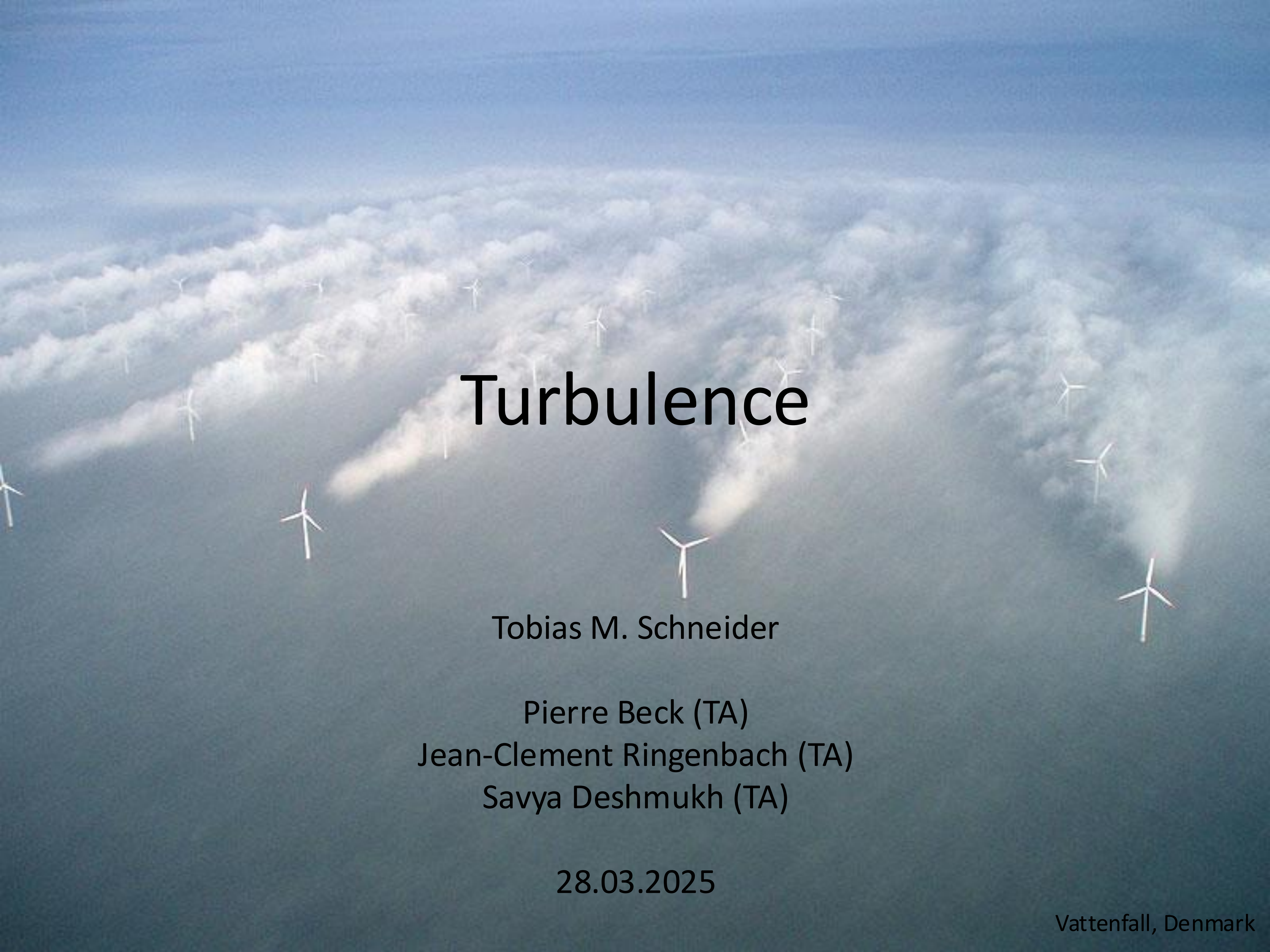

Turbulence

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28.03.2025

Vattenfall, Denmark

Kolmogorov K41 theory - roadmap

Navier-Stokes equations

Energy transport

3 additional hypotheses

H1: restored symmetries

H2: self-similar scaling

H3: finite dissipation

Statistics

E(k) <-> correllations

Karman-Howarth-Monin

Kolmogorov four-fifth law

$$\langle(\delta v_{||}(\underline{r},\underline{l}))^3\rangle = -\frac{4}{5}\epsilon l$$

Kolmogorov spectrum

$$E(k) \sim \epsilon^{2/3}\ k^{-5/3}$$

The three hypotheses

H1: restored symmetries

In the limit of infinite Reynolds numbers, all the possible symmetries of the Navier-Stokes equations, usually broken by the mechanism producing the turbulence, are restored at small scales and away from boundaries.
(Especially: homogeneity and isotropy)

H2: self-similar scaling

Under the same assumption as H1, the turbulent flow is 'self-similar' at small scales, i.e it possesses a unique exponent h such that

$$\delta \underline{v}(\underline{r}, \lambda \underline{l}) \text{ and } \lambda^h \delta \underline{v}(\underline{r}, \underline{l}) \quad \forall \lambda \in \mathbb{R}_+$$

Have the same moments / pdf for all $l \text{ and } \lambda l << l_0$

H3: finite dissipation limit

Under the same assumption as in H1, the turbulent flow has a finite non-vanishing mean rate of dissipation per unit mass ε.

$$Re \to \infty \text{ or } \nu \to 0$$

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Today

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