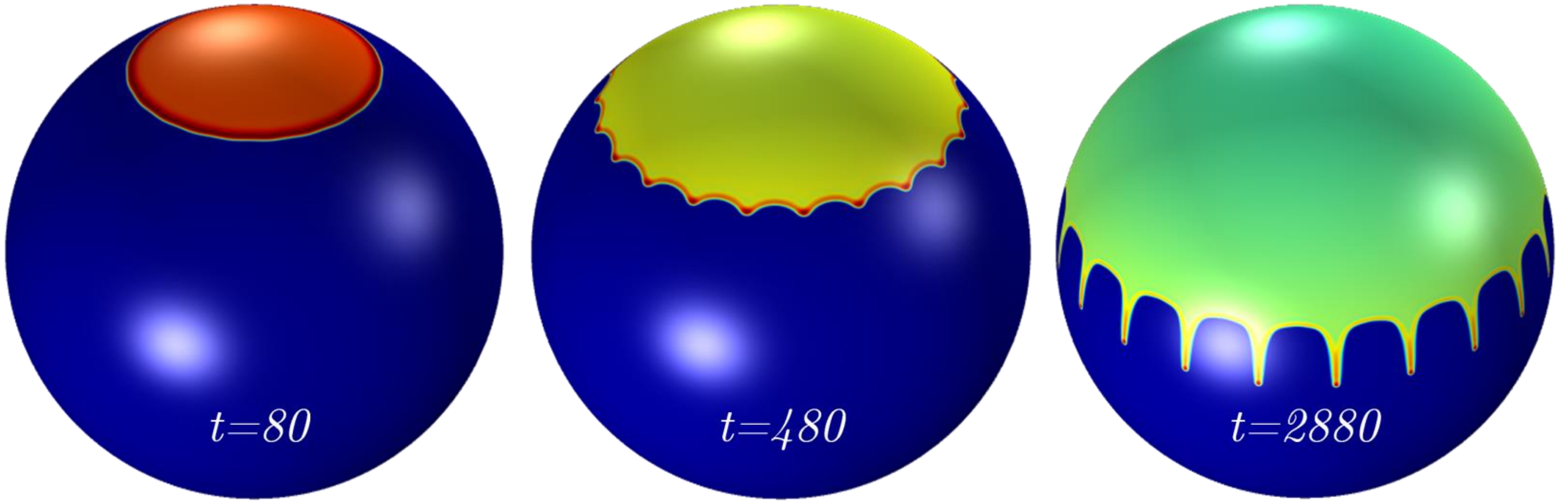
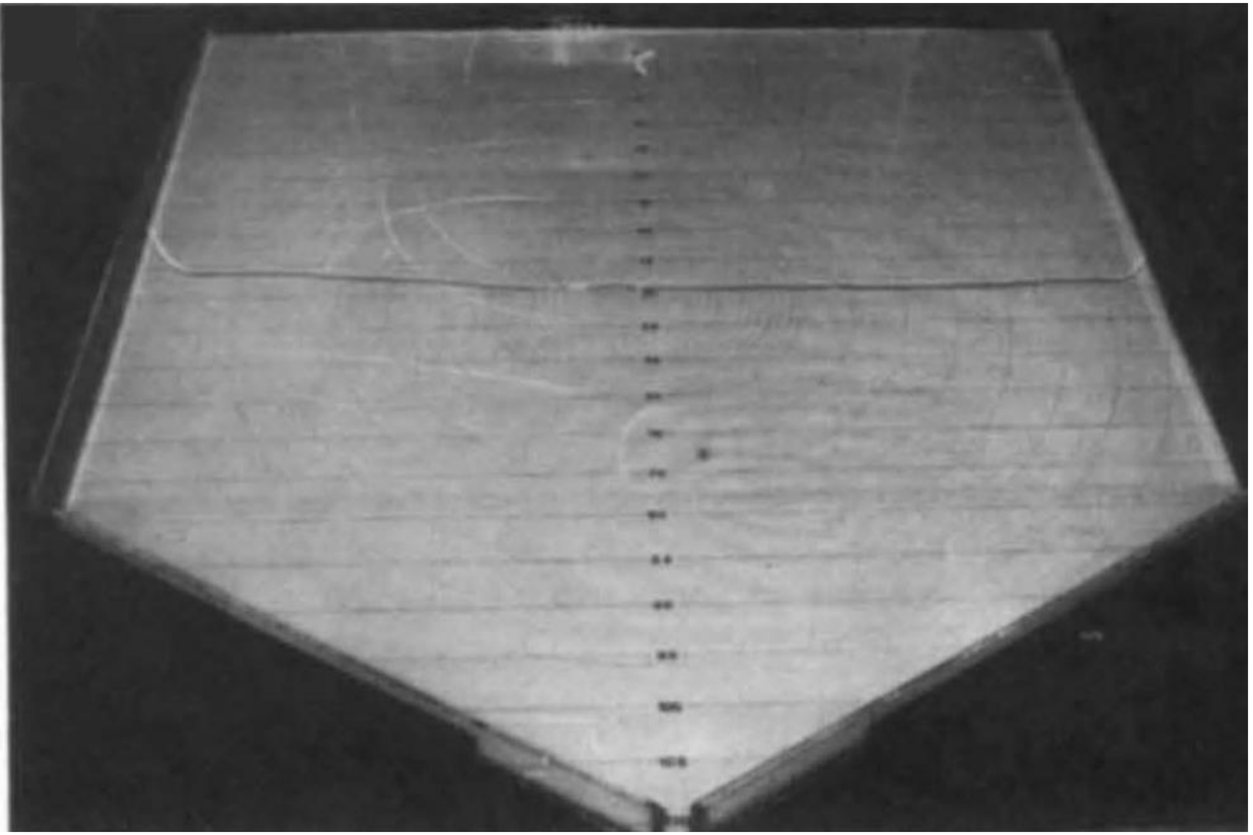
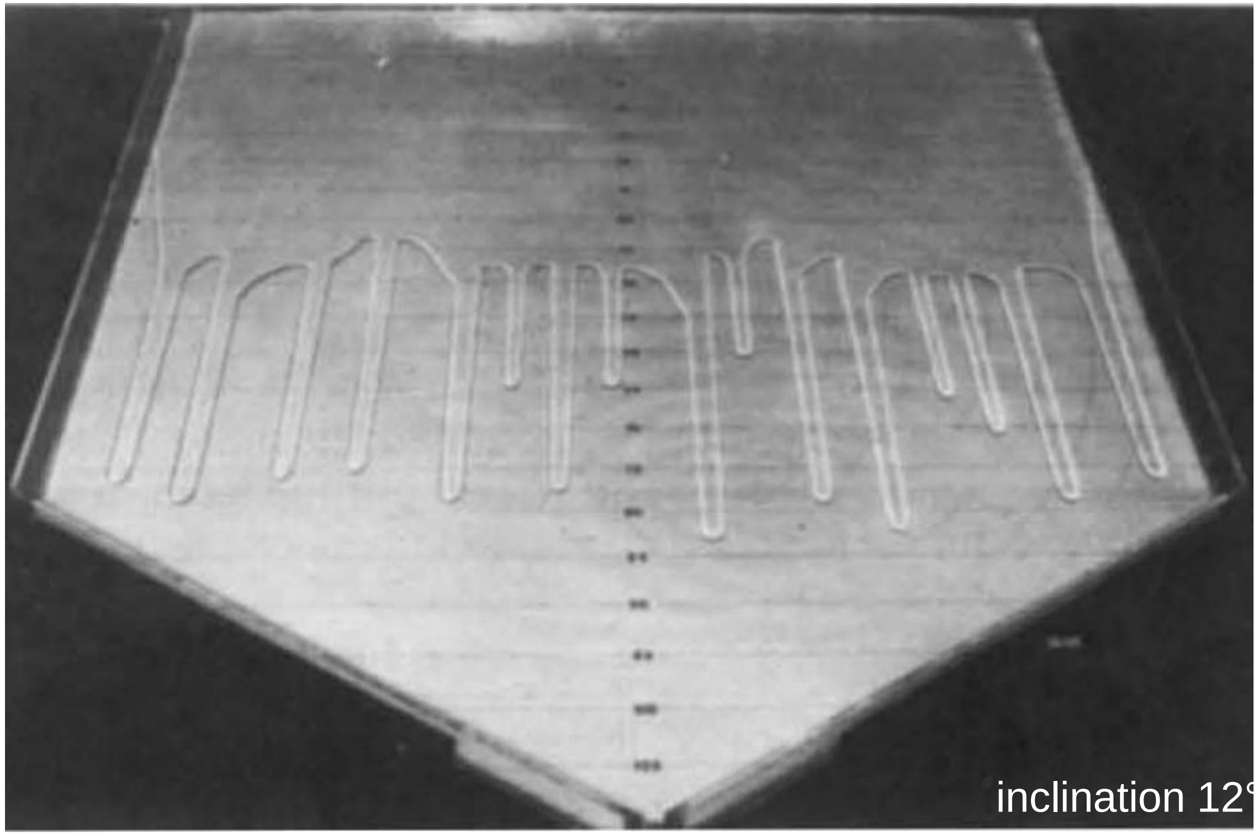
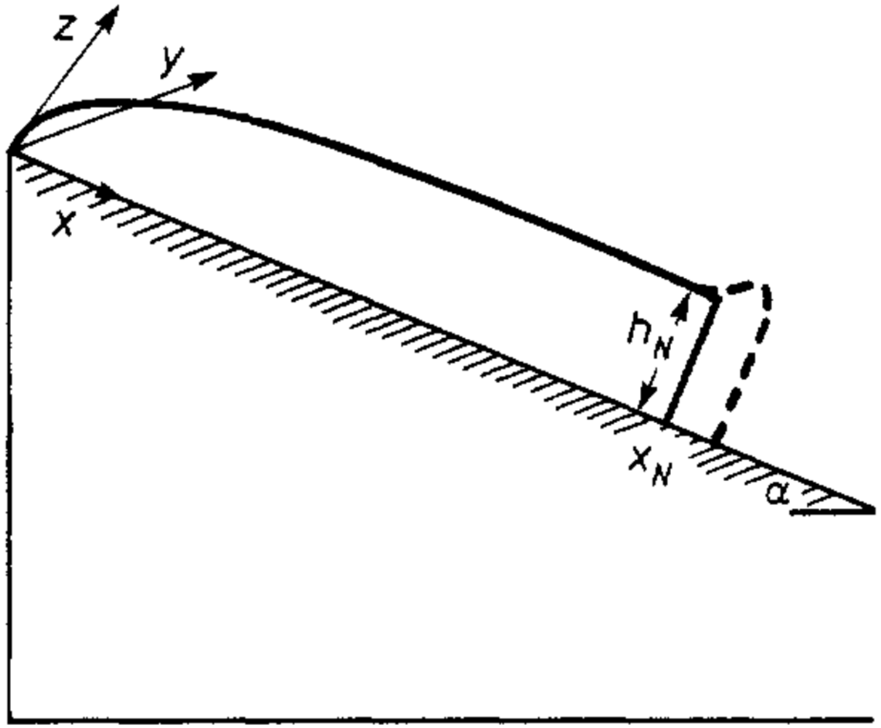
Contact line driven fingering



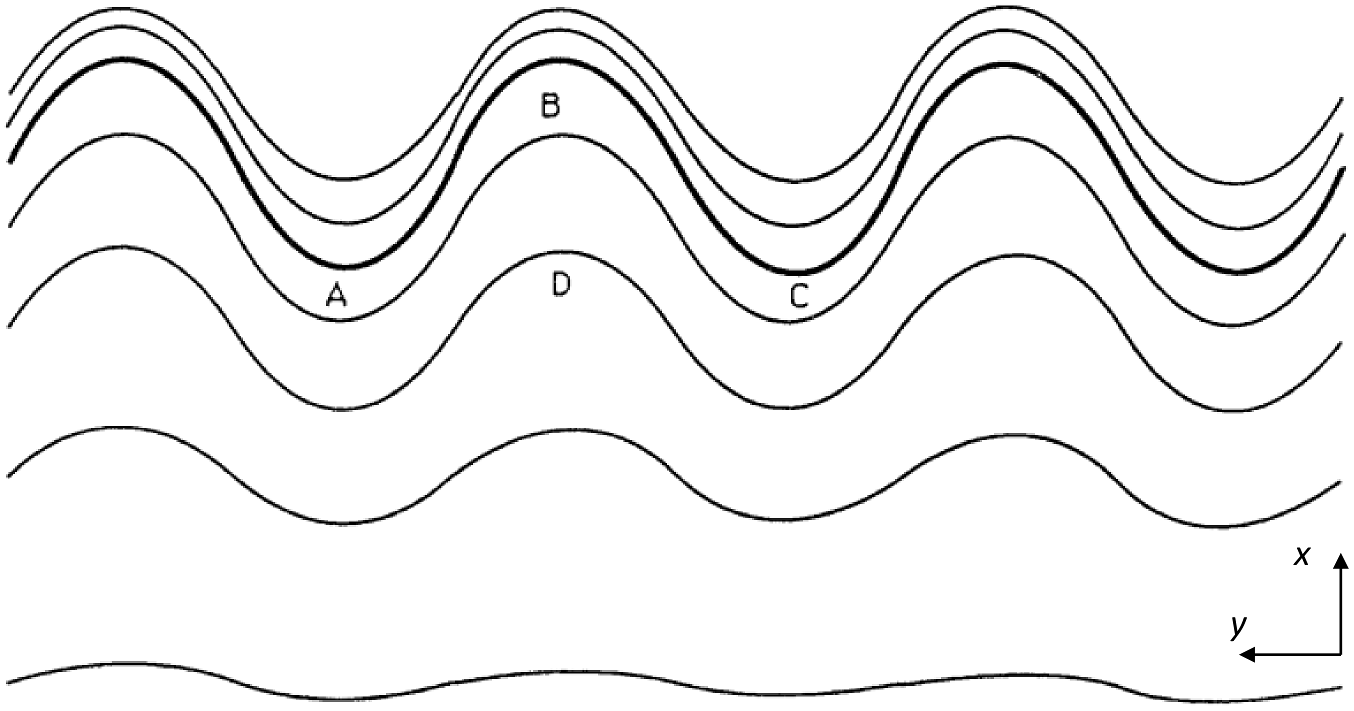




Huppert, "Flow and instability of a viscous current down a slope." *Nature* **300** (1982).



Troian, Herbolzheimer, Safran, and Joanny, “Fingering instabilities of driven spreading films.” *Europhys. Lett.* **10**:1 (1989).



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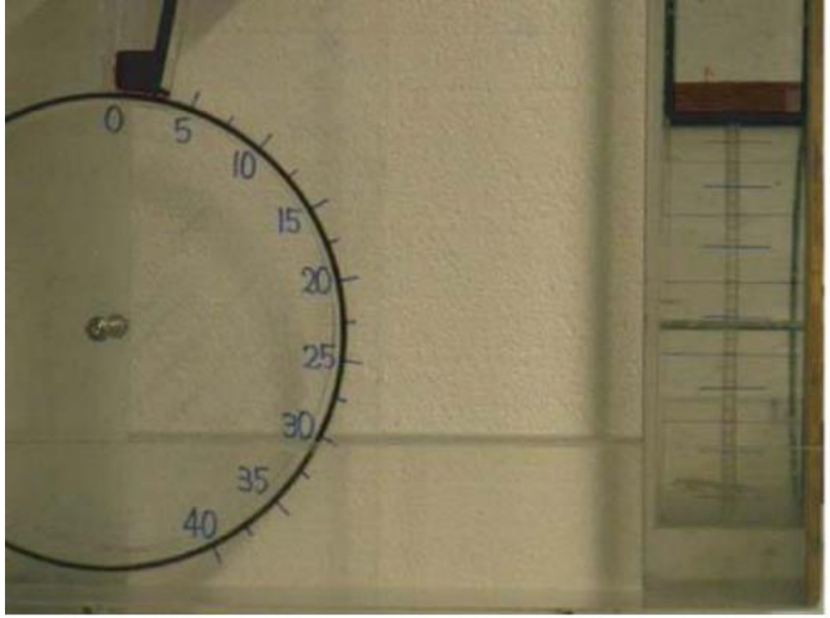
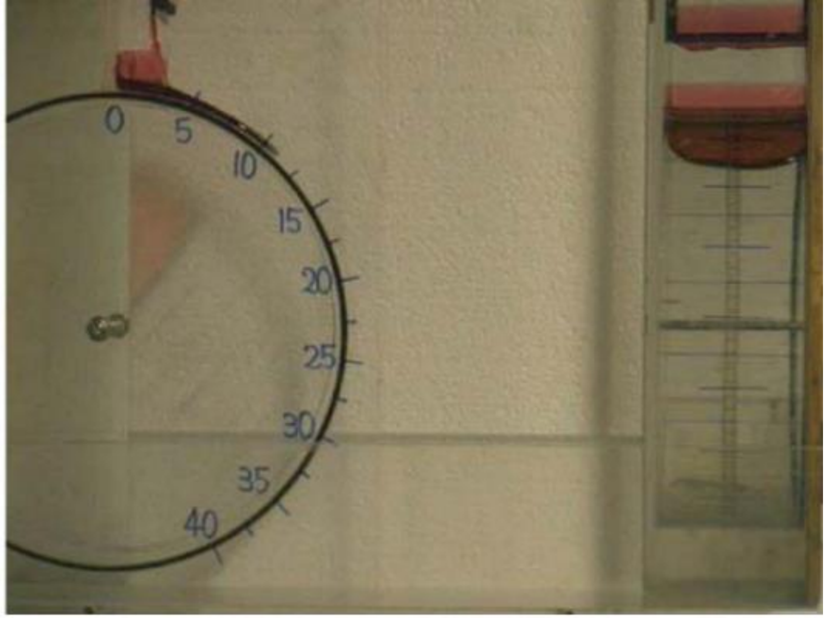
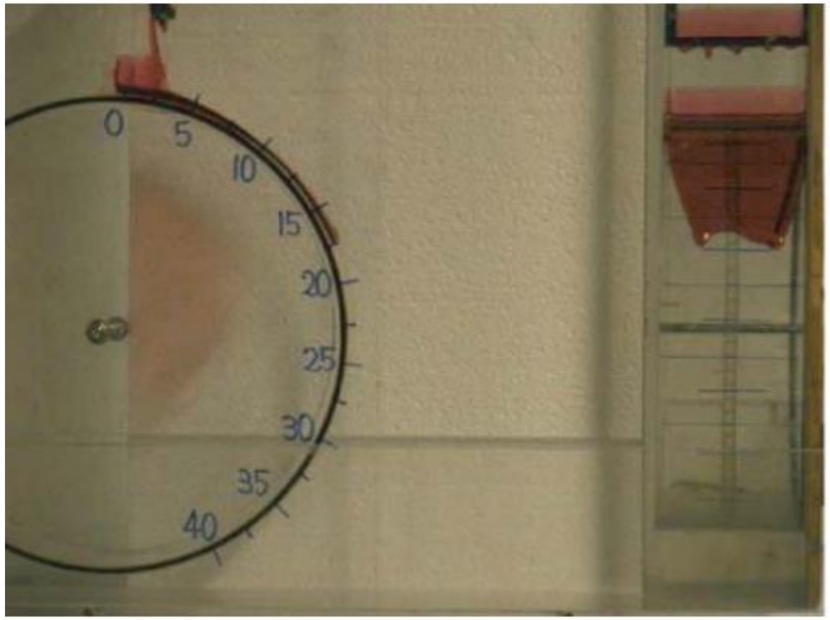


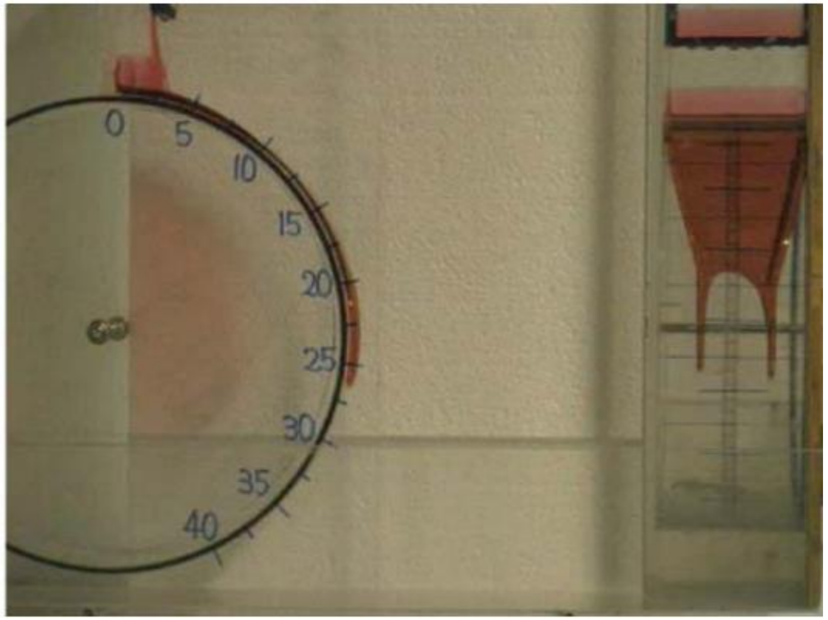
(*a*) $t = 0$



(*b*) $t = 0.7$ s

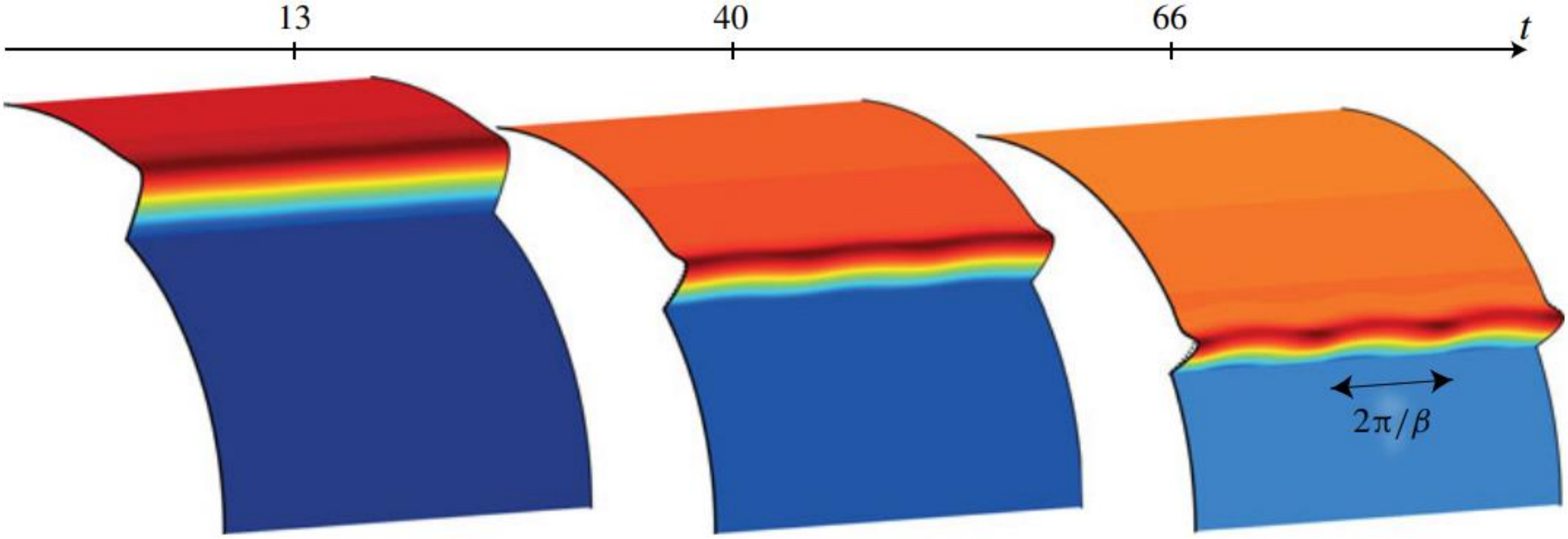


(*c*) $t = 1.5$ s



(*d*) $t = 2.2$ s

Takagi and Huppert, “Flow and instability of thin films on a cylinder and sphere.” *J. Fluid Mech.* **647** (2010).

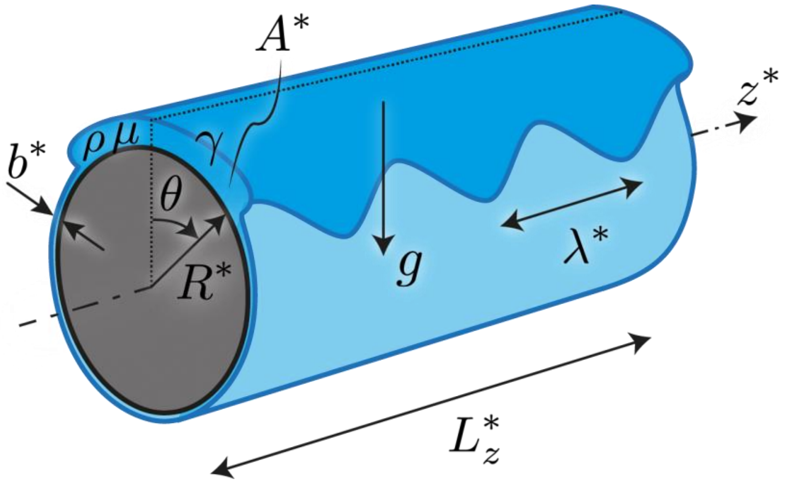


Balestra, Badaoui, Ducimetière, and Gallaire, “Fingering instability on curved substrates: optimal initial film and substrate perturbations.” *J. Fluid Mech.* **868** (2019)

Drainage solution

$$Bo = \rho g A^{3/2}/(\gamma R)$$

$$\delta = \sqrt{A}/R \ll 1$$

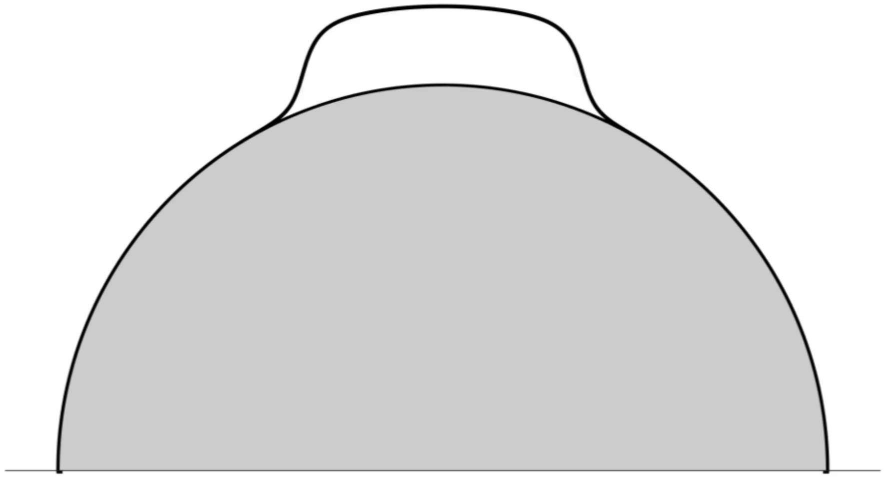


Newtonian fluid
Precursor film

Lubrication equation (1D)

$$H_t + \frac{1}{3}\left\{ H^3 \left[\underbrace{\frac{\delta^4}{Bo}(H_\theta + H_{\theta\theta\theta})}_{\mathbf{I}} - \underbrace{\delta \cos\theta H_\theta}_{\mathbf{II}} + \underbrace{\sin\theta}_{\mathbf{III}} \right] \right\}_\theta = 0$$

Initial condition $H_0(\theta) = c_1\{1 - \tanh[c_2(\theta - \phi)]\} + b$



Effect of the *curved* geometry

Forces depend on space and time

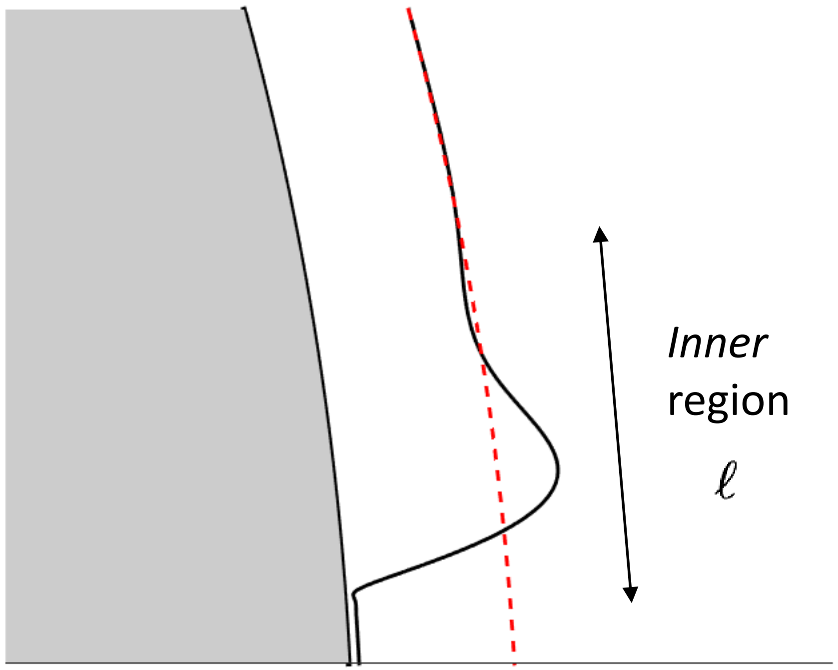
$$H_t + \frac{1}{3}\left\{ H^3 \left[\underbrace{\frac{\delta^4}{Bo}(H_\theta + H_{\theta\theta\theta})}_{\text{I}} - \underbrace{\delta\cos\theta H_\theta}_{\text{II}} + \underbrace{\sin\theta}_{\text{III}} \right] \right\}_\theta =$$

Short times: drainage term negligible

$$\ell_c = \left(\frac{\delta}{Bo\cos\theta_N}\right)^{1/2}$$

Late times: var. hydro pressure term negligible

$$\ell = \left(\frac{\delta H_N}{Bo\sin\theta_N}\right)^{1/3} = \left(\frac{\delta^2}{Bo\theta_N\sin\theta_N}\right)^{1/3} \qquad \ell \sim \left(\frac{2}{3}Bo\,t\right)^{-1/3}$$



Nonmodal analysis ➡ **Optimal transient growth analysis**

Capture disturbance growth also for asymptotically stable systems (Bertozzi & Brenner 1997)

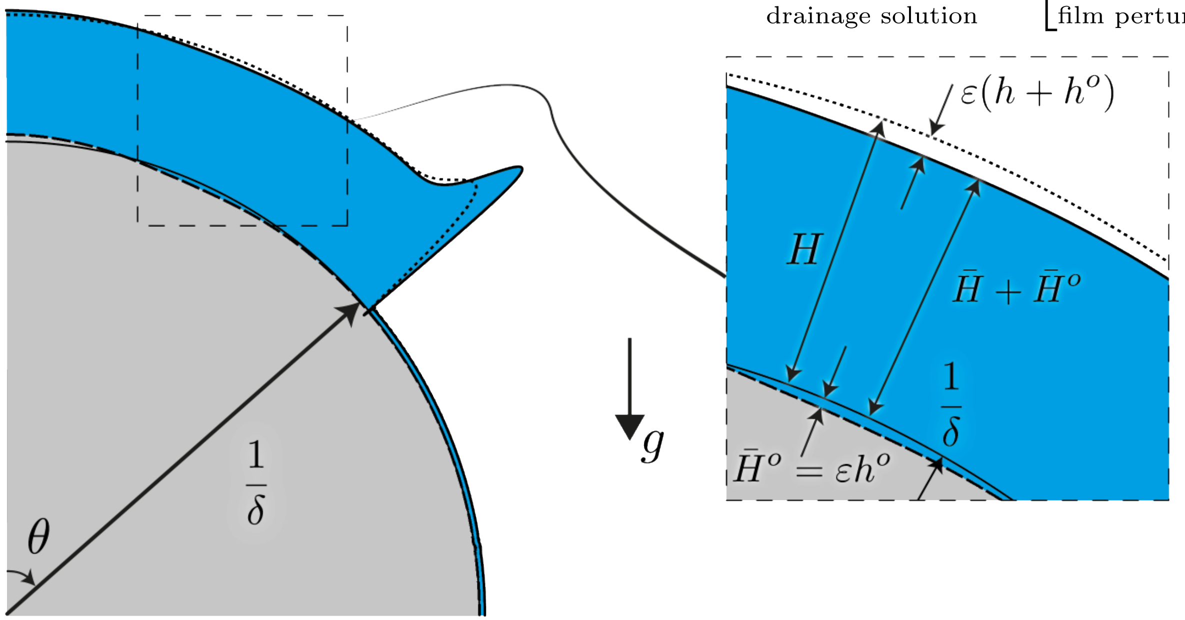
Optimal substrate perturbations

Lubrication equation

$$\bar{H}_t + \frac{1}{3}\left\{\bar{H}^3\left[\frac{\delta^2}{Bo}\bar{\kappa}_\theta - \delta\cos\theta(\bar{H}_\theta + \bar{H}^o_\theta) + \sin\theta\right]\right\}_\theta + \frac{1}{3\delta^2}\left\{\bar{H}^3\left[\frac{\delta^2}{Bo}\bar{\kappa}_z - \delta\cos\theta(\bar{H}_z + \bar{H}^o_z) - \delta^2\sin\theta\bar{H}\bar{H}^o_{\theta z}\right]\right\}_z = 0$$

Free-surface elevation decomposition

$$\bar{H}(\theta,z,t)+\bar{H}^o(\theta,z) = \underbrace{H(\theta,t)}_{\text{drainage solution}} + \varepsilon\left[\underbrace{\hat{h}(\theta,z,t)}_{\text{film perturbation}} + \underbrace{\hat{h}^o(\theta,z)}_{\text{substrate perturbation}}\right], \quad \varepsilon \ll 1$$



Periodic assumption in the axial direction

$$\hat{h}(\theta,z,t) = h(\theta,t)\exp(i\beta z) + c.c.,$$
$$\hat{h}^o(\theta,z) = h^o(\theta)\exp(i\beta z) + c.c.,$$

forcing

Linear disturbance equation

$$h_t + L(H,\beta,Bo,\delta)h = -L^o(H,\beta,Bo,\delta)h^o$$

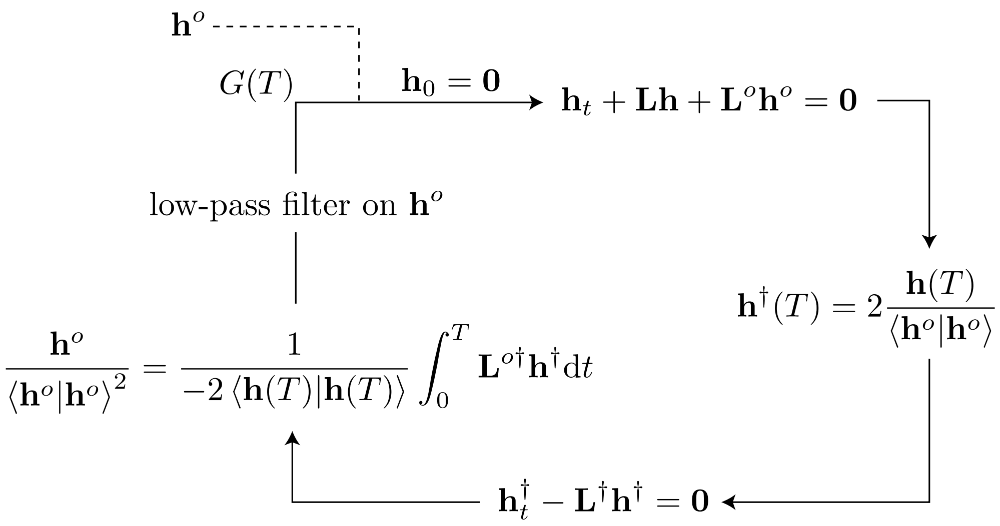
Transient growth analysis

Disturbance gain

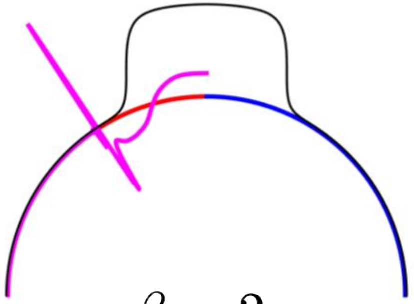
$$G(T) = \frac{E(T)}{E^o} = \frac{\langle h(T)|h(T)\rangle}{\langle h^o|h^o\rangle}$$

Linear operators are **space** and **time** dependent $\mathbf{L}(H, \beta, Bo, \delta)$ and $\mathrm{L}^o(H, \beta, Bo, \delta)$

Iterative approach using **direct** and **adjoint systems**



Types of optimal *substrate* perturbations



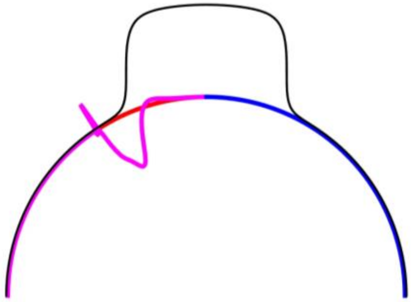
$\beta = 2$

optimal
substrate perturbation

response
evolution

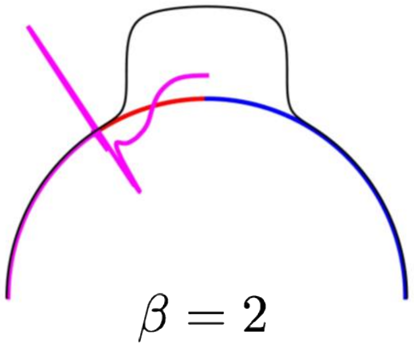
Short times T = 13

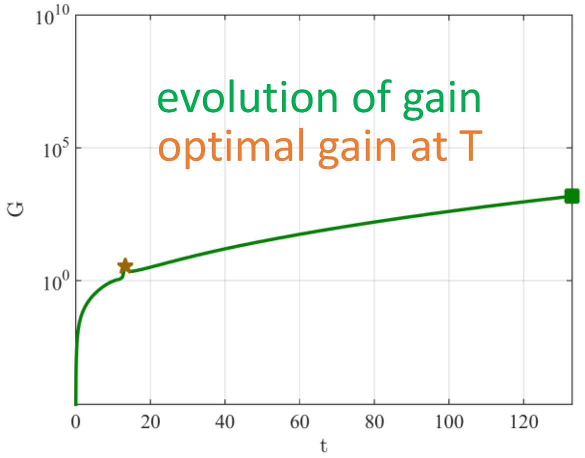
Front type

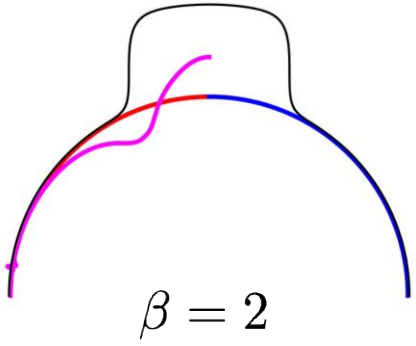


$\beta = 6$

Types of optimal *substrate* perturbations







Short times T = 13

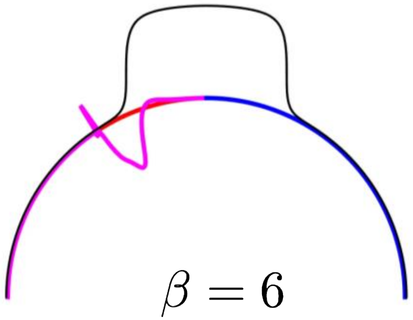
Front type

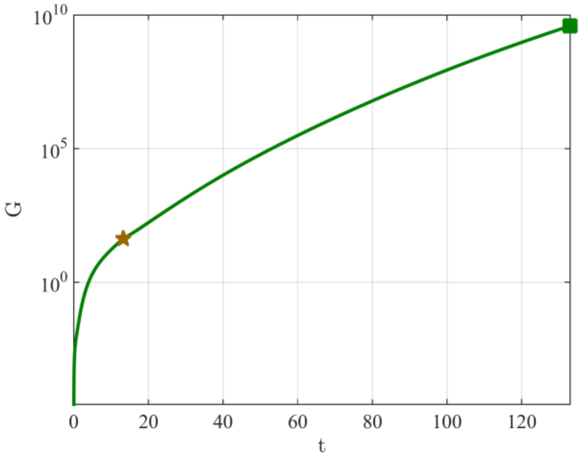
response evolution

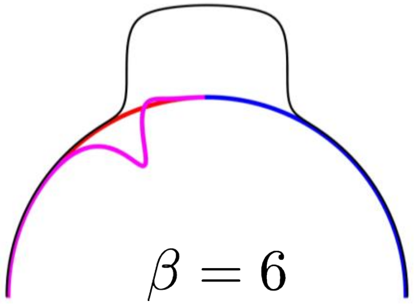
optimal substrate perturbation

Late times T = 120

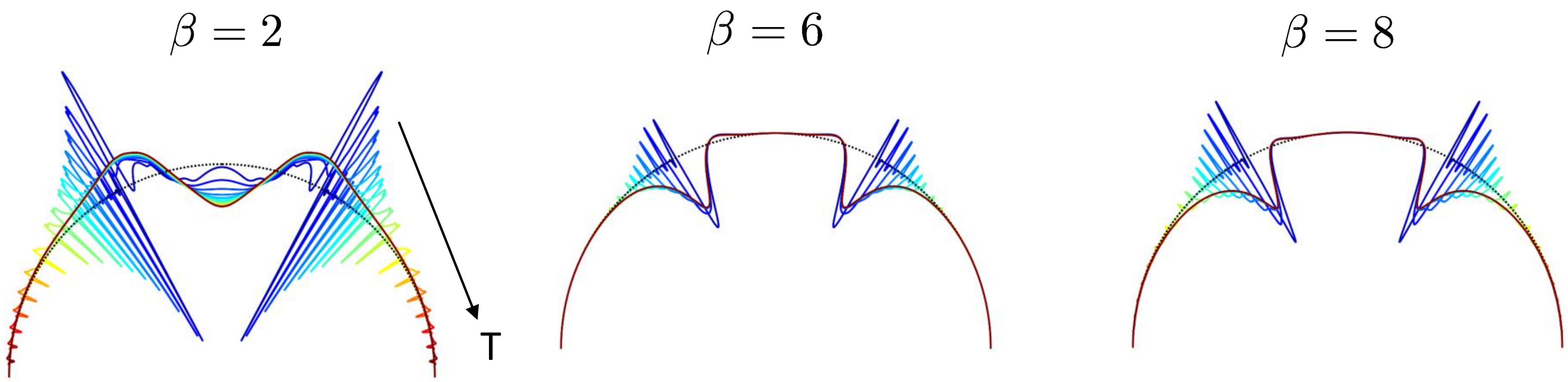
Bump type





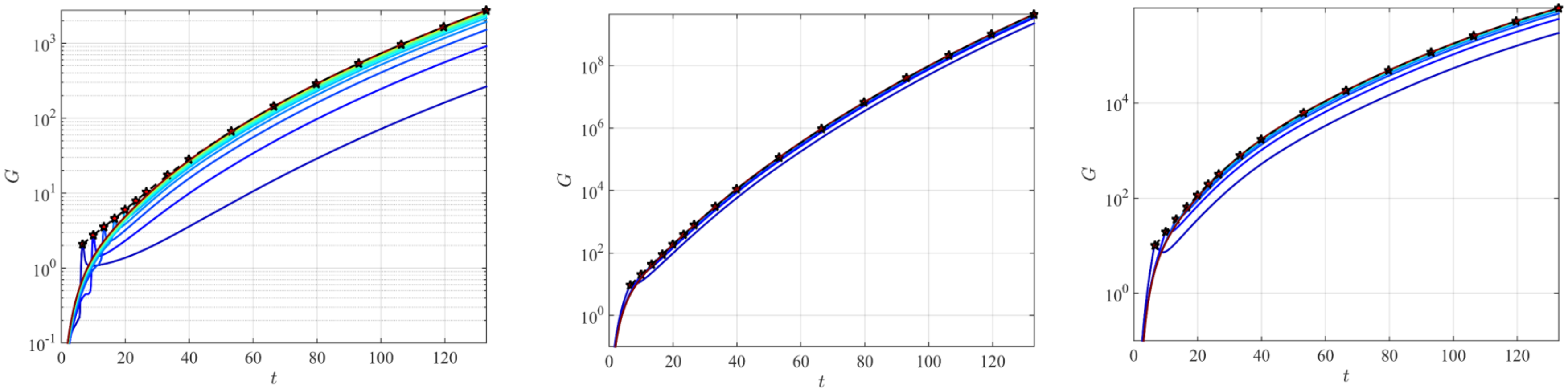


Effect of time horizon



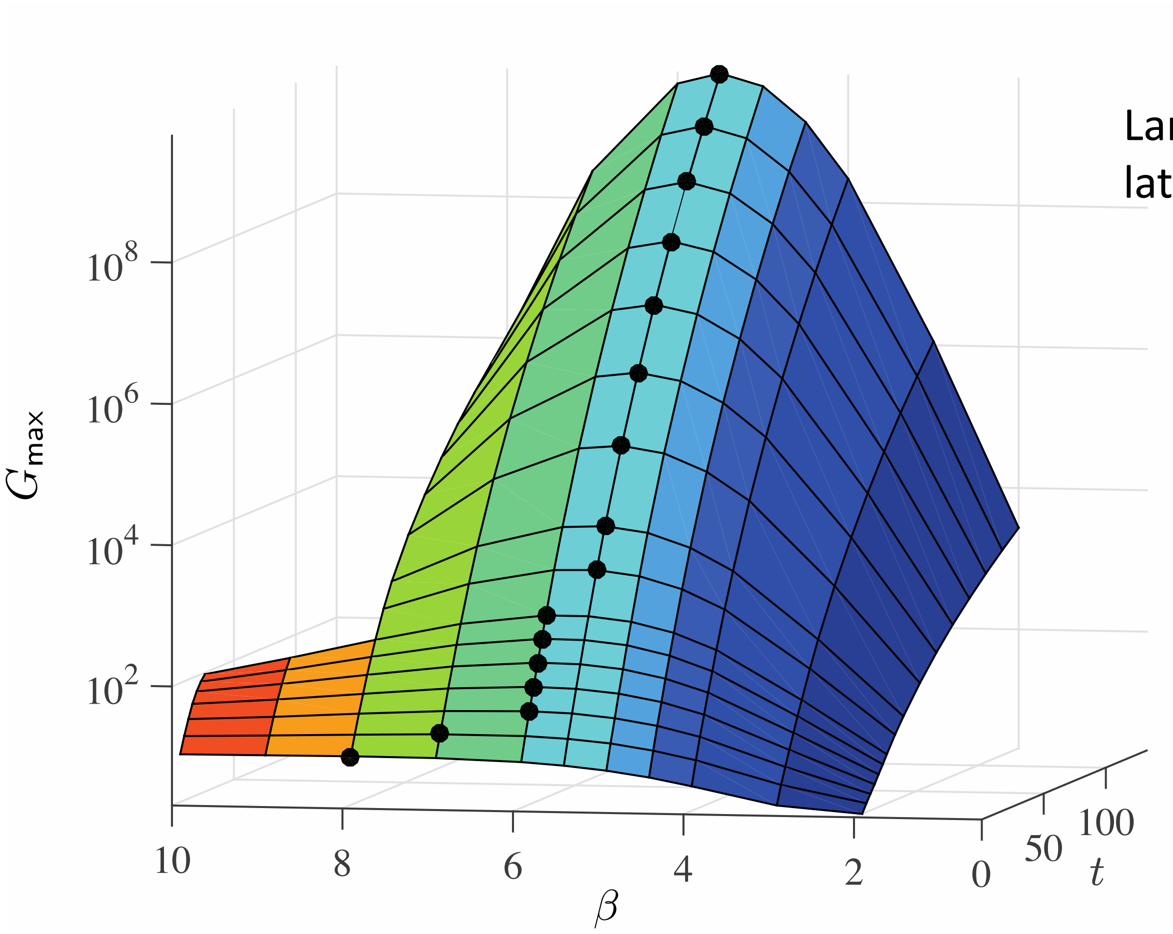
Bump location does not depend on beta

Most time independent optima substrate yields largest gain



Gain envelope

Envelopes of optimal gains



Large optimal gain at late times

Optimal wavenumber depends on optimization time

- *Large* wavenumbers at *short* times
- *Small* wavenumbers at *late* times

EPFL

Thanks!

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