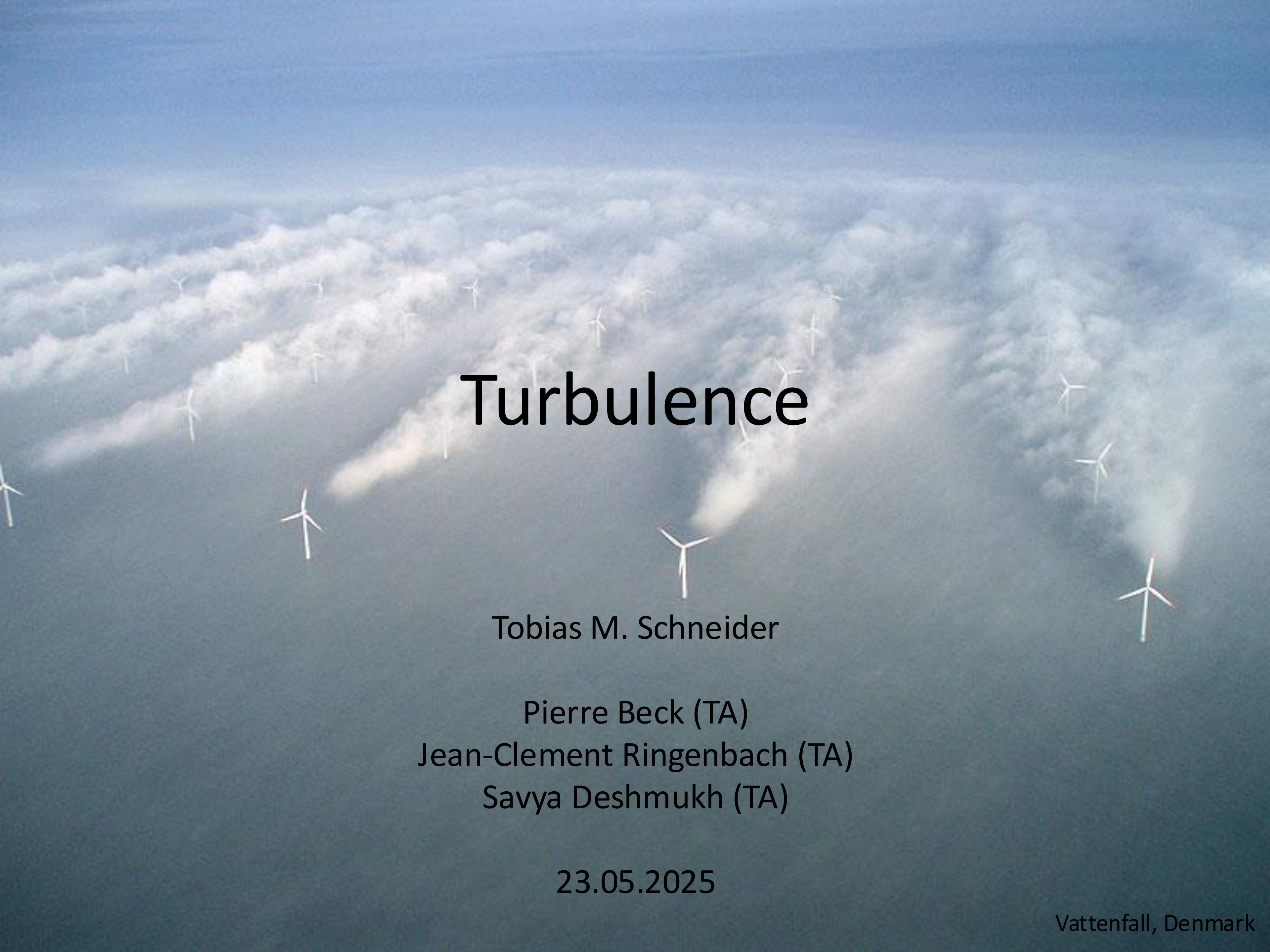

Turbulence

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Savya Deshmukh (TA)

23.05.2025

Vattenfall, Denmark

Logistics for the remaining week

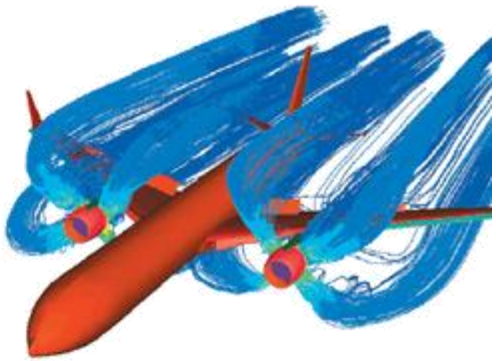
Reminder: Project deadline May 30 17h00 sharp

May 23rd: RANS & LES – last class of the semester (!!!)

Week of May 26th: if possible, one extra Q&A session
-> likely: Monday, May 26, 17h15 [TBC, Moodle]

Ch. 9 – Simulation and Models

Engineering problems involving turbulence - examples



Typical tasks

- determine forces (drag, lift,…)
- design geometries optimizing forces
- …

Features of the flow:

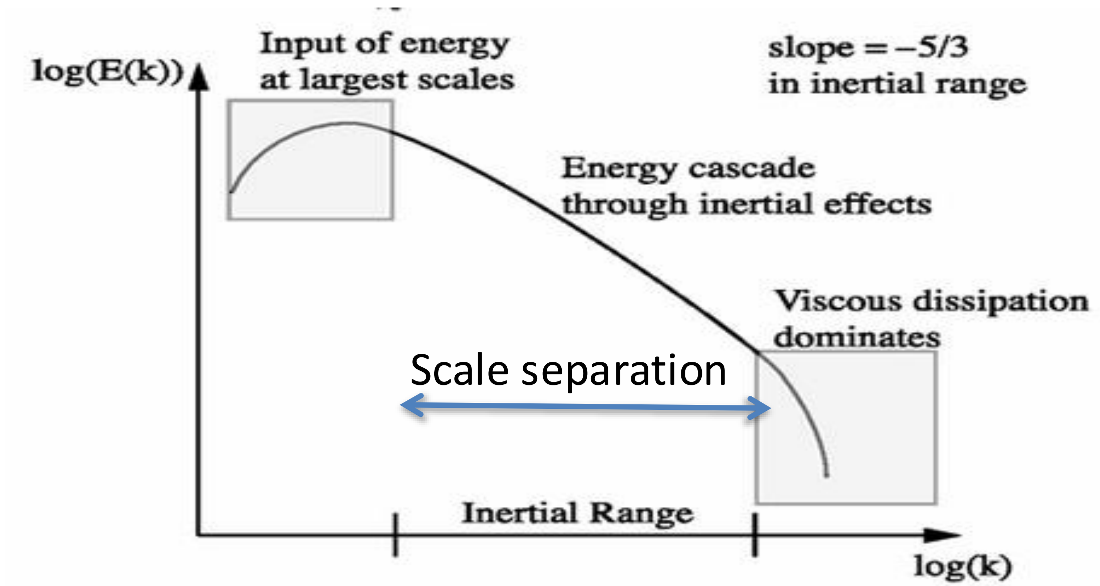
- high Reynolds number
- complex geometries
- …

Tools

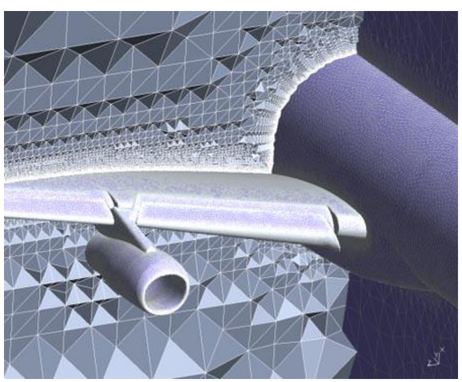
- experiments (full-size / models)
- computer simulation CFD – *Computational Fluid Dynamics*

Question: How to simulate the flow?

What about direct numerical simulation (DNS)?



$$\frac{l_0}{\eta} \sim Re^{3/4}$$



No of grid points: $N \sim Re^{9/4}$

→ Full DNS is impossible for most applications!!

We need to 'model' part of the turbulent dynamics!

Reynolds averaged Navier-Stokes (RANS)

- eddy viscosity
  - k-ε model(s)
- Reynolds-stress models

Large-eddy simulations (LES)

- various sub-grid-models

Model at all spatial scales
Computationally cheap

Model only at small scales
Computationally much more expensive

The plan for the last class

9.1 Introduction and motivation – Why models?

9.2 RANS Concepts

Reynolds decomposition and equations

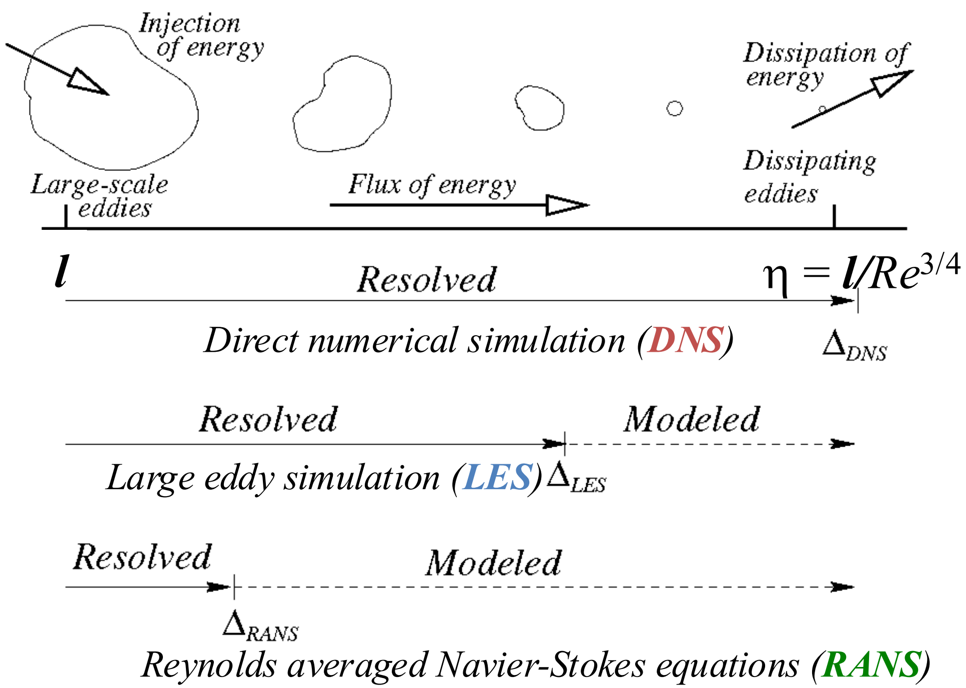
- closure problem

eddy viscosity models

- mixing length
- k-ε model
- Reynolds stress models

9.3 LES (very short)

Hierarchy of simulation approaches



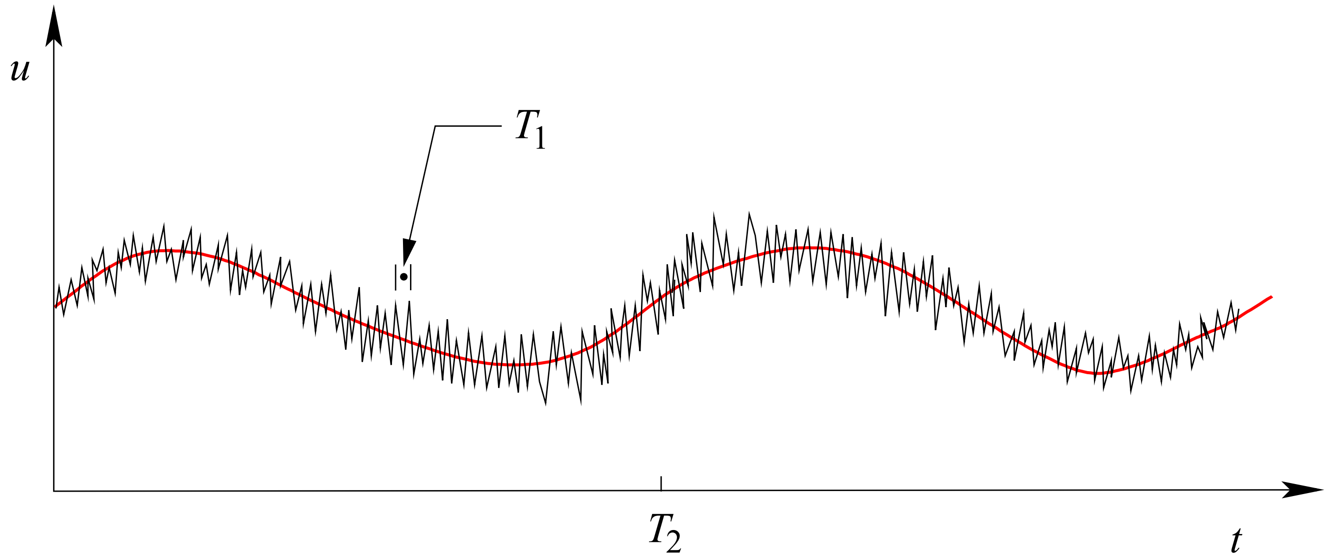
DNS: Direct numerical simulation
- no model assumptions
- resolving all scales

LES: Large-eddy simulation
- resolve large scales
- model assumptions for small scales

RANS: Reynolds averaged Navier Stokes simulation
- model at basically all scales
- solve for mean flow

RANS – the aim

Aim: Develop equation for the mean flow



RANS concepts

Reynolds decomposition

$$\underline{u}(\underline{x}, t) = \overline{\underline{u}}(\underline{x}, t) + \underline{u}'(\underline{x}, t)$$

Mean fluctuations

Reynolds averaged Navier-Stokes for the **mean** flow

$$\partial_t \overline{\underline{u}} + \overline{\underline{u}} \cdot \nabla \overline{\underline{u}} = -\nabla \overline{p} + \nu \nabla^2 \overline{\underline{u}} - \nabla \cdot \tau$$

$$\nabla \cdot \overline{\underline{u}} = 0 \qquad \tau_{ij} = < u_i' u_j' >$$

Reynolds stress tensor

Closure problem:

10 unknown fields: $\overline{\underline{u}}, p, \tau$ but only 4 equations

→ Models for *Reynolds stress tensor* required $\tau \leftrightarrow \overline{\underline{u}}$

The k-ε model

Evolution of turbulent energy k: (exact)

production

redistribution

$$k_t + \overline{u}_j \frac{\partial k}{\partial x_j} = -\overline{u_i' u_j'} \frac{\partial \overline{u}_i}{\partial x_j} - \varepsilon + \frac{\partial}{\partial x_j}\left(\nu \frac{\partial k}{\partial x_j} - \overline{p' u_j'} - \frac{1}{2}\overline{u_i' u_i' u_j'} \right)$$

dissipation

$$\varepsilon = 2\nu \overline{s_{ij}' s_{ij}'}$$

Model the redistribution terms:

$$\frac{\partial k}{\partial t} + \overline{u}_j \frac{\partial k}{\partial x_j} = -\overline{u_i' u_j'} \frac{\partial \overline{u}_i}{\partial x_j} - \varepsilon + \frac{\partial}{\partial x_j}\left[(\nu + \nu_T / \sigma_k) \frac{\partial k}{\partial x_j} \right]$$

Diffusive energy flux!!

The k-ε model

The ε equation (modeling of 6 higher order correlations)

$$\frac{\partial \varepsilon}{\partial t} + \bar{u}_j \frac{\partial \varepsilon}{\partial x_j} = -C_{\varepsilon 1} \frac{\varepsilon}{k} \overline{u_i' u_j'} \frac{\partial \bar{u}_i}{\partial x_j} - C_{\varepsilon 2} \frac{\varepsilon^2}{k} + \frac{\partial}{\partial x_j} \left[(\nu + \nu_T / \sigma_\varepsilon) \frac{\partial \varepsilon}{\partial x_j} \right]$$

Closing the k-ε model by

$$\nu_T = C_\nu \frac{k^2}{\varepsilon}$$

The k-ε model (Fluent 'standard')

$$\nabla \cdot \overline{u} = 0,$$

$$\overline{u}_t + \overline{u} \cdot \nabla \overline{u} = -\nabla \overline{p} + \nabla \cdot [(\nu + \nu_T)\nabla \overline{u}],$$

$$k_t + \overline{u} \cdot \nabla k = P - \varepsilon + \nabla \cdot [(\nu + \nu_T/\sigma_k)\nabla k],$$

$$\varepsilon_t + \overline{u} \cdot \nabla \varepsilon = C_{\varepsilon 1}\frac{\varepsilon}{k}P - C_{\varepsilon 2}\frac{\varepsilon^2}{k} + \nabla \cdot [(\nu + \nu_T/\sigma_\varepsilon)\nabla \varepsilon]$$

$$P = -\overline{u_i' u_j'}\frac{\partial \overline{u}_i}{\partial x_j}$$

$$-\overline{u_i' u_j'} = 2\nu_T \overline{s_{ij}} - \frac{2}{3}k\delta_{ij}$$

$$\nu_T = C_\nu \frac{k^2}{\varepsilon}$$

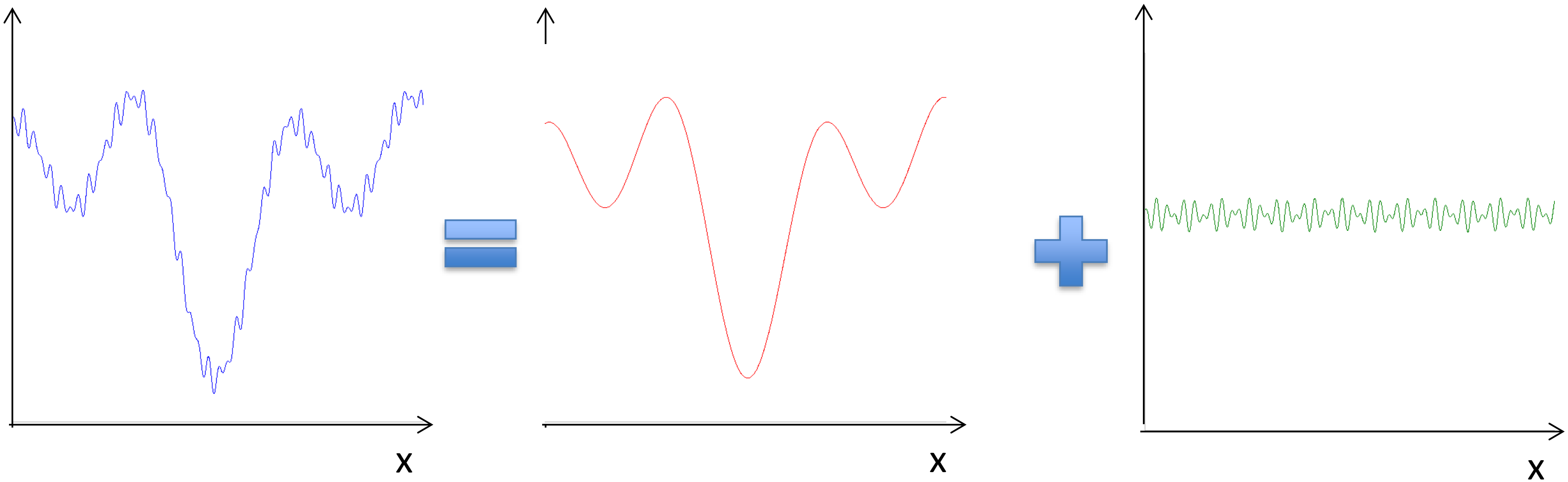
$$C_\nu = 0.09, \quad C_{\varepsilon 1} = 1.44, \quad C_{\varepsilon 2} = 1.92, \quad \sigma_k = 1.0, \quad \sigma_\varepsilon = 1.3$$

Fluent Doc

Note: Pope version: molecular viscosity $\nu = 0$

Large eddy simulations

Idea: - Decompose the flow into large scales and small scales
- Fully resolve the dynamics of large scales
- Only model on the small scales



Full signal

Low (spatial) frequencies
Large scales (resolved)

High (spatial) frequencies
Small scales (modeled)

Note: **Different** decompositions

RANS: Mean + fluctuations
LES: Large scales + small scales

LES equations

$$\tilde{u}_t + \nabla \cdot (\tilde{u}\tilde{u}) = -\nabla \tilde{p} + \nu \Delta \tilde{u} - \nabla \cdot \tau_{SGS}$$

Sub-grid stress

$$\tau_{SGS,ij} \equiv L_{ij} + C_{ij} + R_{ij}$$

$$L_{ij} \equiv \widetilde{\tilde{u}\tilde{v}} - \tilde{u}\tilde{v} \qquad \left(= \widetilde{\tilde{u}_i \tilde{u}_j} - \tilde{u}_i \tilde{u}_j \right)$$ Leonard stress

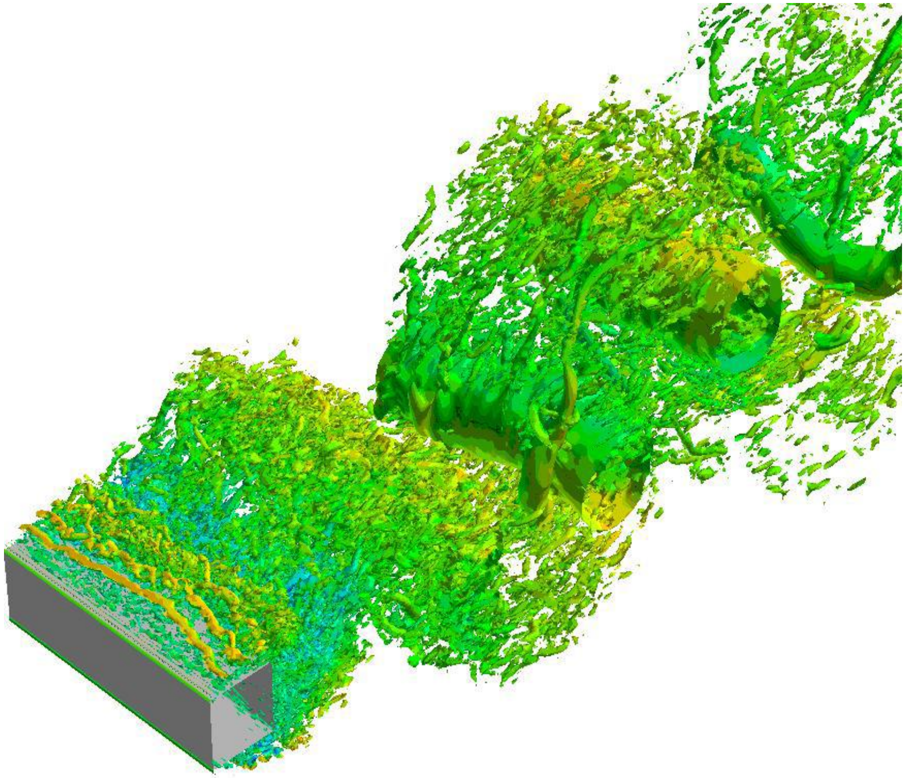
$$C_{ij} \equiv \widetilde{\tilde{u}_i u_j'} + \widetilde{\tilde{u}_i u_j'} \qquad \left(= \widetilde{\tilde{u}_i u_j'} + \widetilde{\tilde{u}_i u_j'} \right)$$ Cross stress

$$R_{ij} \equiv \widetilde{u'v'} \qquad \left(= \widetilde{u_i' u_j'} \right),$$ Reynolds stress

Smagorinsky sub-grid-model

$$\boldsymbol{\tau}_{SGS} = -2\nu_{SGS}\widetilde{\boldsymbol{S}} \qquad \nu_{SGS} = (C_S\Delta)^2|\widetilde{\boldsymbol{S}}|$$

Example of a LES



3D unsteady SVV-LES (Minguez et al. 2013),
Mesh O(10^6) points
500h supercomputer Nec SX8, GENCI

Summary turbulence theory

3 hypotheses

H1: restored symmetries

H2: self-similar scaling

H3: finite dissipation

Navier-Stokes equations

Energy transport

Statistics

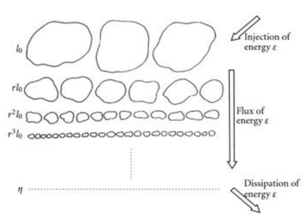
E(k) <-> correlations

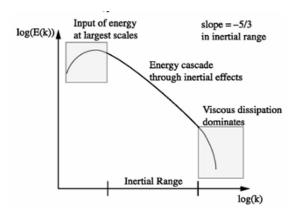
K41 theory, including spectrum

$$E(k) \sim \epsilon^{2/3} \, k^{-5/3}$$

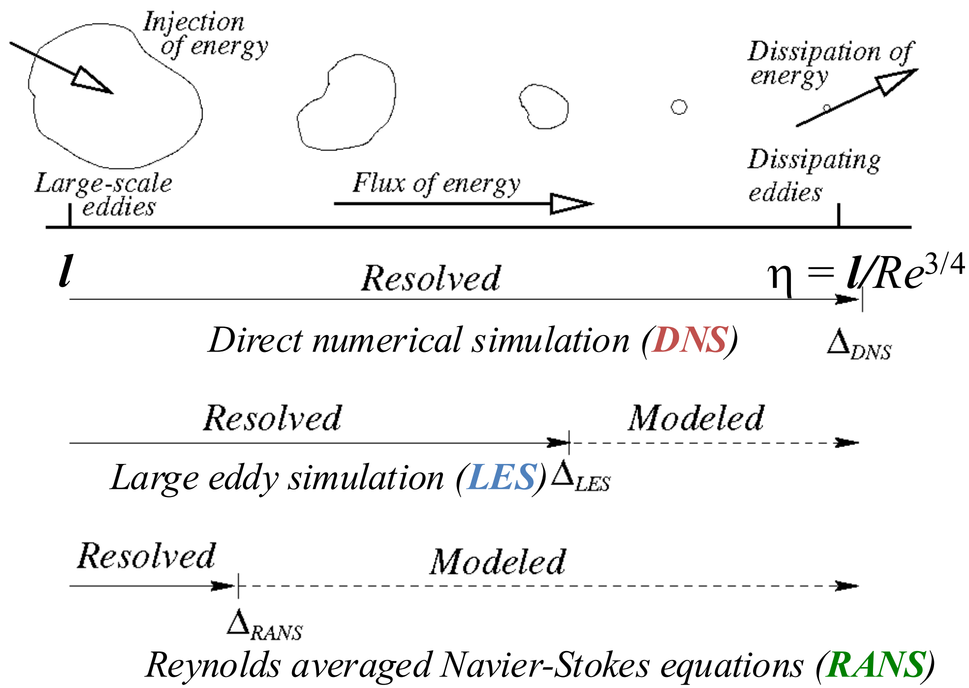
Physical picture

- Richardson cascade
- characteristic scales





Simulation approaches



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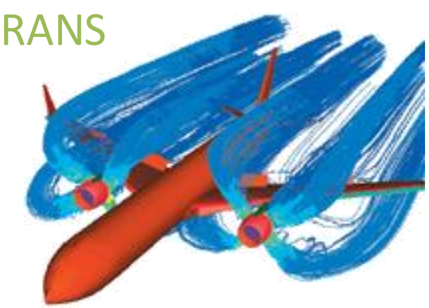
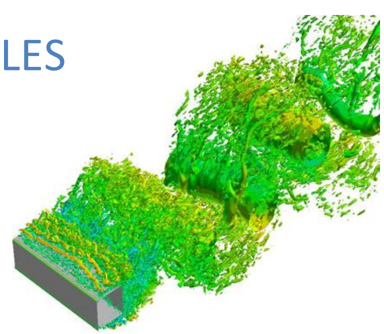
RANS



LES



Have a great summer break!!