Reactor Kinetics and Control

Reactor Control

We have seen that k_{eff} must be equal to 1 for the reactor to be critical (= constant power).

However, different mechanisms can cause variation of k_{eff}:

1. Changes in composition of the reactor core due to fissions and activation (i.e. neutron capture) effectively change the material composition of the reactor, and thus the cross sections. Temperature and density changes during common transients (like a power ramp) due to changes in reactor load, maintenance etc. can also influence cross sections
   → These effects need to be compensated as to maintain $k_{eff} = 1$
2. The position of control rods or the concentration of neutron poisons can change over time. For instance, sometimes it is necessary to increase/decrease power or switch off reactor.
3. Undesired accidental situation can lead to sudden changes of k_{eff}
   → Automatic reactor shutdown needs to be guaranteed.

Indeed, the reactor power needs to be closely monitored and regulated, but how to study the time-dependent behaviour of the neutron population (and thus the power) in the reactor?

Reactor Kinetics – Point Kinetics

In general, one seeks to determine $\Phi(\vec{r}, E, t)$ the time- and energy-dependent neutron flux from the time-dependent diffusion or transport equation (solved numerically).

A good approximation for mono-energetic, uniform systems (i.e. uniform cross sections) is the so-called ***point kinetics*** model that studies the neutron population $N(t)$ in the whole reactor

$$N(t) = \int_{V_{tot}} n(\vec{r}, t) dV$$

- It assumes that the shape of the neutron population does not change with time
- Does not describe spatial effects in large complex systems but it is very useful to understand the global time-dependent behaviour.

One particular case can be solved analytically, i.e. a step change in k_{eff} starting from critical conditions. This will lead us to the ***Reactivity Equation (or Inhour Equation)***

Point Kinetics (w/o Delayed Neutrons)

We start by going back to the neutron balance equation this time written for the whole neutron population (i.e. P, A, L integrated over whole core). Let's start with an infinite core, without production and no external source from t=0 onward:

$$\frac{dN(t)}{dt} = -A(t) = -\Sigma_a \bar{v} N(t)$$

where $\bar{v}$ is the average neutron velocity. The solution is an exponential:

$$N(t) = N_0 \exp(-\Sigma_a \bar{v}\, t)$$

I.e. in a purely absorbing infinite medium, the neutron population decreases with a characteristic time l_∞ called the neutron mean lifetime:

$$l_\infty = 1/\Sigma_a \bar{v}$$

Point Kinetics (w/o Delayed Neutrons)

Let's now consider production always for an infinite medium:

$$\frac{dN(t)}{dt} = P - A = \Sigma_f \bar{\nu} N(t) - \Sigma_a \bar{\nu} N(t) = \frac{k_\infty - 1}{l_\infty} N(t)$$

The solution is always an exponential:

$$N(t) = N_0 \exp\left(\frac{k_\infty - 1}{l_\infty} t\right)$$

The population grows exponentially if $k_\infty > 1$ and decreases if $k_\infty < 1$ as expected

Point Kinetics (w/o Delayed Neutrons)

Let's now consider a finite medium, where we assume that the leakages L can be expressed as proportional to the absorption with a coefficient of proportionality Γ :

$$\frac{dN(t)}{dt} = P - A - L = \Sigma_f \bar{\nu} N - \Sigma_a \bar{\nu} N - \Gamma \Sigma_a \bar{\nu} N \ = \Sigma_f \bar{\nu} N - (1+\Gamma)\Sigma_a \bar{\nu} N$$

We can say that

$$P_{NL} = 1 \ - \frac{\Gamma \Sigma_a \bar{\nu} N}{\Sigma_a \bar{\nu} N + \Gamma \Sigma_a \bar{\nu} N} = 1 - \frac{\Gamma}{1+\Gamma} = \frac{1}{1+\Gamma} \rightarrow 1 + \Gamma = \frac{1}{P_{NL}}$$

We can rearrange the equation as:

$$\frac{dN(t)}{dt} = \Sigma_f \bar{\nu} N - (1+\Gamma)\Sigma_a \bar{\nu} N = \frac{P_{NL} k_\infty - 1}{l_\infty P_{NL}} N(t) = \frac{k-1}{l} N(t)$$

Where $k = P_{NL} k_\infty$ is the finite multiplication factor and $l = l_\infty P_{NL}$ is the finite neutron lifetime

Point Kinetics w/o Delayed Neutrons

$$N(t) = N(0)\exp\left[\frac{k_{eff}-1}{\ell}t\right] = N(0)\exp\left[\frac{t}{T}\right]$$

where $T = \frac{\ell}{k_{eff}-1}$

As before, the solution is an exponential with period T (i.e. the time it takes for the population to increase by a factor equal to e)

Starting from initial condition $N(0)$

- If $k_{eff} > 1 \ldots N\uparrow$ (supercritical system)
- If $k_{eff} = 1 \ldots N = N(0)$ (critical system)
- If $k_{eff} < 1 \ldots N\downarrow$ (subcritical system)

The rate of change of N depends on how large is $k_{eff} - 1$ but also on ℓ which is a property of the system → the period could be small even for small variations of k_{eff} if ℓ is very small…

Prompt Period of a Reactor

For a thermal reactor $\ell \approx t_d$ (diffusion time, average time as thermal neutron before absorption) and values of t_d for different moderators are small, ranging from 10^{-2} to 10^{-4}sec.

	t_d
H_2O	$2{,}1 \cdot 10^{-4}$ sec.
D_2O	$1{,}4 \cdot 10^{-1}$ sec.
Be	$3{,}9 \cdot 10^{-3}$ sec.
Graphite	$1{,}7 \cdot 10^{-2}$ sec.

For example:

- Assume change in k_{eff} from 1.000 to 1.0015 (150 pcm) and $\ell = 10^{-3}$
- The period $T = \frac{\ell}{k_{eff}-1} = \frac{2}{3}$ sec. $\Rightarrow N(t) = N(0)\exp\left[\frac{3}{2}t\right]$
- The neutron population will increase by a factor of $e^{\frac{3}{2}} = 4.5$ in only 1 sec.!! ***Reactors would be practically impossible to control…***

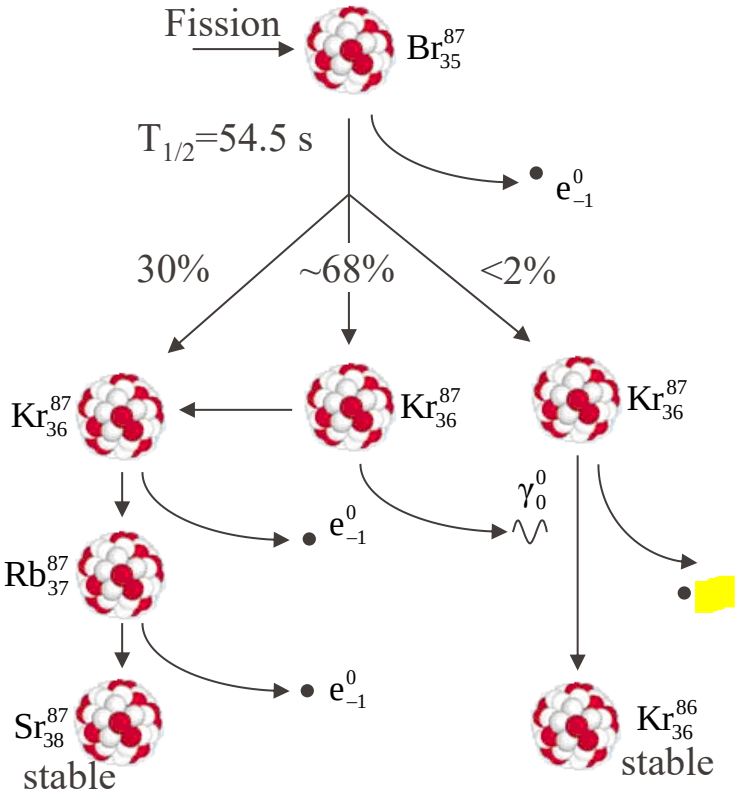
Control of the Chain Reaction - Delayed Neutrons

In a real reactor (luckily!):

- Not all neutrons are ***prompt*** (i.e. emitted immediately with fission)
- A small fraction is ***delayed*** (i.e. it is produced later by disintegration of some of the FP's).

Even if the fraction of delayed neutrons is small, it turns out that it is necessary for the control of the chain reaction

- It makes the response of a reactor which becomes slightly supercritical much slower!
- For small changes in k_{eff}, it makes the mean neutron lifetime significantly longer due to the long decay time of the precursors.



Delayed Neutron Parameters (Reminder)

Group number	Precursor nuclei	Average half-life (s)	Delayed neutron fraction – β_i (%)
1	^{87}Br, ^{142}Cs	55.72	0.021
2	^{137}I, ^{88}Br	22.72	0.140
3	^{138}I, ^{89}Br, $^{(93,94)}$Rb	6.22	0.126
4	^{139}I, $^{(93,94)}$Kr, ^{143}Xe, $^{(90,92)}$Br	2.30	0.252
5	^{140}I, ^{145}Cs	0.61	0.074
6	(Br, Rb, As etc.)	0.23	0.027
			0.640

- ~6-8 groups of precursors can be identified based on the half-life
- Average energy of delayed neutrons is smaller than prompt's: E_{avg} ~ 0.4MeV
- β_i depend on nuclide, e.g. $\beta = \sum\beta_i = 0.21\%$ for $^{239}_{94}\mathrm{Pu}$ or $\beta = 0.26\%$ for $^{233}_{92}\mathrm{U}$

Point Kinetics with Delayed Neutrons

Fraction β of neutrons in reactor are delayed, so the total amount of neutrons produced per s:

$$\underbrace{P \cdot (1-\beta)}_{Prompt\ Sources} + \underbrace{\sum_{i=1}^{6} \left[\underbrace{\lambda_i}_{Decay\ constant} \times \underbrace{C_i(t)}_{Precursor\ density\ at\ time\ t} \right]}_{Delayed\ Sources}$$

We can then rewrite neutron population balance:

$$\frac{dN}{dt} = P(t)(1-\beta) - [A(t) + L(t)] + \sum_{i=1}^{6} \lambda_i C_i(t)$$

With the same substitutions done for the prompt case we obtain:

$$\frac{dN}{dt} = \frac{(1-\beta)k_{\text{eff}}(t) - 1}{\ell} \times N(t) + \sum_{i=1}^{6} \lambda_i C_i(t)$$

We need another set of equations to solve for C_i!

Point Kinetics with Delayed Neutrons (contd.)

Precursor equations (recall radioactive decay with production term):

$$\frac{dC_i(t)}{dt} = \underbrace{\beta_i P(t)}_{\text{Precursor Production}} - \underbrace{\lambda_i C_i(t)}_{\text{Precursor Decay}}$$

$$\begin{cases} \dfrac{dC_i(t)}{dt} + \lambda_i C_i(t) = \beta_i k_{\text{eff}} \cdot [A(t) + L(t)] = \beta_i \dfrac{k_{\text{eff}}}{\ell} N(t) \\ \dfrac{dN}{dt} = \dfrac{(1-\beta)k_{\text{eff}}(t) - 1}{\ell} \times N(t) + \sum_{i=1}^{6} \lambda_i C_i(t) \end{cases}$$

Reactivity or ρ

The effective multiplication factor gives a measure of the change in neutron population in two subsequent generations

$$k_{\text{eff}} = \frac{\text{Production}}{\text{Absorption + Leakage}}$$

In kinetics it is often more convenient to express the ***reactivity*** ($\boldsymbol{\rho}$)

$$\rho = \frac{k_{\text{eff}} - 1}{k_{\text{eff}}} \Rightarrow \begin{cases} \rho > 0 & \text{i.e. super-critical} \\ \rho = 0 & \text{i.e. critical} \\ \rho < 0 & \text{i.e. sub-critical} \end{cases}$$

which measures the relative change of k_{eff} in terms of its deviation from criticality (k_{eff}=1)

→ ***ρ (unitless) is expressed either in %, pcm (per cent mille = 10^{-5}) or in dollars ($)***

Point Kinetics with Delayed Neutrons (contd.)

Substitutions: $\frac{k_{\text{eff}}-1}{k_{\text{eff}}} = \underbrace{\rho}_{\text{Reactivity}}$ and $\frac{\ell}{k_{\text{eff}}} = \underbrace{\Lambda}_{\text{Prompt Neutron Generation Time}}$

Point Kinetics Equations w/ delayed neutrons

$$\begin{cases} \dfrac{dC_i(t)}{dt} + \lambda_i C_i(t) = \dfrac{\beta_i}{\Lambda} N(t), \quad i = 1, \ldots 6 \\ \dfrac{dN}{dt} = \dfrac{\rho(t)-\beta}{\Lambda} N(t) + \displaystyle\sum_{i=1}^{6} \lambda_i C_i(t) \end{cases}$$

Step Change in Reactivity - The Roots ω_j

Solution of point kinetics equations is complicated. An important application of point kinetics equations is the case of a step change in reactivity starting from a previously critical condition

- Constant ρ ($\pm$) introduced at t = 0 e.g. due to a very quick movement of a control rod
- Illustrative case, analytical solution possible with method of Laplace transforms (but we will not deal with the derivations)

In this case the solution is a sum of seven exponential (for 6 precursor groups):

$$N(t) = \sum_{i=1}^{7} A_j \exp(\omega_j t)$$

Where the exponents ω_j are the 7 roots of the reactivity equation

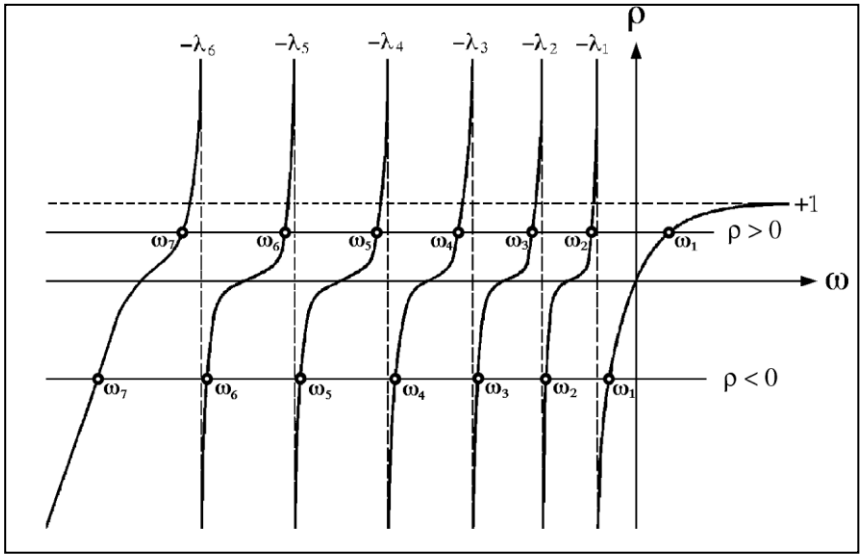
$$\rho = \Lambda\omega + \sum_i \frac{\beta_i \omega}{(\omega + \lambda_i)}$$

Step Change in Reactivity – Graphical Solution

$$\rho = \Lambda\omega + \sum_i \frac{\beta_i \omega}{(\omega + \lambda_i)}$$

One may study the Reactivity Eqn. graphically and obtain already quite a lot of information:

- For positive ρ (supercritical reactor), one value of ω is positive, the others are negative and the corresponding exponentials disappear in time.
- For negative ρ (subcritical reactor), all 7 roots ω are negative, but the first root is smaller so the other 6 exponentials decay faster in time.



NB: in the chart, the reactivity is expressed as fraction of Beta (ρ equal to 1 corresponds to ρ equal to Beta)

Point Kinetics with Delayed Neutrons (contd.)

One can obtain a simplified solution if one considers a single precursor group with average half life of 9s (from previous table).

In this case then the solution for neutron and precursor population are:

$$C(t) = N_0 \left[\frac{\beta}{\Lambda\lambda} \exp\left(\frac{\lambda\rho}{\beta - \rho}\, t \right) + \frac{\rho\beta}{(\beta-\rho)^2} \exp\left(-\frac{\beta - \rho}{\Lambda}\, t \right) \right]$$

$$N(t) = N_0 \left[\frac{\beta}{\beta - \rho} \exp\left(\frac{\lambda\rho}{\beta - \rho}\, t \right) - \frac{\rho}{\beta - \rho} \exp\left(-\frac{\beta - \rho}{\Lambda}\, t \right) \right]$$

First root ω_1

Second root ω_2

Point Kinetics with Delayed Neutrons (contd.) - Shutdown

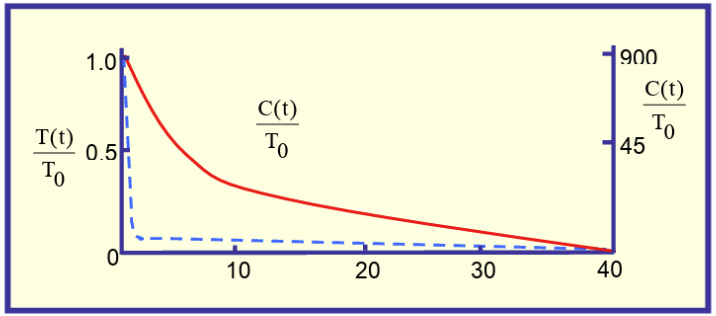
Assume β=0.0065, $\Lambda = 10^{-4}$ and $\lambda = 0.076\ s^{-1}$. Let ρ = -0.05. We obtain:

$$C(t) = N_0[855\exp(-0.067t) + 0.1\exp(-565t)]$$

$$N(t) = N_0[0.12\exp(-0.067t) + 0.88\exp(-565t)]$$

88% of the neutron population is gone almost immediately (0.01 s) while the precursor population is virtually unchanged.

Both the remaining neutrons and the precursors will die off at a **15 s period (T = 1/0.067**).



Neutron and precursor population following a change in reactivity of -0.05 from an initially critical state – one-group model.

Point Kinetics with Delayed Neutrons (contd.) – Supercritical

Let ρ = +0.0015 (same value we used for case without delayed neutrons). We obtain:

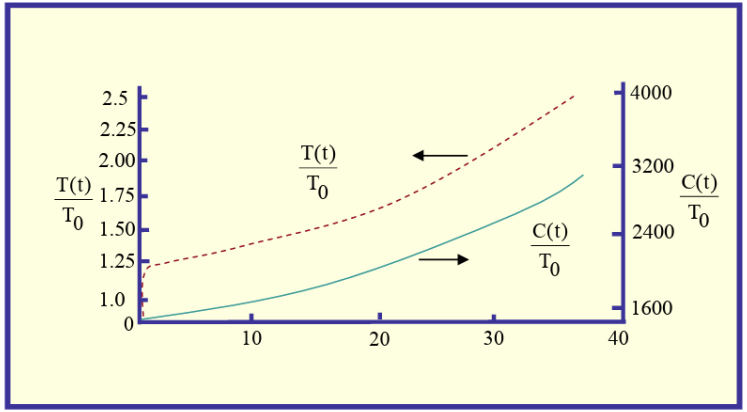
$$C(t) = N_0[855\exp(0.02t) + 0.39\exp(-20t)]$$
$$N(t) = N_0[1.3\exp(0.02t) - 0.3\exp(-20t)]$$

The result is very similar in shape to the one given in the previous slide. The initial rapid rise of the prompt neutrons is called the “prompt jump.”

It is the start of a neutron runaway but it can’t continue because ρ<<β and the delayed neutrons are needed in order to sustain the chain reaction.

Both the remaining neutrons and the precursors will grow at a **50 s period (T = 1/0.02)**!

Much larger preriod than what we saw for case without delayed neutrons! This is why for ρ<<β reactors can be controlled.



Neutron and precursor population following a change in reactivity of +0.0015 from an initially critical state: One-group model.

Point Kinetics with Delayed Neutrons (contd.) – Supercritical

- Indeed, for very small values of $\rho << \beta$, the roots and periods are
- Thus, for small reactivities the reactor period (~ T_1) is governed almost completely by the delayed neutrons!

- This might seem surprising as delayed neutrons are a small fraction…
- … but their weighted average lifetime (dominated by the long decay of the precursors) is much larger than the prompt neutron lifetime.

- In a sense one can think about the mean neutron generation time as:

$$\bar{l_d} = (1-\beta) l_d + \sum_i^6 \frac{\beta_i}{\lambda_i} \approx \sum_i^6 \frac{\beta_i}{\lambda_i}$$

Point Kinetics with Delayed Neutrons (contd.) – Super prompt-critical

Let ρ = +0.0115. Recall that we assumed β=0.0065. We obtain:

$$C(t) = N_0[855\exp(-0.17t) + 2.9\exp(50t)]$$

$$N(t) = N_0[-1.3\exp(-0.17t) + 2.3\exp(50t)]$$

The reactor power rises by a factor of 340 in the first tenth of a second!

Point Kinetics with Delayed Neutrons (contd.) – Super prompt-critical

When $\rho \geq \beta$ the reactor is said to be ***prompt critical.*** Indeed:

$$\rho = \beta \qquad \rightarrow \qquad (1-\beta)\mathrm{k}_{eff} = 1,$$

i.e. the reactor is critical on prompt neutrons alone!

For $\rho \geq \beta$ thus the reactor period becomes small. The roots and periods tend to:

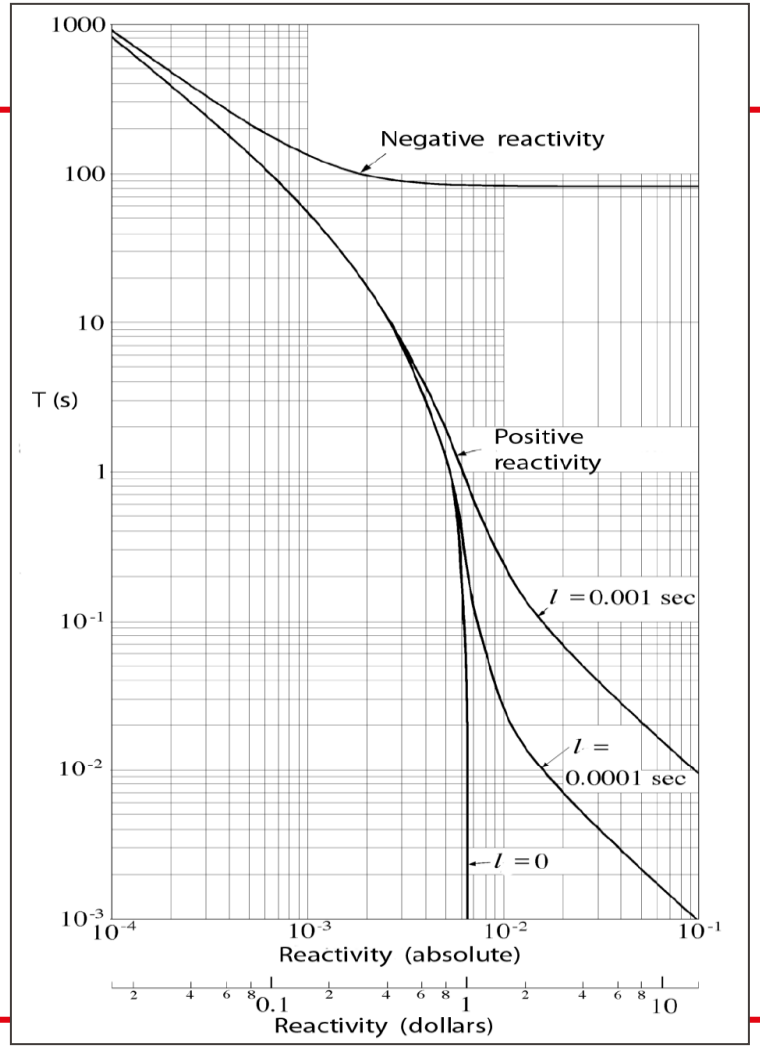
$$\omega_2 = -\frac{\beta - \rho}{\Lambda} \approx \frac{\rho}{\Lambda} = \frac{k-1}{l} \qquad T_2 = \frac{1}{|\omega_2|} = \frac{l}{k-1}$$

$$\omega_1 = \frac{\lambda\rho}{\beta - \rho} \approx -\lambda = -0.076 \ \ s^{-1} \qquad T_1 = \frac{1}{|\omega_1|} \approx 13.15\ s$$

The rise is dominated by the much shorter positive period T_2, analogous to the one we get w/o delayed neutrons!

Period as function of ρ

- Reactivity Equation can give, for a given set of fissile nuclides a value of ω_1 and thus T for each value of ρ
- ρ needs to be << β (for positive ρ)
  - ***Prompt criticality*** needs to be avoided at all costs!
  - For small reactivities, Λ no longer important and the period is dominated by delayed neutrons lifetime
- $\rho = \beta$ defines reactivity unit of 1 dollar ($)
  - ρ usually few cents (¢)
- For large negative ρ (reactor shutdown), T ≈ 80s
  - Period of 1st precursor group



Reactivity Variations

In reality, reactivity changes are not sudden and constant, i.e. not “step functions”

- ρ changes continuously as function of time
- Various types of “feedbacks” with different “time constants” are involved (e.g. time for power change to affect temperature,…)

Short-term effects:

- Fuel temperature (*Doppler effect*), < 1 sec
- Moderator temperature, secs – mins
- Fraction of liquid moderator, secs (boiling, bubble formation, effect on density…)

Medium-term effects:

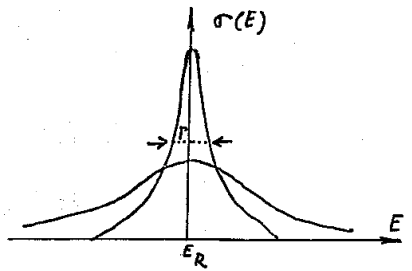
- Principal effect: Fission product Xe^{135} in a thermal reactor, hours - days

Long-term effects:

- Fuel composition changes with irradiation (burnup), days - months
- Important mostly for power reactors (the largest the power the more the fuel is changing)

Short-term Effects: Doppler (Fuel Temp.) Coeff

Resonances in cross-sections are not lines: distribution in energy due to the thermal agitation (statistical distribution) of the nuclei.



When $T_f \uparrow$, U^{238} resonances broadened due to increased thermal agitation of nuclei and the effective resonance integral ↑ (i.ie more difficult for the neutrons to surpass the resonances)

$$p \downarrow \Longrightarrow k_\infty = \eta\ fp\ \varepsilon \downarrow$$

Fuel Temperature Coefficient of Reactivity (*Doppler Coeff.*)

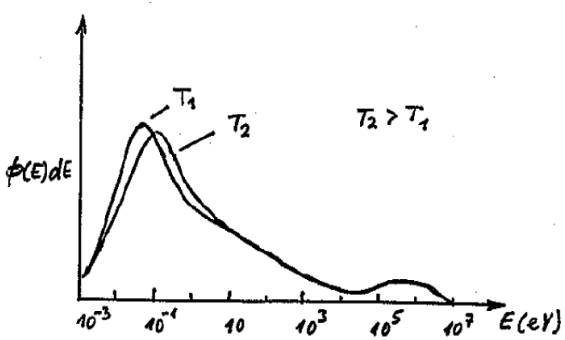
- α_c always negative → helps stability of power control!
- almost immediate, crucial for “inherent safety”

$$\alpha_f = \frac{\partial \rho}{\partial T_f} \approx \frac{\partial k_{\text{eff}}}{\partial T_f}$$

Short-term Effects: Moderator Temp./Density Coeff.

Moderator Temp. Coefficient

$$\alpha_m = \frac{\partial \rho}{\partial T_m} \cong \frac{\partial k_{eff}}{\partial T_m}$$



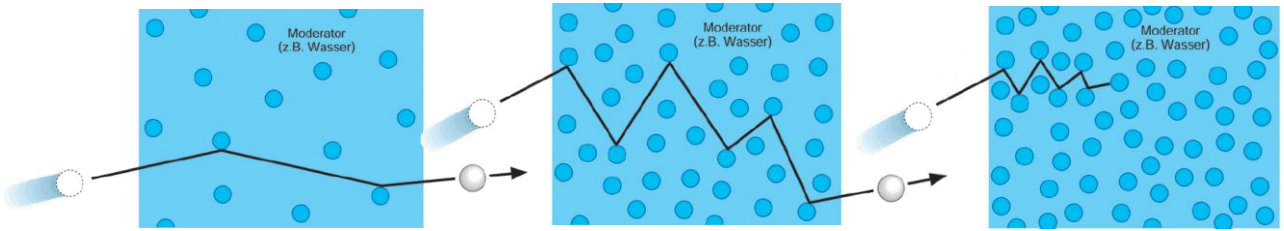
- For a solid moderator, e.g. graphite, small effects mainly due to shift in neutron spectrum (i.e. the thermal equilibrium is at higher T)
- For a liquid moderator (coolant), a change in temperature is ***a change in density***
  - "Under-moderation" crucial for safety → reactors designed so that this effect is negative

Moderator Density or Void Coefficient

- Density can change also due to boiling
- α_V must be negative
- Strongly positive α_V is very dangerous (Chernobyl!)

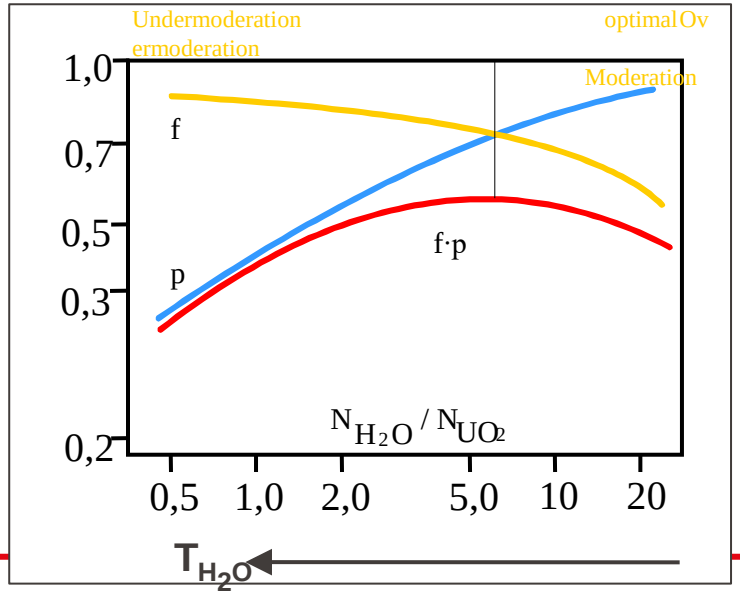
$$\alpha_V = \frac{\partial \rho}{\partial V} \cong \frac{\partial k_{eff}}{\partial V}$$

Short-term Effects: Moderator Temperature Coeff.



In k_∞, only f and p are significantly affected by change of moderator density: two competing effects!

Reactors are operated in ***under-moderated*** region: when moderator density decreases, the multiplication factor decreases.



Comments

- One can use the reactivity coefficients as a first approximation, but this does not give the true “dynamic” behaviour.
- One needs proper time-dependent “modelling” of the power reactor (including the secondary cooling system), with “coupling” between neutronics, thermal-hydraulics, mechanics, system

→ Safety studies: numerical simulation and analysis of hypothetical accident situations

- In general, if all the α’s are negative, reactor “inherently” safe from viewpoint of automatic shutdown

- Calculation of α’s generally very delicate
  - Compensation of individual effects, e.g. sodium α_v , or α_m in HTR (graphite)
  - Necessary to carry out “checks” on power reactor before start-up.

Consequences for Reactor Control

- Strongly negative α's demand large reactivity reserve
  - Complex control system (economics aspect)

  - After reactor shut-down, one needs to be able to compensate the important $\Delta\rho$'s corresponding to different reactor states:

    (1) "Hot full power" (HFP) → T_f ↑, T_m ↑

    ⇕

    (2) "Hot zero power" (HZP) → T_f ↓, T_m ↑

    ⇕

    (3) "Cold zero power" (CZP) → T_f ↓, T_m ↓

- For such considerations, one may use: $\Delta\rho \cong \alpha_{'c}\Delta T_c + \alpha_M \Delta T_M + \alpha_V \Delta V + \ldots\ldots$