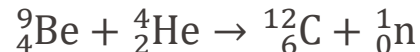


**Nuclear Physics II –
Nuclear Reactions,
Cross-section, Neutron Flux**

- A nuclear reaction occurs when an incoming particle interacts with a nucleus.
- The incoming particle can be **another nuclide** or a **fundamental particle** (n, p, α, γ).

- **Example** of a common reaction used in neutron sources:



- Similar to chemical reactions but with key differences:
 - Chemical reactions involve the **rearrangement of electrons** in atoms.
 - Nuclear reactions involve **changes within the nucleus**, leading to the **transformation of elements** with **much larger amounts of energy involved** (either released or required).

- Nuclear reactions have the general form

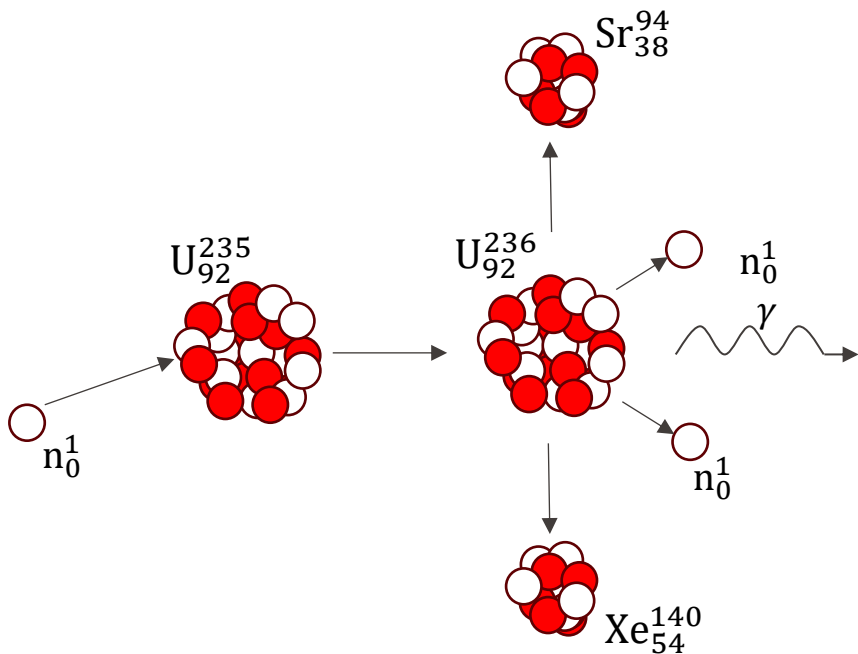
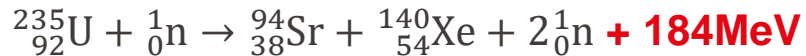


- Main conservation laws to consider:

- Number of nucleons $\rightarrow A_1 + A_2 = A_3 + A_4$
- Electric charge $\rightarrow Z_1 + Z_2 = Z_3 + Z_4$

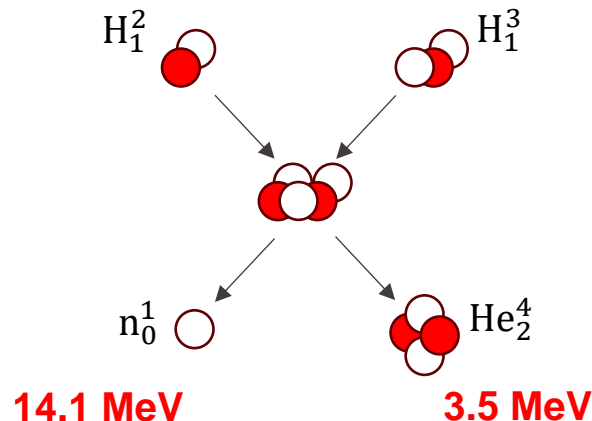
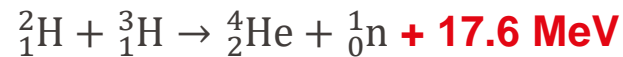
- NOTE:

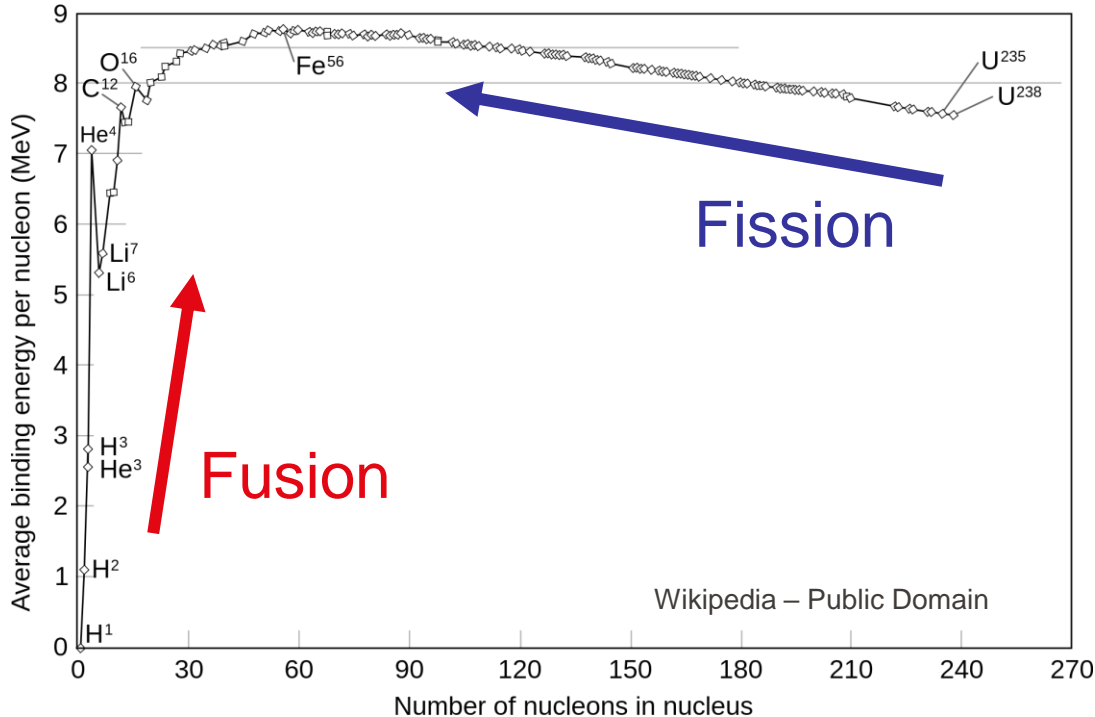
- **Alternative notations for reactions involving “basic” particles** $\rightarrow {}^9_4\text{Be}(\alpha, n) {}^{12}_6\text{C}$
- Radioactivity can be considered a particular example of a nuclear reaction with single reactant



- Large **energy density**:
 - ~200 MeV per fission vs. ~4 eV per CH_4 molecule combustion (~50M times more)
- It produces additional neutrons:
 - Each fission neutron can induce further fissions, leading to a self-sustaining **chain reaction**.

- Even higher energy efficiency than fission:
 - ~200 MeV per fission reaction, but **~0.85 MeV per nucleon**.
 - ~17.6 MeV per fusion reaction, but **~3.5 MeV per nucleon**.
- Need to overcome the Coulomb barrier:
 - In the Sun, **gravitational confinement** creates high pressure enabling fusion at "only" ~15 million K
 - Future reactors will require ~100 million K and **magnetic confinement**.



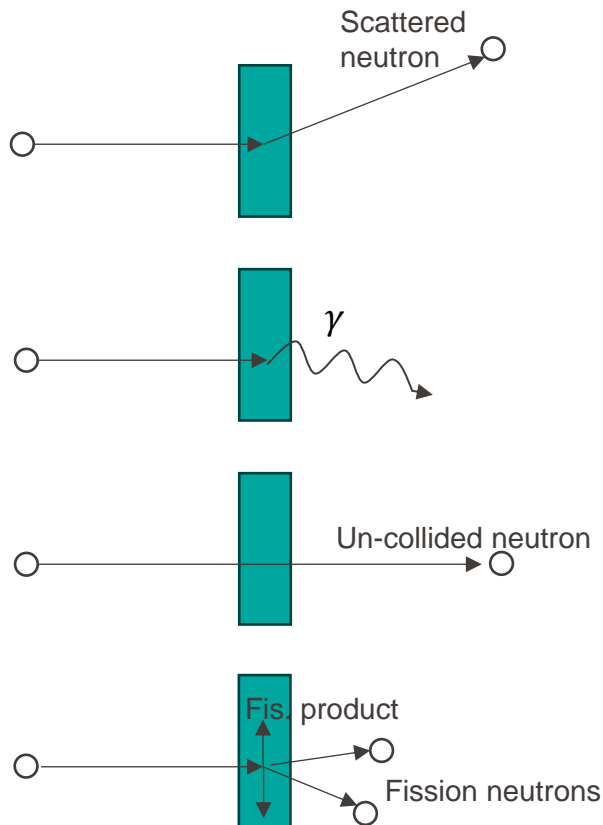


- The mass of a nucleus is slightly less than the sum of its protons and neutrons.

- This difference Δm is the **mass defect** and the **binding energy** is given by:

$$\Delta E = \Delta m \cdot c^2$$

- **$\Delta E/A$ – Binding Energy per Nucleon** measures how strongly nucleons are bound together.
- Higher $\Delta E/A \rightarrow$ More stable nucleus (more energy required to break it apart).
- Reactions resulting in a shift towards the maximum $\Delta E/A$ release energy, i.e. the products are more stable.



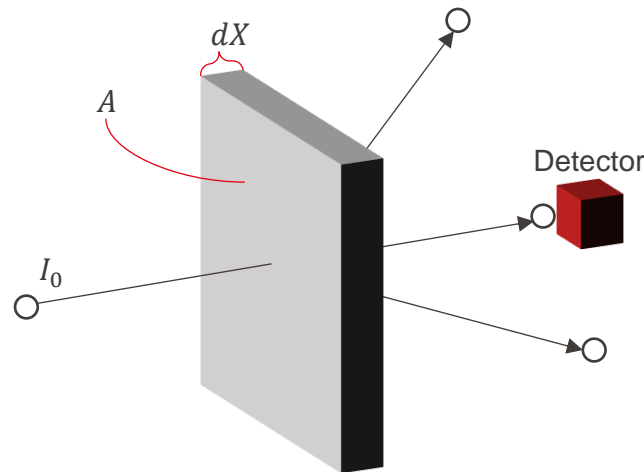
- Nuclear reactions are **probabilistic** in nature!
- E.g., a neutron approaching a nucleus can experience different fate:
 - **No interaction!**
 - **Elastic scattering** – The neutron bounces off, losing some energy.
 - **Inelastic scattering** – The neutron excites the nucleus, which emits gamma radiation.
 - **Capture (n,γ)** – The neutron is absorbed, and a gamma ray is emitted, i.e. mass number increases by 1
$${}^A_Z\text{X} + {}^1_0\text{n} \rightarrow {}^{A+1}_Z\text{X} + \gamma$$
 - **Fission (n,f)** – The neutron splits the nucleus, releasing energy and more neutrons.
 - **Other reactions** – The neutron may induce an (n,α), (n,p), (n,2n) or other reactions.

Imagine the following setup:

- A beam of monoenergetic neutrons with density n , speed v and intensity $I_0 = nv$ strikes a thin slab of material.
- The slab has area A , thickness X and nuclide density N .
- A detector measures # of neutrons that pass through $I(X)$

How many reactions or collisions have occurred?

- **Number of reactions:** $-dI \cdot A = (I_0 - I(dX)) \cdot A = \sigma I N A dX$
- **Reaction rate** (per unit volume per second): $R = -\frac{dI}{dx} = \sigma NI = \Sigma I$



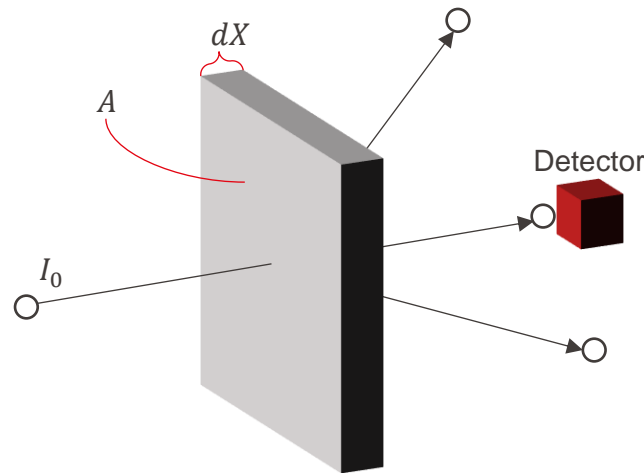
- Cross-section (sometimes written as XS) quantifies the probability of a neutron interaction.

Microscopic cross-section σ

- Probability of interaction per nucleus
- Effective area for interaction
- Measured in barns ($1 \text{ barn} = 10^{-24} \text{ cm}^2$)

Macroscopic cross-section $\Sigma = \sigma N \text{ (cm}^{-1}\text{)}$

- Probability of interaction per cm of material.
- Accounts for all nuclei in the material.
- For a mixture: $\Sigma = \sum_j (N_j \sigma_j)$.



- Each reaction type has a different cross-section!

- Total σ_t : sum of scattering and absorption $\sigma_t = \sigma_s + \sigma_a = \underbrace{\sigma_{s,e} + \sigma_{s,i}}_{\text{scattering}} + \underbrace{\sigma_c + \sigma_f + \sigma_p + \sigma_\alpha + \dots}_{\text{absorption}}$

From the definition of reaction rate: $-\frac{dI}{dx} = \sigma NI \rightarrow -\frac{dI}{I} = \sigma N dx$

We can integrate between 0 and x, obtaining the **beam intensity as function of penetration depth**:

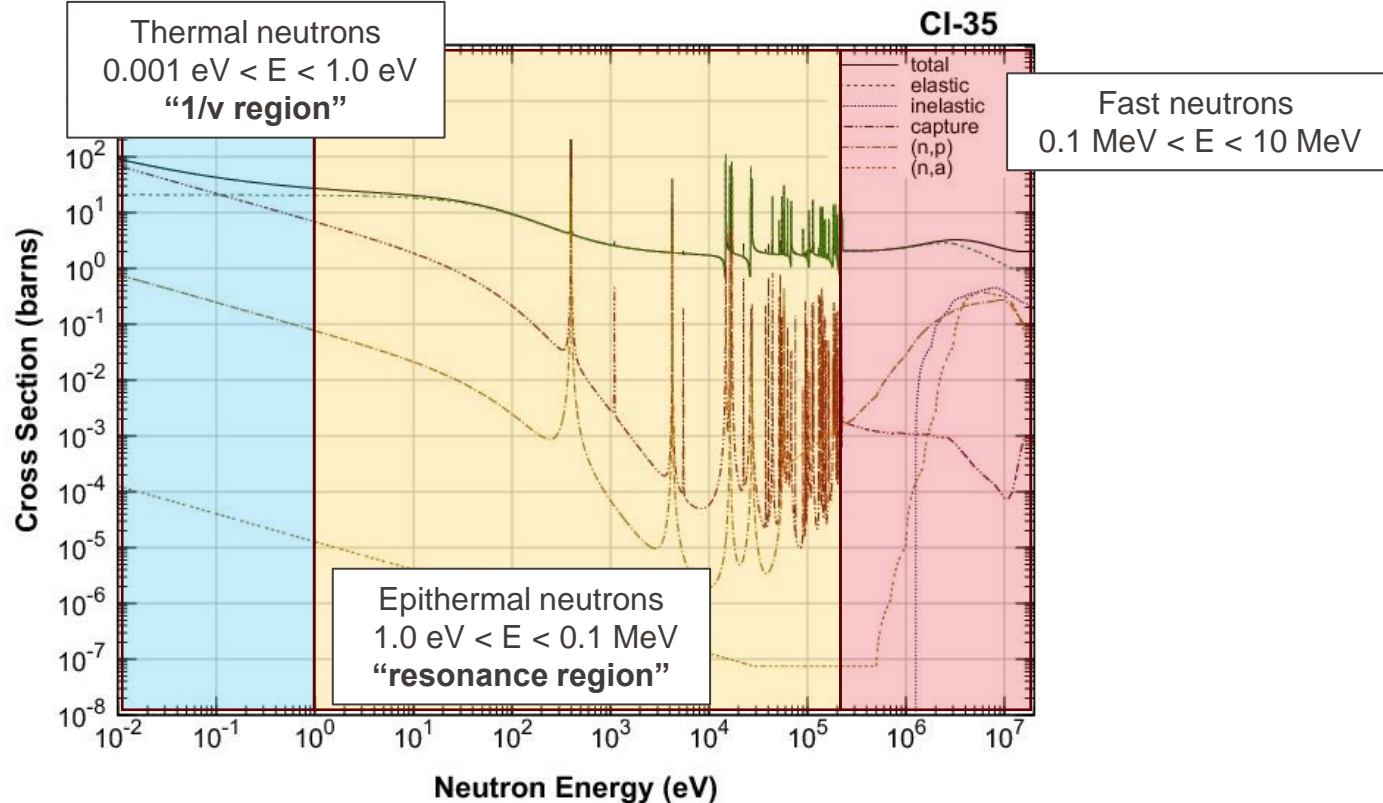
$$I(x) = I_0 \exp(-N\sigma x) = I_0 \exp(-\Sigma x)$$

Probabilistic interpretation:

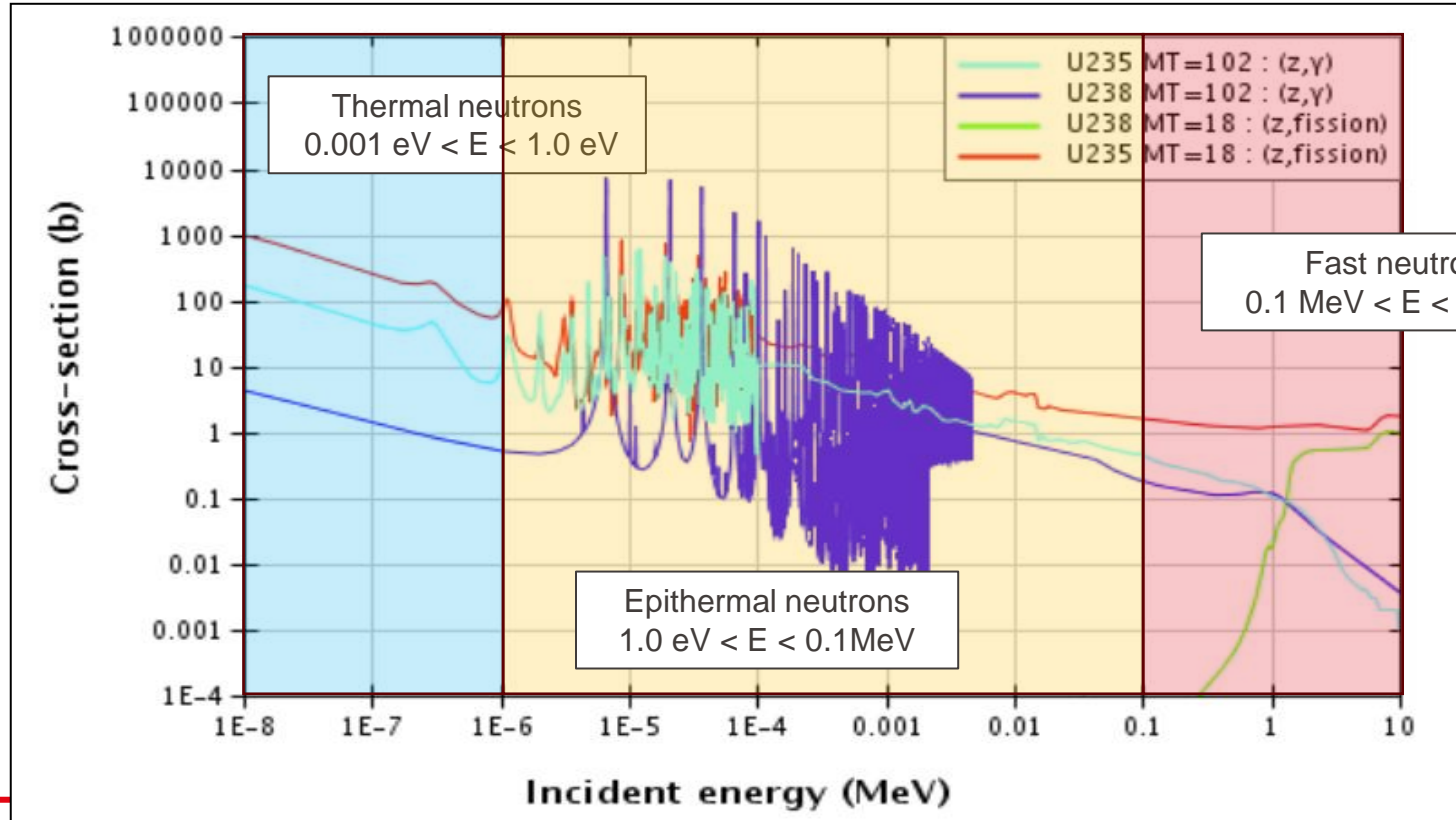
1. $-\frac{dI}{I} = \Sigma dx$ is the probability a neutrons that has survived until x will collide in dx
2. $\frac{I(x)}{I_0} = \exp(-\Sigma x)$ is the probability that a neutron has survived without collision until x
3. From the previous two we can derive that the **mean free path (i.e. the mean distance traveled by a neutron between collisions) is the inverse of the macroscopic cross-section**:

$$\lambda = \int_0^{\infty} xp(x) = \int_0^{\infty} x \Sigma \exp(-\Sigma x) dx = 1/\Sigma$$

- Example, Cl-35 XS



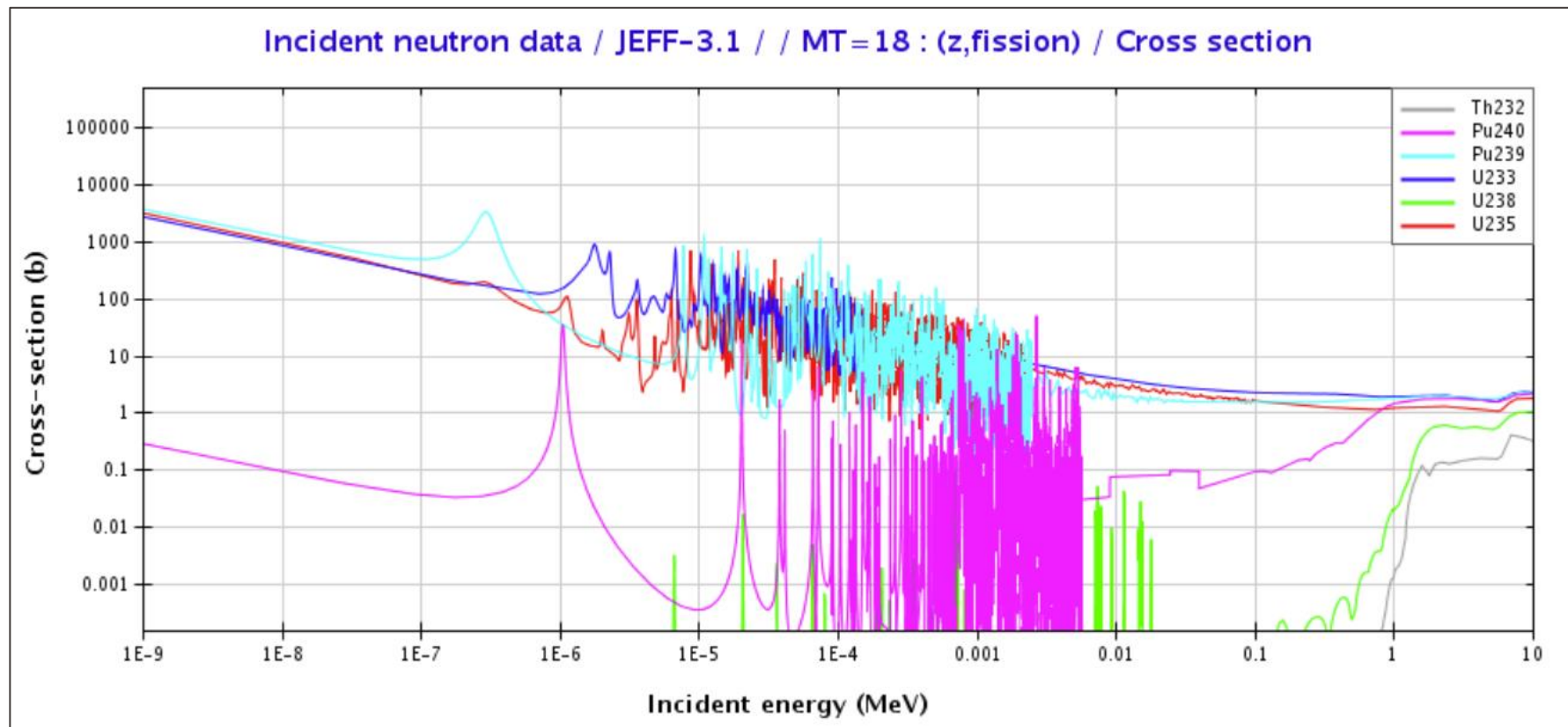
- Example, U-238 and U-235 fission and capture cross-sections

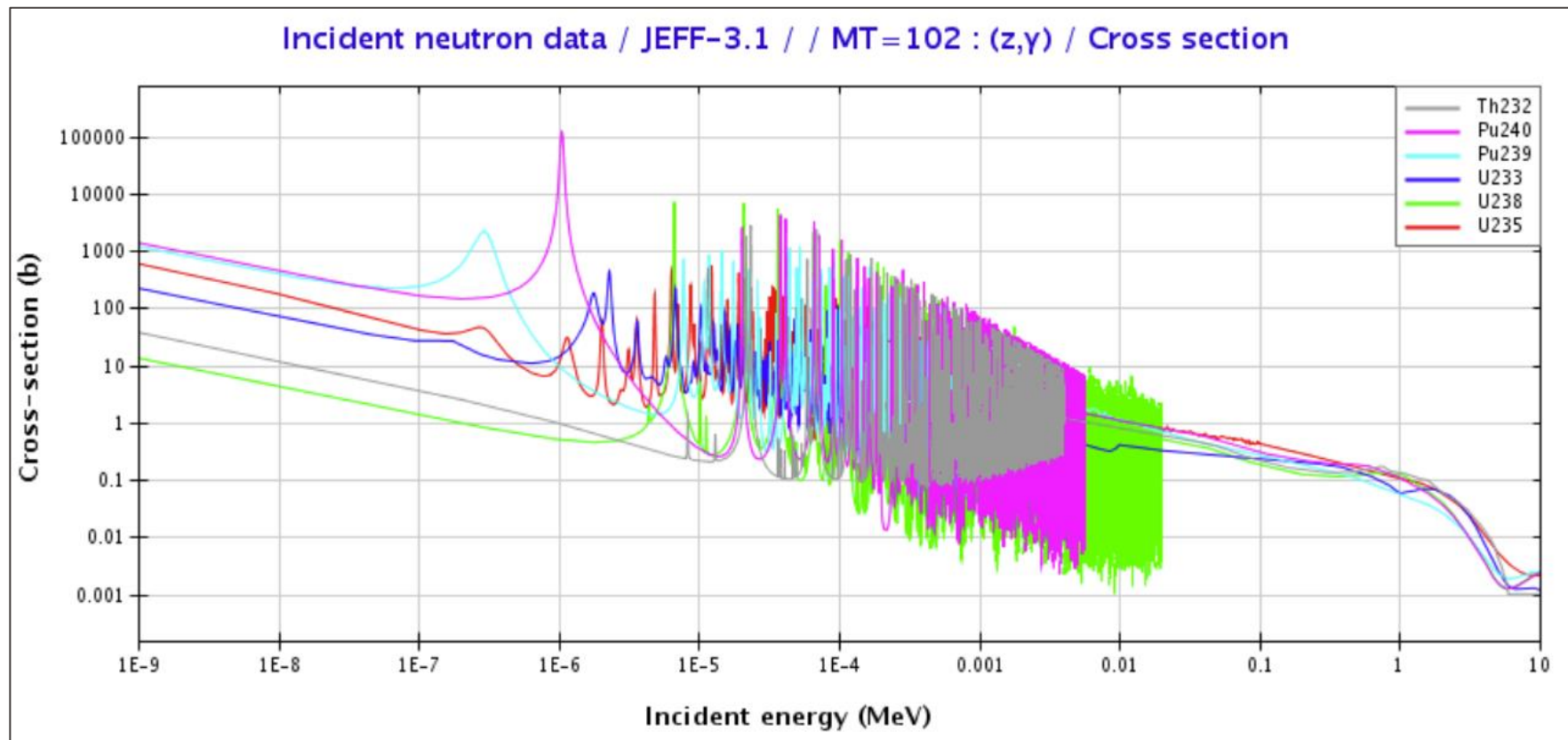


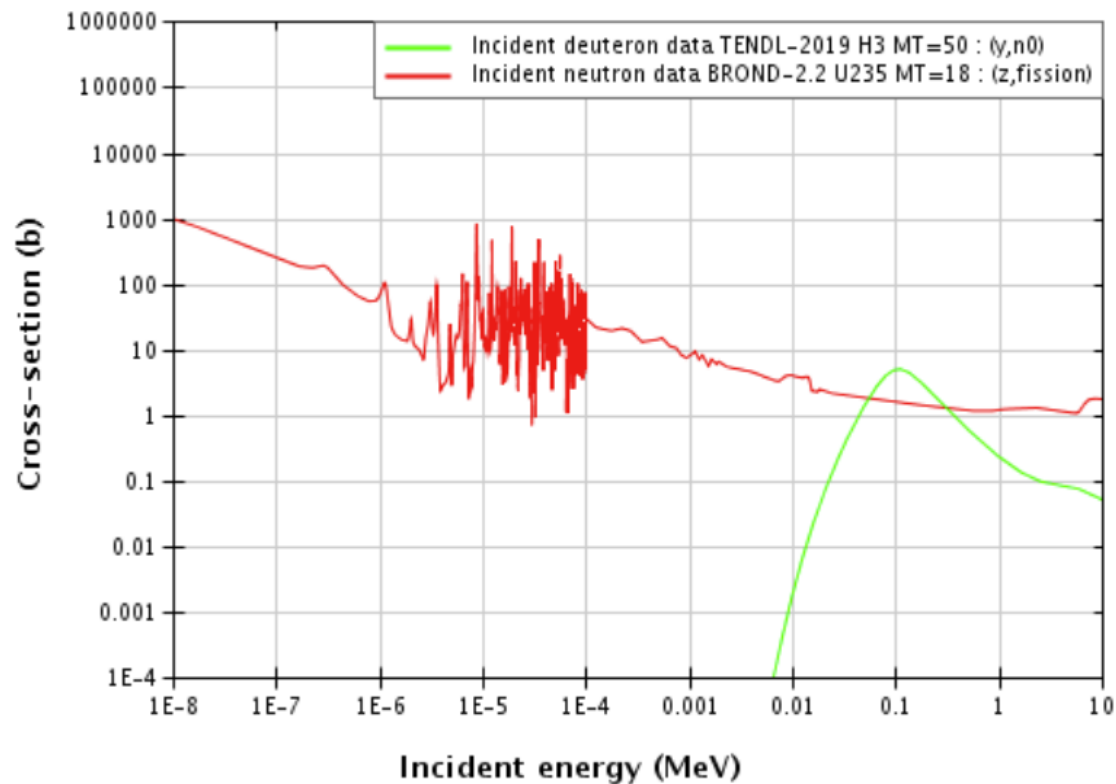
- Nuclear fuel is any **fissionable** material, i.e. a material capable of producing heat from nuclear reactions.
- A fissionable nuclide that can have fission with thermal neutrons with high probability is called **fissile** e.g. ^{233}U , ^{235}U , ^{239}Pu , or ^{241}Pu
- Certain isotopes (often fissionable at high neutron energies) are **fertile**: they provide artificial fissionable isotopes through neutron capture, e.g. ^{232}Th , ^{238}U , or ^{240}Pu



| | Fissiles U^{235} , Pu^{239} , U^{233} | Fertiles U^{238} , Th^{232} |
|------------------|--|---|
| Thermal fissions | significant | insignificant |
| Fast fissions | weak | weak |
| Captures | parasitic | useful (new fissile is produced) |





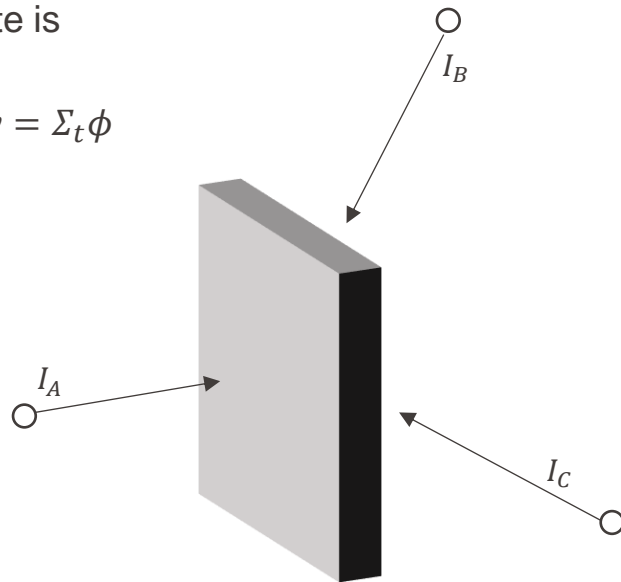


- For neutrons with multiple direction I_A , I_B , etc. the total reaction rate is

$$R = IN\sigma_t = \Sigma_t(n_A + n_B + \dots)v = \Sigma_t \cdot \sum n_i v = \Sigma_t \phi$$

where $\phi = \sum n_i v = nv$ is the **neutron flux** and has units of $\frac{\#}{\text{cm}^2 \text{s}}$

- The neutron flux can be interpreted as the number of neutrons crossing unit area of the medium in unit time



- Despite the name, the **flux is a scalar (non-directional) quantity**.

- The microscopic XS of a nucleus for a given reaction depends on:
 1. Nothing, it is constant
 2. The material density
 3. The energy of the incoming neutron
 4. Both energy and density

- A material has a neutron XS of $3.50 \times 10^{-24} \text{ cm}^2/\text{nuclei}$ and contains $4.20 \times 10^{23} \text{ nuclei/cm}^3$
 1. What is the macroscopic cross section?
 2. What is the mean free path?
 3. If neutrons impinge perpendicularly on a slab of the material that is 3.0 cm thick what fraction of them will penetrate the slab without a collision?
 4. What part will collide before 1.5 cm?

- What is the total macroscopic XS of UO_2 that has been enriched to 4%?
Assume:

- $\sigma_{25} = 607.5 \text{ b}$
- $\sigma_{28} = 11.8 \text{ b}$
- $\sigma_0 = 3.8 \text{ b}$
- $\rho_{\text{UO}_2} = 10.5 \text{ g/cm}^3$

- A nuclear reactor of cylindrical shape ($H = 3.5$ m, $R = 1.5$ m) operates at 1 GW. Compute the macroscopic XS of fission if the neutron flux is homogeneous and equal to 10^{13} n/cm²/s
 - $\phi = 10^{13} \text{ cm}^{-2} \text{ s}^{-1}$
 - $E_f = 200 \text{ MeV}$ ($1 \text{ eV} = 1.6 * 10^{-19} \text{ J}$)
 - $P = 1 \text{ GW}$

- A reactor with same dimension and power of that of Ex.4, operates instead with a non-homogeneous flux, given by

$$\phi(x, R) = \phi_0 \cos\left(\frac{\pi x}{H}\right) J_0\left(\frac{a_{01} r}{R}\right)$$

where x varies between $-H/2$ and $H/2$ and

- Compute the new fission total XS if

- $\phi_0 = 1 * 10^{19} cm^{-2} s^{-1}$
- $\int_0^R J_0\left(\frac{a_{01} r}{R}\right) dr = \frac{R^2}{a_{01}^2}, a_0 = 2.405$

- Compute also the uranium enrichment assuming that the reactor core's volume is 55.4% fuel and the rest is water and if
 - $\sigma_{f,U} = 580 \text{ b}$