

## Exercise Set 5: Probability Distributions

Let the random variable  $X$  be distributed according to the probability density function (PDF)

$$\rho(x) \equiv \frac{2}{\pi} \frac{1}{(1+x^2)^2} \quad (1)$$

for  $x \in \mathbb{R}$ . The cumulative probability of  $X$  is

$$R(x) = \frac{1}{\pi} \left[ \frac{x}{1+x^2} + \tan^{-1}(x) + \frac{\pi}{2} \right]. \quad (2)$$

### 1 Moments of a Distribution

- a) Show that  $\rho(x)$  is a PDF and its cumulative probability is  $R(x)$ .
- b) Calculate the mean and variance of  $X$ .
- c) Plot  $\rho(x)$  and compare it to a normal distribution of equal mean and variance.
- d) If possible, find the skewness and flatness of  $X$ .
- e) Describe (in words) the higher order moments of  $X$ .

### 2 Transforming Probability Distributions

Consider the random variable  $Y$  on  $y \in [0 : \infty)$ , defined by

$$Y = \exp(X), \quad (3)$$

with  $X$  distributed according to some PDF  $\rho(x)$  with cumulative probability  $R(x)$ .

- a) Find the cumulative probability  $F_Y(y) \equiv \text{Prob}\{Y < y\}$ .
- b) Find the PDF  $f_Y(y)$  of  $Y$ .
- c) Determine the cumulative probability and PDF for  $Z = \exp(-X)$ .