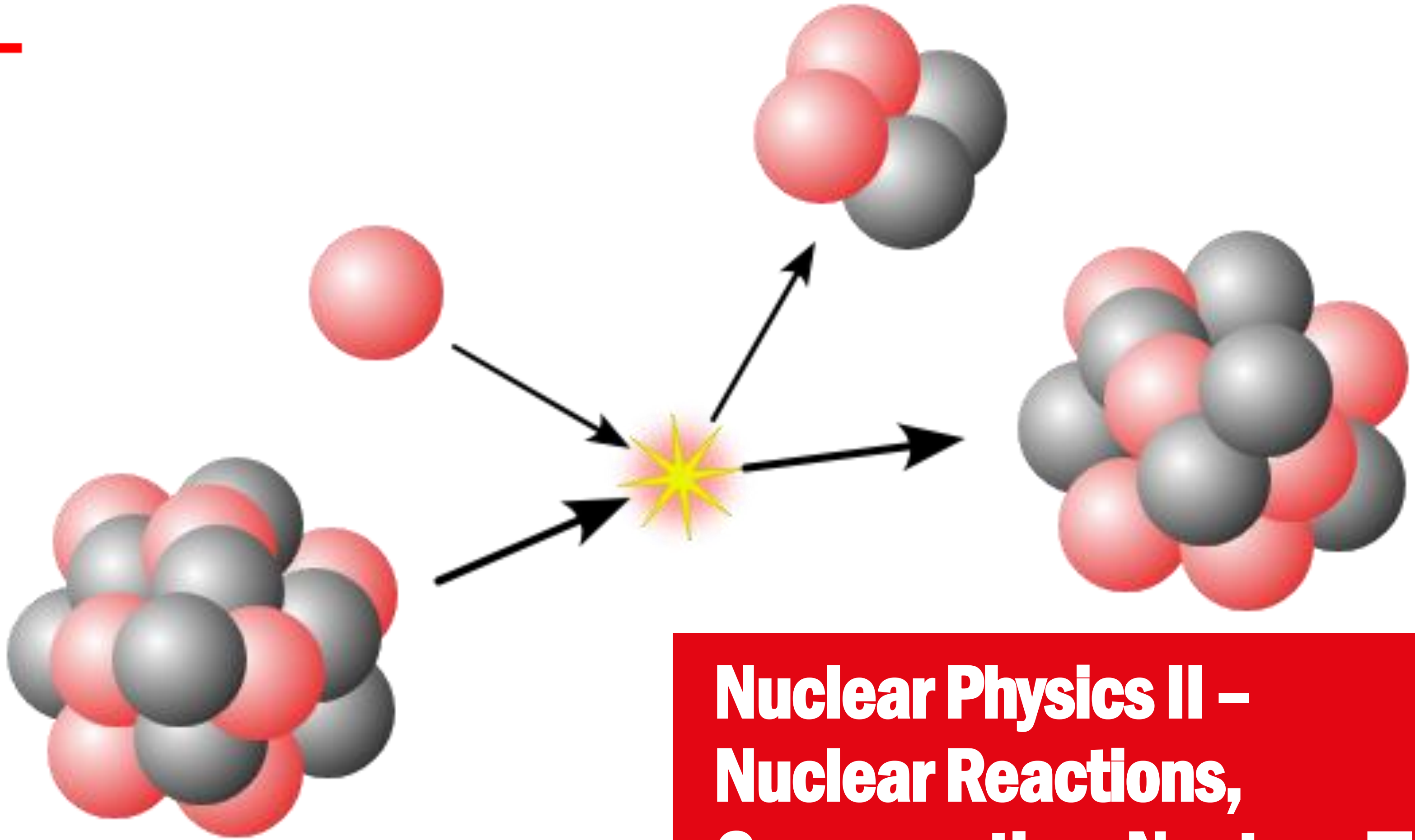
EPFL



Nuclear Physics II – Nuclear Reactions, Cross-section, Neutron Flux

Nuclear Reactions

- A nuclear reaction occurs when an incoming particle interacts with a nucleus.
- The incoming particle can be **another nuclide** or **a fundamental particle (n, p, α, γ).**
- **Example** of a common reaction used in neutron sources:

$$ {}^{9}_{4}\mathrm{Be} + {}^{4}_{2}\mathrm{He} \rightarrow {}^{12}_{6}\mathrm{C} + {}^{1}_{0}\mathrm{n} $$

- Similar to chemical reactions but with key differences:
  - Chemical reactions involve the **rearrangement of electrons** in atoms.
  - Nuclear reactions involve **changes within the nucleus**, leading to the **transformation of elements** with **much larger amounts of energy involved** (either released or required).

Balancing Nuclear Reactions

- Nuclear reactions have the general form

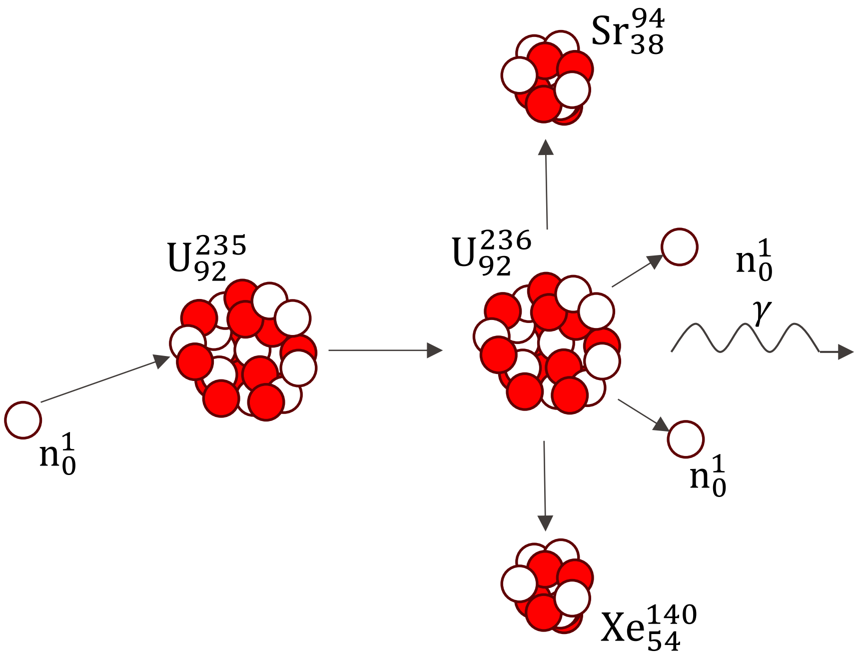
$$^{A_1}_{Z_1}X_1 + {}^{A_2}_{Z_2}X_2 \rightarrow {}^{A_3}_{Z_3}X_3 + {}^{A_4}_{Z_4}X_4$$

- Main conservation laws to consider:
  - Number of nucleons → $A_1 + A_2 = A_3 + A_4$
  - Electric charge → $Z_1 + Z_2 = Z_3 + Z_4$

- NOTE:
  - **Alternative notations for reactions involving "basic" particles →** $\mathbf{{}^{9}_{4}Be(\alpha, n)\ {}^{12}_{6}C}$
  - Radioactivity can be considered a particular example of a nuclear reaction with single reactant

Fission Reaction

$$ {}^{235}_{92}\mathrm{U} + {}^{1}_{0}\mathrm{n} \rightarrow {}^{94}_{38}\mathrm{Sr} + {}^{140}_{54}\mathrm{Xe} + 2\,{}^{1}_{0}\mathrm{n} + 184\mathrm{MeV} $$



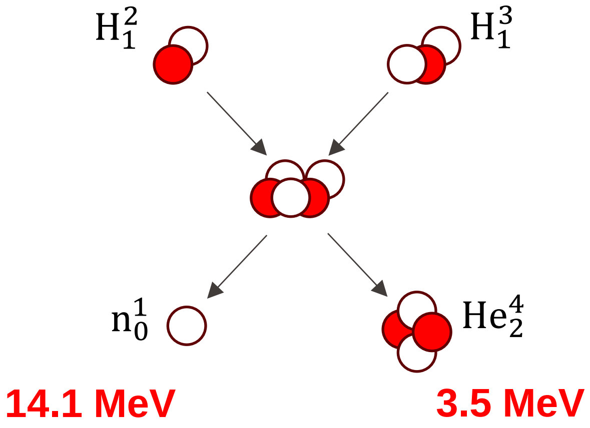
- Large **energy density**:
  - ~200 MeV per fission vs. ~4 eV per CH_4 molecule combustion (~50M times more)
- It produces additional neutrons:
  - Each fission neutron can induce further fissions, leading to a self-sustaining **chain reaction.**

Fusion Reaction

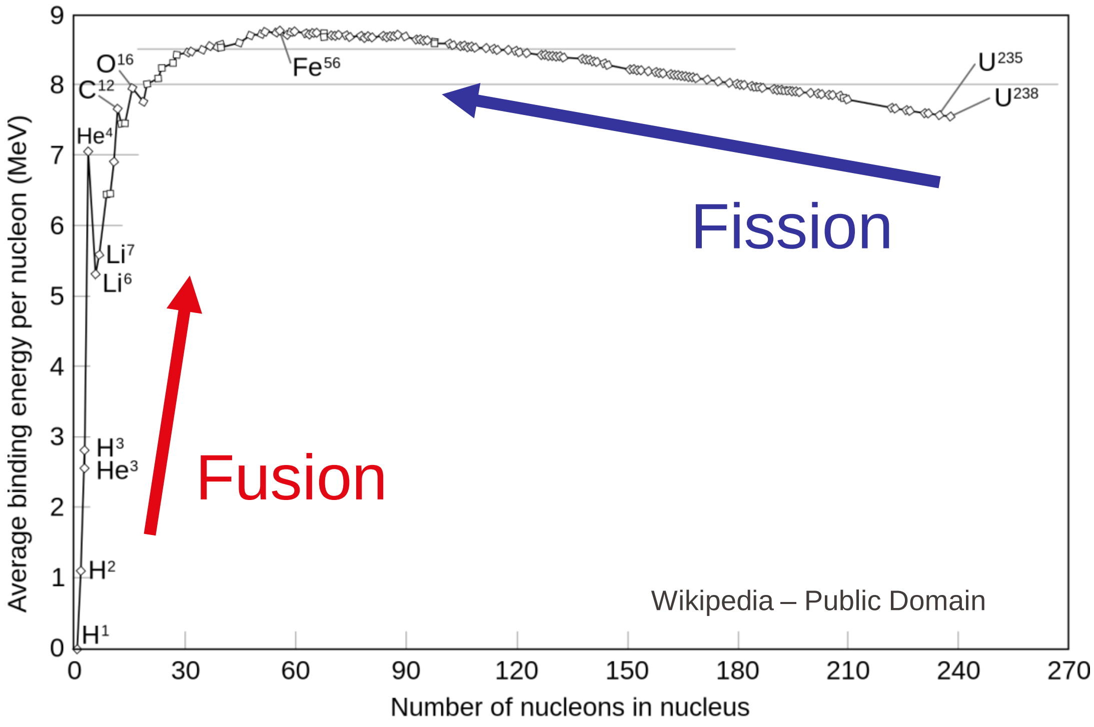
$${}^{2}_{1}\mathrm{H} + {}^{3}_{1}\mathrm{H} \rightarrow {}^{4}_{2}\mathrm{He} + {}^{1}_{0}\mathrm{n}$$ **+ 17.6 MeV**

- Even higher energy efficiency than fission:
  - ~200 MeV per fission reaction, but **~0.85 MeV per nucleon**.
  - ~17.6 MeV per fusion reaction, but **~3.5 MeV per nucleon**.

- Need to overcome the Coulomb barrier:
  - In the Sun, **gravitational confinement** creates high pressure enabling fusion at ”only” ~15 million K
  - Future reactors will require ~100 million K and **magnetic confinement**.



Average Binding Energy per Nucleon

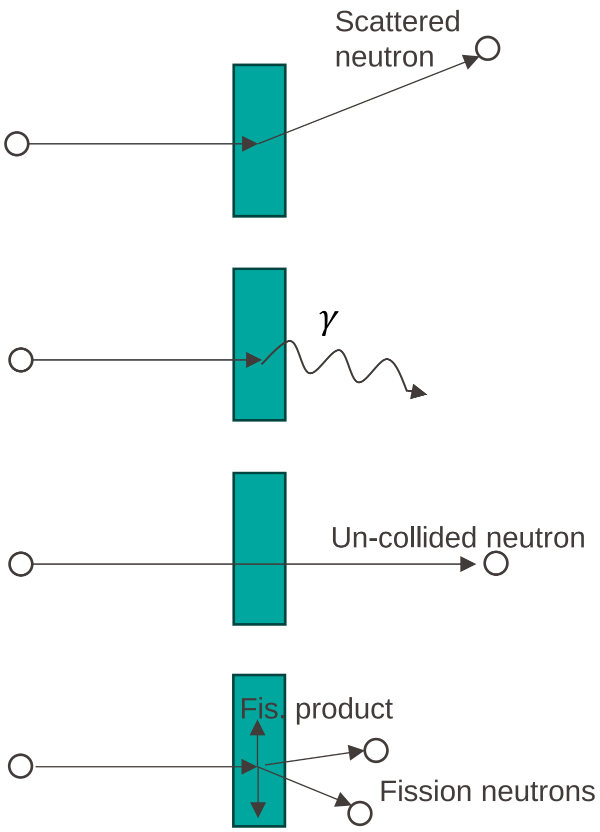


- **The mass of a nucleus is slightly less than the sum of its protons and neutrons.**
- This difference Δm is the **mass defect** and the **binding energy** is given by:

$$\Delta E = \Delta m \cdot c^2$$

- **$\Delta E/A$ – Binding Energy per Nucleon** measures how strongly nucleons are bound together.
- Higher ΔE/A → More stable nucleus (more energy required to break it apart).
- Reactions resulting in a shift towards the maximum $\Delta E/A$ release energy, i.e. the products are more stable.

Neutron Interactions with Matter



- Nuclear reactions are **probabilistic** in nature!

- E.g., a neutron approaching a nucleus can experience different fate:
  - **No interaction!**
  - **Elastic scattering** – The neutron bounces off, losing some energy.
  - **Inelastic scattering** – The neutron excites the nucleus, which emits gamma radiation.
  - **Capture (n,γ)** – The neutron is absorbed, and a gamma ray is emitted, i.e. mass number increases by 1

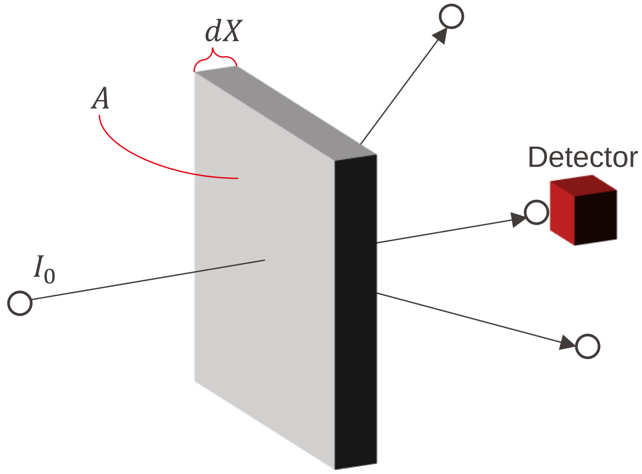
$$ {}^{A}_{Z}X + {}^{1}_{0}n \rightarrow {}^{A+1}_{Z}X + \gamma $$

  - **Fission (n,f)** – The neutron splits the nucleus, releasing energy and more neutrons.
  - **Other reactions** – The neutron may induce an (n,α), (n,p), (n,2n) or other reactions.

How to Quantify Reactions? Cross-Section (1)

Imagine the following setup:

- A beam of monoenergetic neutrons with density n, speed v and intensity $I_0 = nv$ strikes a thin slab of material.
- The slab has area A, thickness X and nuclide density N.
- A detector measures # of neutrons that pass through $I(X)$



How many reactions or collisions have occurred?

- **Number of reactions:** $-dI \cdot A = \left(I_0 - I(dX)\right) \cdot A = \sigma\ I\ N\ A\ dX$
- **Reaction rate** (per unit volume per second): $R = -\frac{dI}{dx} = \sigma N I = \Sigma I$

How to Quantify Reactions? Cross-Section (2)

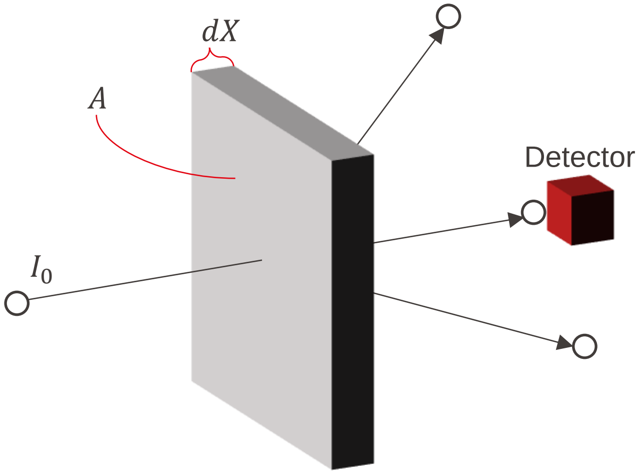
- **Cross-section <u>(sometimes written as *XS*)</u> quantifies the probability** of a neutron interaction.

Microscopic cross-section σ

- Probability of interaction per nucleus
- Effective area for interaction
- Measured in barns (1 barn = $10^{-24} cm^2$)

Macroscopic cross-section $\Sigma = \sigma N \ \ (cm^{-1})$

- Probability of interaction per cm of material.
- Accounts for all nuclei in the material.
- For a mixture: $\Sigma = \sum_j (N_j \sigma_j)$.



- <u>**Each reaction type has a different cross-section!**</u>
  - Total σ_t: sum of scattering and absorption $\sigma_t = \sigma_s + \sigma_a = \underbrace{\sigma_{s,e} + \sigma_{s,i}}_{\text{scattering}} + \underbrace{\sigma_c + \sigma_f + \sigma_p + \sigma_\alpha + \cdots}_{\text{absoprtion}}$

How to Quantify Reactions? Cross-Section (3)

From the definition of reaction rate: $-\frac{dI}{dx} = \sigma N I \rightarrow -\frac{dI}{I} = \sigma\, N\, dx$

We can integrate between 0 and x, obtaining the **beam intensity as function of penetration depth**:

$$\boldsymbol{I(x) = I_0\, exp(-N\sigma x) = I_0\, exp(-\Sigma x)}$$

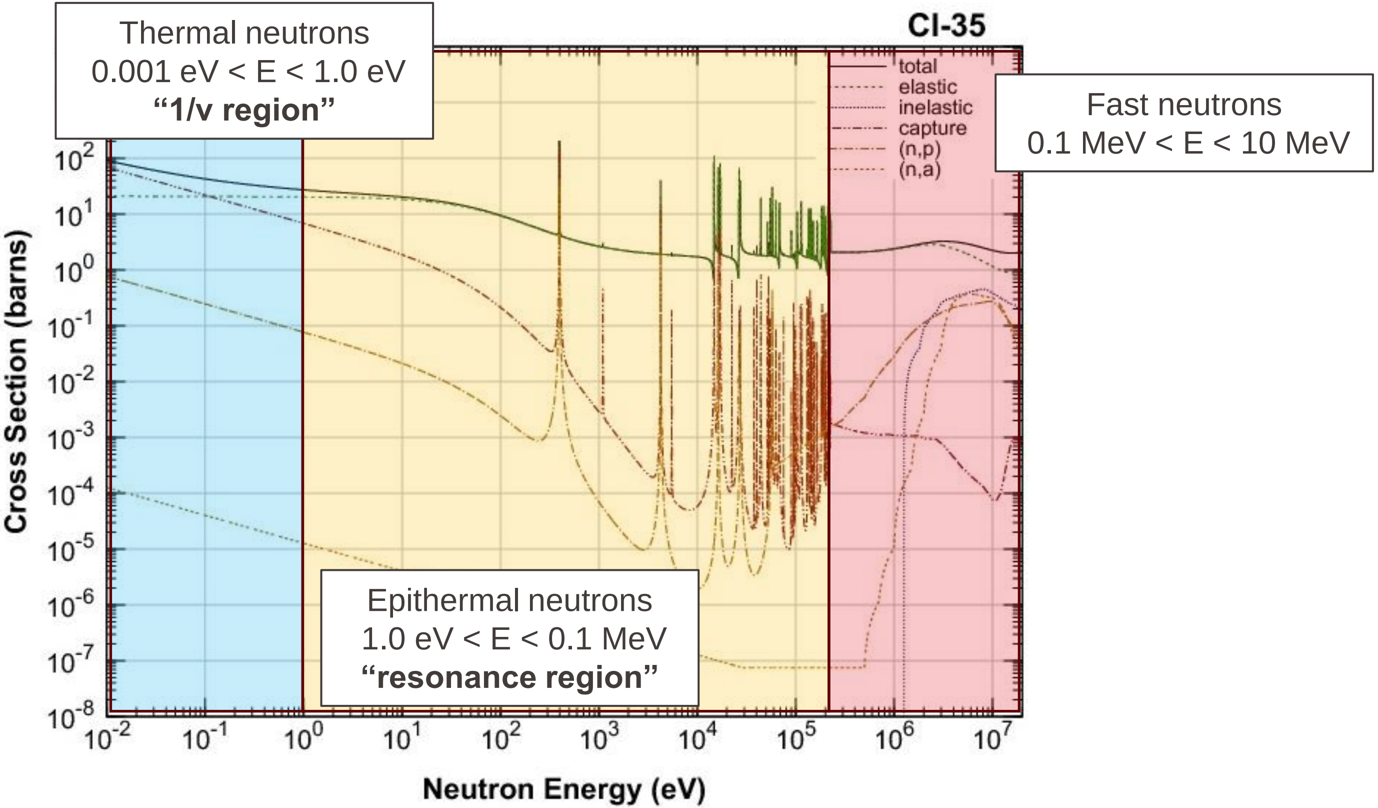
Probabilistic interpretation:

1. $-\frac{dI}{I} = \Sigma\, dx$ is the probability a neutrons that has survived until x will collide in dx
2. $\frac{I(x)}{I_0} = exp(-\Sigma x)$ is the probability that a neutron has survived without collision until x
3. From the previous two we can derive that the **mean free path (i.e. the mean distance traveled by a neutron between collisions) is the inverse of the macroscopic cross-section**:

$$\boldsymbol{\lambda = \int_0^\infty x p(x) = \int_0^\infty x\, \Sigma\, exp(-\Sigma x) dx = 1/\Sigma}$$

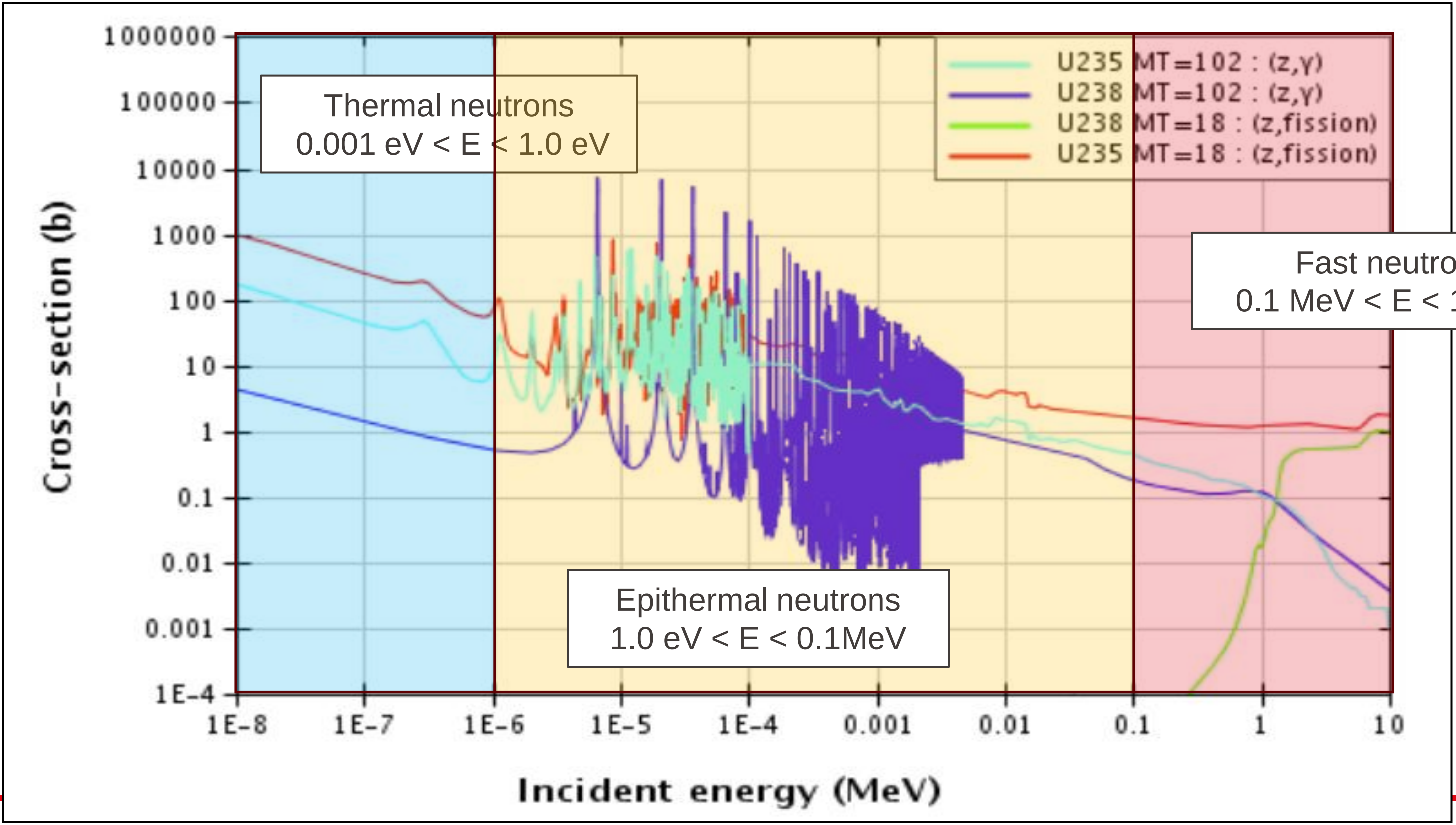
Energy Dependence of Cross-Sections

- Example, Cl-35 XS



Energy Dependence of Cross-Sections

- Example, U-238 and U-238 fission and capture cross-sections



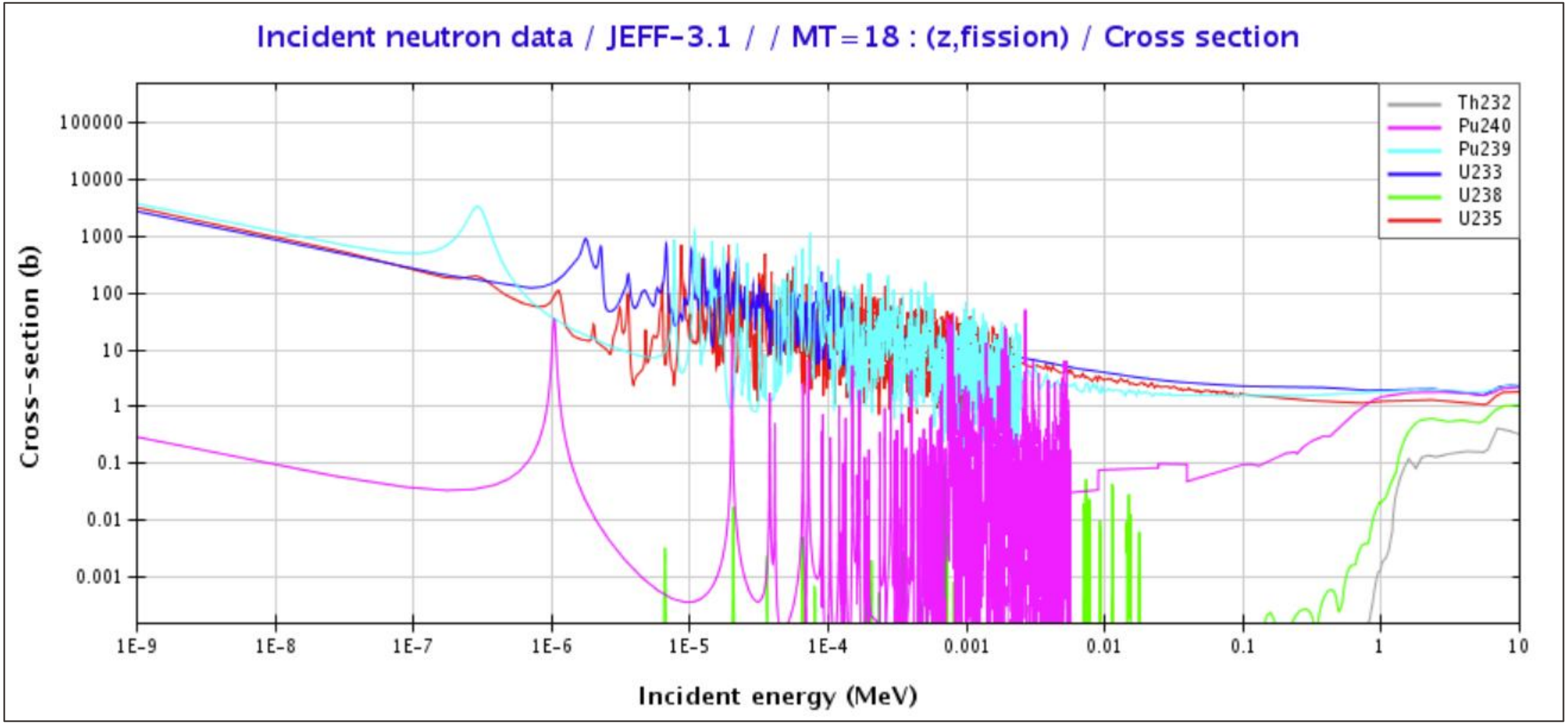
Nuclear Fuel – Fissile vs Fertile

- Nuclear fuel is any ***fissionable*** material, i.e. a material capable of producing heat from nuclear reactions.
- A fissionable nuclide that can have fission with thermal neutrons with high probability is called ***fissile*** e.g. ^{233}U, ^{235}U, ^{239}Pu, or ^{241}Pu
- Certain isotopes (often fissionable at high neutron energies) are ***fertile***: they provide artificial fissionable isotopes through neutron capture, e.g. ^{232}Th, ^{238}U, or ^{240}Pu

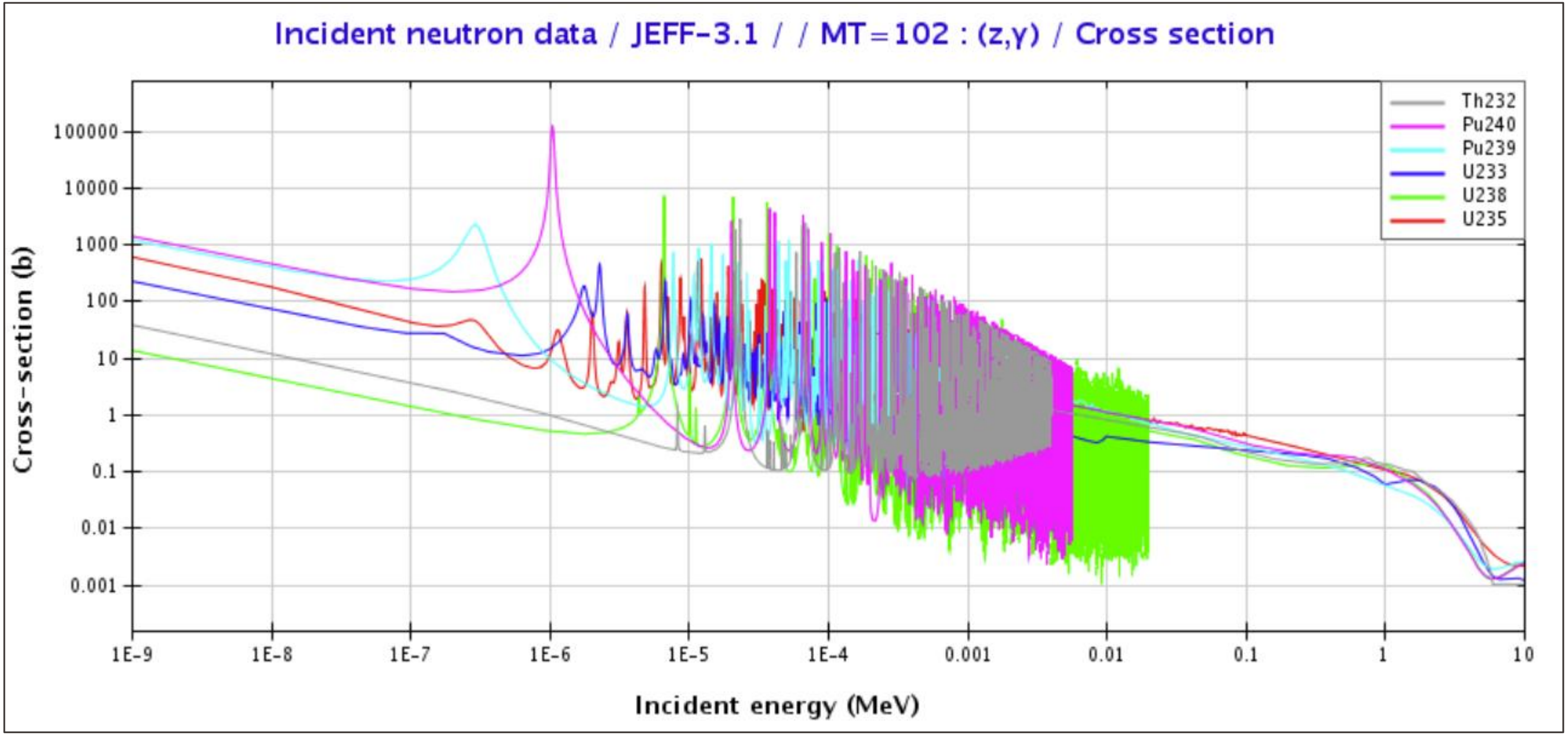
$$^{232}_{90}\mathrm{Th} \rightarrow {}^{233}_{92}\mathrm{U} \text{ or } {}^{238}_{92}\mathrm{U} \rightarrow {}^{239}_{94}\mathrm{Pu}$$

	Fissiles U^{235}, Pu^{239}, U^{233}	**Fertiles** U^{238}, Th^{232}
Thermal fissions	significant	insignificant
Fast fissions	weak	weak
Captures	parasitic	useful (new fissile is produced)

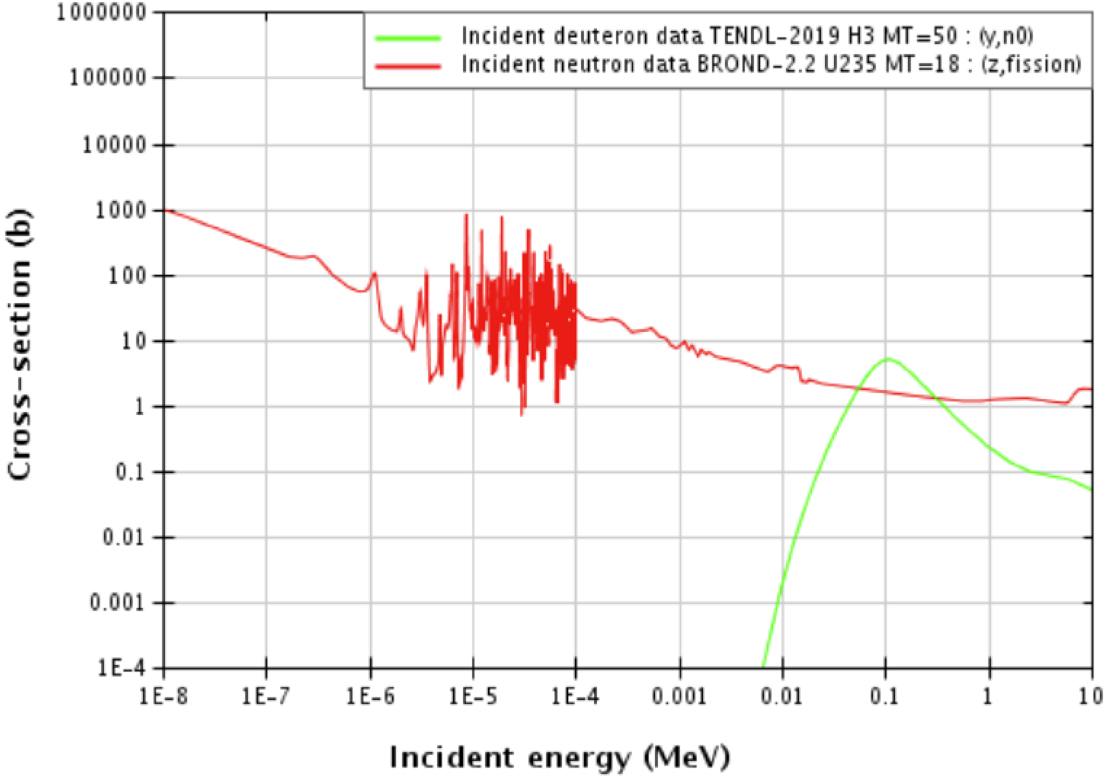
Fission cross sections for fissile and fertile isotopes



Capture cross sections for fissile and fertile isotopes



Comparison of Fission and Fusion XS

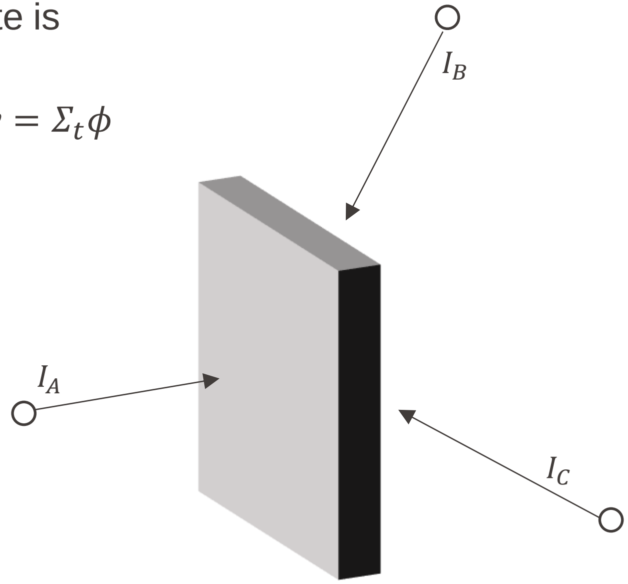


Neutron Flux

- For neutrons with multiple direction I_A, I_B, etc. the total reaction rate is

$$\mathrm{R} = IN\sigma_t = \Sigma_t(n_A + n_B + \cdots)v = \Sigma_t \cdot \sum n_i v = \Sigma_t \phi$$

where $\phi = \sum n_i v = nv$ is the **neutron flux and has units of** $\frac{\#}{\boldsymbol{cm^2 s}}$



- The neutron flux can be interpreted as the number of neutrons crossing unit area of the medium in unit time

- Despite the name, the **flux is a scalar (non-directional) quantity.**

Exercise 1

- The microscopic XS of a nucleus for a given reaction depends on:

1. Nothing, it is constant
2. The material density
3. The energy of the incoming neutron
4. Both energy and density

Exercise 1

- The microscopic XS of a nucleus for a given reaction depends on:

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  2. The material density
  3. The energy of the incoming neutron
  4. Both energy and density

- It is the **macrosopic XS** that depends on medium density!

Exercise 2

- A material has a neutron XS of 3.50e -24 cm^2/nuclei and contains 4.20e23 nuclei/cm^3

1. What is the macroscopic cross section?
2. What is the mean free path?
3. If neutrons impinge perpendicularly on a slab of the material that is 3.0 cm thick what fraction of them will penetrate the slab without a collision?
4. What part will collide before 1.5 cm?

Exercise 2 - Solution

1. $$\Sigma = \sigma N = 1.47\ cm^{-1}$$

2. $$\lambda = \frac{1}{\Sigma} = 0.680\ cm$$

3. $$\frac{I}{I_0} = e^{-\Sigma x} = 1.21\ \%$$

4. $$\mathrm{P} = 1 - e^{-\Sigma x} = 89\ \%$$

Exercise 3

- What is the total macroscopic XS of UO_2 that has been enriched to 4%? Assume:

  - $\sigma_{25} = 607.5\ b$
  - $\sigma_{28} = 11.8\ b$
  - $\sigma_{O} = 3.8\ b$
  - $\rho_{UO_2} = 10.5\ g/cm^3$

Exercise 3 - Solution

1. Compute the total microscopic XS of uranium (4%)

$$\sigma_U = \epsilon * 607.5 + (1-\epsilon) * 11.8 = 35.63\ b$$

2. Compute the microscopic XS of UO_2 (two nuclei of oxygen per uranium)

$$\sigma_{UO_2} = \sigma_U + 2 * \sigma_O = 43.23\ b$$

3. Compute the macroscopic XS of UO_2

$$\Sigma_{UO_2} = \frac{\rho_{UO_2} * N_A}{238 + 2 * 16} * \sigma_{UO_2} = 1.06\ cm^{-1}$$

Exercise 4

- A nuclear reactor of cylidrical shape (H = 3.5 m, R = 1.5 m) operates at 1 GW. Compute the macroscopic XS of fission if the neutron flux is homogeneous and equal to 10e13 n/cm^2/s

  - $\phi = 10^{13}\ cm^{-2}s^{-1}$
  - $E_f = 200\ MeV$ (1 eV = $1.6 * 10^{-19}$ J)
  - $P = 1\ GW$

Exercise 4 - Solution

1. Compute the reaction rate

$$R.R. = \frac{P}{E_f V} = 1.26 \ * 10^{12} \ \frac{fissions}{cm^3}$$

2. Compute the macroscopic XS

$$\Sigma_f = \frac{R.R}{\phi} = 0.126 \ cm^{-1}$$

Exercise 5 (Difficult!)

- A reactor with same dimension and power of that of Ex.4, operates instead with a non-homogeneous flux, given by

$$\phi(x, R) = \phi_0 \cos\left(\frac{\pi x}{H}\right) J_0\left(\frac{a_{o1} r}{R}\right)$$

where x varies between –H/2 and H/2 and

- Compute the new fission total XS if
  - $\phi_0 = 1 * 10^{19} cm^{-2} s^{-1}$
  - $\int_0^R J_0\left(\frac{a_{01} r}{R}\right) dr = \frac{R^2}{a_{01}^2}, a_0 = 2.405$

Exercise 5 (Difficult!)

- Compute also the uranium enrichment assuming that the reactor core's volume is 55.4% fuel and the rest is water and if

  - $\sigma_{f,U} = 580\ b$

Exercise 5 - Solution

1. Compute the total R.R

$$R.R. = \frac{P}{E_f} = 3.125 * 10^{19} \ \frac{fission}{s}$$

2. Compute the total macroscopic XS

$$R.R. = \int_V \Sigma_f \ \phi dV = \Sigma_f \int_V \phi \ dV$$

$$\int_V \phi dV = \phi_0 \int_{2\pi} d\theta \ \int_0^R J_0\left(\frac{a_{01} r}{R}\right) dR \int_{-\frac{H}{2}}^{\frac{H}{2}} \cos\left(\frac{\pi x}{H}\right) dx$$

Exercise 5 - Solution

$$\Sigma_f = \frac{R.R.}{\int_V \phi dV} = 0.57\ cm^{-1}$$

2. Compute the uranium enrichment

$$\Sigma_{f_{tot}} = 0.554 * \Sigma_{f,UO_2} + (1 - 0.554) * \Sigma_{f,H_2O} = 0.544 * \Sigma_{f,UO_2}$$

$$\Sigma_{f_{UO2}} = \rho_{UO_2} * \frac{N_A}{238 + 2 * 16} * (\epsilon\sigma_{f,235} + (1-\epsilon)\sigma_{f,238} + 2 * \sigma_{f,O})$$

$$\epsilon = \frac{\Sigma_{f,tot} * 0.554}{\rho_{UO_2} * \frac{N_A}{238 + 2 * 16} * \sigma_{f,235}} = 2.32\%$$