ANNEXE II

OPÉRATEURS DIFFÉRENTIELS

1. Coordonnées cartésiennes orthonormées.

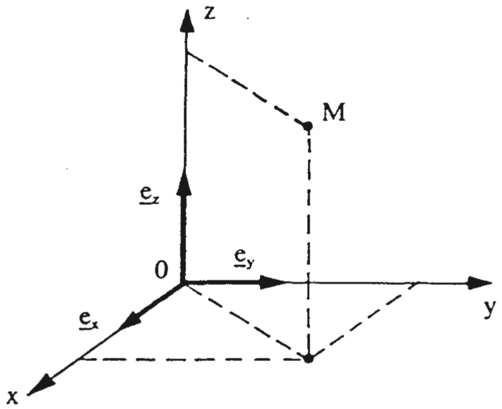

Figure 1 - Coordonnées cartésiennes orthonormées.

1.1. Coordonnées.

Un point M a pour coordonnées x_i (i = 1, 2, 3), notées également x, y, z, dans la base $\underline{e}_i$ (i = 1, 2, 3), notée également ($\underline{e}_x$, $\underline{e}_y$, $\underline{e}_z$) . Le vecteur position $\underline{OM}$ s'écrit :

$$\underline{OM} = x_i\, \underline{e}_i = x\, \underline{e}_x + y\, \underline{e}_y + z\, \underline{e}_z \quad .$$

1.2. Champ scalaire *b* ($\underline{x}$, *t*) .

$$\underline{\text{grad}}\ b = \frac{\partial b}{\partial x_i}\, \underline{e}_i$$

$$\Delta b = \text{div}\ (\underline{\text{grad}}\ b) = \frac{\partial^2 b}{\partial x_i\, \partial x_i}$$

$$\frac{db}{dt} = \frac{\partial b}{\partial t} + \underline{U} \cdot \underline{\text{grad}}\ b = \frac{\partial b}{\partial t} + U_i\, \frac{\partial b}{\partial x_i}$$

1.3. Champ de vecteurs $\underline{b}$ ($\underline{x}$, *t*) .

$$\underline{b}\ (\underline{x}\ , t) = b_i\ (\underline{x}\ , t)\ \underline{e}_i$$

ou

$$\underline{b}(\underline{x}, t) = b_x(\underline{x}, t)\,\underline{e}_x + b_y(\underline{x}, t)\,\underline{e}_y + b_z(\underline{x}, t)\,\underline{e}_z$$

$$\operatorname{div} \underline{b} = \frac{\partial b_i}{\partial x_i}$$

$$\Delta \underline{b} = \operatorname{div}(\underline{\underline{\operatorname{grad} b}}) = \frac{\partial^2 b_i}{\partial x_j\,\partial x_j}\,\underline{e}_i$$

$$\underline{\operatorname{rot}}\ \underline{b} = \varepsilon_{ijk}\frac{\partial b_k}{\partial x_j}\,\underline{e}_i$$

ou

$$\underline{\operatorname{rot}}\ \underline{b} = \left(\frac{\partial b_z}{\partial y} - \frac{\partial b_y}{\partial z}\right)\underline{e}_x + \left(\frac{\partial b_x}{\partial z} - \frac{\partial b_z}{\partial x}\right)\underline{e}_y + \left(\frac{\partial b_y}{\partial x} - \frac{\partial b_x}{\partial y}\right)\underline{e}_z$$

$$\underline{\underline{\operatorname{grad} b}} = \frac{\partial b_i}{\partial x_j}\,\underline{e}_i \otimes \underline{e}_j$$

ou

$$\underline{\underline{\operatorname{grad} b}} = \begin{pmatrix} \dfrac{\partial b_x}{\partial x} & \dfrac{\partial b_x}{\partial y} & \dfrac{\partial b_x}{\partial z} \\[2ex] \dfrac{\partial b_y}{\partial x} & \dfrac{\partial b_y}{\partial y} & \dfrac{\partial b_y}{\partial z} \\[2ex] \dfrac{\partial b_z}{\partial x} & \dfrac{\partial b_z}{\partial y} & \dfrac{\partial b_z}{\partial z} \end{pmatrix}$$

$$\frac{d\underline{b}}{dt} = \frac{\partial \underline{b}}{\partial t} + \underline{\underline{\operatorname{grad} b}}\,.\,\underline{U} = \left(\frac{\partial}{\partial t} + U_j\frac{\partial}{\partial x_j}\right) b_i\,\underline{e}_i$$

$$\underline{\underline{d}} = d_{ij}\,\underline{e}_i \otimes \underline{e}_j = \frac{1}{2}\left(\underline{\underline{\operatorname{grad} U}} + {}^t\underline{\underline{\operatorname{grad} U}}\right) = \frac{1}{2}\left(\frac{\partial U_i}{\partial x_j} + \frac{\partial U_j}{\partial x_i}\right)\underline{e}_i \otimes \underline{e}_j$$

1.4. Champ de tenseurs du 2ème ordre $\underline{\underline{b}}(\underline{x}, t)$.

$$\underline{\underline{b}}(\underline{x}, t) = b_{ij}\,\underline{e}_i \otimes \underline{e}_j$$

$$\operatorname{div} \underline{\underline{b}} = \frac{\partial b_{ij}}{\partial x_j}\,\underline{e}_i$$

2. *Coordonnées cylindriques.*

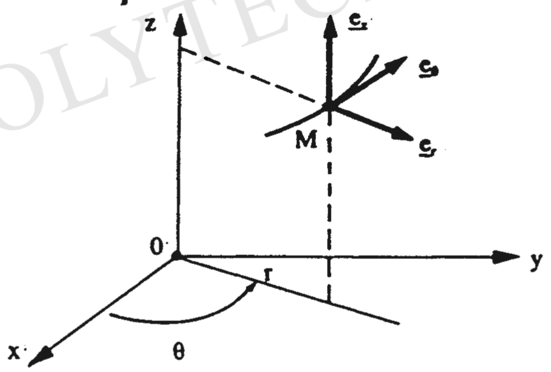

Figure 2 - Coordonnées cylindriques.

2.1. *« Coordonnées ».*

Un point M a pour « coordonnées » cylindriques r, θ et z, la base orthonormée locale correspondante étant notée $\underline{e}_r$, $\underline{e}_\theta$, $\underline{e}_z$. Le déplacement élémentaire $\underline{dM}$ s'écrit :

$$\underline{dM} = dr\,\underline{e}_r + r\,d\theta\,\underline{e}_\theta + dz\,\underline{e}_z \quad .$$

2.2. *Champ scalaire b ($\underline{x}$, t) .*

$$b(\underline{x}, t) = b(r, \theta, z, t)$$

$$\underline{\mathrm{grad}}\, b = \frac{\partial b}{\partial r}\underline{e}_r + \frac{1}{r}\frac{\partial b}{\partial \theta}\underline{e}_\theta + \frac{\partial b}{\partial z}\underline{e}_z$$

$$\Delta b = \mathrm{div}\,(\underline{\mathrm{grad}}\, b) = \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial b}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2 b}{\partial \theta^2} + \frac{\partial^2 b}{\partial z^2}$$

$$\frac{db}{dt} = \frac{\partial b}{\partial t} + \underline{U}\,.\,\underline{\mathrm{grad}}\, b = \left(\frac{\partial}{\partial t} + U_r\frac{\partial}{\partial r} + \frac{U_\theta}{r}\frac{\partial}{\partial \theta} + U_z\frac{\partial}{\partial z}\right) b$$

2.3. *Champ de vecteurs $\underline{b}$ ($\underline{x}$, t) .*

Les composantes de $\underline{b}$ ($\underline{x}$, t) dans la base locale sont b_r , b_θ , b_z de telle sorte que :

$$\underline{b}(\underline{x}, t) = b_r(r, \theta, z, t)\,\underline{e}_r + b_\theta(r, \theta, z, t)\,\underline{e}_\theta + b_z(r, \theta, z, t)\,\underline{e}_z \quad .$$

$$\mathrm{div}\,\underline{b} = \frac{1}{r}\frac{\partial}{\partial r}(r\,b_r) + \frac{1}{r}\frac{\partial b_\theta}{\partial \theta} + \frac{\partial b_z}{\partial z}$$

$$\Delta \underline{b} = \mathrm{div}\,(\underline{\underline{\mathrm{grad}\, b}}) = \left[\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial}{\partial r}(r\, b_r) \right) + \frac{1}{r^2}\frac{\partial^2 b_r}{\partial \theta^2} - \frac{2}{r^2}\frac{\partial b_\theta}{\partial \theta} + \frac{\partial^2 b_r}{\partial z^2} \right] \underline{e}_r +$$

$$\left[\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial}{\partial r}(r\, b_\theta) \right) + \frac{1}{r^2}\frac{\partial^2 b_\theta}{\partial \theta^2} + \frac{2}{r^2}\frac{\partial b_r}{\partial \theta} + \frac{\partial^2 b_\theta}{\partial z^2} \right] \underline{e}_\theta + \Delta b_z\, \underline{e}_z$$

$$\underline{\mathrm{rot}}\ \underline{b} = \left(\frac{1}{r}\frac{\partial b_z}{\partial \theta} - \frac{\partial b_\theta}{\partial z} \right) \underline{e}_r + \left(\frac{\partial b_r}{\partial z} - \frac{\partial b_z}{\partial r} \right) \underline{e}_\theta + \left(\frac{1}{r}\frac{\partial\, (r\, b_\theta)}{\partial r} - \frac{1}{r}\frac{\partial b_r}{\partial \theta} \right) \underline{e}_z$$

$$\underline{\underline{\mathrm{grad}\ b}} = \begin{pmatrix} \dfrac{\partial b_r}{\partial r} & \dfrac{1}{r}\left(\dfrac{\partial b_r}{\partial \theta} - b_\theta \right) & \dfrac{\partial b_r}{\partial z} \\[2ex] \dfrac{\partial b_\theta}{\partial r} & \dfrac{1}{r}\left(\dfrac{\partial b_\theta}{\partial \theta} + b_r \right) & \dfrac{\partial b_\theta}{\partial z} \\[2ex] \dfrac{\partial b_z}{\partial r} & \dfrac{1}{r}\dfrac{\partial b_z}{\partial \theta} & \dfrac{\partial b_z}{\partial z} \end{pmatrix}$$

$$\frac{d\underline{b}}{dt} = \frac{\partial \underline{b}}{\partial t} + \underline{\underline{\mathrm{grad}\ b}}\,.\,\underline{U} = \left(\frac{db_r}{dt} - \frac{b_\theta\, U_\theta}{r} \right) \underline{e}_r + \left(\frac{db_\theta}{dt} + \frac{b_r\, U_\theta}{r} \right) \underline{e}_\theta + \frac{db_z}{dt}\, \underline{e}_z$$

$$\underline{\underline{d}} = \frac{1}{2}\,(\underline{\underline{\mathrm{grad}\ U}} + {}^t\underline{\underline{\mathrm{grad}\ U}})$$

$$d_{rr} = \frac{\partial U_r}{\partial r} \qquad d_{\theta\theta} = \frac{1}{r}\frac{\partial U_\theta}{\partial \theta} + \frac{U_r}{r} \qquad d_{zz} = \frac{\partial U_z}{\partial z}$$

$$d_{\theta z} = \frac{1}{2}\left(\frac{1}{r}\frac{\partial U_z}{\partial \theta} + \frac{\partial U_\theta}{\partial z} \right) \qquad d_{zr} = \frac{1}{2}\left(\frac{\partial U_r}{\partial z} + \frac{\partial U_z}{\partial r} \right)$$

$$d_{r\theta} = \frac{1}{2}\left(r\,\frac{\partial}{\partial r}\left(\frac{U_\theta}{r}\right) + \frac{1}{r}\frac{\partial U_r}{\partial \theta} \right)$$

2.4. Champ de tenseurs du $2^{\grave{e}me}$ ordre symétriques $\underline{\underline{b}}\,(\underline{x}\,,\,t)$.

$$\underline{\underline{b}}\,(\underline{x}\,,\,t) = b_{ij}\,(r\,,\,\theta\,,\,z\,,\,t)\ \underline{e}_i \otimes \underline{e}_j$$

$$\mathrm{div}\ \underline{\underline{b}} = \left(\frac{\partial b_{rr}}{\partial r} + \frac{1}{r}\frac{\partial b_{r\theta}}{\partial \theta} + \frac{\partial b_{rz}}{\partial z} + \frac{b_{rr} - b_{\theta\theta}}{r} \right) \underline{e}_r$$

$$+ \left(\frac{\partial b_{\theta r}}{\partial r} + \frac{1}{r}\frac{\partial b_{\theta\theta}}{\partial \theta} + \frac{\partial b_{\theta z}}{\partial z} + 2\,\frac{b_{r\theta}}{r} \right) \underline{e}_\theta$$

$$+ \left(\frac{\partial b_{zr}}{\partial r} + \frac{1}{r}\frac{\partial b_{z\theta}}{\partial \theta} + \frac{\partial b_{zz}}{\partial z} + \frac{b_{zr}}{r} \right) \underline{e}_z$$

3. Coordonnées sphériques.

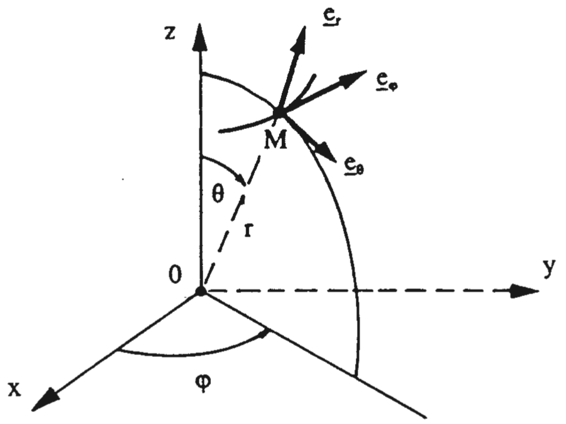

Figure 3 - Coordonnées sphériques.

3.1. « Coordonnées ».

Un point M a pour « coordonnées » sphériques r , θ . φ , la base orthonormée locale correspondante étant notée $\underline{e}_r$, $\underline{e}_\theta$, $\underline{e}_\varphi$. Le déplacement élémentaire $d\underline{M}$ s'écrit :

$$d\underline{M} = dr\,\underline{e}_r + r\,d\theta\,\underline{e}_\theta + r\sin\theta\,d\varphi\,\underline{e}_\varphi\,.$$

3.2. Champ scalaire $b\,(\underline{x}\,,t)$.

$$b\,(\underline{x}\,,t) = b\,(r\,,\theta\,,\varphi\,,t)$$

$$\underline{\mathrm{grad}}\;b = \frac{\partial b}{\partial r}\,\underline{e}_r + \frac{1}{r}\frac{\partial b}{\partial \theta}\,\underline{e}_\theta + \frac{1}{r\sin\theta}\frac{\partial b}{\partial \varphi}\,\underline{e}_\varphi$$

$$\Delta b = \mathrm{div}\,(\underline{\mathrm{grad}}\;b) = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial b}{\partial r}\right) + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\,\frac{\partial b}{\partial\theta}\right) + \frac{1}{r^2\sin^2\theta}\frac{\partial^2 b}{\partial\varphi^2}$$

$$\frac{db}{dt} = \frac{\partial b}{\partial t} + \underline{U}\,.\,\underline{\mathrm{grad}}\;b = \left(\frac{\partial}{\partial t} + U_r\frac{\partial}{\partial r} + \frac{U_\theta}{r}\frac{\partial}{\partial\theta} + \frac{U_\varphi}{r\sin\theta}\frac{\partial}{\partial\varphi}\right)b$$

3.3. Champ de vecteurs $\underline{b}\,(\underline{x}\,,\,t)$.

Les composantes de $\underline{b}\,(\underline{x}\,,\,t)$ dans la base locale sont $b_r\,,\,b_\theta\,,\,b_\varphi$ de telle sorte que :

$$\underline{b}\,(\underline{x}\,,\,t) = b_r\,(r\,,\theta\,,\varphi\,,t)\,\underline{e}_r + b_\theta\,(r\,,\theta\,,\varphi\,,t)\,\underline{e}_\theta + b_\varphi\,(r\,,\theta\,,\varphi\,,t)\,\underline{e}_\varphi \quad .$$

$$\operatorname{div}\underline{b} = \frac{1}{r^2}\frac{\partial}{\partial r}(r^2\,b_r) + \frac{1}{r\sin\theta}\frac{\partial}{\partial\theta}(\sin\theta\,b_\theta) + \frac{1}{r\sin\theta}\frac{\partial b_\varphi}{\partial\varphi}$$

$$\Delta\underline{b} = \left[\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial b_r}{\partial r}\right) + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\,\frac{\partial b_r}{\partial\theta}\right) + \frac{1}{r^2\sin^2\theta}\frac{\partial^2 b_r}{\partial\varphi^2} - \frac{2}{r^2}\left(b_r + \frac{1}{\sin\theta}\frac{\partial}{\partial\theta}(b_\theta\sin\theta) + \frac{1}{\sin\theta}\frac{\partial b_\varphi}{\partial\varphi}\right)\right]\underline{e}_r$$

$$+\left[\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial b_\theta}{\partial r}\right) + \frac{1}{r^2}\frac{\partial}{\partial\theta}\left(\frac{1}{\sin\theta}\frac{\partial}{\partial\theta}(\sin\theta\,b_\theta)\right) + \frac{1}{r^2\sin^2\theta}\frac{\partial^2 b_\theta}{\partial\varphi^2} + \frac{2}{r^2}\left(\frac{\partial b_r}{\partial\theta} - \frac{\cos\theta}{\sin^2\theta}\frac{\partial b_\varphi}{\partial\varphi}\right)\right]\underline{e}_\theta$$

$$+\left[\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial b_\varphi}{\partial r}\right) + \frac{1}{r^2}\frac{\partial}{\partial\theta}\left(\frac{1}{\sin\theta}\frac{\partial}{\partial\theta}(\sin\theta\,b_\varphi)\right) + \frac{1}{r^2\sin^2\theta}\frac{\partial^2 b_\varphi}{\partial\varphi^2} + \frac{2}{r^2\sin\theta}\left(\frac{\partial b_r}{\partial\varphi} + \operatorname{cotg}\theta\,\frac{\partial b_\theta}{\partial\varphi}\right)\right]\underline{e}_\varphi$$

$$\underline{\operatorname{rot}}\,\underline{b} = \frac{1}{r\sin\theta}\left(\frac{\partial}{\partial\theta}(b_\varphi\sin\theta) - \frac{\partial b_\theta}{\partial\varphi}\right)\underline{e}_r + \left(\frac{1}{r\sin\theta}\frac{\partial b_r}{\partial\varphi} - \frac{1}{r}\frac{\partial}{\partial r}(r\,b_\varphi)\right)\underline{e}_\theta + \left(\frac{1}{r}\frac{\partial}{\partial r}(r\,b_\theta) - \frac{1}{r}\frac{\partial b_r}{\partial\theta}\right)\underline{e}_\varphi$$

$$\underline{\underline{\operatorname{grad}}\,b} = \begin{pmatrix} \dfrac{\partial b_r}{\partial r} & \dfrac{1}{r}\left(\dfrac{\partial b_r}{\partial\theta} - b_\theta\right) & \dfrac{1}{r}\left(\dfrac{1}{\sin\theta}\dfrac{\partial b_r}{\partial\varphi} - b_\varphi\right) \\[2ex] \dfrac{\partial b_\theta}{\partial r} & \dfrac{1}{r}\left(\dfrac{\partial b_\theta}{\partial\theta} + b_r\right) & \dfrac{1}{r}\left(\dfrac{1}{\sin\theta}\dfrac{\partial b_\theta}{\partial\varphi} - \operatorname{cotg}\theta\,b_\varphi\right) \\[2ex] \dfrac{\partial b_\varphi}{\partial r} & \dfrac{1}{r}\dfrac{\partial b_\varphi}{\partial\theta} & \dfrac{1}{r}\left(\dfrac{1}{\sin\theta}\dfrac{\partial b_\varphi}{\partial\varphi} + \operatorname{cotg}\theta\,b_\theta + b_r\right) \end{pmatrix}$$

$$\frac{d\underline{b}}{dt} = \frac{\partial\underline{b}}{\partial t} + \underline{\underline{\operatorname{grad}}\,b}\,.\,\underline{U} = \left(\frac{db_r}{dt} - \frac{b_\theta\,U_\theta + b_\varphi\,U_\varphi}{r}\right)\underline{e}_r$$

$$+\left(\frac{db_\theta}{dt} + \frac{b_r\,U_\theta}{r} - \operatorname{cotg}\theta\,\frac{b_\varphi\,U_\varphi}{r}\right)\underline{e}_\theta$$

$$+\left(\frac{db_\varphi}{dt} + \operatorname{cotg}\theta\,\frac{b_\theta\,U_\varphi}{r} + \frac{b_r\,U_\varphi}{r}\right)\underline{e}_\varphi$$

$$\underline{\underline{d}} = \frac{1}{2}(\underline{\underline{\text{grad}\,U}} + {}^t\underline{\underline{\text{grad}\,U}})$$

$$d_{rr} = \frac{\partial U_r}{\partial r} \qquad d_{\theta\theta} = \frac{1}{r}\frac{\partial U_\theta}{\partial \theta} + \frac{U_r}{r}$$

$$d_{\varphi\varphi} = \frac{1}{r\sin\theta}\frac{\partial U_\varphi}{\partial\varphi} + \frac{U_\theta}{r}\operatorname{cotg}\theta + \frac{U_r}{r}$$

$$d_{\theta\varphi} = \frac{1}{2}\left[\frac{\sin\theta}{r}\frac{\partial}{\partial\theta}\left(\frac{U_\varphi}{\sin\theta}\right) + \frac{1}{r\sin\theta}\frac{\partial U_\theta}{\partial\varphi}\right]$$

$$d_{\varphi r} = \frac{1}{2}\left[\frac{1}{r\sin\theta}\frac{\partial U_r}{\partial\varphi} + r\frac{\partial}{\partial r}\left(\frac{U_\varphi}{r}\right)\right]$$

$$d_{r\theta} = \frac{1}{2}\left[r\frac{\partial}{\partial r}\left(\frac{U_\theta}{r}\right) + \frac{1}{r}\frac{\partial U_r}{\partial\theta}\right]$$

3.4. Champ de tenseurs du 2ème ordre symétriques $\underline{\underline{b}}\,(\underline{x}\,,t)$.

$$\underline{\underline{b}}\,(\underline{x}\,,t) = b_{ij}\,(r\,,\theta\,,\varphi\,,t)\,\underline{e}_i \otimes \underline{e}_j$$

$$\begin{aligned}\operatorname{div}\underline{\underline{b}}\,(r\,,\theta\,,\varphi) = {} & \left(\frac{\partial b_{rr}}{\partial r} + \frac{1}{r}\frac{\partial b_{r\theta}}{\partial\theta} + \frac{1}{r\sin\theta}\frac{\partial b_{r\varphi}}{\partial\varphi} + \frac{1}{r}(2b_{rr} - b_{\theta\theta} - b_{\varphi\varphi} + b_{r\theta}\operatorname{cotg}\theta)\right)\underline{e}_r \\ & + \left(\frac{\partial b_{\theta r}}{\partial r} + \frac{1}{r}\frac{\partial b_{\theta\theta}}{\partial\theta} + \frac{1}{r\sin\theta}\frac{\partial b_{\theta\varphi}}{\partial\varphi} + \frac{1}{r}\left[(b_{\theta\theta} - b_{\varphi\varphi})\operatorname{cotg}\theta + 3\,b_{r\theta}\right]\right)\underline{e}_\theta \\ & + \left(\frac{\partial b_{\varphi r}}{\partial r} + \frac{1}{r}\frac{\partial b_{\varphi\theta}}{\partial\theta} + \frac{1}{r\sin\theta}\frac{\partial b_{\varphi\varphi}}{\partial\varphi} + \frac{1}{r}(3b_{r\varphi} + 2b_{\theta\varphi}\operatorname{cotg}\theta)\right)\underline{e}_\varphi\end{aligned}$$

4. Identités vectorielles et tensorielles usuelles.

Les symboles f, $\underline{U}$ et $\underline{\underline{T}}$ désignent respectivement un champ scalaire, un champ de vecteurs et un champ de tenseurs du 2ème ordre.

$$\mathrm{div}\,(\underline{\mathrm{rot}}\,\underline{U}) = 0$$

$$\mathrm{div}\,(\underline{\mathrm{grad}}\,f) = \Delta f$$

$$\mathrm{div}\,(\Delta\,\underline{U}) = \Delta\,(\mathrm{div}\,\underline{U})$$

$$\mathrm{div}\,(f\,\underline{U}) = f\,\mathrm{div}\,\underline{U} + \underline{U}\,.\,\underline{\mathrm{grad}}\,f$$

$$\mathrm{div}\,(\underline{U} \wedge \underline{V}) = \underline{V}\,.\,\underline{\mathrm{rot}}\,\underline{U} - \underline{U}\,.\,\underline{\mathrm{rot}}\,\underline{V}$$

$$\mathrm{div}\,(\underline{\underline{\mathrm{grad}\,U}}) = \Delta\,\underline{U}$$

$$\mathrm{div}\,({}^t\,\underline{\underline{\mathrm{grad}\,U}}) = \underline{\mathrm{grad}}\,(\mathrm{div}\,\underline{U})$$

$$\mathrm{div}\,(\underline{U} \otimes \underline{V}) = \underline{U}\,\mathrm{div}\,\underline{V} + \underline{\underline{\mathrm{grad}\,U}}\,.\,\underline{V}$$

$$\mathrm{div}\,(f\,\underline{\underline{T}}) = f\,\mathrm{div}\,\underline{\underline{T}} + \underline{\underline{T}}\,.\,\underline{\mathrm{grad}}\,f$$

$$\mathrm{div}\,(\underline{\underline{T}}\,.\,\underline{U}) = \mathrm{div}\,{}^t\underline{\underline{T}}\,.\,\underline{U} + \underline{\underline{T}} : \underline{\underline{\mathrm{grad}\,U}}$$

$$\mathrm{div}\,(f\,\underline{\underline{1}}) = \underline{\mathrm{grad}}\,f$$

$$\underline{\mathrm{rot}}\,(\underline{\mathrm{grad}}\,f) = 0$$

$$\underline{\mathrm{rot}}\,(\underline{\mathrm{rot}}\,\underline{U}) = \underline{\mathrm{grad}}\,(\mathrm{div}\,\underline{U}) - \Delta\,\underline{U}$$

$$\underline{\mathrm{rot}}\,(\Delta\,\underline{U}) = \Delta\,(\underline{\mathrm{rot}}\,\underline{U})$$

$$\underline{\mathrm{rot}}\,(f\,\underline{U}) = f\,\underline{\mathrm{rot}}\,\underline{U} + \underline{\mathrm{grad}}\,f \wedge \underline{U}$$

$$\underline{\mathrm{rot}}\,(\underline{U} \wedge \underline{V}) = \underline{\underline{\mathrm{grad}\,U}}\,.\,\underline{V} + \underline{U}\,\mathrm{div}\,\underline{V} - \underline{V}\,\mathrm{div}\,\underline{U} - \underline{\underline{\mathrm{grad}\,V}}\,.\,\underline{U}$$

$$\underline{\mathrm{grad}}\,(f\,g) = f\,\underline{\mathrm{grad}}\,g + g\,\underline{\mathrm{grad}}\,f$$

$$\underline{\mathrm{grad}}\,(\underline{U}\,.\,\underline{V}) = \underline{\underline{\mathrm{grad}\,U}}\,.\,\underline{V} + \underline{\underline{\mathrm{grad}\,V}}\,.\,\underline{U} + \underline{U} \wedge \underline{\mathrm{rot}}\,\underline{V} + \underline{V} \wedge \underline{\mathrm{rot}}\,\underline{U}$$

$$\underline{\underline{\mathrm{grad}\,U}}\,.\,\underline{U} = \underline{\mathrm{rot}}\,\underline{U} \wedge \underline{U} + \underline{\mathrm{grad}}\,\frac{U^2}{2}\ .$$