ME-467: Turbulence 28 February 2025
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Exercise Set 1: Symmetries

Consider the diffusion equation

$$\frac{\partial T}{\partial t} = \frac{\partial^2 T}{\partial x^2} + s(x,t) \tag{1}$$

on the interval $x \in [-L/2 : L/2)$ with periodic boundary conditions and some source function $s(x,t)$. The temperature $T(x,t)$ has initial conditions

$$T(x, t=0) = T_0(x). \tag{2}$$

1 Symmetries of the System

The symmetries of the system depend on the function $s(x,t)$.

a) Assume s is of the form $s(t)$, i.e. it does not depend on x. What are the symmetries of the system?

b) Under which condition (on s) is the system symmetric under reflection?

2 Symmetries of the Solution

a) Let $s(x,t) = s(t)$ and $T_0(x) = const$. Use a symmetry to show that $T(x,t)$ is constant with respect to x for all times.

b) Let $s(x,t) = 0$ and $T_0(x) = \cos\left(\frac{4\pi x}{L}\right)$. Without solving the diffusion equation, what can you say about the shape of $T(t)$ at $t > 0$?

c) Let $s(x,t) = 0.01 \cdot \cos\left(\frac{2\pi x}{L}\right)$ and $T_0(x) = \cos\left(\frac{4\pi x}{L}\right)$. How does this change your answer from b)? Solve the equation to validate your predictions.