Questions and Exercises

Questions

Why is nuclear energy beneficial w/ respect to fossil energy?

A. Much lower carbon footprint
B. Reduced waste problems
C. Reduced deployment time

Which of the following is true

A. Most of the mass of an atom is concentrated in the nucleus.
B. The mass of an atom is approximately equally split among electrons and nucleons.
C. Most of the mass of an atom is carried by the electrons

Which of the following is true

A. The molar mass of an atom is the mass in kg of 1 atom
B. The molar mass is equal to the atomic weight in g times the Avogadro number
C. The value of the molar mass of an atom (in g) is equal to the value of the mass weight (in amu)

At high atomic number, stable isotopes tend to have

A. $Z > N$
B. $N > Z$
C. $N \sim Z$

Questions

How can we normally use a nuclear reaction to produce energy ?

A. Fusion of 2 light isotopes
B. Fission of 1 heavy isotope
C. Fusion of 2 heavy isotopes

Why U-235 fission liberates energy?

A. Because the binding energy of U-235 is lower than the sum of the binding energy of the fission products
B. Because the binding energy of U-235 is higher than the sum of the binding energy of the fission products
C. Because the neutrons released by fission carry a significant kinetic energy

Which laws generally describes the time evolution of N radioactive atoms (decay + production)?

A. $dN/dt = N \lambda$
B. $dN/dt = N \lambda + R$
C. $dN/dt = -N \lambda + R$
D. $dN/dt = -N \lambda$

What is the activity A of N radioactive nucleus for which the decay constant is λ

A. $A = N \lambda$
B. $A = N / \lambda$
C. $A = N \exp(\lambda t)$

Questions

What is the absorption reaction rate (#/s) of U-238 (atomic concentration *N*) in a reactor of volume *V*, with neutron flux ϕ

A. $RR = -\sum_a \phi V$

B. $RR = \sum_a \phi V$

C. $RR = \sigma_a N \phi V$

D. $RR = \sigma_a N \phi / V$

What are the most important average results of a fission reaction?

A. 2 specific fission products and approximately 2-3 neutrons per fission

B. 2 specific fission products and approximately 5 neutrons per fission

C. A distribution of fission products and approximately 2-3 neutrons per fissions

What is the average energy released per fission of U-235 ?

A. 0.2 MeV

B. 200 eV

C. 200 MeV

D. 300 keV

Questions

Where is most of the fission energy released?

A. Directly in the water by the neutrons

B. In the fuel itself due to the slowing down of fission products

C. In the cladding due to gamma heating

D. In the cladding due to the radioactivity created by neutron capture

Why is the FP radioactivity a problem?

A. FPs decay with very long half lives and thus they represent the major problem for storage

B. The energy release from the radioactive decay cannot be recovered to produce electicity

C. The radioactivity of FP continues also when the reactor is switched off

Why is a self-sustained fission chain reaction possible in fission reactors?

A. Because fission reactions produce more than 2 neutrons per fission on average

B. Because every neutron produced by fission is absorbed in a new fissile isotope

C. Because radioactive fission products emit neutrons

Questions

In which energy range are fission neutrons mostly produced?

A. $10^{-3} – 10^{0}$ eV

B. $10^{0} – 10^{4}$ eV

C. $10^{4} – 10^{7}$ eV

In which energy range is neutron fissioning of Uranium-235 most efficient?

A. $10^{-3} – 10^{0}$ eV

B. $10^{0} – 10^{4}$ eV

C. $10^{4} – 10^{7}$ eV

And that of Uranium-238?

A. $10^{-3} – 10^{0}$ eV

B. $10^{0} – 10^{4}$ eV

C. $10^{4} – 10^{7}$ eV

Questions

Coolant and moderator – Which of the following is true?

A. A moderator is a material of low mass number whose purpose is to slow down neutrons to increase the probability of neutron interaction with fuel

B. A coolant is a fluid circulating through the reactor to remove the fission heat

C. Light water acts as both coolant and moderator in a LWR

What are the major differences between PWR and BWR?

A. BWRs operate at ambient pressure

B. In PWRs , water mainly remains below saturation temperature

C. PWRs do not have a secondary loop for energy conversion

What is the thermal efficiency of a typical LWR?

A. 10%

B. 33%

C. 52%

D. 67%

Questions

What is the typical core outlet temperature in a PWR?

A. 250 °C

B. 120 °C

C. 320 °C

And what is the typical pressure in the primary circuit of a PWR?

A. 150 bar / 15 Mpa

B. 15 bar / 1.5 Mpa

C. 1 bar / 0.1 MPa

What is the role of the pressurizer?10%

A. It stabilizes the pressure in a PWR

B. It stabilizes the presure in a BWR

C. It injects high pressure water in the primary circuit

Questions

What is the correct formula for the multiplication factor k?

A. $k = P/(L)$

B. $k = P/(A+L)$

C. $k = K_{inf} \cdot P_{NL}$

D. $k = A/(A+L)$

Under which circumstances do we term a reactor supercritical?

A. $K_{eff} > 1.0$

B. $K_{eff} = 1.0$

C. $K_{eff} < 1.0$

Why does the size of a reacor affects its multiplication factor?

A. Because it affects the leakages, so that a larger reactor corresponds to higher leakages

B. Because it affects the leakages, so that a larger reactor corresponds to lower leakages

C. Because it affects how many neutrons can enter in the reactor from outside

Questions

What does the termp p in the 4 factor formula describe

A. The probability that neutrons survive the slowing down through the cross section resonance region

B. The production bonus due to fast fission

C. The relative absorption in fuel material

What is the most important slowing down mechanism for neutrons in a thermal reactor?

A. Inelastic scattering

B. Elastic scattering on heavy nuclei

C. Elastic scattering on light nuclei

D. Absorption

Questions

What is a typical linear heat generation rate for a PWR/BWR?

A. 10 W/cm

B. 300 W/cm

C. 30 kW/m

Which is the largest resistance heat finds flowing from fuel to coolant?

A. Coolant

B. Cladding

C. Gap between fuel and cladding

D. Fuel itself

Which of the following quantities are directly limited by the risk of fuel melting?

A. Specific power

B. Linear power

C. Reactivity

D. keff

Exercise: Concentration of a fission product

- The fission product of I^{131} has a half-life of 8.05 days and is produced in fission with a yield of 2.9% (that is, 0.029 atoms of I^{131} are produced per fission). Calculate the equilibrium activity (in Bq) of this radionuclide in a reactor operating at 3300 MW. Assume 200 MeV per fission.

Solution

- Radioactive decay law with constant source R

$$\frac{dN(t)}{dt} = -\lambda N(t) + R$$

- At equilibrium $\frac{dN(t)}{dt} = 0$, thus

$$A_{eq} = \lambda N_{eq} = R$$

$$R = \frac{\left(3300\ MW \cdot 10^6 \frac{W}{MW}\right)}{200 \cdot 10^6 \frac{eV}{fission} \cdot 1.6 \cdot 10^{-19} \frac{J}{ev}} \cdot 0.029 \cong 2.99 \cdot 10^{18}\ Bq \cong 8.08 \cdot 10^7\ Curie$$

Exercise: ^{197}Au foil activation in a reactor

- The radioisotope ^{198}Au ($T_{1/2}$~ 64.8 h) can be produced by the activation of gold (100% ^{197}Au) in a nuclear reactor by a neutron capture. Consider a gold foil of mass 0.1 g, which is placed at the center of the core of an MTR (Material Testing Reactor) during 12 hours of continuous reactor operation. At the end of this irradiation period, it is extracted and its activity measured to be 33.3 GBq. What is:

a) the theoretically maximum activity which the foil could have acquired in the reactor

b) the irradiation duration necessary such that 80% of this maximal value may be reached?

Solution

- Solving radioactive decay law with constant source R (previous exercise) one gets: $A(t) = R(1 - e^{-\lambda t})$
- The maximum activity is for $t = \infty$ or at equilibrium $\frac{dN(t)}{dt} = 0 \rightarrow A_{eq} = R$

a) To find R we use the fact that we know the activity of the sample after 12 hours

$$R = A_{eq} = \frac{A(12hr)}{(1 - e^{-\lambda \cdot 12})} = \frac{33.3 \cdot 10^9}{(1 - e^{-\frac{\ln(2)}{64.8} \cdot 12})} = 2.76 \cdot 10^{11}$$

b) $A(t_{80\%}) = 0.8 * R = R(1 - e^{-\lambda t}) \rightarrow e^{-\lambda t} = 0.2 \rightarrow t_{80\%} = -\frac{\ln(0.2)}{\ln(2)} 64.8 = 150.4\ hr$

Exercise: 4-factor formula for k∞

A reactor core has the following specifications:

- Volumetric composition: U...32%, Zr...8%, Fe...2%, H_2O...58%.
- The enrichment of the uranium is 3% [η = 1.83 (for 3% enr. U)].
- The macroscopic thermal cross-sections (Σ_a) are: U...0.5553 cm^{-1}, Zr...0.0077 cm^{-1}, Fe...0.2145 cm^{-1}, H_2O...0.0176 cm^{-1}
- The ratio of neutrons produced by fission in ^{238}U (fast fissions), relative to those produced by ^{235}U fission, is 2.4%.
- Another measurement reveals that the fraction of neutrons slowing down, which are captured in the resonances of ^{238}U, is 31% of the total number of neutrons captured.

Calculate:

a) What is the k∞ value for the reactor core?

b) If these material specifications are applied to a **cubic reactor of side *a*, with $B^2 \sim 3\left(\frac{\pi}{a}\right)^2$**, what are:

   a) the critical size of the reactor (take M^2 = 60 cm2)
   b) the critical mass of uranium and
   c) of ^{235}U for the system (ρ_U = 19 g/cm^3),

Exercise: 4-factor formula for k∞

Solution

$k_\infty = \frac{P}{A} = \varepsilon p f \eta = \frac{P_{th}+P_{fast}}{P_{th}} \cdot \frac{A_{th}}{A} \cdot \frac{A_{th,fuel}}{A_{th}} \cdot \frac{P_{th}}{A_{th,fuel}}$ where we have $\eta = 1.83$ $\quad \varepsilon = \frac{P_{th}+P_{fast}}{P_{th}} = 1 + \frac{P_{fast}}{P_{th}} = 1.024$

$p = \frac{A_{th}}{A} = 1 - 0.31 = 0.69$ $\quad f = \frac{A_{th,fuel}}{A_{th}} = \frac{0.32 \cdot 0.5553}{0.32 \cdot 0.5553 + 0.08 \cdot 0.0077 + 0.02 \cdot 0.2145 + 0.58 \cdot 0.0176} = 0.921612$

a) Therefore $k_\infty = 1.196754$

b) For a cubic reactor $\mathbf{B^2 \sim 3\left(\frac{\pi}{a}\right)^2} = \frac{k_\infty - 1}{M^2} \rightarrow \mathrm{a} = \mathrm{sqrt}\left(\frac{3\pi^2 \mathrm{M}^2}{k_\infty - 1}\right) = 95.02$ cm

The critical mass is $\mathrm{M}_{\mathrm{crit,U}} = 19\frac{\mathrm{g}}{\mathrm{cm}^3} \cdot a^3 \cdot 0.32 = 5.22\ tonnes$

The critical mass of U235 is $\mathrm{M}_{\mathrm{crit,U}^{235}} = \mathrm{M}_{\mathrm{crit,U}} \cdot 0.03 = 156.5\ kg$

Exercise: 6-factor formula

Measurements in a reactor fueled with slightly enriched uranium show that 10% of the fission neutrons leak out of the reactor and 15% are absorbed, both while these neutrons are slowing down to thermal (of the fast absorptions, 25% are fission reactions). 5% of the neutrons that slow down to thermal leak from the reactor. Of the thermal neutrons that do not leak, 82% are absorbed in the fuel and 75% of these are absorbed in the ^{235}U (of the thermal absorptions in ^{235}U, 85.5% are fission reactions). You are given that ν_{th} (^{235}U) = 2.418 and ν_{fast} = 2.667 for this reactor.

a) What is k_{eff} for this system?

b) What are the six factors for this system? Recalculate k_{eff} with the 6-factor formula.

NOTE that: $$P_{NL} = \frac{A}{A+L} = P_{NL}^{f}\, P_{NL}^{th} = \frac{A_f+A_t+L_t}{A_f+A_t+L_t+L_f} \cdot \frac{A_f+A_t}{A_f+A_t+L_t} = \frac{A_f+A_t}{A_f+A_t+L_t+L_f} = \frac{A}{A+L}$$

Exercise: 6-factor formula

NOTE that:

$$P_{NL} = \frac{A}{A+L} = P_{NL}^{f}\, P_{NL}^{th} = \frac{A_f + A_t + L_t}{A_f + A_t + L_t + L_f} \cdot \frac{A_f + A_t}{A_f + A_t + L_t} = \frac{A_f + A_t}{A_f + A_t + L_t + L_f} = \frac{A}{A+L}$$

$$k_{eff} = k_{\infty} \cdot P_{NL} = \frac{P}{A+L} = \varepsilon p f \eta\, P_{NL} = \frac{P_{th} + P_f}{P_{th}} \cdot \frac{A_{th}}{A} \cdot \frac{A_{th,fuel}}{A_{th}} \cdot \frac{P_{th}}{A_{th,fuel}} \cdot \frac{A}{A+L}$$

Solution

We split leakages L and absorption A into fast (f) and thermal (th). Note that (inconveniently) the same symbol P is traditionally used for the non-lekage (P_{NL}) probability and for the neutron production (P) due to fission.

Out of the total neutrons (for a critical system always equal to A+L), 0.1 neutrons (in fraction terms) escape while fast . The fast non-leakage probability is:

$$P_{NL}^{f} = \frac{A_f + A_{th} + L_{th}}{A_f + A_{th} + L_{th} + L_f} = 1 - \frac{L_f}{A+L} = 1 - 0.1 = 0.9$$

Another 0.15 neutrons are absorbed while fast and 0.25 of this make fission. So the fraction of “fast” fission neutrons produced is:

$$\frac{P_f}{A+L} = \frac{A_f}{A+L} \frac{P_{fast}}{A_f} = 0.15 \cdot 0.25 \cdot 2.667 = 0.1$$

The fraction of neutrons that survives for thermal range is

$$\frac{A_{th} + L_{th}}{A+L} = 1 - 0.1 - 0.15 = 0.75$$

Exercise: 6-factor formula

Of these thermalized neutrons, 0.95 don't leak while thermal, 0.82 are absorbed in the fuel, where 0.75 are absorbed in the U235, and 0.855 of these make fission with 2.418 neutrons per fission. So, the thermal fission neutron fraction is:

$$\frac{P_{th}}{A+L} = \frac{A_{th}+L_{th}}{A+L}\frac{A_{th}}{A_{th}+L_{th}}\frac{A_{th,fuel}}{A_{th}}\frac{P_{th}}{A_{th,fuel}} = 0.75\cdot 0.95\cdot 0.82\cdot 0.75\cdot 0.855\cdot 2.418 = 0.9059$$

Therefore, the fast fission bonus factor is:

$$\epsilon = 1+\frac{P_f}{P_{th}} = 1+\frac{0.100}{0.9059} = 1.1104$$

For the fuel utilization factor we know from the text that:

$$f = \frac{A_{th,fuel}}{A_{th}} = 0.82$$

Always from the text we can derive the fuel multiplication factor:

$$\eta = \frac{P_{th}}{A_{th,fuel}} = \frac{A_{th,U235}}{A_{th},fuel}\frac{P_{th}}{A_{th,U235}} = 0.75\cdot 0.855\cdot 2.418 = 1.55$$

And for the resonance scape probability we have:

$$p = \frac{A_{th}}{A} = \frac{\frac{A_{th}}{A+L}}{\frac{A_f}{A+L}+\frac{A_{th}}{A+L}} = \frac{\frac{A_{th}+L_{th}}{A+L}\frac{A_{th}}{A_{th}+L_{th}}}{\frac{A_f}{A+L}+\frac{A_{th}+L_{th}}{A+L}\frac{A_{th}}{A_{th}+L_{th}}} = \frac{(1-0.1-0.15)\cdot 0.95}{0.15+(1-0.1-0.15)\cdot 0.95} = 0.8261$$

Exercise: 6-factor formula

For the thermal non-leakage probability we use the fact that:

$$\mathrm{P_{NL}} = \frac{A}{A+L} = \frac{A_{th}}{A+L} + \frac{A_f}{A+L} = 0.75 \cdot 0.95 + 0.15 = 0.8625$$

And that by definition $\mathrm{P_{NL}} = \mathrm{P}_{\mathrm{NL},f} \cdot \mathrm{P}_{\mathrm{NL},th}$, thus:

$$\mathrm{P}_{\mathrm{NL},th} = \frac{0.8625}{0.9} = 0.9583$$

Finally the multiplication factor is :

$$k = \mathrm{k}_\infty \cdot \mathrm{P_{NL}} = \frac{\mathrm{P}}{\mathrm{A+L}} = \varepsilon\mathrm{pf}\eta\mathrm{P}_{\mathrm{NL},f}\mathrm{P}_{\mathrm{NL,th}} = 1.1104 \cdot 0.8261 \cdot 0.82 \cdot 1.55 \cdot\ 0.9 \cdot 0.9583 = 1.0055$$

Exercise: An example nuclear power plant

- Consider a 900 MWe PWR.

a) The efficiency for the conversion of heat into electricity is equal to 33%; how much is the nominal thermal power? How many fissions per second are necessary to produce this thermal power?

b) The core contains 157 fuel assemblies (0.215 m× 0.215 m square cross-section, and 3.658 m height); what size is its volume and its equivalent radius if it is transformed into a cylinder? It contains 82 t of uranium oxide (density: 10 300 kg/m^3); how large is the volume of fuel, and what fraction of the total is occupied by the fuel?

c) How large is the power per unit of volume of core, and per unit of volume of fuel?

d) The uranium of the fuel is enriched up to 3%. Calculate the number of uranium-235 atoms per unit of volume. Using 582 barns for the uranium-235 fission microscopic cross-section, and neglecting fission of the uranium-238 atoms, calculate the macroscopic fission cross-section of the fuel. Consequently, how large is the mean neutron flux in the fuel?

e) Assuming that the neutrons are monokinetic, with a velocity equal to 3100 ms−1, calculate the mean neutron density in the fuel. Compare this with the number of atoms per unit of volume.

Exercise: An example nuclear power plant

- $\mathrm{Q_{th}} = \frac{Q_e}{\eta_{th}} = \frac{900}{0.33} = 2'727.272\ \ MW$
- $\#_{fiss} = \frac{Q_{th}}{200 \cdot 10^6 \frac{eV}{fission} \cdot 1.6 \cdot 10^{-19} \frac{J}{ev}} = 8.522725 \cdot 10^{19}$

- $V_{core} = (0.215 \cdot 0.215) \cdot 3.658 \cdot 157 = 26.54\, m^3$
- $R_{eq} = \sqrt{\frac{V_{core}}{\pi H}} = 1.5196$
- $V_{fuel} = \frac{M_{fuel}}{\rho_{fuel}} = \frac{82000\ kg}{10300 \frac{kg}{m^3}} = 7.96\, m^3$
- $\frac{V_{fuel}}{V_{core}} = 0.299$

- $\frac{\mathrm{Q_{th}}}{V_{core}} = 102 \frac{MW}{m^3}$
- $\frac{\mathrm{Q_{th}}}{V_{fuel}} = 342 \frac{MW}{m^3}$

Exercise: An example nuclear power plant

- $M_U = 0.03 \cdot 235 + 0.97 \cdot 238 = 237.91\ \frac{g}{mol}$
- $M_{UO_2} = 0.03 \cdot 235 + 0.97 \cdot 238 + 2 \cdot 16 = 269.91\,\frac{g}{mol}$
- $\rho_U = \frac{M_U}{M_{UO_2}}\rho_{UO_2} = 9'078.85\ \frac{kg}{m^3} = 9'078'850\ \frac{g}{m^3}$
- $N_{235} = at_{235}\% \cdot \frac{\rho_U}{M_U} \cdot N_A = 6.89 \cdot 10^{26}\,\frac{at}{m^3}$
- $\mathrm{Q_{th}} = N_{235}\Sigma_{f,235}\phi \cdot E_{fission} \cdot V_{fuel} \rightarrow$
- $\phi = \frac{\mathrm{Q_{th}}}{E_{fission}}\ \frac{1}{V_{fuel}}\,\frac{1}{N_{235}\Sigma_{f,235}} = 8.522725 \cdot 10^{19}\ \frac{fiss}{s}\ \cdot \frac{1}{7.96\ m^3}\,\frac{1}{40.098\ m^{-1}} = 2.67 \cdot 10^{17}\,\frac{\#}{m^2 s}$

Exercise: dimension a thermal reactor

- Assume the following specifications:
  - Power Q = 3000 MW(th)
  - Moderator/fuel ratio V_{H_2O}/V_F = 1.9
  - Inlet coolant temperature T_i = 290 C
  - Outlet coolant temperature $T_{out,max}$ =330 C

- Find:
  - Linear heat rate q'_{max} assuming a fuel conductivity $k_F = 2.25 \frac{W}{m \cdot K}$ and a safety margin factor of 0.6
  - Surface heat flux q''_{max} using a DNBR of 1.3 and a q''_{crit} of 150 W/cm^2
  - Fuel radius
  - Lattice pitch
  - Core volume and dimensions
  - Core-averaged power density
  - Number of fuel elements
  - Coolant mass flow rate
  - Mean coolant velocity.