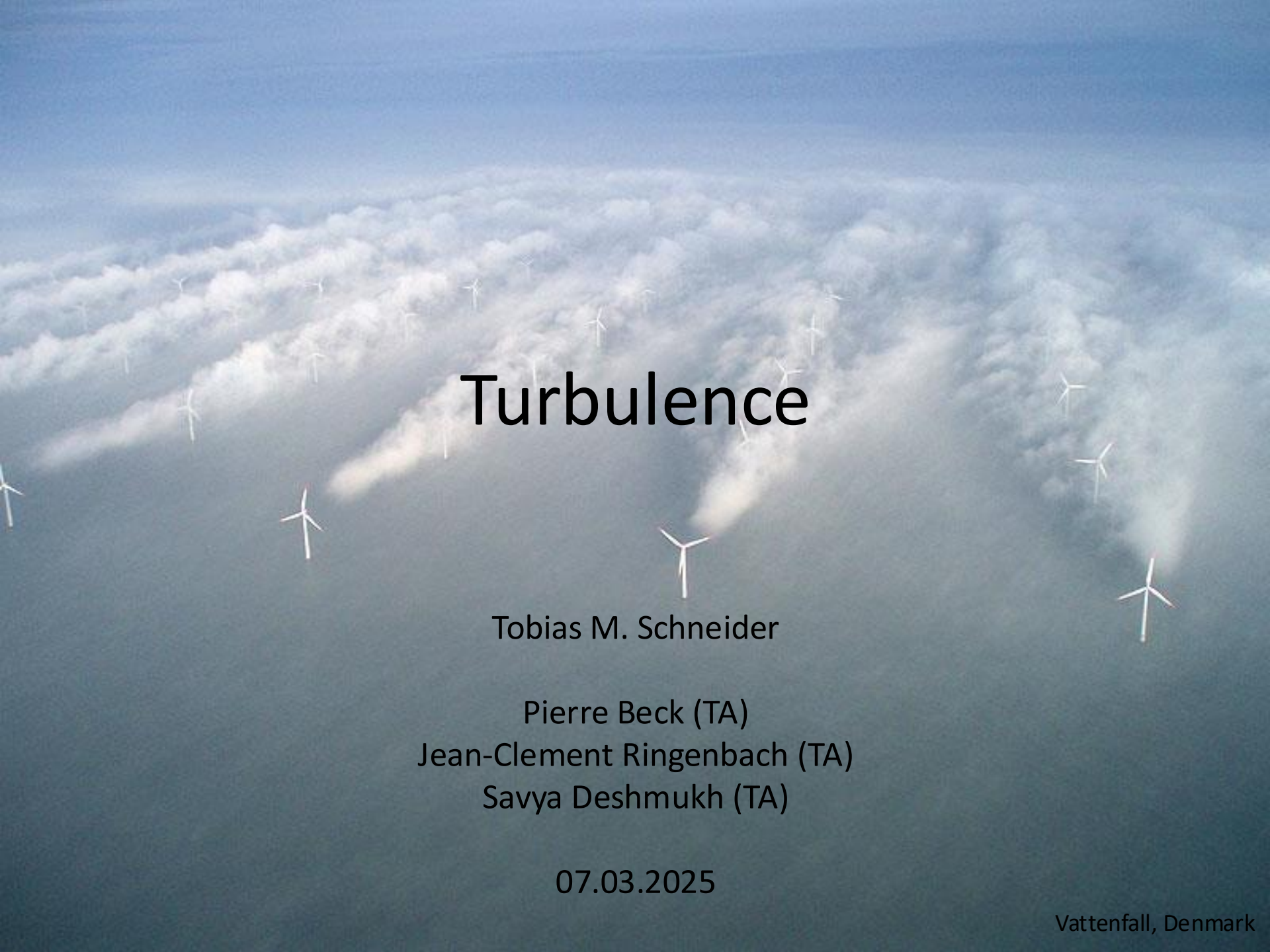

# Turbulence

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07.03.2025

Vattenfall, Denmark

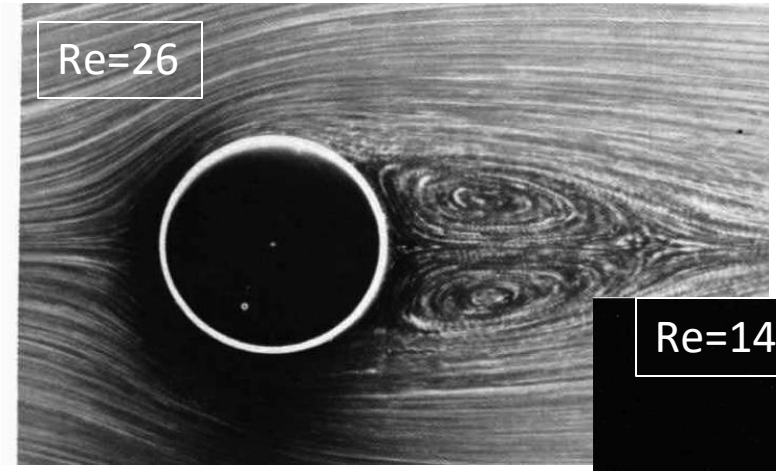
# Plan for today

1. **Finish: Global conservation laws (Frisch Ch. 2.3)**
2. **Scale-by-scale energy budget (Frisch Ch. 2.4)**

3. **Probabilistic description of turbulence – why?**

**Note: First exercise session today 14-15**

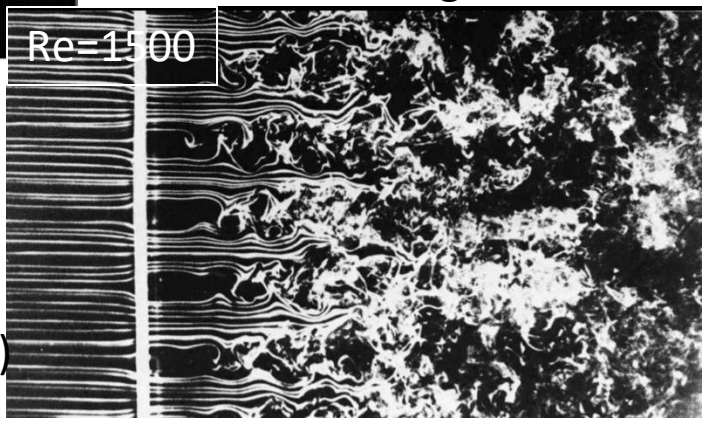
# Turbulence and symmetries




Reynolds number increases


**Symmetry breaking**

Behind a grid



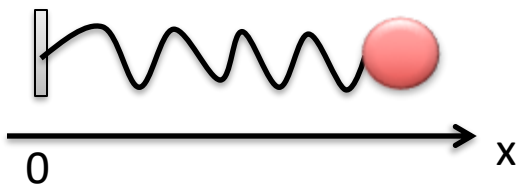

**Symmetries recovered**
(in a **statistical** sense, far from boundaries)

Van Dyke, An Album of Fluid Motion

Fully developed turbulence

# Symmetries of dynamical equations

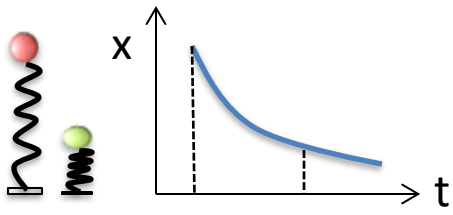
Example: overdamped particle in a quadratic potential (1D)




$$\dot{x} = -x; \quad x(t=0) = x_0$$
$$x(t) = x_0\, e^{-t} = f^t(x_0)$$

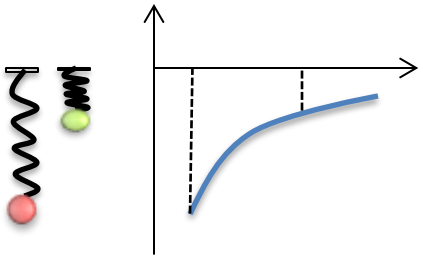
Symmetry: $g: \quad x \to -x$ (reflection)

Transforms solutions into solutions



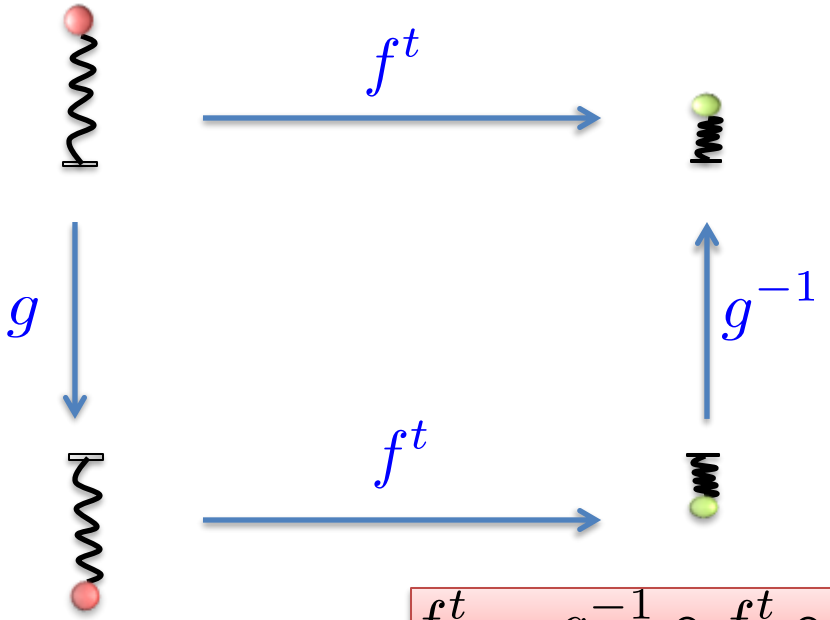

$$x(t) = x_0 e^{-t}$$

$g$



$$\hat{x}(t) = \hat{x}_0 e^{-t} = (-x_0)e^{-t} = -x(t)$$

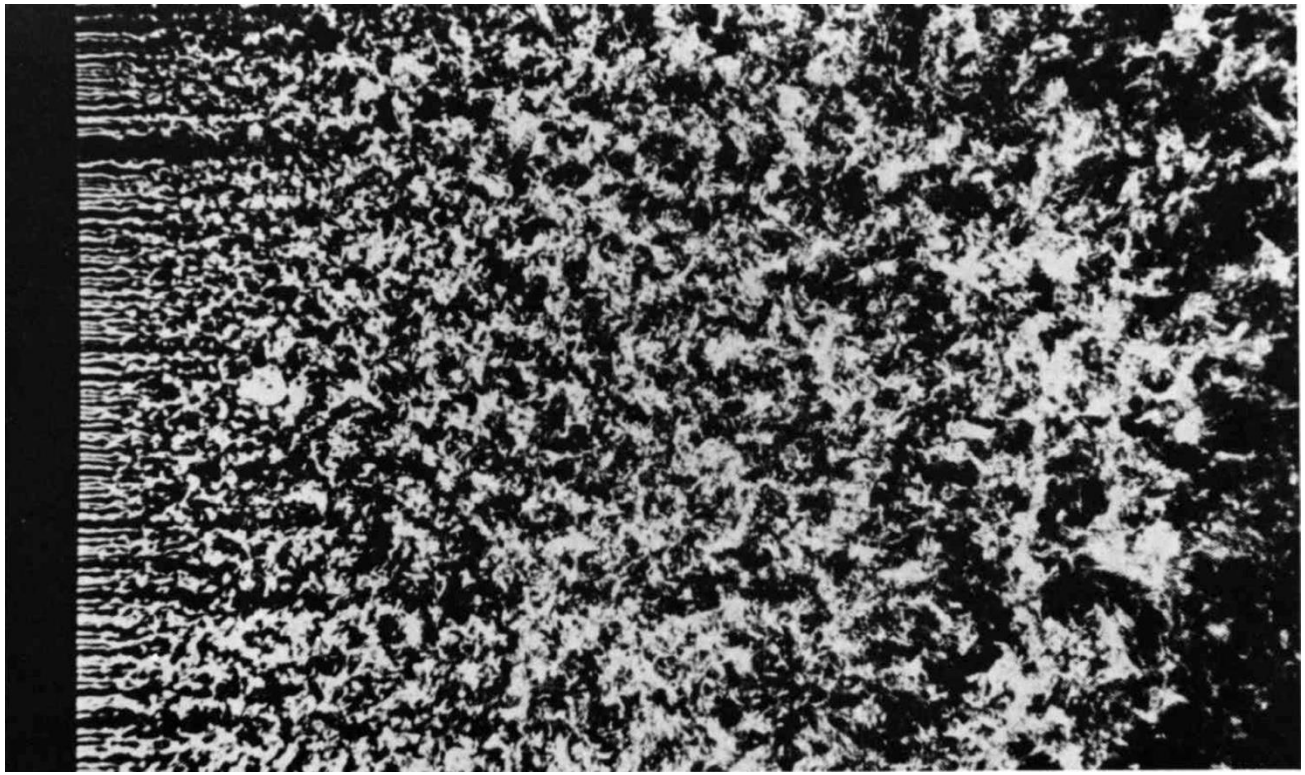
Symmetry and time evolution commute




$$f^t = g^{-1} \circ f^t \circ g$$

# Filtering in space

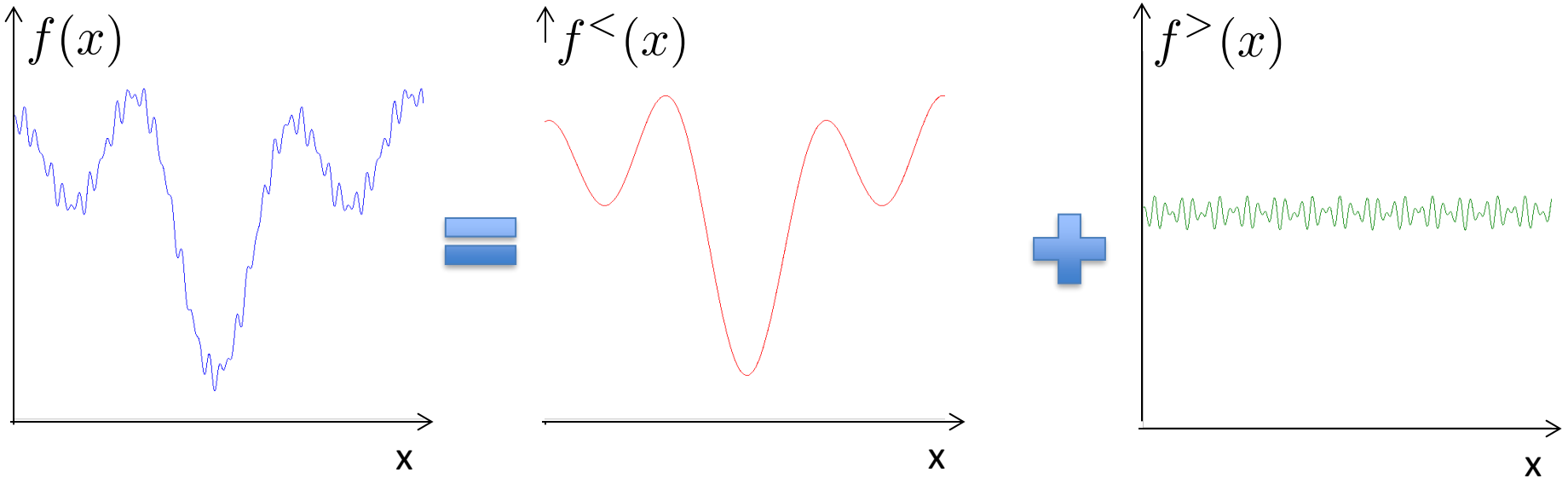
Grid turbulence



Observe: Structures on many different scales

Van Dyke, Album of Fluid Motion

# Filtering in space




Full signal

Low (spatial) frequencies
Large scales

High (spatial) frequencies
Small scales