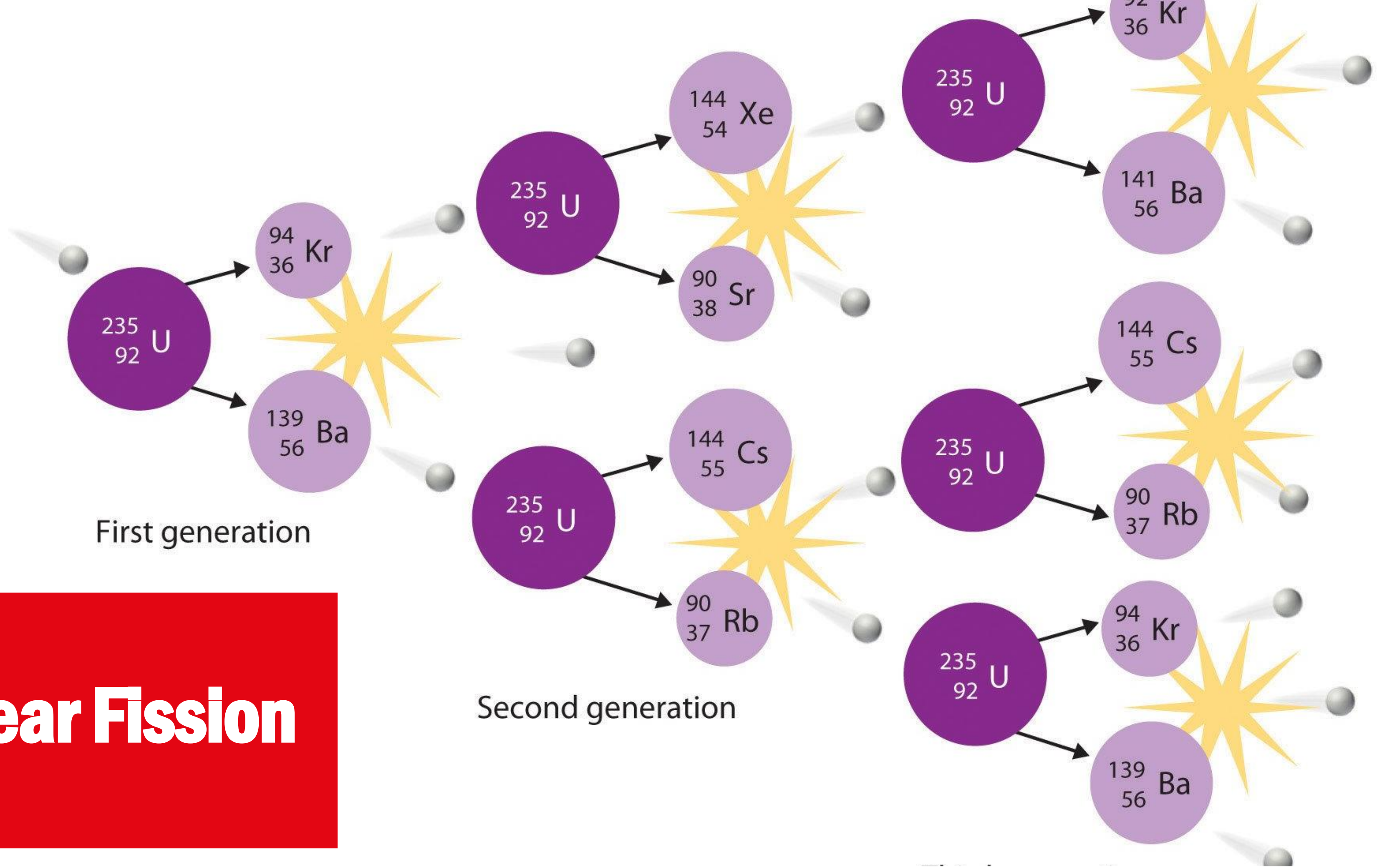
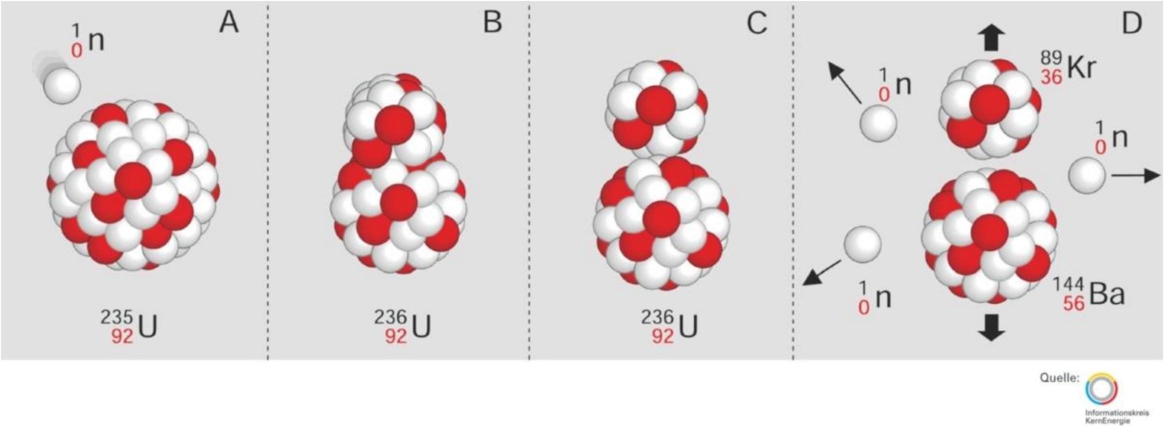
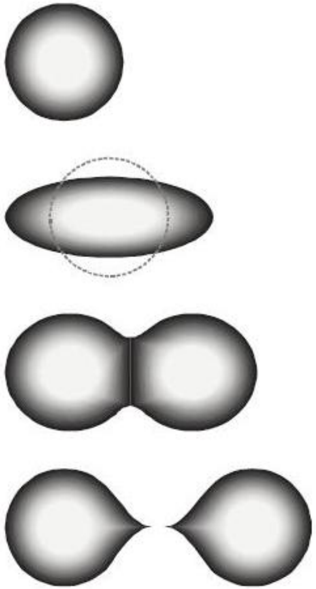
Nuclear Fission



Physics of the Fission Reaction

$$^{235}_{92}\mathrm{U} + ^{1}_{0}\mathrm{n} \rightarrow ^{89}_{36}\mathrm{Kr} + ^{144}_{56}\mathrm{Ba} + 3\,^{1}_{0}\mathrm{n} + \gamma$$





Stages of nuclear fission according to the Liquid Drop Model of the nucleus

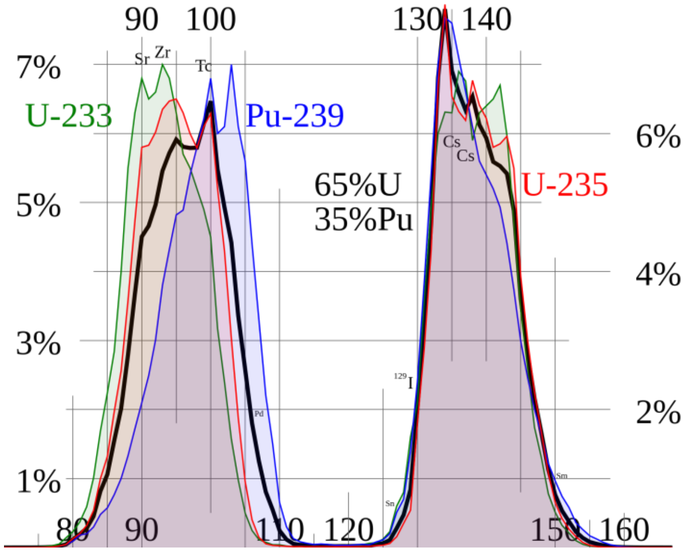
Fission Products – Fission yield

- Fission events release two fission products (ternary fission possible but less probable), a certain number of neutrons, one or more "prompt gamma rays" and large amount of kinetic energy.

- FPs are distributed with a certain **fission yield $y(A_i)$:** the number of fission products of mass number A_i per 100 fissions.

- Fission yield double hump curve:
  - $y(A)$ as a function of A
  - For 100 fissions: $\sum y(A)$ = 200
  - The curve depends on the fissile material
  - Most probable, FPs with $A = 94, 140$

$$^{235}_{92}\mathrm{U} + ^{1}_{0}\mathrm{n} \rightarrow ^{94}_{38}\mathrm{Sr} + ^{140}_{54}\mathrm{Xe} + 2\,^{1}_{0}\mathrm{n}$$



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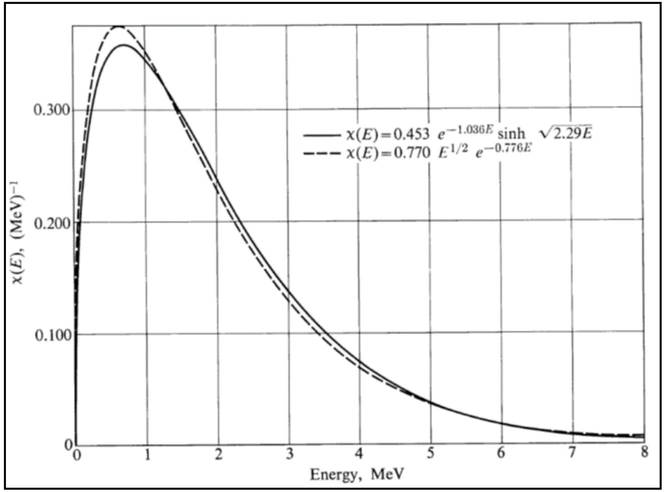
Fission Neutrons Energy

- The number ν of neutrons created per fission varies (from 0 to 5).
- The average $\bar{\nu}$ ranges from 2.4 to 2.9 depending on fissile material.
- Fission neutrons are mostly fast but their energy varies with a spectrum $\chi(\mathrm{E})$ – average E of $\sim 2\mathrm{MeV}$.

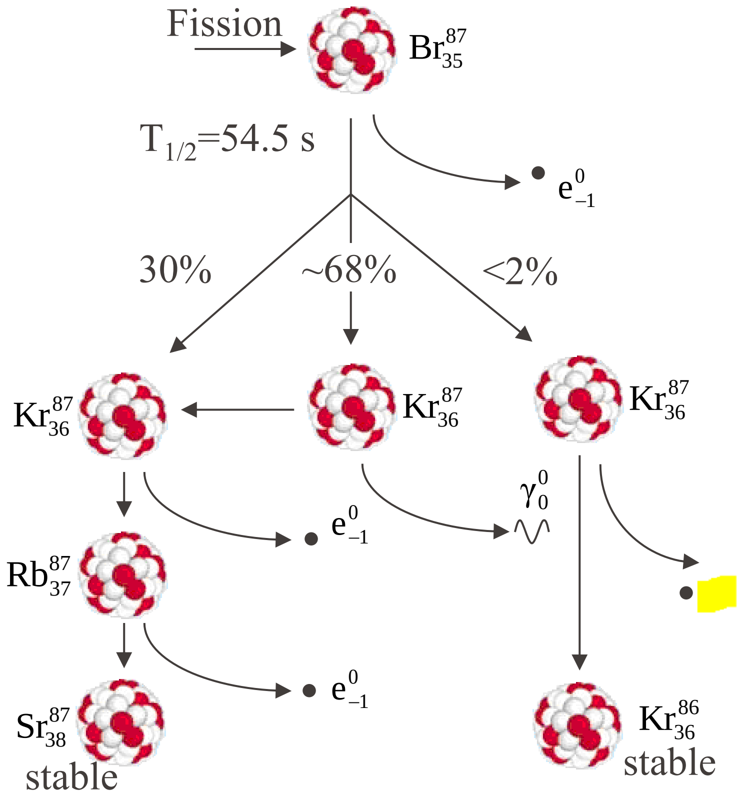


Delayed Neutrons

- Small fraction of the neutrons ($\beta \approx 0.6\%$ for $^{235}_{92}U$) are not ***prompt*** but ***delayed***, i.e. they are not emitted immediately with fission.

  - Emitted by many different FPs called "***precursors***"

  - Small fraction of total population, but necessary for control of the chain reaction

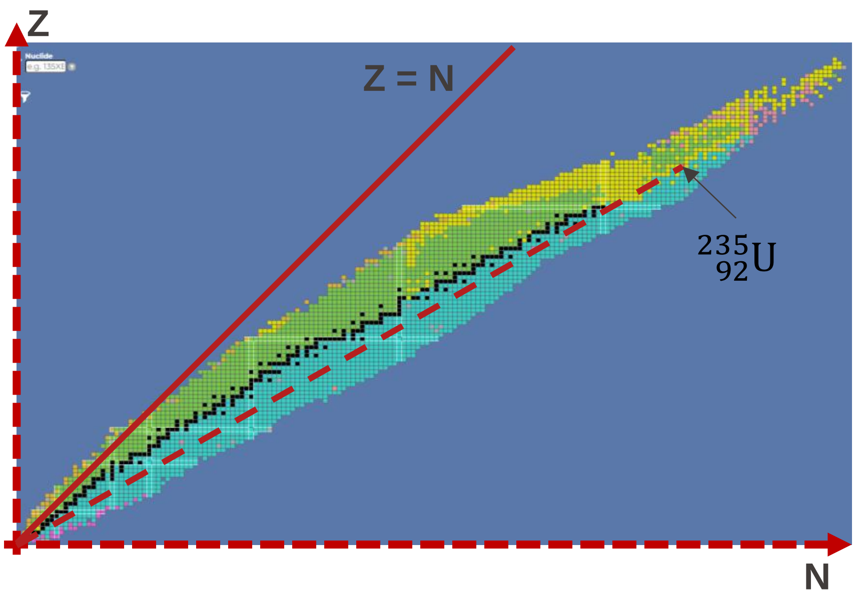


Delayed Neutron Parameters

Group number	Precursor nuclei	Average half-life (s)	Delayed neutron fraction – β_i (%)
1	^{87}Br, ^{142}Cs	55.72	0.021
2	^{137}I, ^{88}Br	22.72	0.140
3	^{138}I, ^{89}Br, $^{(93,94)}Rb$	6.22	0.126
4	^{139}I, $^{(93,94)}Kr$, ^{143}Xe, $^{(90,92)}Br$	2.30	0.252
5	^{140}I, ^{145}Cs	0.61	0.074
6	(Br, Rb, As etc.)	0.23	0.027
			0.640

- ~6-8 groups of precursors can be identified based on the half-life
- Average energy of delayed neutrons is smaller than prompt's: E_{avg} ~ 0.4MeV
- β_i depend on nuclide, e.g. $\beta = \sum \beta_i = 0.21\%$ for $^{239}_{94}Pu$ or $\beta = 0.26\%$ for $^{233}_{92}U$

Fission Products Radioactivity



- $^{235}_{92}U$ has an excess of neutrons: they will be found in the FPs (minus the few ones released).
- Thus, FPs are unstable and decay usually by β^-. For instance:

$$^{140}_{54}Xe \xrightarrow{16s} {}^{140}_{55}Cs \xrightarrow{66s} {}^{140}_{56}Ba \xrightarrow{12.8d} {}^{140}_{57}La \xrightarrow{40h} {}^{140}_{58}Ce\ (\text{stable})$$

- The radioactivity of FPs provides additional heat (compensates loss of neutrinos) but it is problematic:
  - Radiation protection (irradiated fuel) + contamination
  - Residual heat after reactor shutdown

Detailed Energy Balance

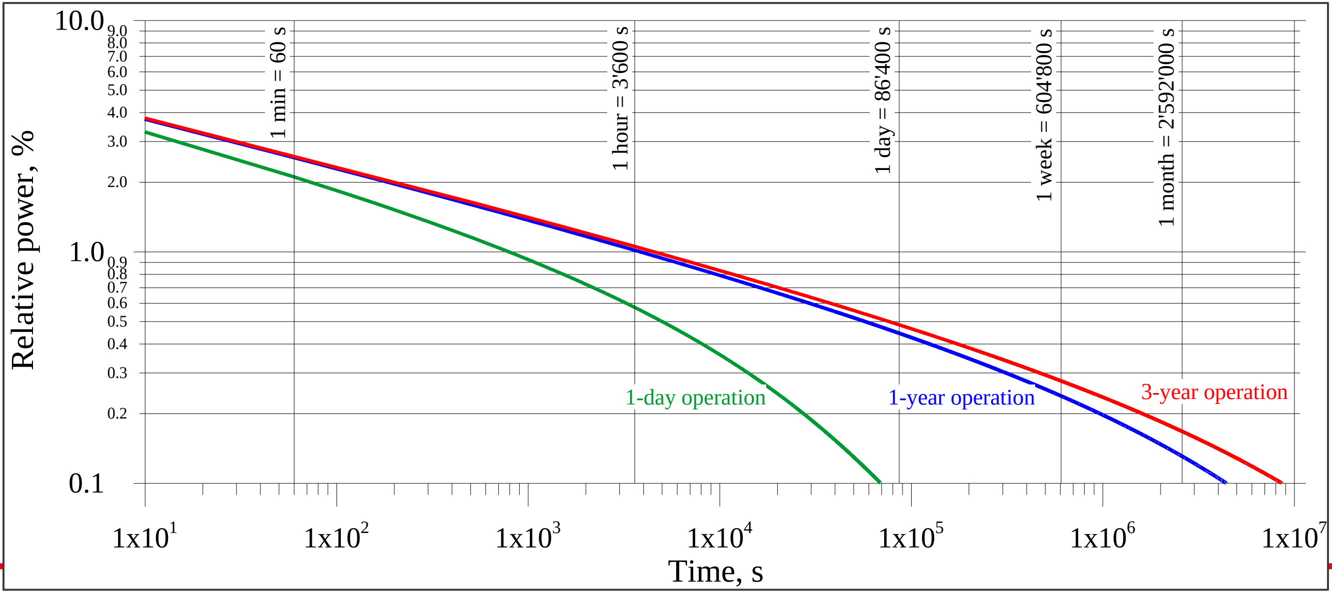
- The energy release is roughly **200 MeV**

  - FPs and β^- slowed down in the fuel and their energy appears in form of heat.
  - Neutrons are slowed down mostly outside the fuel.
  - Neutrons also cause neutron capture which provide additional (not in table) energy (5 to 10 MeV) in form of γ rays.
  - Neutrinos leave reactor entirely.
  - Overall, **<u>180-200 MeV can be recovered!</u>**

Components	*Energy (MeV)*
FP's kinetic energy	168
n's kinetic energy	5
Prompt γ	7
FP-radioactivity (β^-, γ)	15 (8+7)
Neutrinos (non-interacting)	12
TOTAL	**~ 207**

Decay Heat Removal

- After reactor shutdown only FP radioactivity remains – approximately ~7% of rated power:
  - Slow decrease, ~1% after 1 day.
  - Decay heat removal for sufficiently long time must be properly foreseen (Fukushima accident)
- Curves representing power decrease (relative to nominal) 10s from shutdown:
- Depend on operation time due to the slow buildup of long-lived FPs and transuranic elements

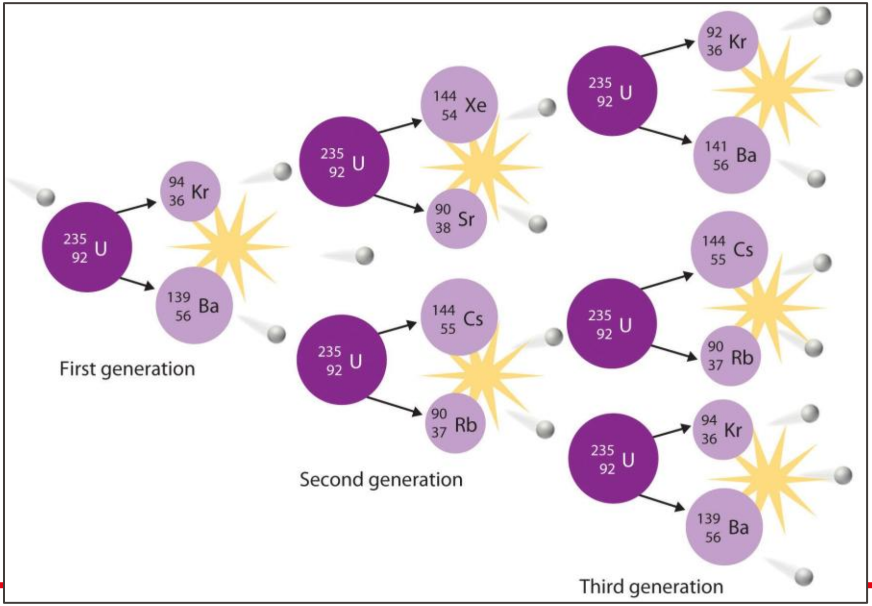


Chain Reaction

- When a nucleus fissions, more than one neutron is emitted.
- As $\bar{\nu} > 2$ it is possible to design the core materials, so that these fission neutrons induce further fissions with the release of more neutrons, inducing further fissions and so on... This sequence of events is called a ***chain reaction.***

- If each neutron was “useful” (i.e. it induced further fissions) then the chain reaction would be strongly divergent…
- … in practice, certain neutrons are lost due to captures & leakages.

- **A nuclear reactor is a device in which geometry and materials are arranged so that a stable self-sustained fission chain reaction can occur in a controlled manner!**



Multiplication Factor

- For a reactor with neutron production rate P, absorption rate (A) and leakages (L), we can define the multiplication factor as:

$$\mathbf{k} = \textbf{Multiplication Factor} = \frac{\text{The number of fissions in generation i}}{\text{The number of fissions in previous generation (i} - \text{1)}} = \frac{\mathrm{P}}{\mathrm{A} + \mathrm{L}}$$

- If one fission leads to more than one fission the fission rate increases in time exponentially
  - Such a reactor is called **supercritical and k >1**
- If one fission leads to less than one fission the fission rate decreases in time and eventually the chain reaction stops
  - Such a reactor is called **subcritical and k < 1**
- If one fission leads on average to exactly one other fission the chain reaction is self-sustained.
  - Such reactor is **called critical and k =1**

Multiplication Factor

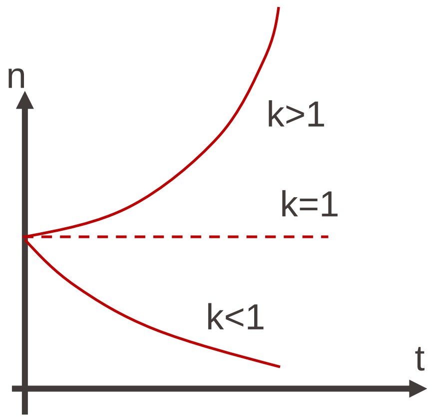
- Suppose to start with n_0 neutrons. After the first generation one has kn_0, then k^2n_0, k^3n_0 etc… until one has $k^i n_0$ at the ith generation.

- If we define an average neutron lifetime l, then each generation is born at $t = i \cdot l$ and we can estimate that the population will grow as

$$n(t) = n_0 k^{t/l}$$

- Using the fact that $|k - 1| << 1$ and using Taylor's expansion we obtain:

$$n(t) = n_0 \exp\left[\frac{(k-1)t}{l}\right]$$



- The initial population evolves as an exponential:
  it increases if k>1; decreases if k<1, constant if k=1

Neutron Balance

- We can start making a first simplified neutron balance. For a self-sustaining or critical reactor, the neutron production rate P must be equal to the sum of the absorption rate (A) plus leakages (L)

$$P = A + L$$

- For a combination of fissile, fertile and structural materials:

$$P_{Fissile} + P_{Fertile} = A_{Fissile} + A_{Fertile} + A_{Parasitic} + L$$

- Let's define the average number of neutrons emitted in fission per neutron absorbed in fuel also called the **fission multiplication factor η or reproduction factor**

$$\eta = \frac{P_{fuel}}{A_{fuel}} = \frac{\bar{\nu}\Sigma_f \phi}{\Sigma_a \phi} = \frac{\bar{\nu}\Sigma_f \phi}{\Sigma_a \phi}$$

- From the balance above we get (note that $A_{fuel} = A_{Fissile} + A_{Fertile}$)

$$\eta = 1 + (A_{Parasitic} + L)/A_{fuel}$$

- For a ***critical*** reactor $\eta > 1$ is imperative. In reality, it needs to be substantially bigger than 1 to compensate for losses…

Reminder: Nuclear Fuel – Fissile vs Fertile

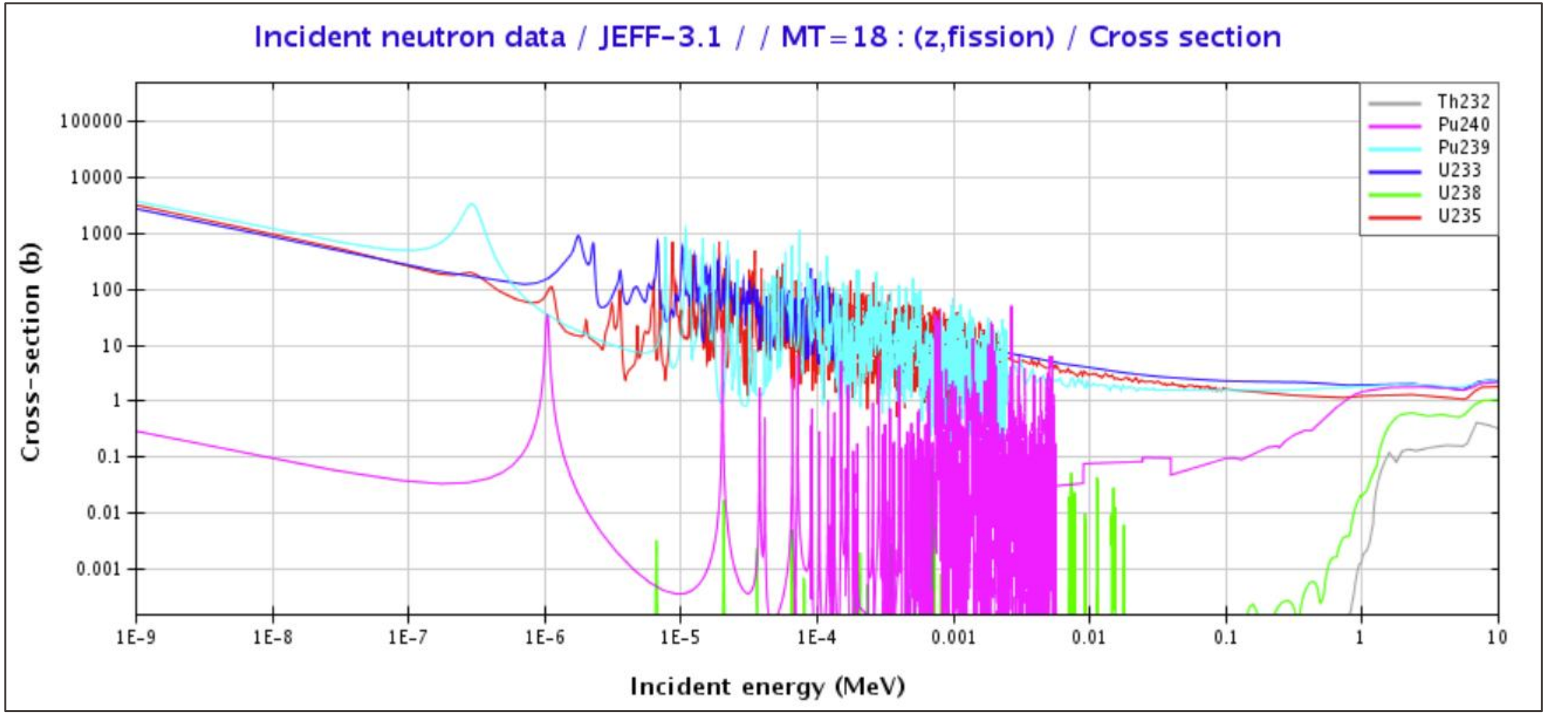
- Nuclear fuel is any ***fissionable*** material, i.e. a material capable of producing heat from nuclear reactions.
- A fissionable nuclide that can have fission with thermal neutrons with high probability is called ***fissile*** e.g. ^{233}U, ^{235}U, ^{239}Pu, or ^{241}Pu
- Certain isotopes (often fissionable at high neutron energies) are ***fertile***: they provide artificial fissionable isotopes through neutron capture, e.g. ^{232}Th, ^{238}U, or ^{240}Pu

$$^{232}_{90}\mathrm{Th} \rightarrow {}^{233}_{92}\mathrm{U} \text{ or } {}^{238}_{92}\mathrm{U} \rightarrow {}^{239}_{94}\mathrm{Pu}$$

	Fissiles **U^{235}, Pu^{239}, U^{233}**	**Fertiles** **U^{238}, Th^{232}**
Thermal fissions	significant	insignificant
Fast fissions	weak	weak
Captures	parasitic	useful (new fissile is produced)

Reproduction Factor η - Fissile

- For only fissile material we have $\eta^{\text{fiss}} = \frac{P_{\text{Fissile}}}{A_{\text{Fissile}}} = \frac{\bar{\nu} N \sigma_f}{N \sigma_a} = \frac{\bar{\nu}\sigma_f}{\sigma_f + \sigma_c} < \nu$



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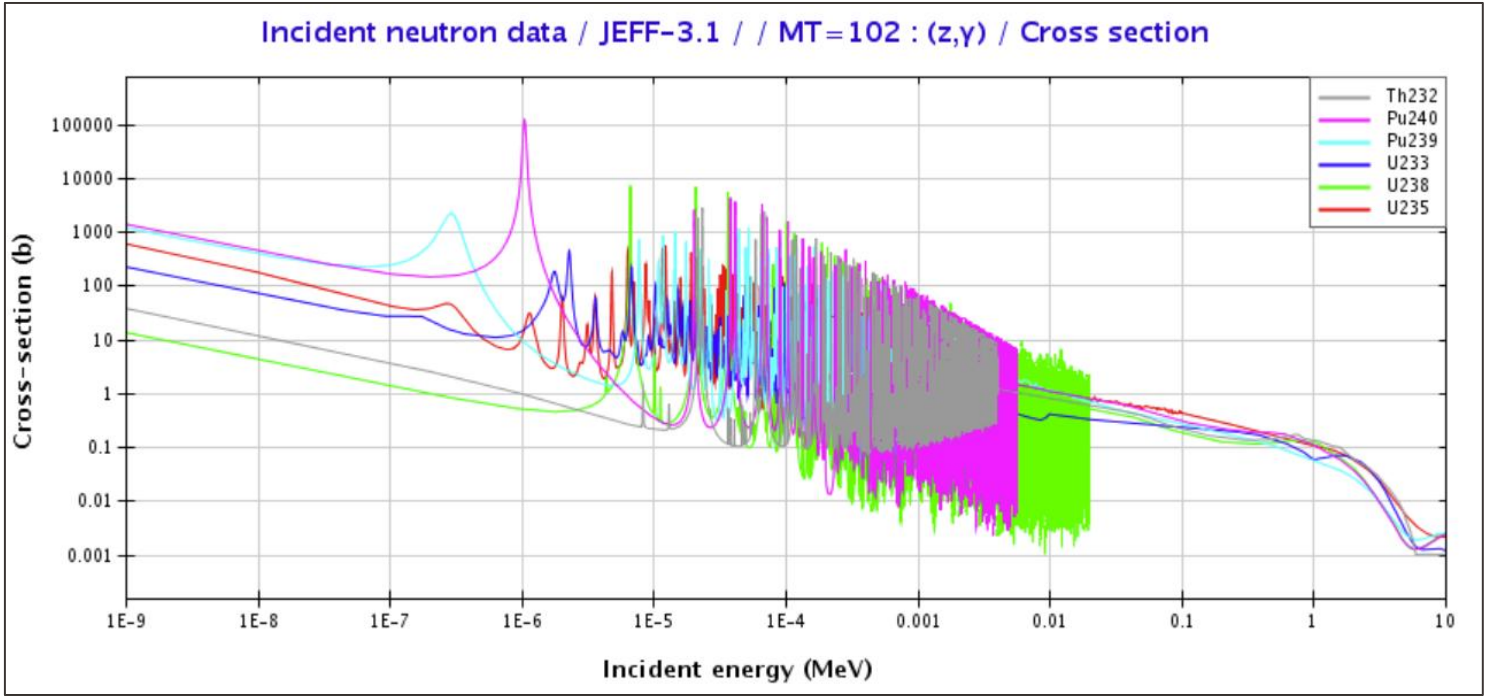

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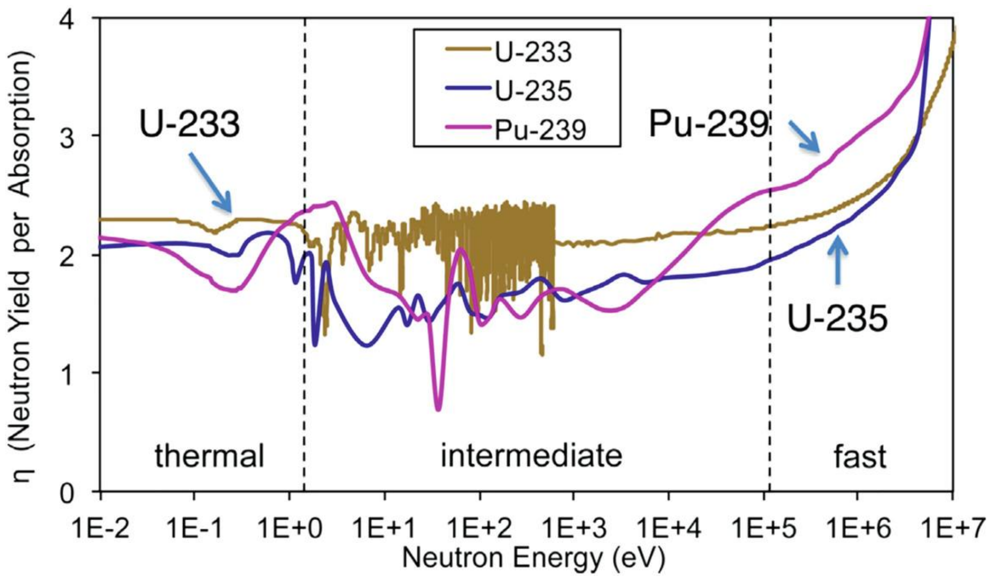
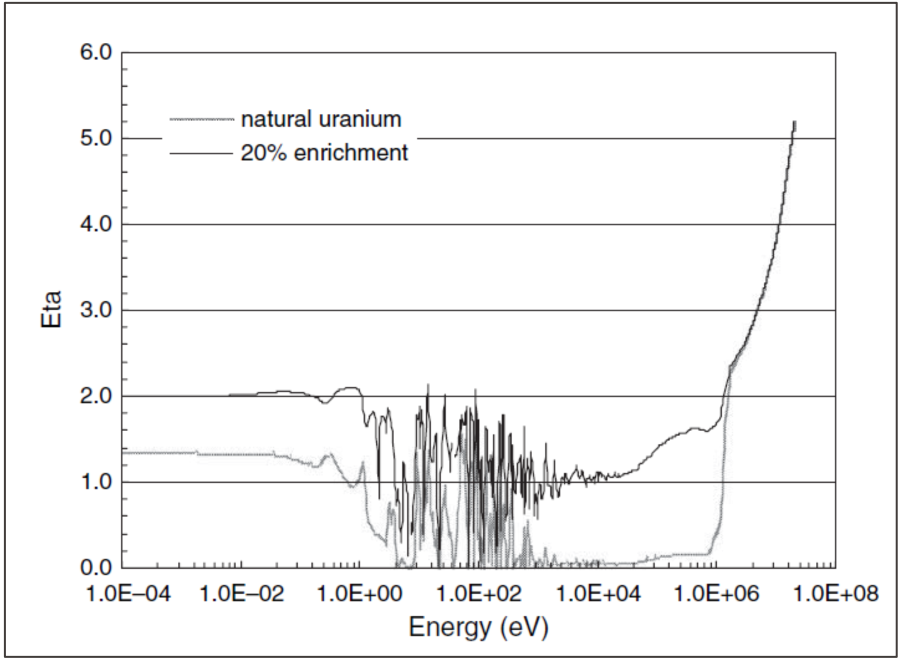


From N.R. Brown et al. “Sustainable Thorium Nuclear Fuel Cycles: A Comparison of Intermediate and Fast Neutron Spectrum Systems”

Reproduction Factor η - Fuel

- The fuel is always made of both fertile and fissile: $\eta = \frac{P_{fuel}}{A_{fuel}} = \frac{\bar{\nu}N^{fiss}\sigma_f^{fiss} + \bar{\nu}N^{fert}\sigma_f^{fert}}{N^{fiss}\sigma_a^{fiss} + N^{fert}\sigma_a^{fert}} = \frac{\tilde{e}\bar{\nu}\sigma_f^{fiss} + (1-\tilde{e})\bar{\nu}\sigma_f^{fert}}{\tilde{e}N^{fiss}\sigma_a^{fiss} + (1-\tilde{e})\sigma_a^{fert}}$

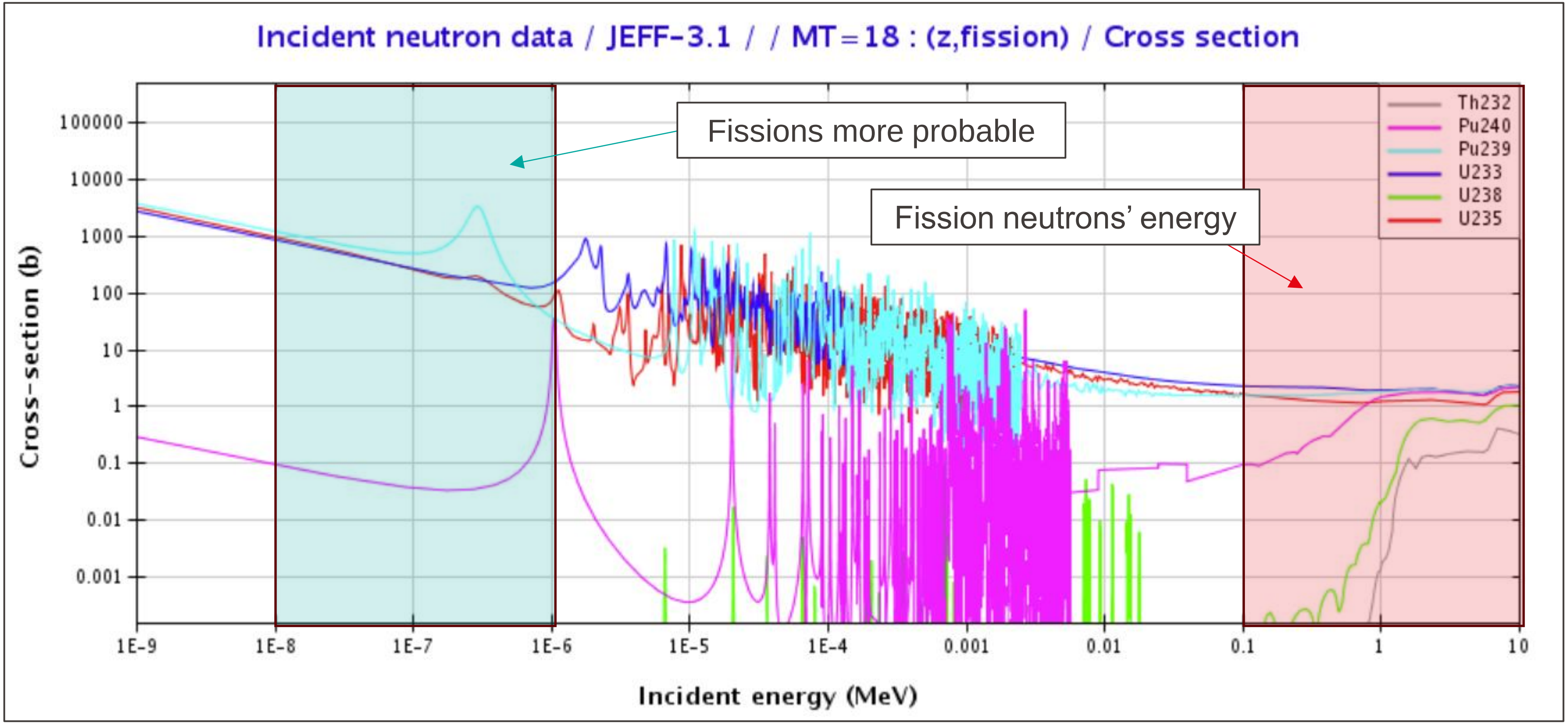


From E.E. Lewis “Fundamentals of Nuclear Reactor Physics”

Reactors built **only in fast or thermal range!**

- **Fast Reactors:**
  - No moderator required.
  - A minimum 10% enrichment is necessary to prevent U-238 scattering from shifting neutrons into the epith. range.
  - XS are smaller at fast energies, so more fissile is needed for fast reactors.
- **Thermal Reactors:**
  - A moderator is essential to slow down neutrons, bypassing the epith. region.
  - Lower enrichment (5%) is acceptable.

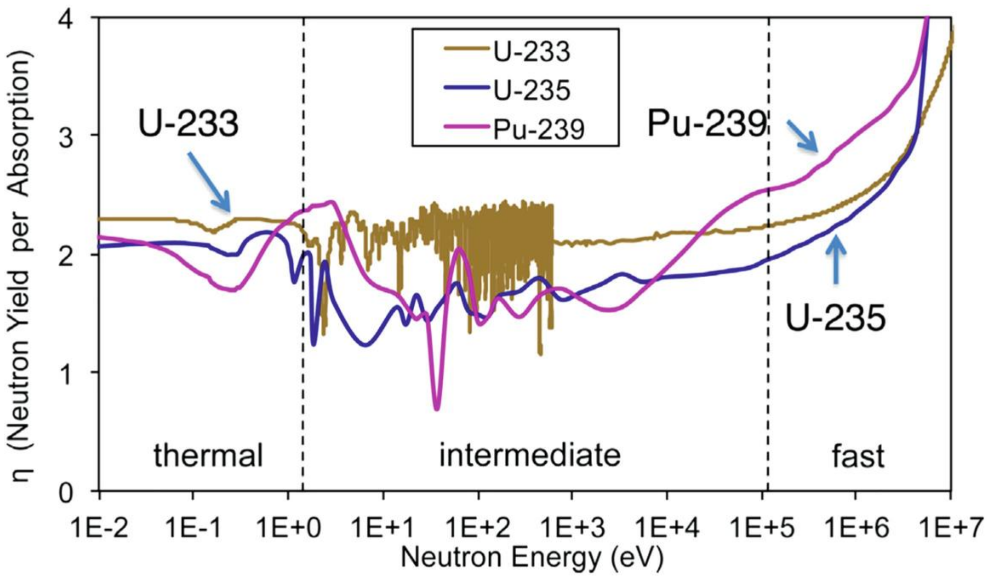
Need for Moderator in a Thermal Reactor



Conversion – Breeding Ratio

- Defining **the conversion or breeding ratio** $\mathbf{C} = \frac{\mathbf{A}_{\boldsymbol{Fertile}}}{\mathbf{A}_{\boldsymbol{Fissile}}}$ and neglecting the fissions in the fertile:

$$\eta^{\text{fiss}} = 1 + C + (\mathrm{A}_{\text{Parasitic}} + \mathrm{L})/\mathrm{A}_{\text{Fissile}}$$

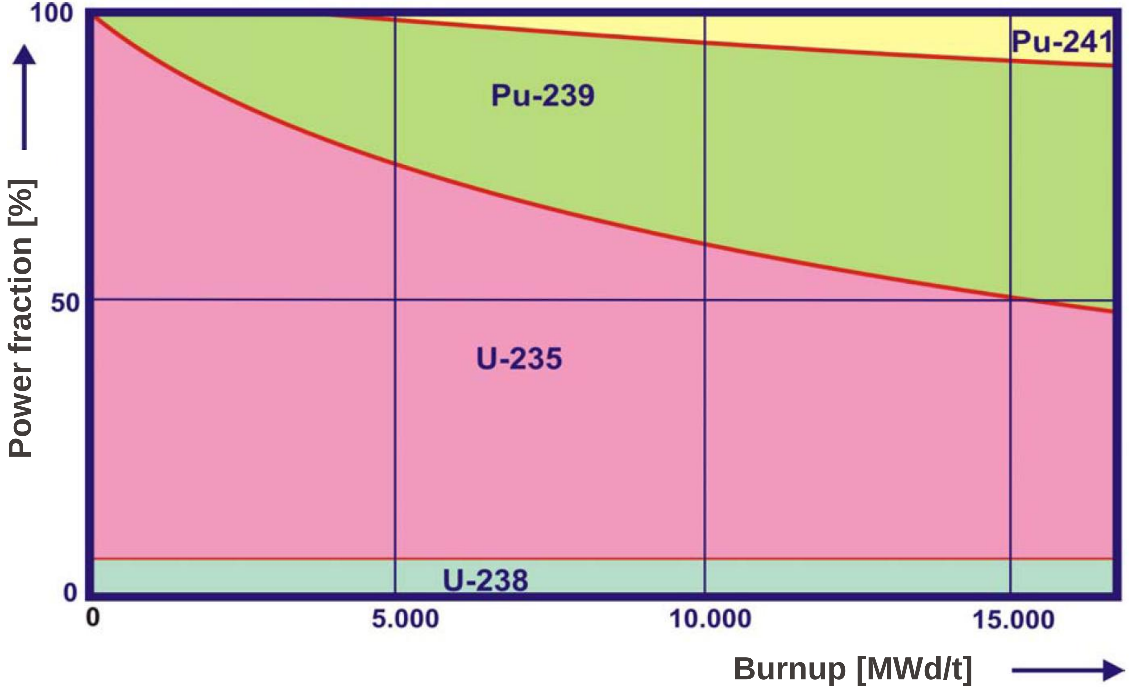


- Solving for C we get

$$\mathrm{C} = \eta^{\text{fiss}} - 1 - (\mathrm{A}_{\text{Parasitic}} + \mathrm{L})/\mathrm{A}_{\text{Fissile}}$$

- Special type of reactor: ***breeder*** or ***iso-breeder*** with C > 1 or = 1, respectively:
  - $\boldsymbol{\eta^{fiss} > 2}$ **is imperative for a breeder**… but it is difficult to obtain in thermal spectrum.
  - **Fast spectrum is optimal for breeding ($\boldsymbol{\eta \gg 2}$), especially for Pu-239**.

Fission Power Redistribution from U to Pu in a LWR



U238 / U235 Cycle (with and without recycling)