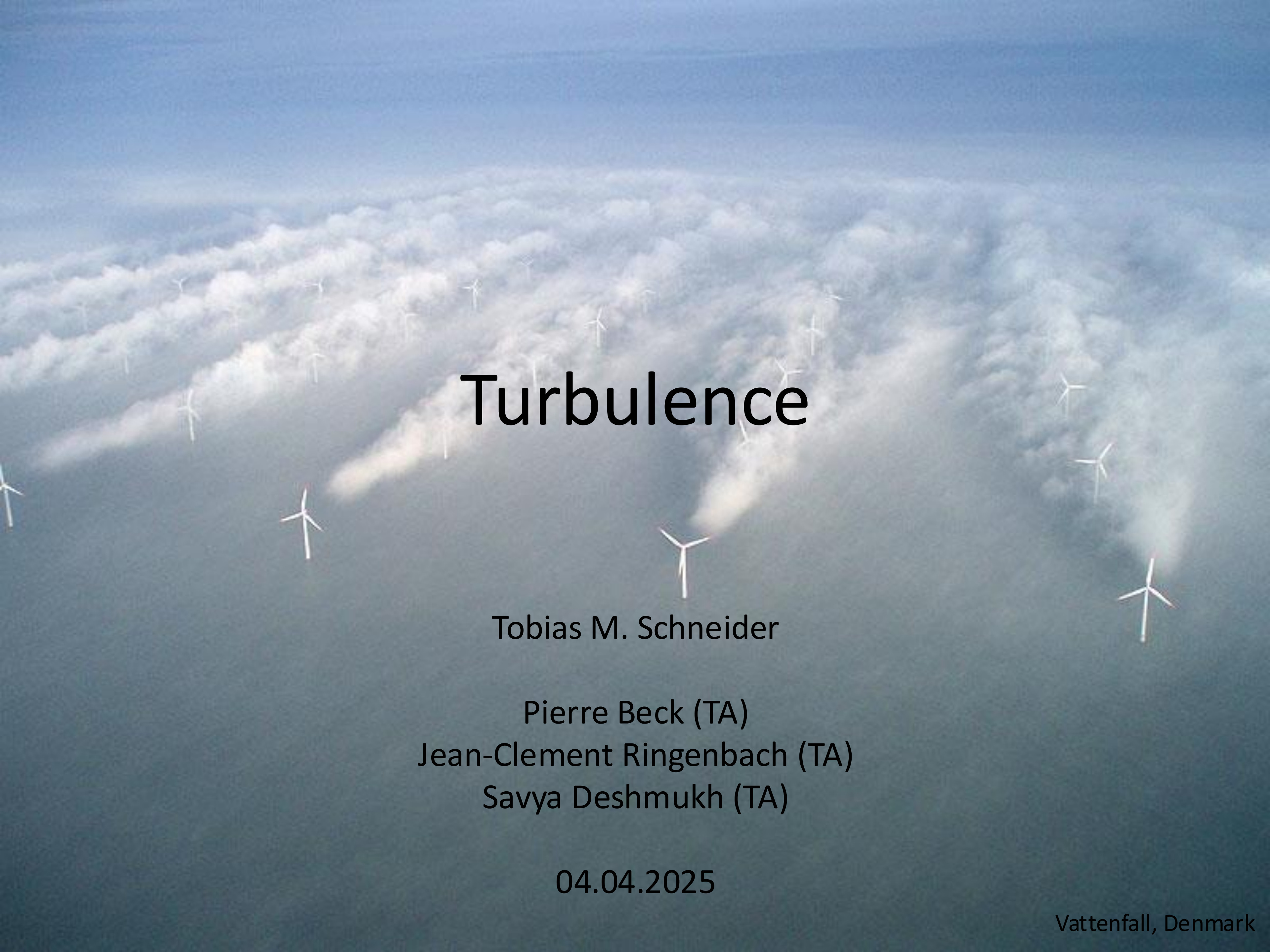

Turbulence

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Vattenfall, Denmark

Kolmogorov K41 theory - roadmap

Navier-Stokes equations

Energy transport

3 additional hypotheses

H1: restored symmetries

H2: self-similar scaling

H3: finite dissipation

Statistics

E(k) <-> correllations

Karman-Howarth-Monin

done

Kolmogorov four-fifth law

$$\langle (\delta v_{||}(\underline{r}, \underline{l}))^3 \rangle = -\frac{4}{5}\epsilon l$$

Kolmogorov spectrum

$$E(k) \sim \epsilon^{2/3} \; k^{-5/3}$$

The three hypotheses

H1: restored symmetries

In the limit of infinite Reynolds numbers, all the possible symmetries of the Navier-Stokes equations, usually broken by the mechanism producing the turbulence, are restored at small scales and away from boundaries.
(Especially: homogeneity and isotropy)

H2: self-similar scaling

Under the same assumption as H1, the turbulent flow is 'self-similar' at small scales, i.e it possesses a unique exponent h such that

$$\delta \underline{v}(\underline{r}, \lambda \underline{l}) \text{ and } \lambda^h \delta \underline{v}(\underline{r}, \underline{l}) \quad \forall \lambda \in \mathbb{R}_+$$

Have the same moments / pdf for all $l \text{ and } \lambda l << l_0$

H3: finite dissipation limit

Under the same assumption as in H1, the turbulent flow has a finite non-vanishing mean rate of dissipation per unit mass ε.

$$Re \to \infty \text{ or } \nu \to 0$$

Kolmogorov's 4/5 law

Situation:
- random, stationary, homogeneous forcing
- homogeneous, isotropic (not stationary) turbulent flow
- limit of infinite Reynolds numbers

Karman-Howard-Monin relation

Energy flux for homogeneous **and** isotropic turbulence

$$\Pi_K = -\frac{1}{6\pi}\int_0^\infty dl\, \frac{sin(Kl)}{l}(1+l\partial_l)(3+l\partial_l)(5+l\partial_l)\frac{S_3(l)}{l}$$

$S_3(l) = \langle(\delta_{||}(\underline{r},\underline{l})^3\rangle$
3rd order structure fct.

Statistically stationary forced turbulence

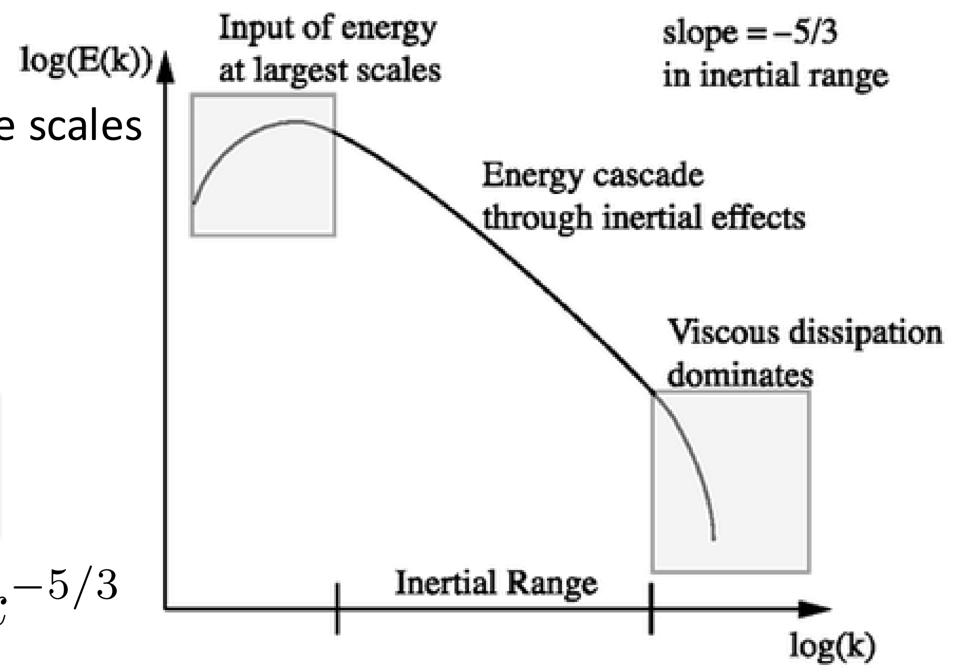
(1) Driving force (energy input) only on large scales

(2) 'steady state' for large times – field is statistically stationary

(3) H3: finite dissipation

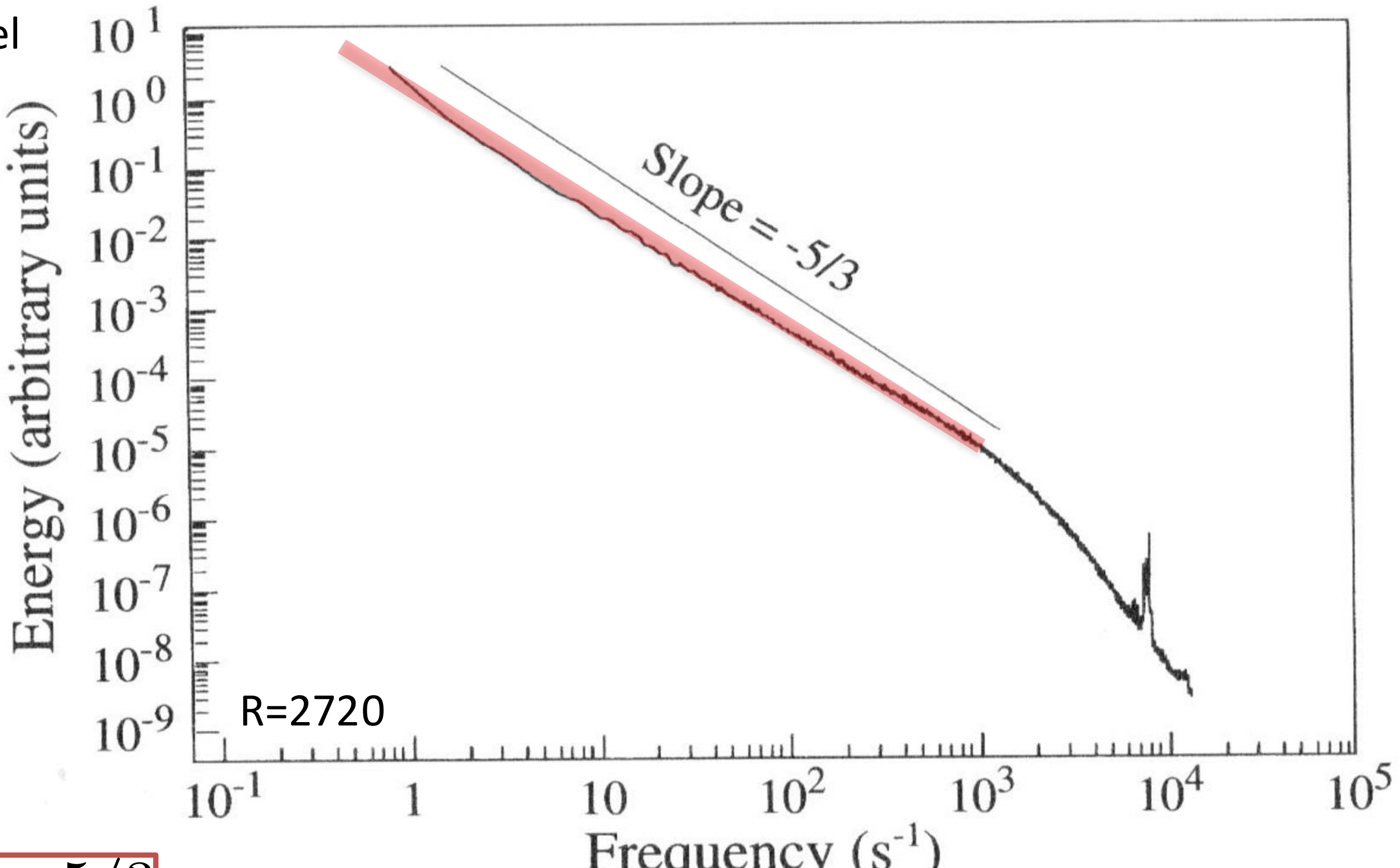
$$\langle(\delta v_{||}(\underline{r},\underline{l}))^3\rangle = -\frac{4}{5}\epsilon l$$

$$E(k) \sim \epsilon^{2/3}\, k^{-5/3}$$



Why 5/3 energy spectra?

ONERA S1 windtunnel



Or wavenumber k (Taylor frozen flow)

$$E(k) \sim k^{-5/3}$$