Exercise: dimension a thermal reactor

- Assume the following specifications:
  - Power Q = 3000 MW(th)
  - Moderator/fuel ratio V_{H_2O}/V_F = 1.9
  - Inlet coolant temperature T_i = 290 C
  - Outlet coolant temperature $T_{out,max}$ =330 C

- Find:
  - Linear heat rate q'_{max} assuming a fuel conductivity $k_F = 2.25 \frac{W}{m \cdot K}$ and a safety margin factor of 0.6
  - Surface heat flux q''_{max} using a DNBR of 1.3 and a q''_{crit} of 150 W/cm^2
  - Fuel radius
  - Lattice pitch
  - Core volume and dimensions
  - Core-averaged power density
  - Number of fuel elements
  - Coolant mass flow rate
  - Mean coolant velocity.

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- Linear heat rate q'_{max} assuming a fuel conductivity $k_F = 2.25 \frac{W}{m \cdot K}$ and a safety margin factor of 0.6
  - Let's assume $T_{melt} = 2973\ K$ and $T_{co} = 290\ C = 563K$ → $\Delta T_{max} = T_{melt} - T_{co} = 2410\ K$
  - We can neglect the temperature jump in gap, clad and coolant → $\Delta T_{max} = \sim \frac{1}{4\pi k_f} q'_{max}$
  - Therefore, $q'_{max} = 4\pi k_f \cdot \Delta T_{max} \sim 68 \frac{kW}{m} = 680 \frac{W}{cm}$
  - We then apply a safety margin of 0.6 to reach a more realistic value of $q'_{max} = 680 \cdot 0.6 = 400 \frac{W}{cm}$

- Surface heat flux q''_{max} using a DNBR of 1.3 and a q''_{crit} of 150 W/cm^2
  - We can use the definition of $DNBR = \frac{q''_{crit}}{q''_{max}} = 1.3$
  - Therefore $q''_{max} = \frac{q''_{crit}}{1.3} \sim 115 \frac{W}{cm^2}$

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- Fuel radius
  - We can use the equivalence between linear heat rate and heat flux at the rod outer surface

$$q''_{max} \cdot 2\pi r_f \cdot dz = q'_{max} \cdot dz$$

  - Therefore $r_f = \frac{q'_{max}}{2\pi q'_{max}} = 0.56 \text{ cm}$

- Lattice pitch
  - Assuming a square lattice with pictch p → $V_m = p^2 - \pi r_f^2$ and $V_f = \pi r_f^2$
  - Knowing the moderator to fuel volume ratio

$$\frac{V_m}{V_f} = \frac{p^2 - \pi r_f^2}{\pi r_f^2} = \frac{p^2}{\pi r_f^2} - 1 = 1.9 \rightarrow p^2 = 2.9 \cdot \pi r_f^2 \rightarrow p = 1.70$$

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- Core volume and dimensions
  - To estimate core volume and dimensions we assume the core to be cylindrical with radius R and height going from –H/2 to H/2 (total height H)
  - We start from the power distribution in a cylindrical core (see lecture on thermal analysis) which follows a modified bessel function along the radius r, and a cosine function along the axial coordinate z:

$$q'''(r,z) = q'''(0,0) \cdot J_0\left(\frac{2.405}{R}r\right) \cdot \cos\left(\frac{\pi}{H}z\right) = q'''_{max} \cdot J_0\left(\frac{2.405}{R}r\right) \cdot \cos\left(\frac{\pi}{H}z\right)$$

  - If we integrate the power density over the whole volume $(dV = 2\pi r \cdot dr \cdot dz)$ we get total core power

$$\int_V q'''(r,z)\, dV = q'''_{max} \cdot \int_0^R J_0\left(\frac{2.405}{R}r\right) \cdot 2\pi r \cdot dr \int_{-\frac{H}{2}}^{\frac{H}{2}} \cos\left(\frac{\pi}{H}z\right) dz$$

  - The two integrals are equal to

$$\int_0^R J_0\left(\frac{2.405}{R}r\right) \cdot 2\pi r \cdot dr = \pi R^2 \cdot \frac{1}{2.32}$$

$$\int_{-\frac{H}{2}}^{\frac{H}{2}} \cos\left(\frac{\pi}{H}z\right) dz = H \cdot \frac{1}{1:57}$$

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- Substituting the two integrals

$$\int_V q'''(r,z)\,dV = Q = q'''_{max} \cdot \pi R^2 \cdot \frac{1}{2.32} H \cdot \frac{1}{1:57} = q'''_{max} \cdot \frac{1}{2.32} \cdot \frac{1}{1:57} \cdot V$$

- We can substitute the maximum linear heat rate for the maximum power density in the unit element of the lattice

$$q'''_{max} \cdot p^2 \cdot dz = q'_{max} dz \rightarrow q'''_{max} = q'_{max}/p^2$$

- And finally obtain the volume:

$$V = \frac{Q \cdot 2.32 \cdot 1.57}{q'_{max}/p^2} = 77.4\ m^3 = 7.74 \cdot 10^7\ cm^3$$

- To obtain the radius and height we assume a H/2R ratio = 1

$$V = \pi R^2 H = 2\pi R^3 = 77.4\ m^3$$

$\boldsymbol{R} = 2.3\ \mathrm{m}$ **and H** $= 4.6\ \mathrm{m}$

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- Core-averaged power density
  - Going back to the formula

$$Q = q'''_{max} \cdot \frac{1}{2.32} \cdot \frac{1}{1{:}57} \cdot V$$

  - We find that:

$$\overline{q'''} = \frac{Q}{V} = q'''_{max} \cdot \frac{1}{2.32} \cdot \frac{1}{1{:}57} = 3.87 \cdot 10^7 \frac{W}{m^3}$$

  - From this we can also find the physical meaning of the $\mathrm{C_r} = 2.32$ and $\mathrm{C_z} = 1.57$ factors, which indeed prove to be the **radial and axial power peaking factor** (i.e. the ratio between maximum and average power)

$$2.32 \cdot 1.57 \cdot \overline{q'''} = q'''_{max} \text{ or } \mathrm{C_r C_z} = \frac{q'''_{max}}{\overline{q'''}}$$

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- Number of fuel elements
  - A single few element has volume $\mathrm{V_{element}} = \mathrm{p^2 H}$
  - Therefore the number of fuel elments N is

$$N = \frac{V_{core}}{V_{element}} = \frac{\pi R^2 H}{p^2 H} = \frac{\pi R^2}{p^2} \sim 58359$$

- Coolant mass flow rate
  - Assuming water heat capacity at the operating temperature to be $c_p = 6.4 \cdot 10^3 \frac{J}{kgC}$ and knowing water change of temperature to be 320-290=30 C, then

$$\dot{m} = \frac{Q}{c_p \Delta T} = 15.6 \frac{t}{s}$$

- Mean coolant velocity
  - The total flow area for the water is $\mathrm{A_{tot}} = N A_m = N\left(p^2 - \pi r_f\right)$
  - Because the volumetric flow rate $\dot{m} = \rho A_{tot}\, \bar{v}$, thus assuming a density of 0.7 g/cm3 = 700 kg/m3

$$\bar{v} = \frac{\dot{m}}{\rho A_{tot}} = 2 \frac{m}{s}$$