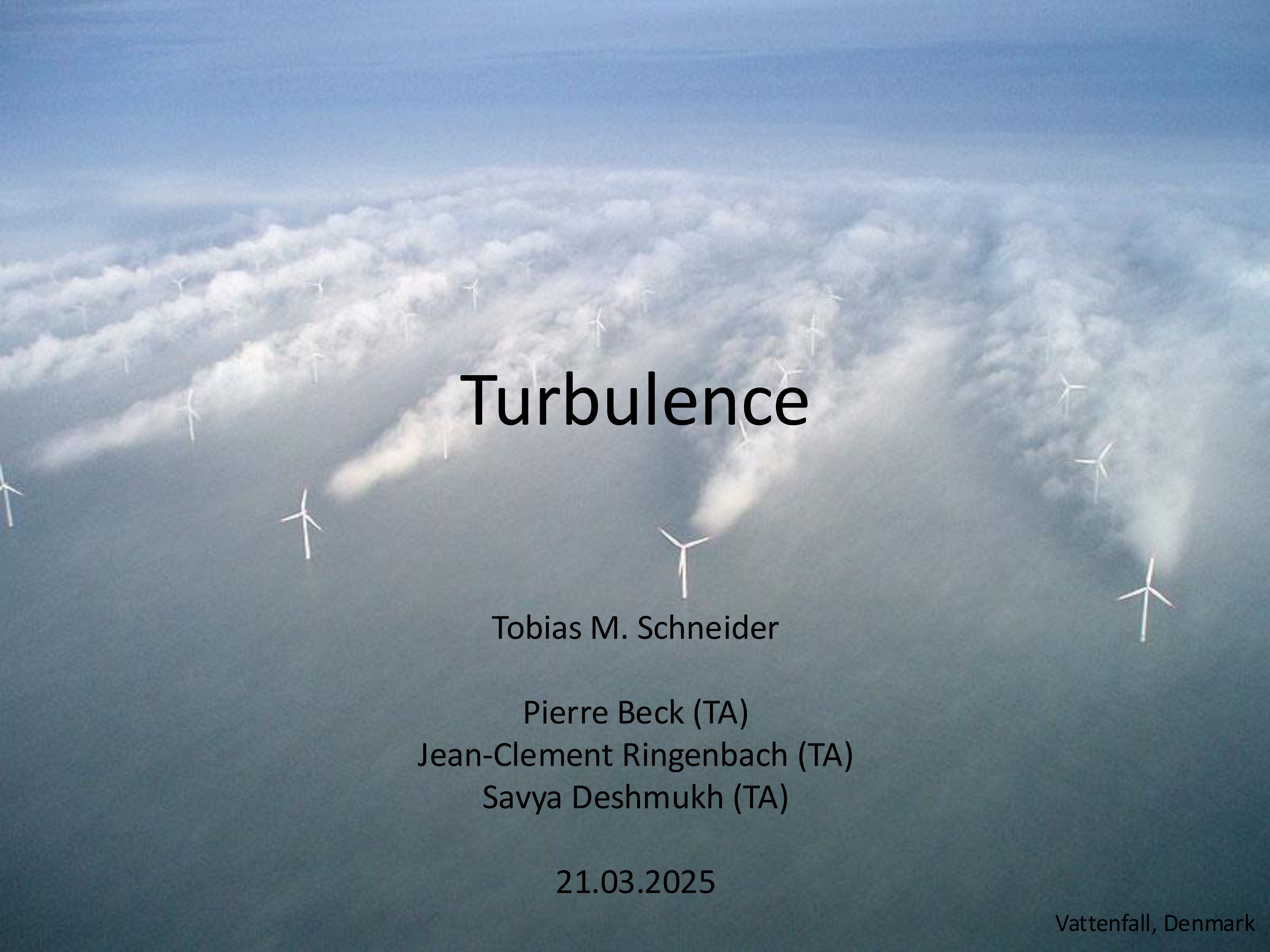

Turbulence

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Plan for today

1. **Towards Kolmogorov theory – Chap 5: Two facts about fully developed turbulence**

Course outline

1. Introduction – fully developed turbulence
2. Symmetries and conservation laws
3. Probabilistic description of turbulence
4. Review: Statistical tools and methods

done

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5. Two experimental laws of fully developed turbulence
6. Kolmogorov's 1941 turbulence theory
7. Phenomenology

8. Intermittency – corrections to K41 theory
9. Modeling and simulation: DNS, RANS, LES,….
10. Transition to turbulence

Towards Kolmogorov theory

Navier-Stokes equations	**Statistics**
Symmetries Conservation laws Scale-by scale energy budget	Wiener formula Correlation fcts – energy spectrum

Aims: - predict turbulent statistics
- describe underlying physical mechanisms (e.g. energy transport)

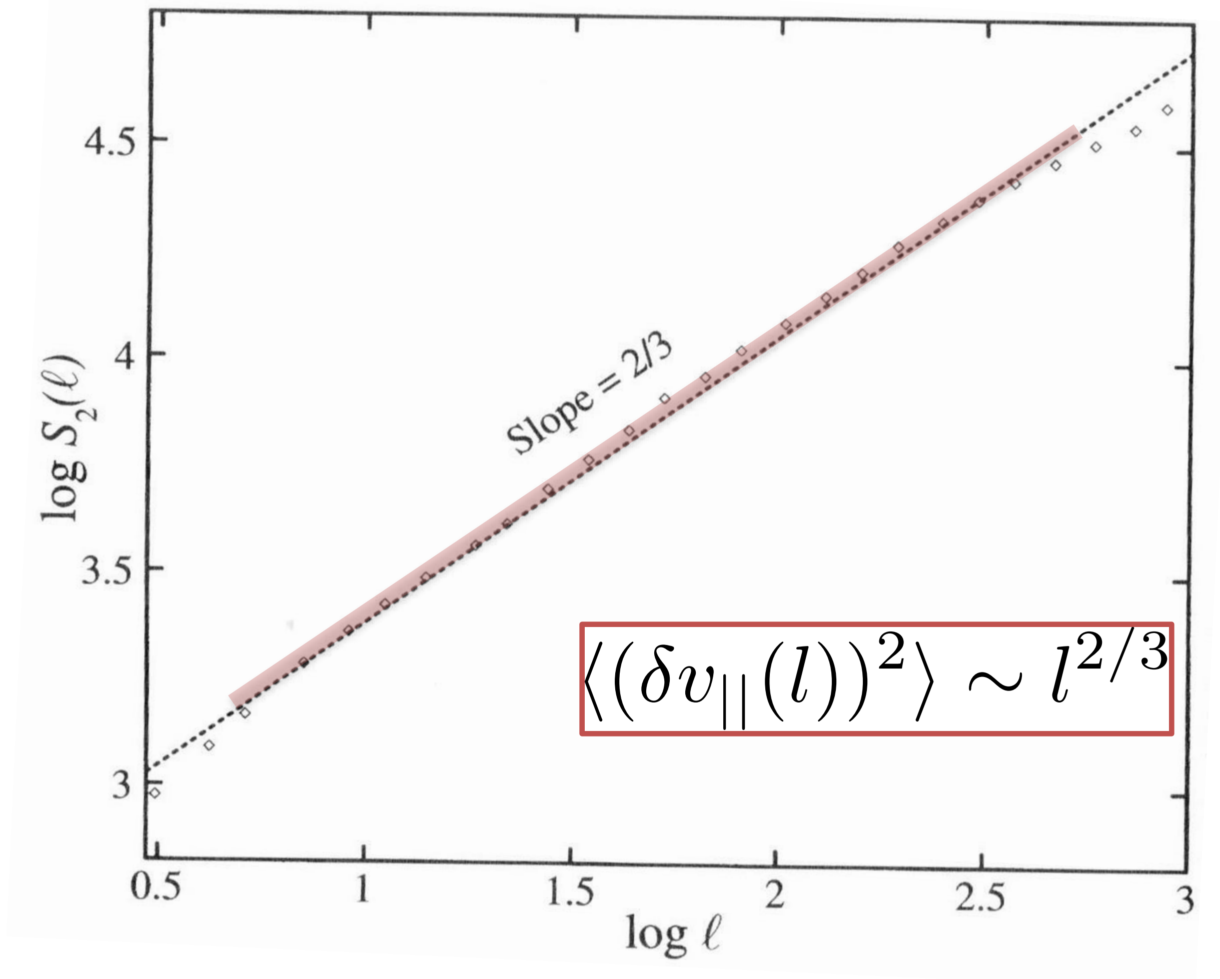
Problem: - no deductive theory from Navier-Stokes
- additional assumption or hypotheses required

→ What are robust features of high Reynolds number turbulence?

Ch. 5: Two experimental laws of fully developed turbulence

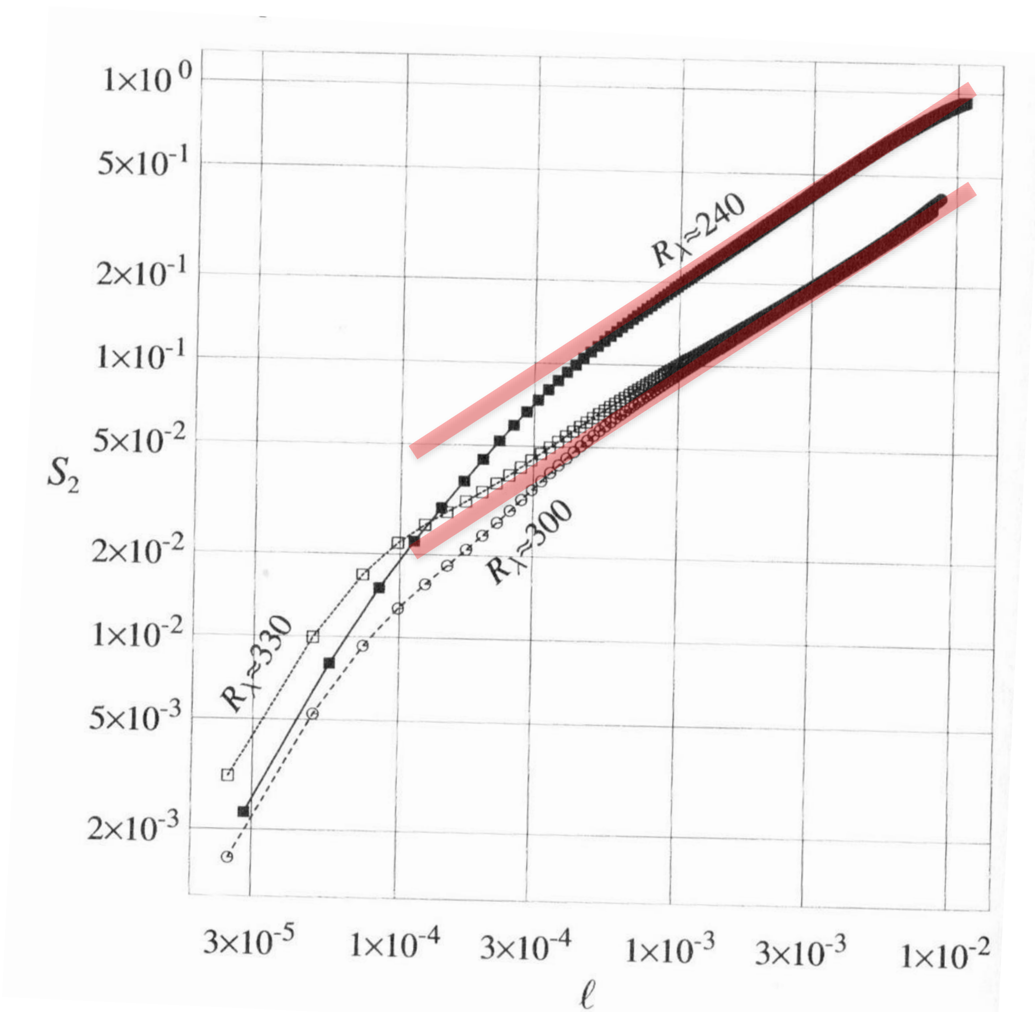
5.1. The two-third law of the second order structure function

ONERA S1 windtunnel



$$\langle (\delta v_{||}(l))^2 \rangle \sim l^{2/3}$$

Transverse structure function

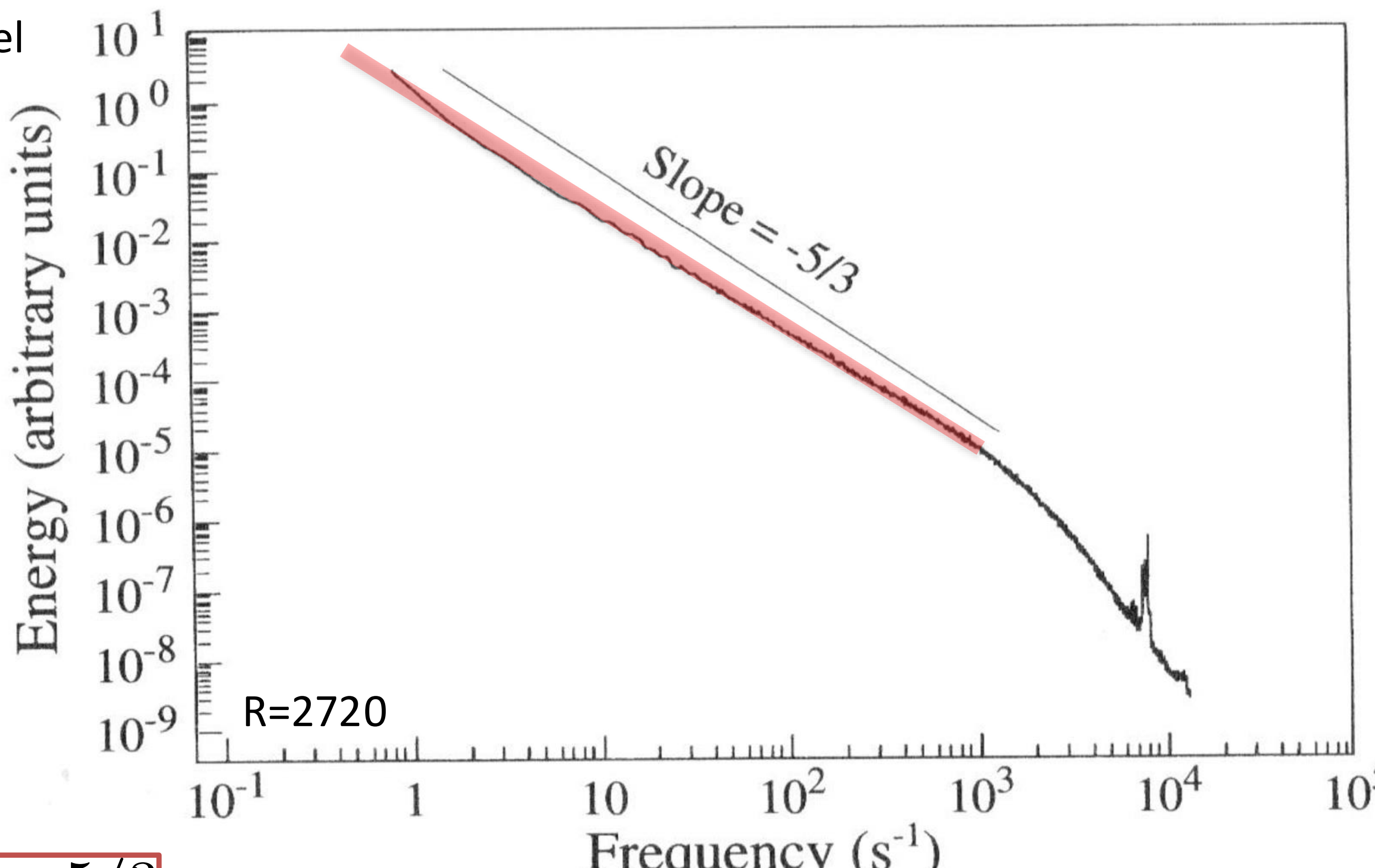


$$\langle(\delta v_\perp(l))^2\rangle \sim l^{2/3}$$

Turbulent jet

Energy spectra

ONERA S1 windtunnel

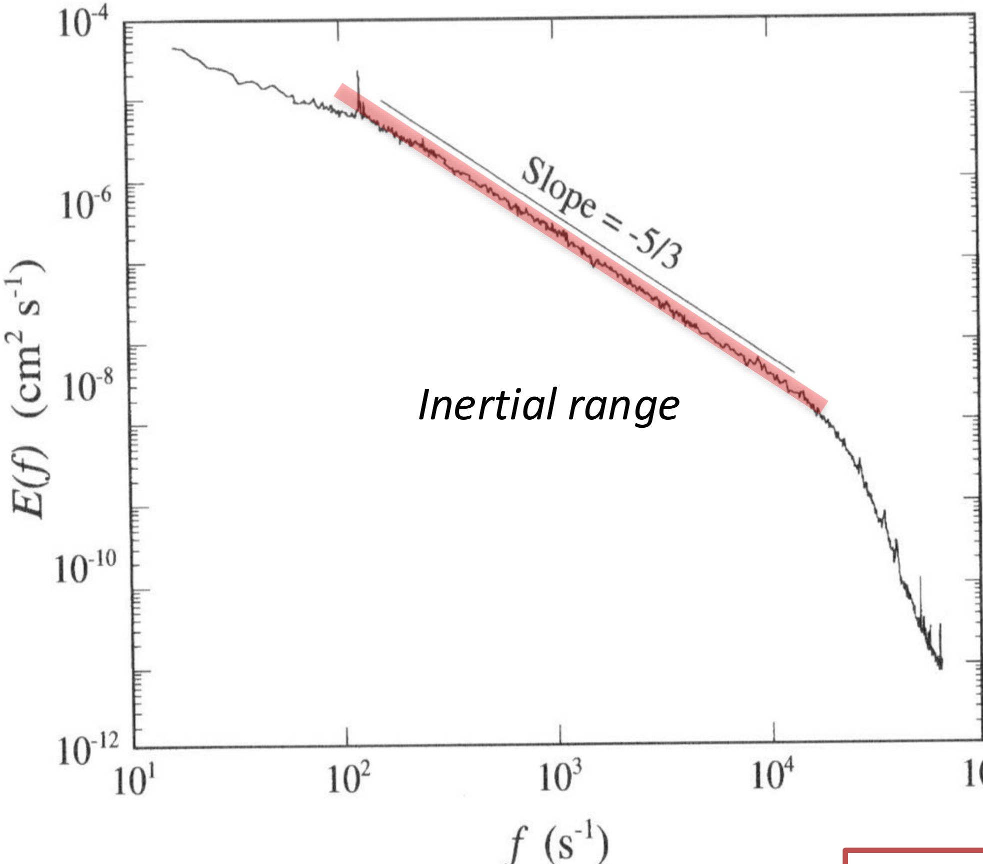


Or wavenumber k (Taylor frozen flow)

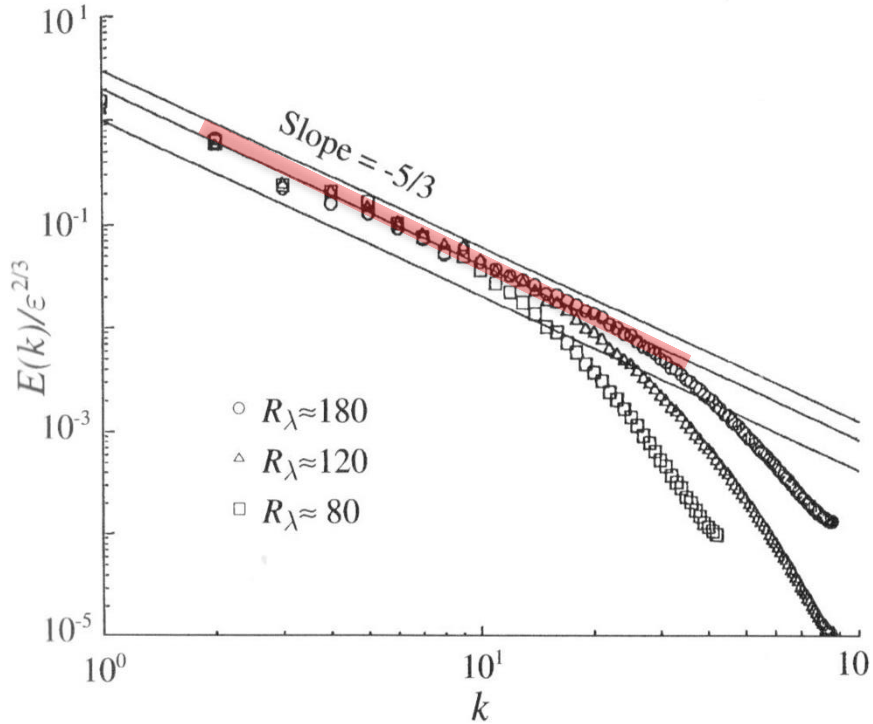
$$E(k) \sim k^{-5/3}$$

Energy spectra

Low-T helium between counter-rotating cylinders



Simulation (periodic box)



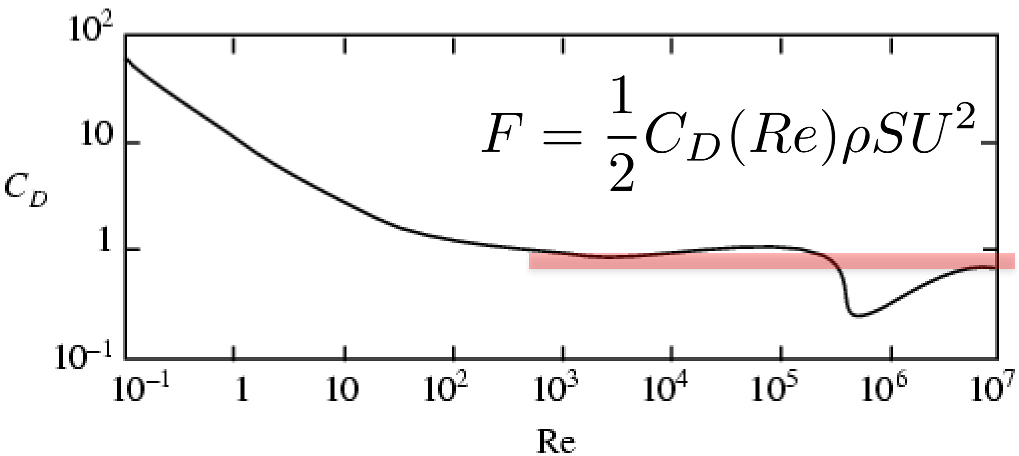
$$E(k) \sim k^{-5/3}$$

2. Law of finite energy dissipation

Evidence: drag coefficient independent of velocity (see Frisch ch. 5.2)

	911 Turbo Coupé
Weights	
Unladen (DIN)	1,595 kg
Unladen (EC)[1)]	1,670 kg
Permissible gross weight	1,990 kg
Performance	
Top speed	315 km/h
0–100 km/h	3.4 secs
Dimensions/aerodynamics	
Length	4,506 mm
Width (with exterior mirrors)	1,880 mm (1,978 m
Height	1,296 mm
Wheelbase	2,450 mm
Luggage compartment volume (German Car Manufacturers' Assoc.)	115 litres
Tank capacity (refill volume)	68 litres
Drag coefficient	0.31

Drag coefficient of a cylinder



$$F = \frac{1}{2} C_D(Re) \rho S U^2$$

$$C_D(Re) \to C_D, Re \to \infty$$

Kolmogorov K41 theory - roadmap

Navier-Stokes equations

Energy transport

3 additional hypotheses

H1: restored symmetries

H2: self-similar scaling

H3: finite dissipation

Statistics

E(k) <-> correllations

Karman-Howarth-Monin

Kolmogorov four-fifth law

$$\langle (\delta v_{||}(\underline{r}, \underline{l}))^3 \rangle = -\frac{4}{5}\epsilon l$$

Kolmogorov spectrum

$$E(k) \sim \epsilon^{2/3}\ k^{-5/3}$$