

Exercise Set 5: Probability Distributions

Let the random variable X be distributed according to the probability density function (PDF)

$$\rho(x) \equiv \frac{2}{\pi} \frac{1}{(1+x^2)^2} \quad (1)$$

for $x \in \mathbb{R}$. The cumulative probability of X is

$$R(x) = \frac{1}{\pi} \left[\frac{x}{1+x^2} + \tan^{-1}(x) + \frac{\pi}{2} \right]. \quad (2)$$

1 Moments of a Distribution

- Show that $\rho(x)$ is a PDF and its cumulative probability is $R(x)$.
- Calculate the mean and variance of X .
- Plot $\rho(x)$ and compare it to a normal distribution of equal mean and variance.
- If possible, find the skewness and flatness of X .
- Describe (in words) the higher order moments of X .

2 Transforming Probability Distributions

Consider the random variable Y on $y \in [0 : \infty)$, defined by

$$Y = \exp(X), \quad (3)$$

with X distributed according to some PDF $\rho(x)$ with cumulative probability $R(x)$.

- Find the cumulative probability $F_Y(y) \equiv \text{Prob}\{Y < y\}$.
- Find the PDF $f_Y(y)$ of Y .
- Determine the cumulative probability and PDF for $Z = \exp(-X)$.