Reactor Thermal Analysis

Reasons for Technological Limits

From previous lectures we have seen that neutronics:

- Provides the conditions for reactor criticality
- Allows determination of flux distributions (i.e. the shape) in the reactor core
- ***But it does not fix the neutron flux level, i.e. the absolute power of the NPP.***

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It's the technological limits which will do that!

- Melting points, failure stress & strain rates, oxidation thickness etc. are all related to the reactor power level over time.
- Analysis of "hottest cell" or "hottest channel" (the one with maximum power density) is necessary to fix all absolute values, for a given margin of safety…
- We will focus only on the thermal analysis of the hottest rod in the reactor.

Linear Heat Rate and Volumetric Heat Density

- First let's introduce a few definitions. For a fuel rod that produces a given power along its height, it is often convenient to consider its average linear heat rate q' $(\frac{W}{m})$.

- Assuming 1D heat transfer (i.e. all heat produced by the pellet moves radially towards the cladding) then we can define an average volumetric heat density q'' $\left(\frac{W}{m^3}\right)$ for this rod so that:

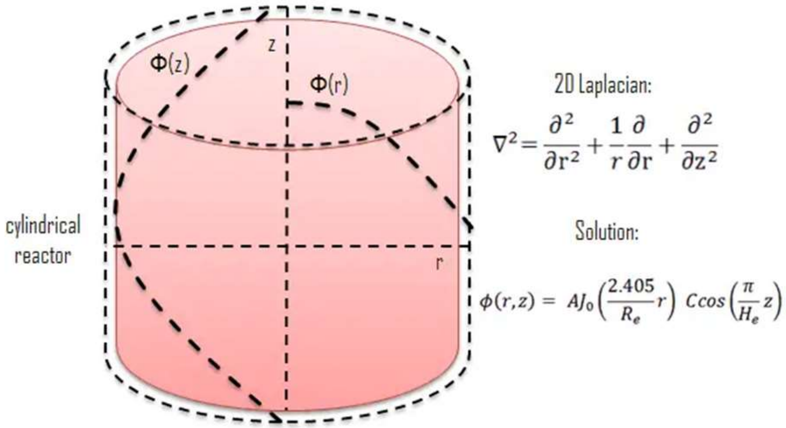
$$q''' \cdot \pi R_{fo}^2 \cdot dH = q' \cdot dH \quad \rightarrow \quad \boldsymbol{q''' = \frac{q'}{\pi R_{fo}^2}}$$

- Also, taking a reference surface (e.g. the outer cladding) we can define a heat flux q'' $(\frac{W}{m^2})$

$$q'' \cdot 2\pi R_{co} \cdot dH = q' \cdot dH \quad \rightarrow \quad \boldsymbol{q'' = \frac{q'}{2\pi R_{co}}}$$

Hottest Channel

Image source: https://nuclear-power.com/



- Most reactor cores are essentially cylindrical in shape.
- For a uniform cylinder with radius R and height H, diffusion theory predicts a neutron flux with cosine axial while the radial profile follows the Bessel function J_0.

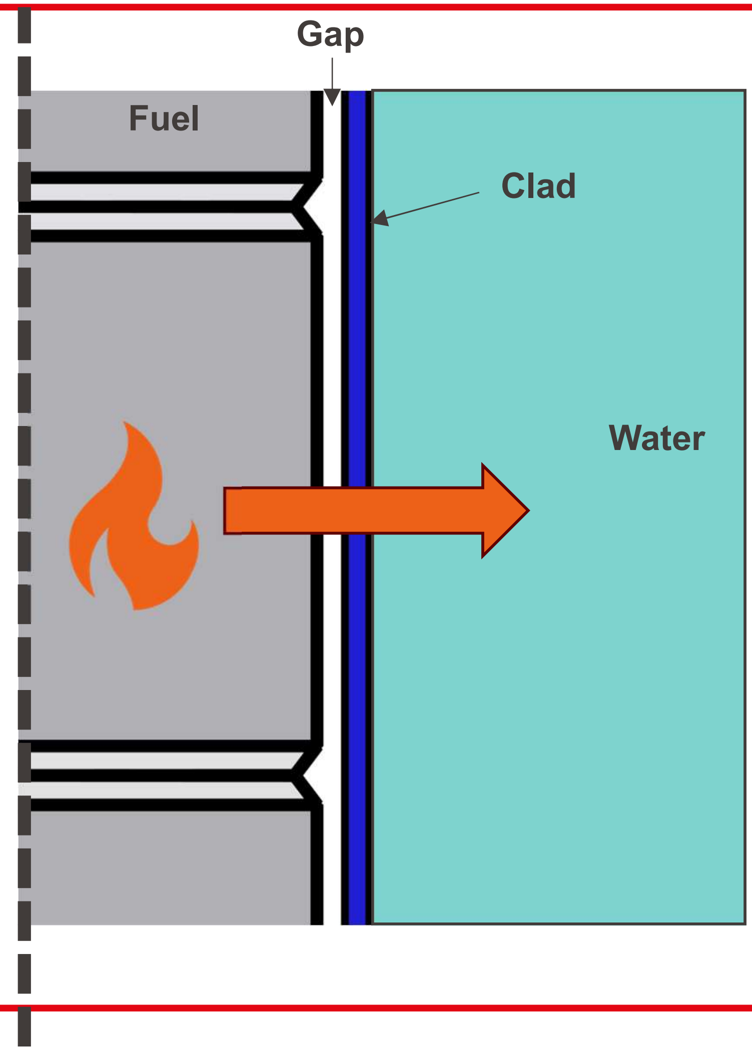
- Neutron flux and power are related by cross sections, thus in our approximation of uniform material (i.e. uniform cross sections) ***power and flux have the same shape***.

$$q'(r,z) = q'(0,0) \cdot J_0\left(2.405 \cdot \frac{r}{R}\right) \cdot \cos\left(\frac{\pi z}{H}\right)$$

- As expected, the hottest channel is at the center ($r = 0$) of the core. The hottest point of this channel is at its center ($z = 0$) .

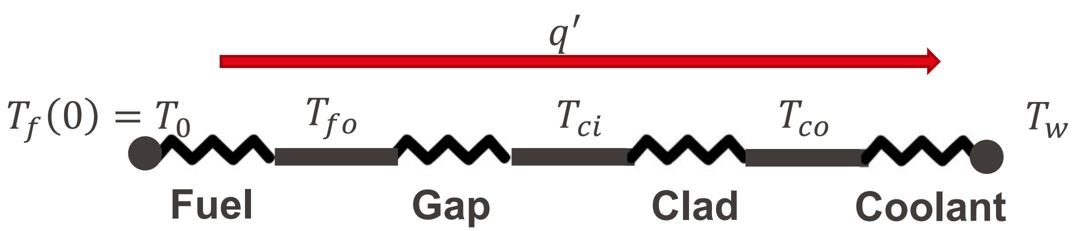
Phenomenology of the Heat Transfer in a Fuel Rod

- The heat produced in the fuel diffuses essentially radially (1D) through the fuel pellet via conduction.
- It is then transmitted across the gap due to a mix of conduction through the gas, radiative heat transfer and conduction through contact points (if gap is closed).
  - Tthe ability of the gap to transmit heat from fuel to clad changes over time due to differential expansions (change in gap size) and fission gas poisoning (Xe and Kr have lower conductivity wrt He).
  - A finite gap conductance is present even in closed-gap conditions because of surface rugosity and imperfect contact.
- Heat then moves across the cladding (conduction) and finally it is removed by the cooling water via forced convection.



Resistance Analogy

- We can interpret this 1D setup as the heat flowing across a series of thermal resistances:



- This causes a total temperature-jump or temperature-drop (with T_{max} at the fuel center) composed of a series of additive temperature differences between each nodes in the resistance series:

$$\Delta T_{total} = T_{max} - T_{cool} = \Delta T_{fuel} + \Delta T_{gap} + \Delta T_{clad} + \Delta T_{cool} = q'(R_{fuel} + R_{gap} + R_{clad} + R_{water})$$

- To find temperature profile and values of thermal resistances, one needs to solve the heat conservation equation (derivations to be considered mostly a bonus material)

1D Cylindrical Heat Transport Equation (Bonus Material)

For a generic control volume V, bounded by a surface S, the heat diffusion equation states:

$$\overbrace{\int_V \frac{\partial \rho c_p T}{\partial t} dV}^{\textit{Rate of change}} = \overbrace{-\oint_S \mathbf{n} \cdot \mathbf{q}'' \, dS}^{\textit{Rate of heat flowing out}} + \overbrace{\int_V q''' \, dV}^{\textit{Rate of generation}}$$

Fourier law relates heat flux $\mathbf{q}''$ and temperature gradient:

$$\mathbf{q}'' = -k \, \nabla T$$

Applying the Green-Gauss theorem and considering steady-state conditions one obtains:

$$\nabla \cdot (k \, \nabla T) + q''' = 0$$

Thin rod approximation: axial conduction can be neglected ($dT/dz \cong 0$), 1D heat exchange:

$$\frac{d}{r dr}\left(rk \frac{dT}{dr} \right) + q''' = 0$$

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Pellet Heat Transfer – Derivation (Bonus Material)

Consider 1D radial heat exchange equation for a fuel pellet:

$$\frac{d}{rdr}\left(rk\frac{dT}{dr}\right) + q''' = 0$$

Assuming q''' to be radially uniform a given height z (does not depend on radial position within rod, not 100%true) one obtains the general solution:

$$r\frac{dT}{dr} = -\frac{q''' r^2}{2k} + A \quad \Rightarrow \quad T(r) = -\frac{q''' r^2}{4k} + A\ln(r) + B$$

We can use appropriate BCs to obtain the two integration constants:

1. $\frac{dT}{dr} = 0$ at r = 0 (axisymmetric condition) → $A = 0$
2. T known at r = R_{fo} (periphery), $T_{fo} = T(R_{fo})$ → $B = T_{fo} + \frac{q'''}{4k_f} R_{fo}^2$

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Pellet Heat Transfer

One obtains a parabolic temperature profile in the fuel: $\boldsymbol{T(r) = \frac{q'''}{4k_f}\left(R_{fo}^2 - r^2\right) + T_{fo}}$

The maximum T is at r=0 and maximum T jump is: $\Delta T_f = T(0) - T_{fo} = \frac{q'''}{4k_f} R_{fo}^2$

Using $q''' \cdot \pi R_{fo}^2 = q'$ one obtains the temperature jump across the pellet:

$$\Delta T_f = T(0) - T_{fo} = T_0 - T_{fo} = \frac{q'}{4\pi k_f}$$

Thus $\boldsymbol{\frac{1}{4\pi k_f}}$ is ***the fuel thermal resistance (mK/W)***

Gap Heat Transfer (Bonus Material)

- Between fuel and cladding there is a small gap filled with helium (relatively low conductivity wrt metals and ceramics).
- Its conductance h_g depends on size, temperature and composition, and typically it changes during irradiation
- Fuel-clad heat exchange typically approximated with conductance model: $q''_g = h_g\left(T_{fo} - T_{ci}\right)$
- Using the fact that $q''_g 2\pi R_g = q'$ one gets:

$$T_{fo} = \frac{q'}{2\pi R_g h_g} + T_{ci}$$

- **Thus $\frac{1}{2\pi R_g h_g}$ is the gap thermal resistance**

Cladding Heat Transfer (Bonus Material)

One can solve the heat diffusion equation with appropriate BCs for the cladding and obtain:

$$T_c(r) = T_{co} + \frac{q'}{2\pi k_c} \ln\left(\frac{R_{co}}{r}\right)$$

And that the temperature jump across the cladding is:

$$\Delta T_c = T_{ci} - T_{co} = \frac{q'}{2\pi k_c} \ln\left(\frac{R_{co}}{R_{ci}}\right)$$

Thus $\frac{1}{2\pi k_c} ln\left(\frac{R_{co}}{R_{ci}}\right)$ is the clad thermal resistance

Coolant Heat Transfer

convective heat transfer coefficient (W/cm^2.C)

For the liquid coolant, one has the Convection Law: $q''_w = h_w\left(T_{co} - T_{w,bulk}\right)$

- h_w depends on properties of the liquid, geometry of the "cell", as also the flow
- One has empirical correlations (forced convection)

Because (in steady state) $q''_w 2\pi R_{co} = q'$ one gets:

$$T_{co} - T_{w,bulk} = \frac{q'}{2\pi R_{co} h_w}$$

Thus $\frac{1}{2\pi R_{co} h_w}$ is the thermal resistance of the coolant

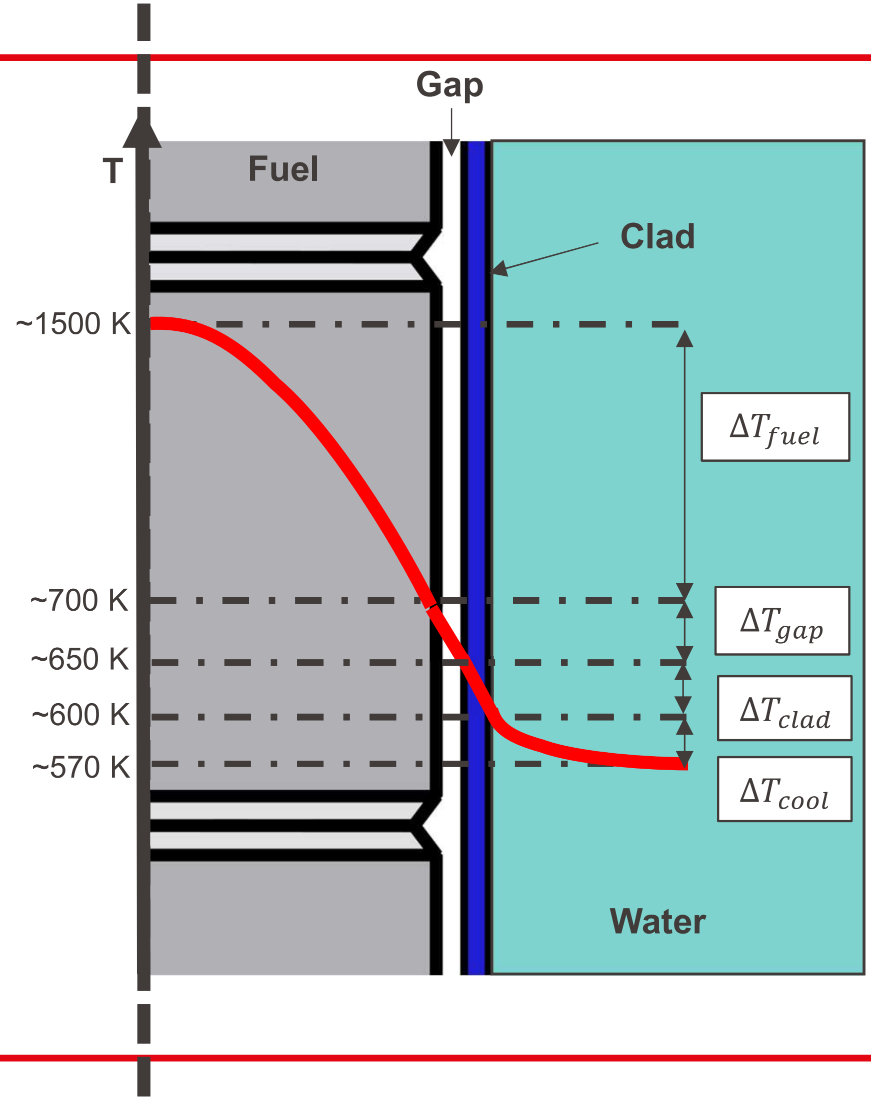
Typical temperature profile (1D)

- Most of the temperature jump between fuel centre and outer cladding takes place in the fuel. At a first approximation we can neglect gap and clad resistances:

$$T_f(0) - T_{co} = \left(\frac{1}{4\pi k_f} + \frac{1}{2\pi h_g R_g} + \frac{1}{2\pi k_c}\ln\left(\frac{R_{co}}{R_{ci}}\right)\right) q'$$

- The additional temperature jump due to clad-water convective heat exchange depends strongly on the heat transfer regime (nucleate boiling, film boiling etc.) and is given by:

$$T_{co} - T_w = \left(\frac{1}{2\pi R_{co} h_w}\right) q'$$



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Characteristics of Fuel Materials (UO_2, UC, U)

- Neglecting gap and clad resistances and assuming to know that $T_{co} = 600K$ we can ***determine the maximum q'_{max} only as a function of fuel conductivity*** as:

$$\Delta T_{max} = \frac{q'_{max}}{4\pi k}$$

Fuel Type	T_{melt} (K)	k (W/mK)	$q'_{max} = 4\pi k\,(T_{melt} - T_{co})$ (kW/m)
UO_2	2973	2-2.5	~60-70
UC	2673	17	~440
U (metal)	1403	30	~300

- UO_2 is the most common fuel material. For UC or metallic U q' can be much higher but:
  - UC not compatible with H_2O (chemical reaction)
  - Metallic U has an additional limit on $(T_0)_{max}$ due to phase change ≈600° C
- In practice one stays quite below these limit levels of q'_{max} to have safety margins
  - e.g. for UO_2, $q' < 45\ kW/m$

Technological Limit for the Fuel

$$\Delta T_{max} = \frac{q'}{4\pi k}$$

- The above formulation *using the* linear power is very useful as:
  - ***It does not depend*** on fuel rod geometry.
  - Two different fuels may have the same $\mathrm{k}(T_{melt} - T_{co})$, and hence the same limit.

- This formula gives a **1st technological constraint** for $\mathrm{q}'_{\mathrm{max}}$ at the hot spot…
- … and, as the radial/axial shape is given, limiting the maximum linear power, in turn, limits the total and average reactor power.
- Once we have fixed the average reactor power density or linear heat rate we can define the specific power w_{sp} which can be used to obtain the fuel mass needed for a given total power:

$$w_{sp} = \frac{\overline{q'''}}{\rho_f} = \frac{\overline{q'}}{\pi R_{fo}^2 \cdot \rho_f} \quad \left(\frac{W}{g}\right)$$

Axial Temperature Distributions

- Fuel maximum temperature is related to the q'_{max} but it depends also on the outer cladding/water temperatures as $\Delta T_{total} = T_{f0} - T_w$
- → How do fuel, cladding and coolant temperatures vary along the height of the core?

- In the hottest radial channel, for an axial segment dz at z the conservation of energy gives

$$\underbrace{\dot{m}}_{\text{Coolant Mass Flow Rate}} \cdot \underbrace{dH}_{\text{Enthalpy increase}} = q'(z)dz \quad \rightarrow \quad \boxed{\dot{m}c_p \frac{dT_w}{dz} = q'(z)}$$

- Main assumptions of single phase & constant pressure → $dH = c_p \cdot dT_w$

Coolant Temperature

- Axial power vary as cosine in a given channel:

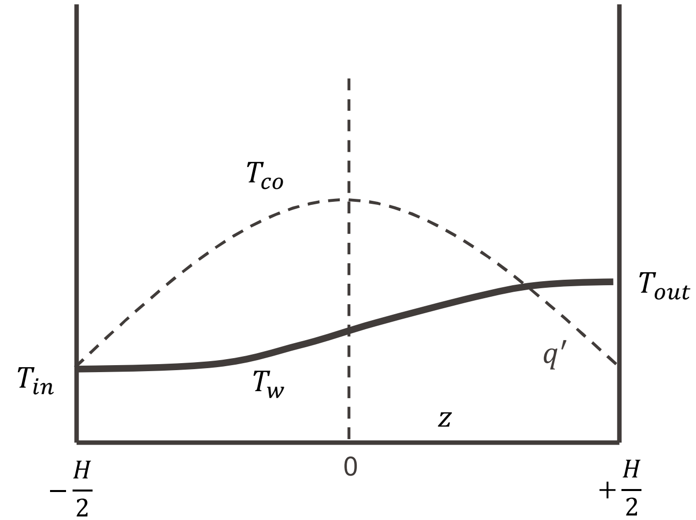
$$q'(z) = q'(0) \cdot \cos\left(\frac{\pi z}{H + 2d}\right)$$

- Integrating energy conservation from –H/2 to z:

$$T_w(z) = T_w(0) + \frac{q'(0)}{\dot{m}c_p}\frac{H + 2d}{\pi} \cdot sin\left(\frac{\pi z}{H + 2d}\right)$$

- Total enthalpy change given by:

$$\dot{m}c_p(T_{out} - T_{in}) = \int_{-H/2}^{+H/2} q'(z) = P_t$$ **(total power in channel)**



Cladding and Fuel Temperatures (Axial Profiles)

$$T_{co}(z) - T_w(z) = \frac{q'(z)}{2\pi R_{co} h_w}$$

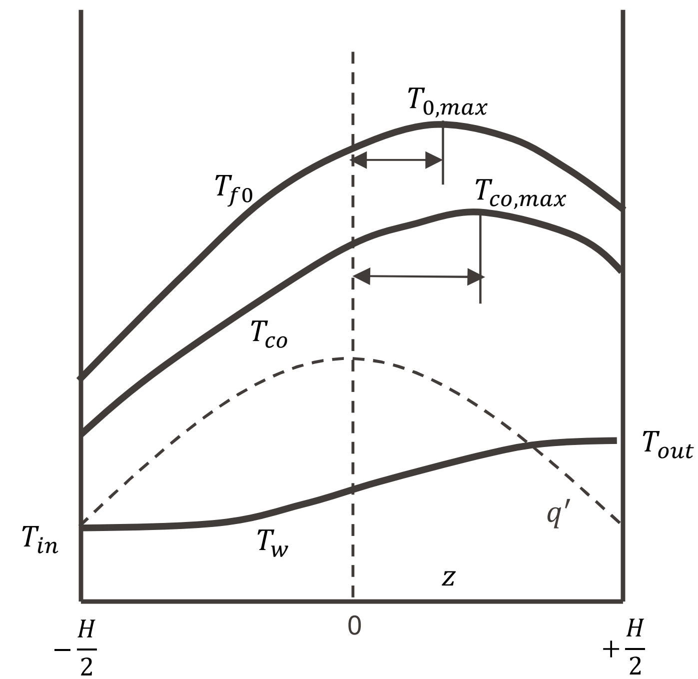
Using the previous relations we have:

$$T_{co}(z) = \frac{q'(0)}{2\pi R_{co} h_w} \cdot \cos\left(\frac{\pi z}{H + 2d}\right) + T_w(z)$$

While for the fuel (at centre, neglecting clad and gap):

$$T_0(z) - T_{co}(z) = \frac{q'}{4\pi k_f}$$

$$T_0(z) = q'(0)\left(\frac{1}{4\pi k_f} + \frac{1}{2\pi R_{co} h_w}\right) \cdot \cos\left(\frac{\pi z}{H + 2d}\right) + T_w(z)$$



Limit for the Cladding

- Maximum T_w is at the outlet as expected (coolant heats up while passing through the core)
- Maxima for T_{co}, T_{f0} are near $z = 0$ with $T_{f0,max}$ somewhat nearer than $T_{co,max}$
  - Hot spot (highest fuel temperature) is not exactly at core centre where we have maximum power because of slight offset due to coolant/cladding temperature… although certainly it is very close to it!

- $T_{co,max}$ depends on T_{in}, h_w, q'. For a given coolant mass flow (and h fixed) and given T_{in}, the limit for the cladding is simply imposed by q'

- ***This gives the second technological constraint on q'***… but which $T_{co,max}$ should we consider?
  - Indeed, $T_{co} << T_{f0}$ and for Zircaloy T_{melt} is relatively high (≈1850 C)

Fuel Rod Cladding Material Selection

It is desirable to choose material with

- good mechanical properties
- good neutronics properties → low neutron absorptions!

Element	σ_a, barn	Application
Mg	0.063	unstable in water → gas cooled, graphite moderated reactors
Al	0.231	unstable in water above 100 ° C → used in research reactors
Ti	6.09	Composition of stainless steel / Austenite (z.B. X6CrNiTi1810)
Cr	3.05	
Fe	2.56	
Ni	4.49	
Co	37.18	${}^{59}_{27}Co + {}^{1}_{0}n \rightarrow {}^{60}_{27}Co^{*} + \gamma$ $T_{1/2}$ = 5.27 a, E_γ = 1.33, 1.17 MeV, **avoid Co!**
Zr	**0.185**	**optimal material for Light Water Reactors (PWR, BWR)**

Low neutron absorption + good mechanical properties

Fuel Rod Cladding Material Selection

- It is also desirable to have good chemical properties when in contact with the chosen coolant.

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- Zr has also good corrosion stability in water due to stable oxide layer formation…
- ***… but the protective oxide layer becomes unstable above ~450 C!***

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- ***Cladding surface oxidations limits in LWRs the coolant outlet temperature at ~350 C thus fixing the second technological constraint on q'***

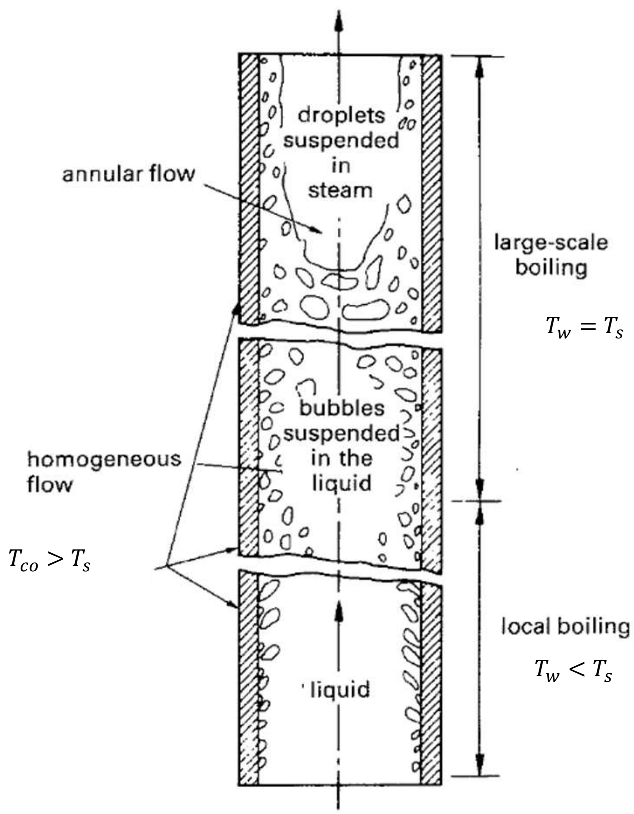
Liquid Coolant Behaviour

- For a single-phase liquid heat flux is given by convection law):

$$q''(R_{co}) = \underbrace{h_w}_{\text{dep. on } \dot{m}} \left(T_{co} - \underbrace{T_w}_{\text{Bulk liquid}} \right)$$

- For high T_{co} there is a complex dependence of q'' on ΔT (see next slide)

- Note that in a PWR, $T_w < T_{sat}$ and in a BWR, $T_w = T_{sat}$ (except in lower part of core)
  - In PWR, even when $T_w < T_{sat}$ one has boiling in the liquid film attached to the cladding (***subcooled nucleate boiling***)
  - Bubbles at the surface migrate into the bulk liquid and the resulting turbulence increases heat transfer (h) due to local boiling, bubble movement/condensation.

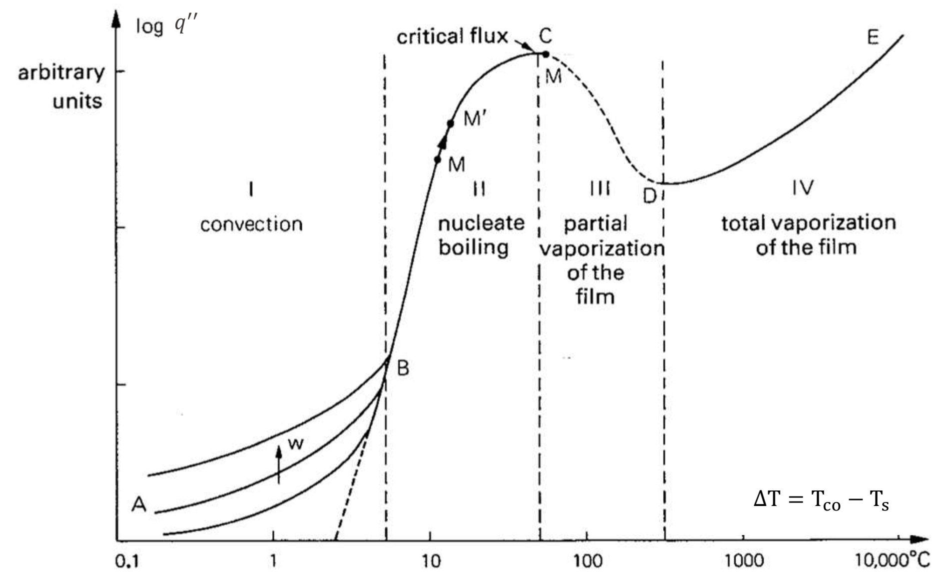


Boiling crisis in PWRs – departure from nucleate boiling

- Region I (AB) - Single-phase convection
  - q'' depends on $\dot{m}$

- Region II (BC) - Nucleate boiling

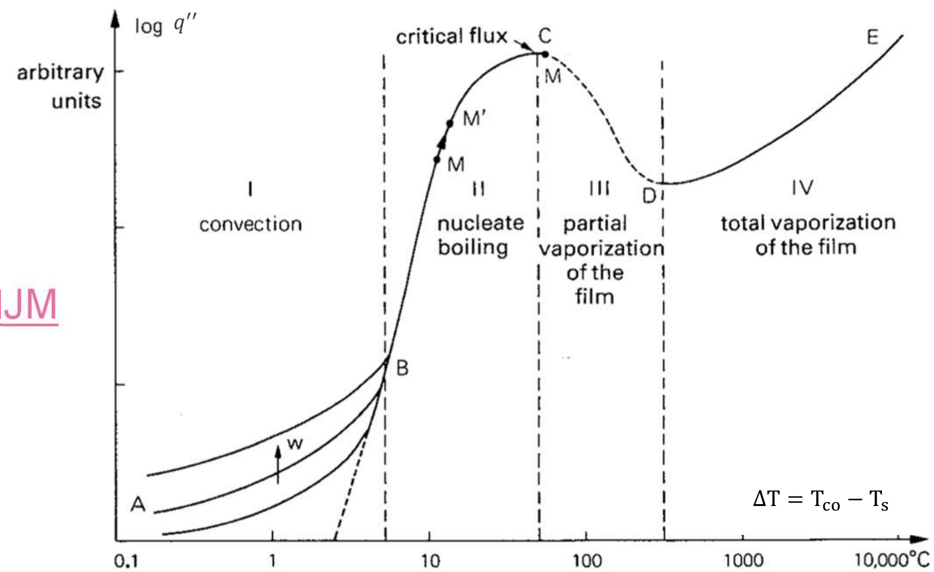
- Region III (CD)
  - Point C: critical heat flux, q''_{cr} formation of vapour film on surface
  - Heat exchange deteriorates sharply (q'' goes down, ΔT goes up)



- Region IV (DE)
  - Point D corresponds to minimal h (vapour film fully formed)
  - Heat transfer by conduction and radiation… q'' goes up again, but ΔT very high!

Boiling crisis in PWRs – departure from nucleate boiling

- Unstable situation at C, as any further increase in q'' produces jump of T_{co} towards E (with the risk of approaching fuel melting)
- Essential to avoid point C at all costs
  - ***Departure from Nuclear Boiling (DNB)***
  - ***Check video from INL at*** https://www.youtube.com/watch?v=5xB_gLfkdJM
- Empirical relations available for q''_{cr}
  - Typically given for $T_w<T_s$ or for $T_w>T_s$, for a given channel geometry, etc.



- Departure from Nucleate Boiling Ratio or DNBR gives the third technological constraint on q' :
  - For PWRs, DNBR ≥1.3 (for BWRs, ≥1.9)

$$DNBR = \frac{q''_{cr}}{q''}$$