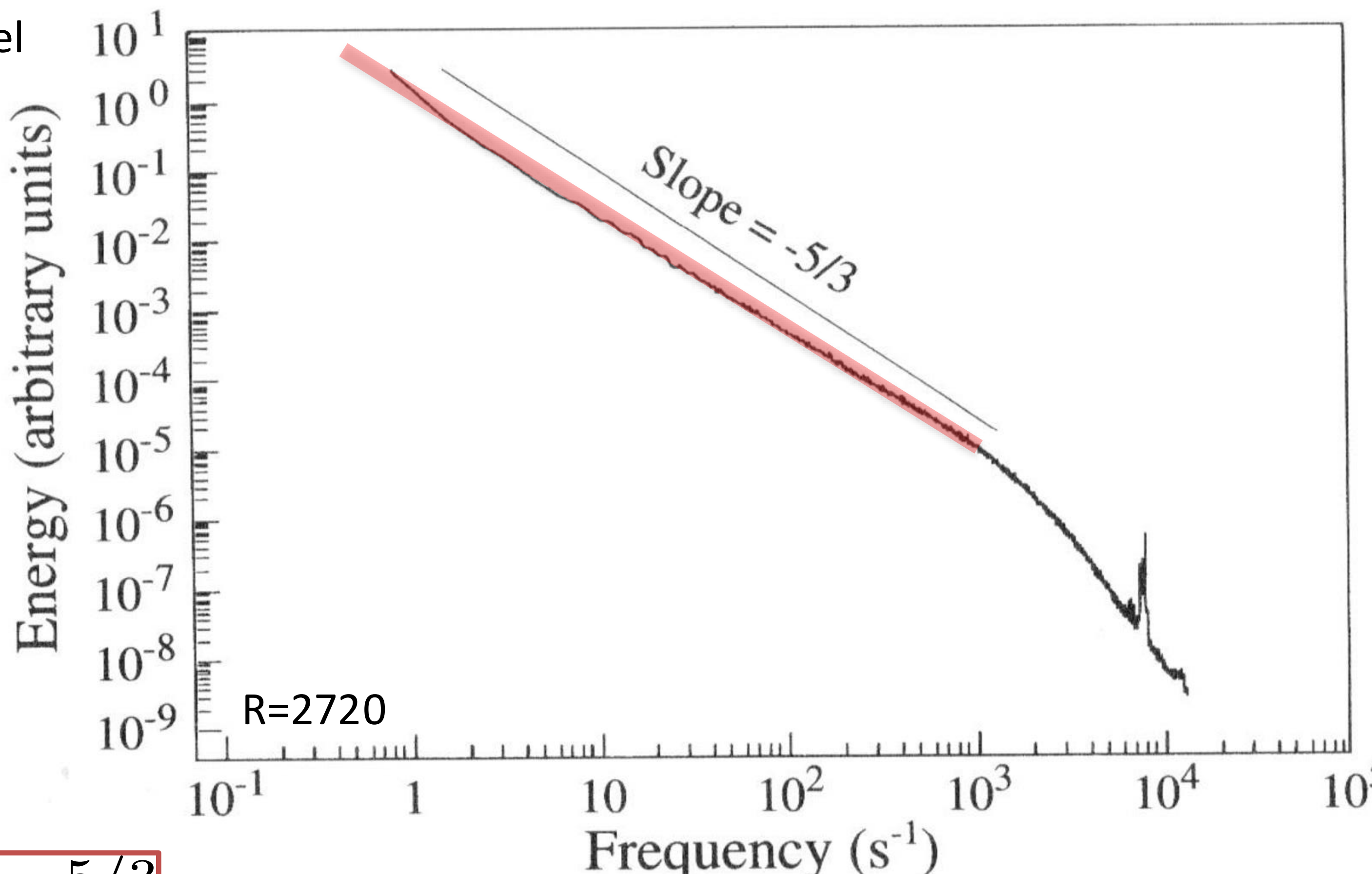


$$\langle (\delta v_{||}(l))^2 \rangle \sim l^{2/3}$$

REMINDER

Energy spectra

ONERA S1 windtunnel



Or wavenumber k (Taylor frozen flow)

$$E(k) \sim k^{-5/3}$$

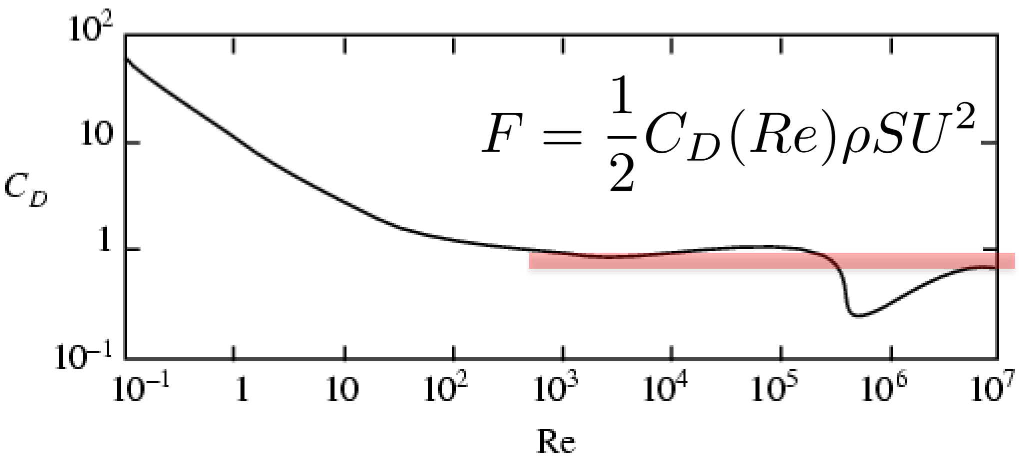
REMINDER

2. Law of finite energy dissipation

Evidence: drag coefficient independent of velocity (see Frisch ch. 5.2)

	911 Turbo Coupé
Weights	
Unladen (DIN)	1,595 kg
Unladen (EC)[1)]	1,670 kg
Permissible gross weight	1,990 kg
Performance	
Top speed	315 km/h
0–100 km/h	3.4 secs
Dimensions/aerodynamics	
Length	4,506 mm
Width (with exterior mirrors)	1,880 mm (1,978 m
Height	1,296 mm
Wheelbase	2,450 mm
Luggage compartment volume (German Car Manufacturers' Assoc.)	115 litres
Tank capacity (refill volume)	68 litres
Drag coefficient	0.31

Drag coefficient of a cylinder



$$C_D(Re) \to C_D, Re \to \infty$$

The three hypotheses

H1: restored symmetries

In the limit of infinite Reynolds numbers, all the possible symmetries of the Navier-Stokes equations, usually broken by the mechanism producing the turbulence, are restored at small scales and away from boundaries.
(Especially: homogeneity and isotropy)

H2: self-similar scaling

Under the same assumption as H1, the turbulent flow is 'self-similar' at small scales, i.e it possesses a unique exponent h such that

$$\delta \underline{v}(\underline{r}, \lambda \underline{l}) \text{ and } \lambda^h \delta \underline{v}(\underline{r}, \underline{l}) \quad \forall \lambda \in \mathbb{R}_+$$

Have the same moments / pdf for all $l \text{ and } \lambda l << l_0$

H3: finite dissipation limit

Under the same assumption as in H1, the turbulent flow has a finite non-vanishing mean rate of dissipation per unit mass ε.

$$Re \to \infty \text{ or } \nu \to 0$$

Kolmogorov K41 theory - roadmap

Navier-Stokes equations

Energy transport

3 additional hypotheses

H1: restored symmetries

H2: self-similar scaling

H3: finite dissipation

Statistics

E(k) <-> correllations

Karman-Howarth-Monin

Kolmogorov four-fifth law

$$\langle (\delta v_{||}(\underline{r},\underline{l}))^3 \rangle = -\frac{4}{5}\epsilon l$$

Kolmogorov spectrum

$$E(k) \sim \epsilon^{2/3}\ k^{-5/3}$$

Kolmogorov's 4/5 law

Situation:
- random, stationary, homogeneous forcing
- homogeneous, isotropic (not stationary) turbulent flow
- limit of infinite Reynolds numbers

Karman-Howard-Monin relation

Energy flux for homogeneous **and** isotropic turbulence

$$\Pi_K = -\frac{1}{6\pi}\int_0^\infty dl\ \frac{sin(Kl)}{l}(1+l\partial_l)(3+l\partial_l)(5+l\partial_l)\frac{S_3(l)}{l}$$

$S_3(l) = \langle(\delta_{||}(\underline{r},\underline{l})^3\rangle$

3rd order structure fct.

Statistically stationary forced turbulence

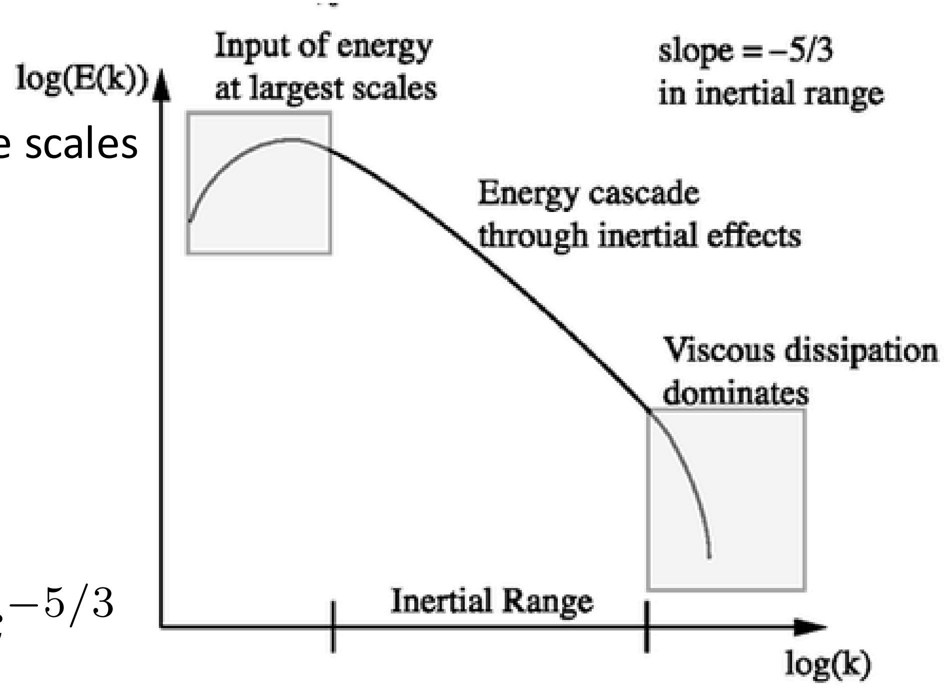
(1) Driving force (energy input) only on large scales

(2) 'steady state' for large times – field is statistically stationary

(3) H3: finite dissipation

$$\langle(\delta v_{||}(\underline{r},\underline{l}))^3\rangle = -\frac{4}{5}\epsilon l$$

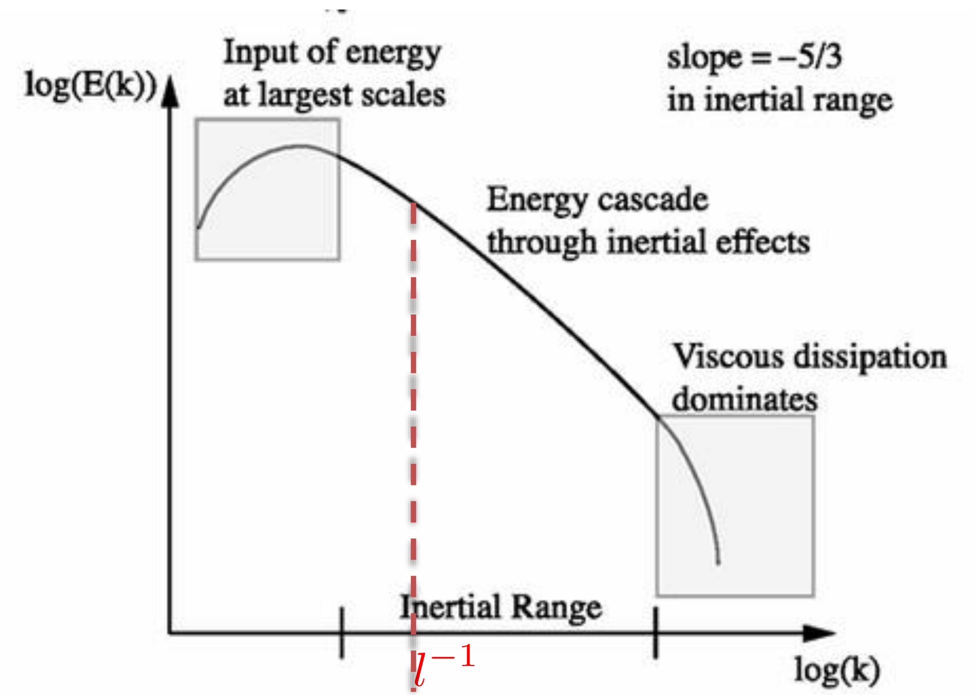
$$E(k) \sim \epsilon^{2/3}\ k^{-5/3}$$



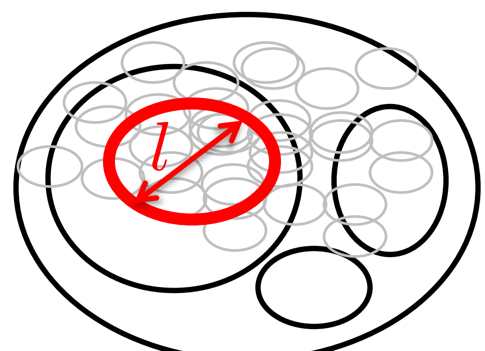
Ch. 7: Phenomenology

Aim: Interpreting the K41 results

Energy spectrum



Turbulent eddies



Length scale l

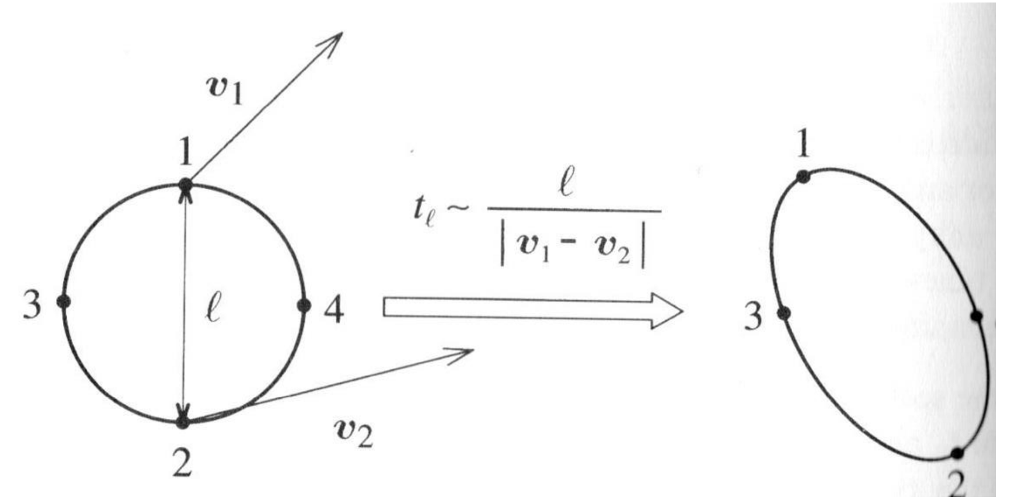
Velocity scale v_l

Time scale $t_l \sim \dfrac{l}{v_l}$

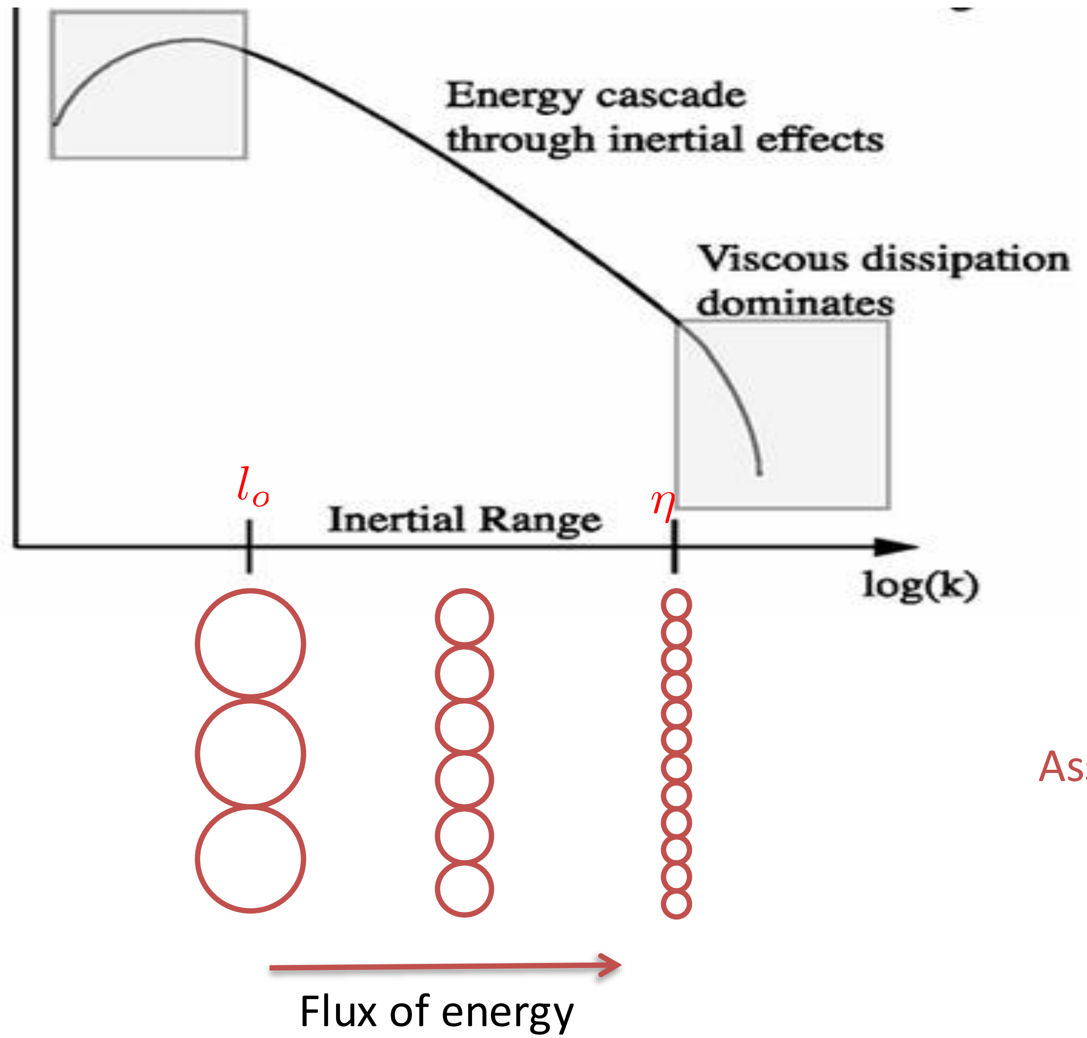
Energy flux

Flux from scale l

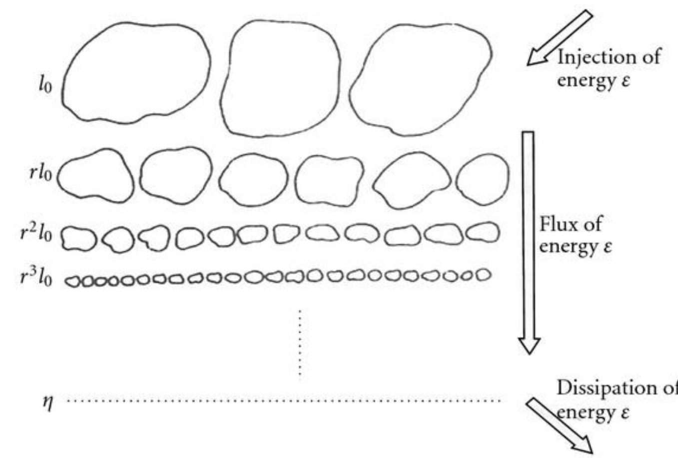
$$\Pi'_l \sim \frac{v_l^2}{t_l} \sim \frac{v_l^3}{l}$$



The Richardson cascade



Typical cartoon
(the cascade)



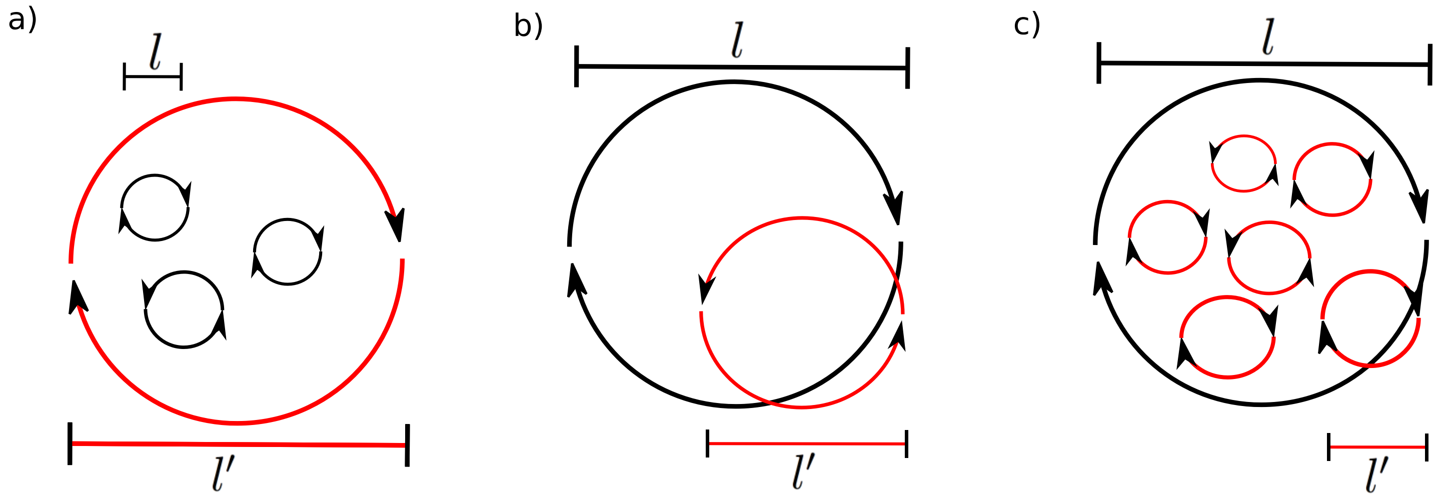
Assumptions

(1) eddies are space-filling

(2) local (in k) energy transport

The Richardson cascade

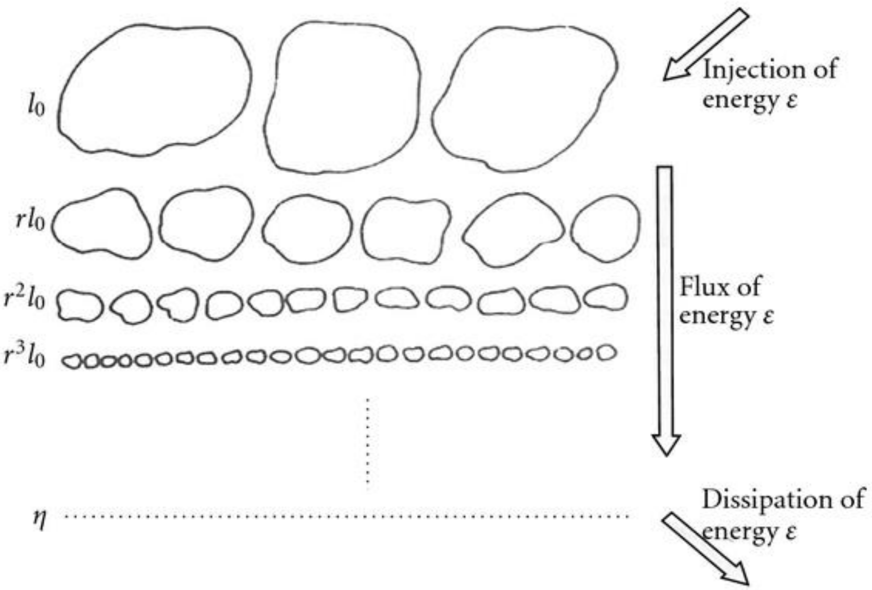
Local energy transfer (see also exercise)



The Richardson cascade

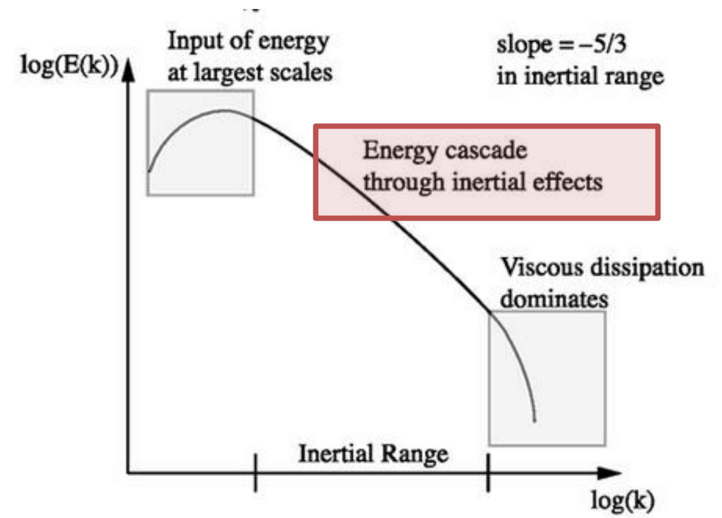
Lewis Richardson, 1922

Big whorls have little whorls,
Which feed on their velocity;
And little whorls have lesser whorls,
And so on to viscosity



Jonathan Swift, "On poetry, a rhapsody"

So, naturalists observe, a flea
Hath smaller fleas that on him prey;
And these have smaller still to bite 'em;
And so proceed ad infinitum.
Thus every poet, in his kind,
Is bit by him that comes behind.



Next: Decaying turbulence

Question: How long does the motion of stirred coffee /tea last?