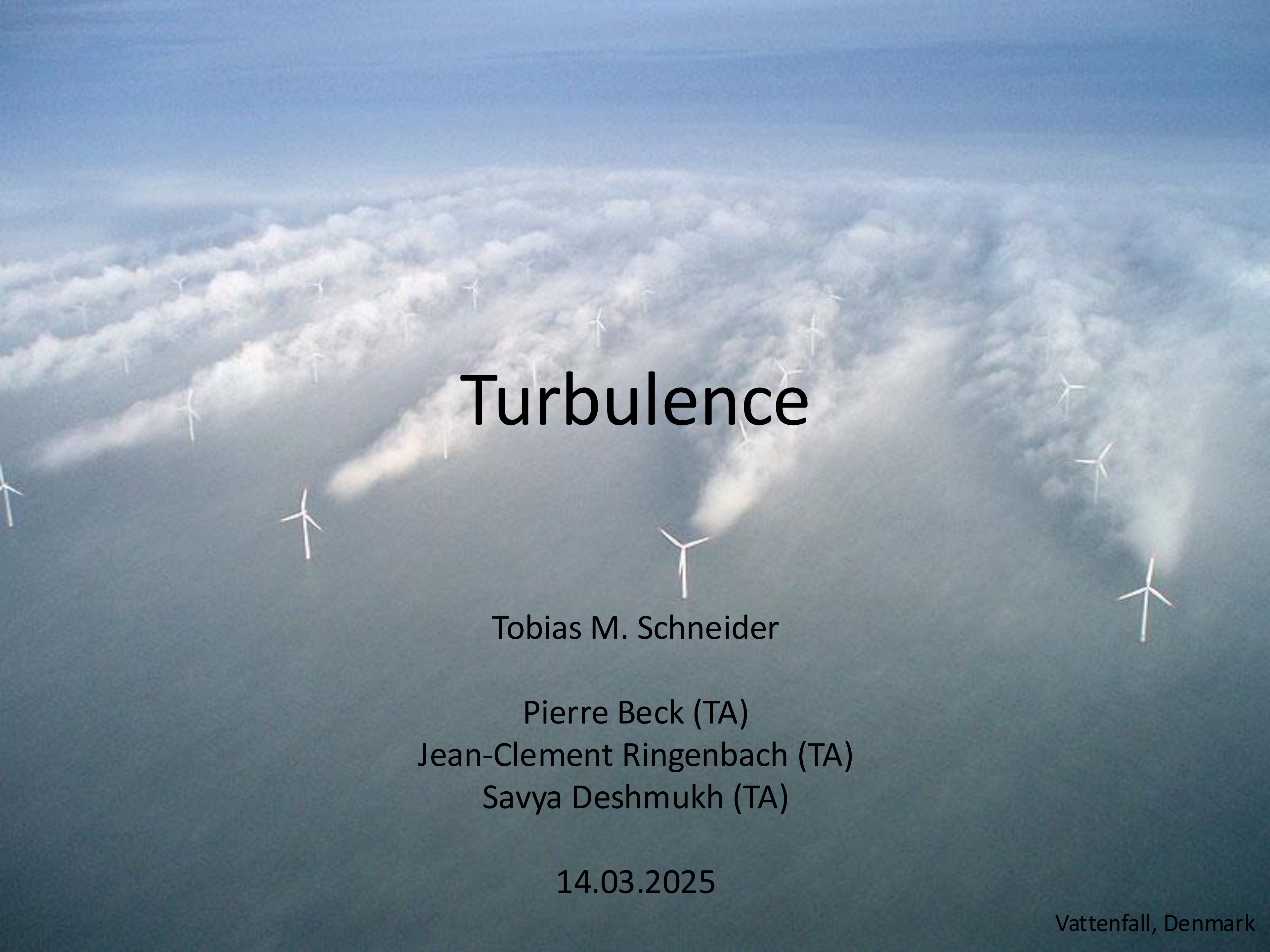

Turbulence

Tobias M. Schneider

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14.03.2025

Vattenfall, Denmark

Plan for today

1. **Probabilistic description of turbulence – why?**

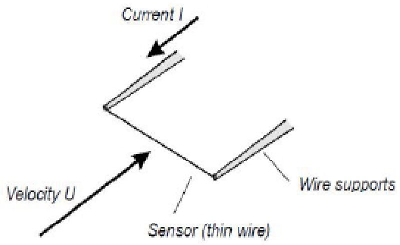
2. **Review of statistical tools and methods**

Note: No exercises today -> postponed to next week

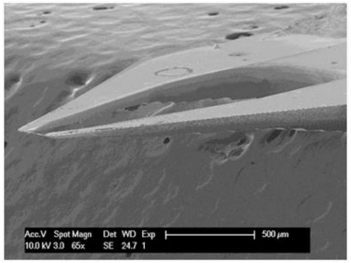
Ch. 3: Probabilistic description of turbulence – why?

ONERA S1 windtunnel

Anemometer – eg. Hot Wire



Schematic

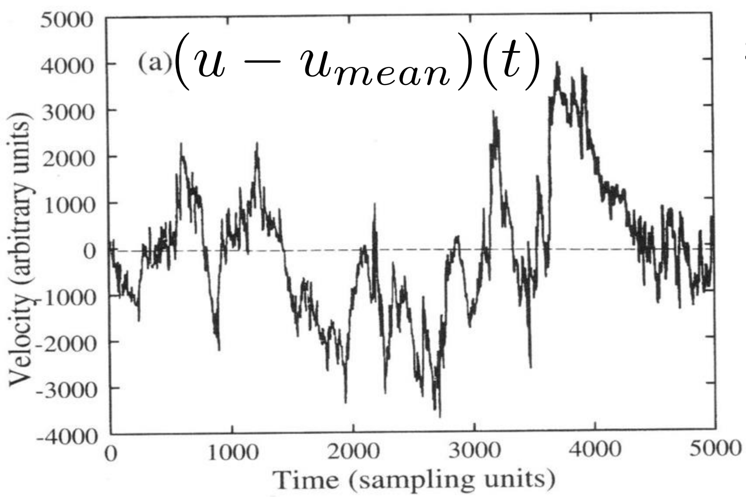


Princeton NSTAP
(Smits Lab)

Collect Data: Measure component of velocity at a fixed location

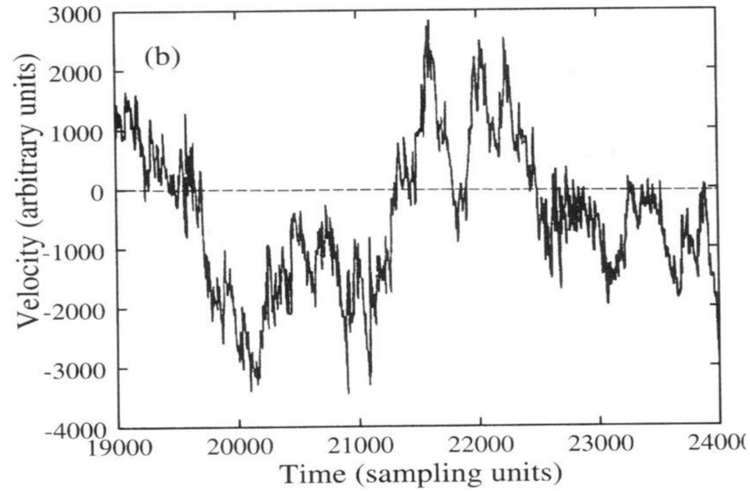
Stochastic data

One second signal



sampled at 5kHz

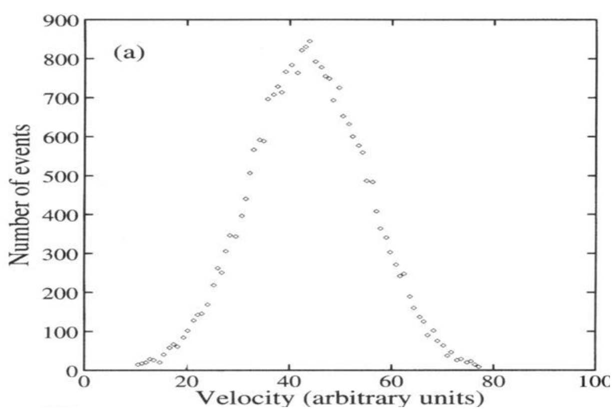
One second signal (later)



(1) disorganized, structure on all (temporal) scales, ‘chaotic’

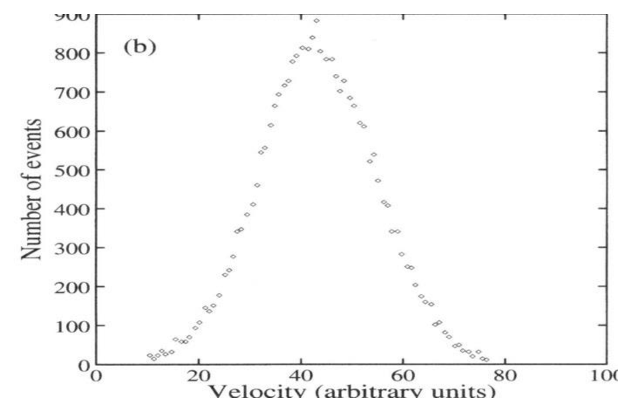
(2) unpredictable in detail

(3) some properties appear reproducible



Histogram of stochastic signal:

Reproducible!!!!!



`Probabilistic description of turbulence’ (GI Taylor, 1930s)

U. Frisch, p. 28-30

Deterministic Chaos

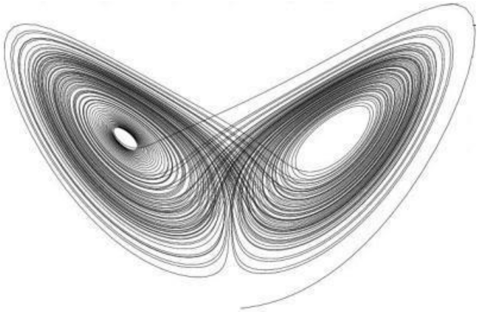
Question: Why a stochastic / probabilistic description when the Navier-Stokes equations are deterministic?

Answer: Chaotic dynamics of nonlinear dynamical systems

$$\rho\left(\frac{\partial \mathbf{v}}{\partial t} + \mathbf{v}\cdot\nabla\mathbf{v}\right) = -\nabla p + \mu\nabla^2\mathbf{v} + \mathbf{f}.$$

Example systems: (1) discrete chaotic maps (see Frisch ch. 3.2)
(2) here: Lorenz equations

$$\frac{\mathrm{d}x}{\mathrm{d}t} = \sigma(y-x),$$
$$\frac{\mathrm{d}y}{\mathrm{d}t} = x(\rho - z) - y,$$
$$\frac{\mathrm{d}z}{\mathrm{d}t} = xy - \beta z.$$



Extremely sensitive dependence on initial conditions limits predictability (butterfly effect)
-> stochastic description despite deterministic (chaotic) dynamics.

Course outline

1. Introduction – fully developed turbulence
2. Symmetries and conservation laws
3. Probabilistic description of turbulence
4. Review: Statistical tools and methods

done

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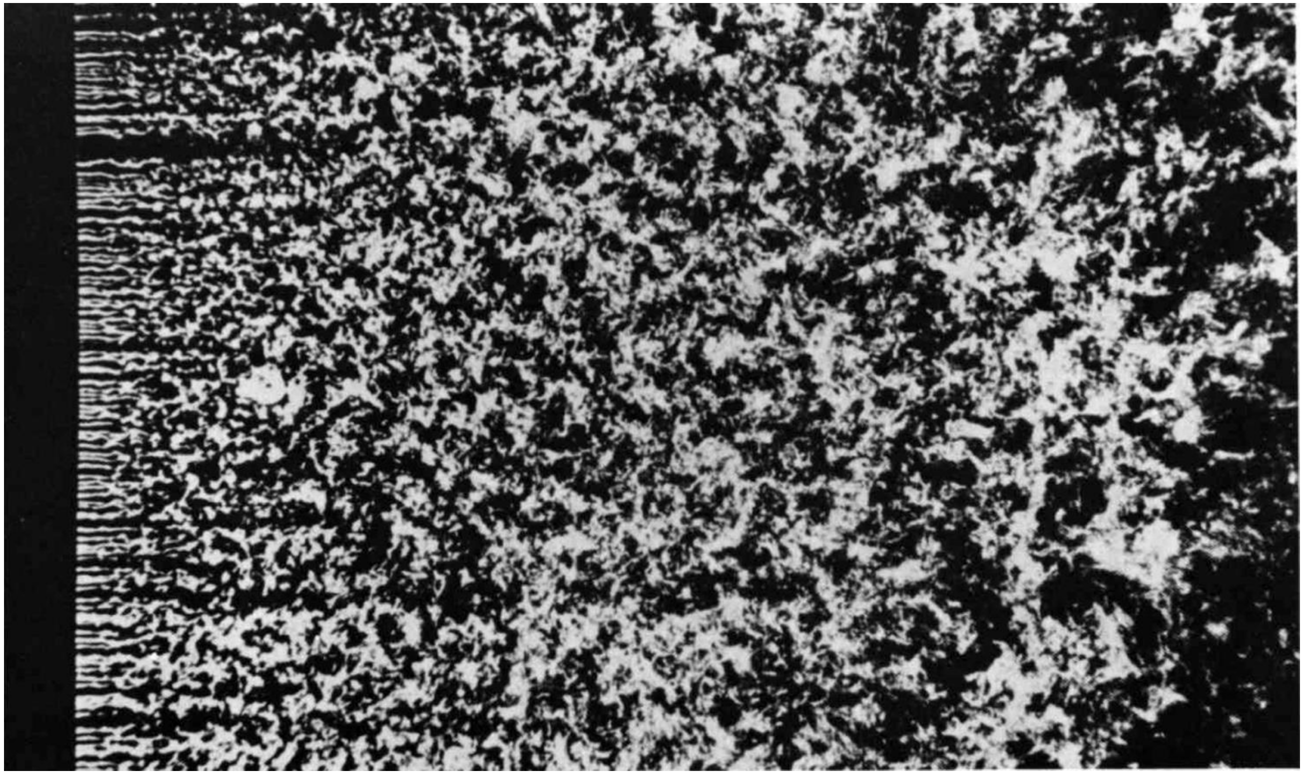
5. Two experimental laws of fully developed turbulence
6. Kolmogorov's 1941 turbulence theory
7. Phenomenology

8. Intermittency – corrections to K41 theory
9. Modeling and simulation: DNS, RANS, LES,….
10. Ergodic Theory and Turbulence

Filtering in space

Grid turbulence



Observe: Structures on many different scales

Van Dyke, Album of Fluid Motion