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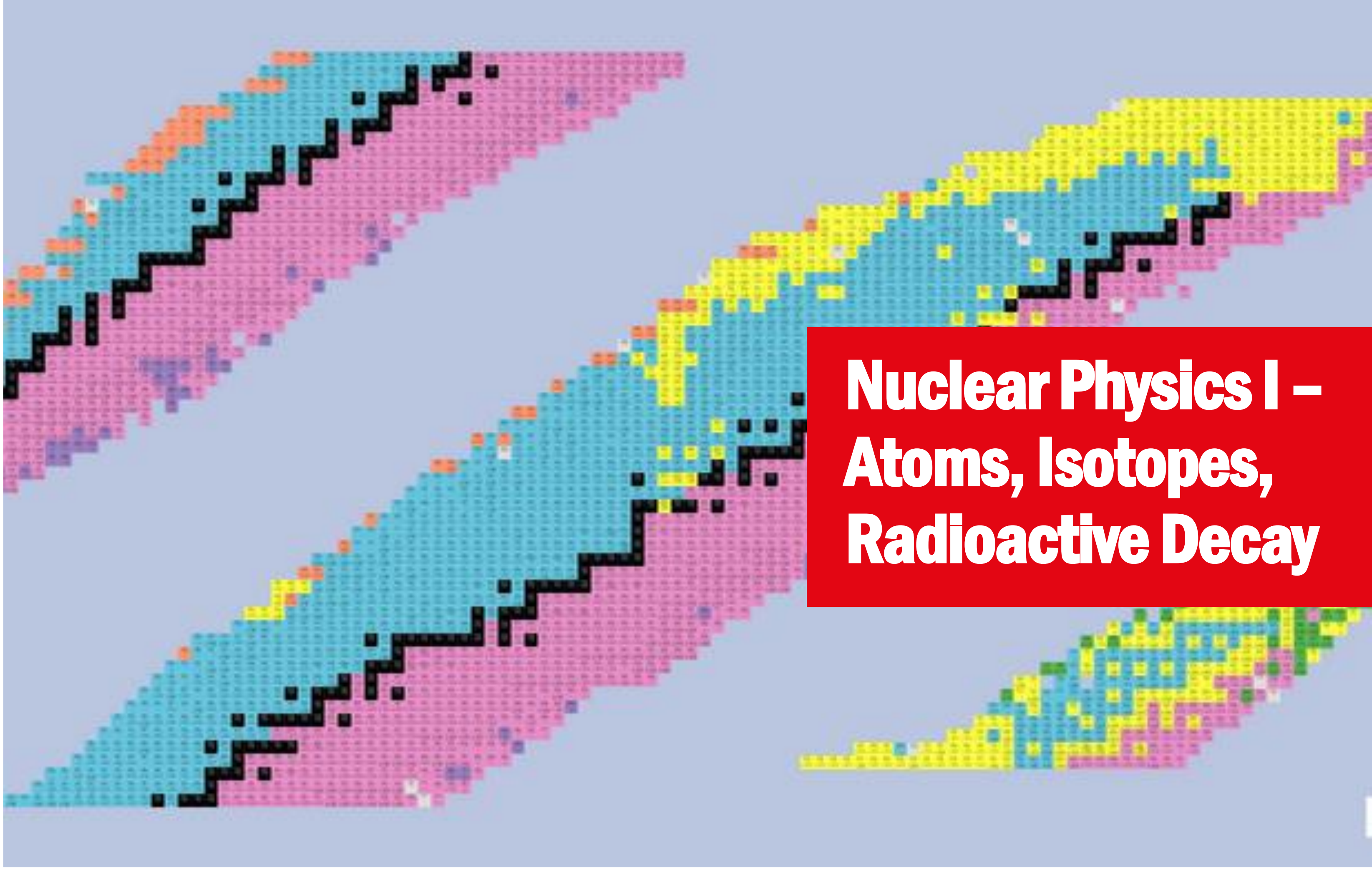
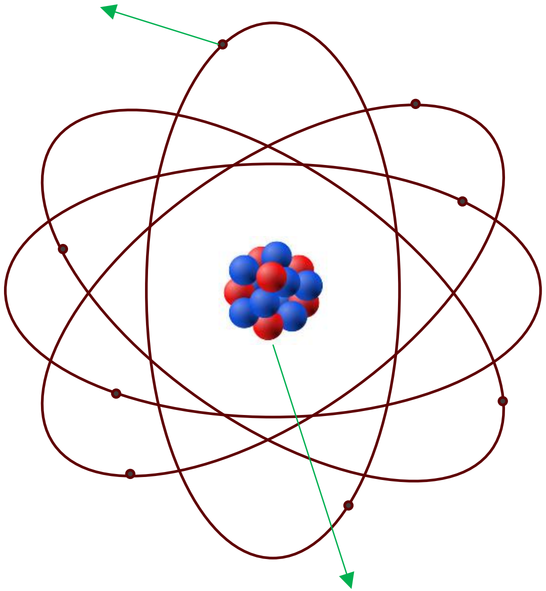


Nuclear Physics I – Atoms, Isotopes, Radioactive Decay

Fundamental Components of the Atom



The Figure is not accurate and not to scale

Basic atomic structure:

- **Electrons (e)**
  - negative charge (q_e ~ -1.6·10^{-19} C)
  - very light (m_e ~ 9.11·10^{-31} kg)
- **Protons (p)**
  - positive charge ($q_p = +|q_e|$)
  - m_p ~ 1800 times larger than m_e (m_p ~ 1.67 ·10^{-27} kg)
- **Neutrons (n)**
  - no charge
  - m_n slightly greater than m_p

Relative sizes – "Classical Dimensions“:

- Nucleus ~ 1 femtometer (1 fm = 10^{-15} m or 10^{-12} cm)
- Atom ~ 0.1 nm or 1 Ångström (1 Å = 10^{-10} meters or 10^{-8} cm)

Reminder on Notation

$$ {}^{A}_{Z}X \rightarrow {}^{16}_{8}O \text{ or } {}^{16}O $$

Z or Atomic Number:

- The number of protons in a nucleus, which also equals the number of electrons in a neutral atom.
- Determines the identity of the element and its position in the periodic table.
- Sometimes omitted when it is clear from the element symbol X.

A or Mass Number:

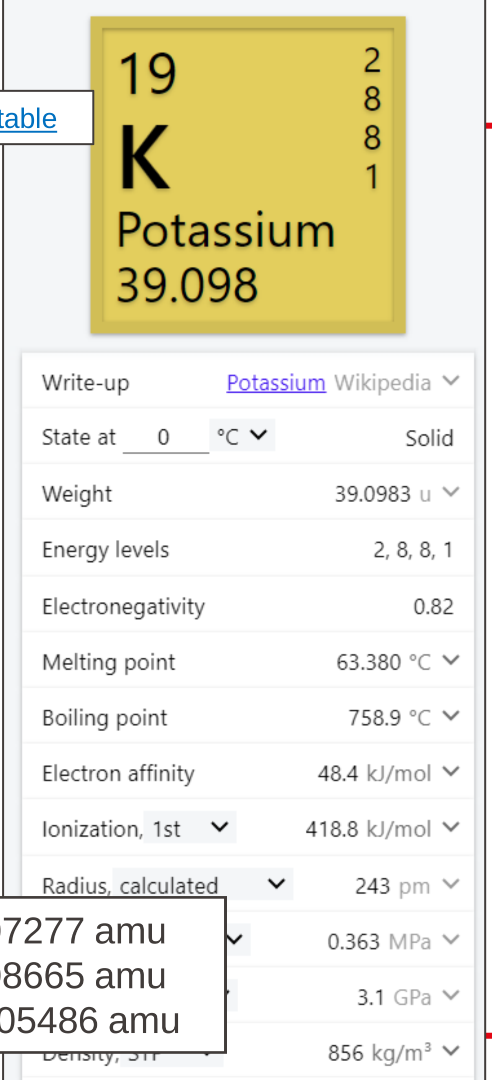
- The total number of nucleons (protons p + neutrons n) in a nucleus.
- Thus, a nucleus is made of Z protons and (A-Z) neutrons.

Reminder on Atomic and Molar Mass

- **Mole:** amount of substance that contains the same number of entities (atoms, molecules) as there are atoms in 12 grams of $^{12}_{6}\mathrm{C}$
- This number is the **Avogrado Number** $N_A = 6.023 \times 10^{23}$
- **Molar mass:** mass in g of 1 mole of an element $\mathrm{M}\left({}^{A}_{Z}\mathrm{X}\right)$. By definition, $\mathrm{M}\left({}^{12}_{6}\mathrm{C}\right)$ is set to 12g.

- **Atomic mass unit (amu):** Defined as 1/12 of the weight of a $^{12}_{6}\mathrm{C}$ atom: 1 amu = $\frac{\mathrm{M}\left({}^{12}_{6}\mathrm{C}\right)}{12 \cdot N_A}$ = 1.66×10^{-27} kg
- Thus, the **atomic mass** of one $^{12}_{6}\mathrm{C}$ atom is exactly $\mathrm{m}\left({}^{12}_{6}\mathrm{C}\right) = 12.00$ amu.
- It follows that $\mathrm{m}\left({}^{A}_{Z}\mathrm{X}\right) = \left|\mathrm{M}\left({}^{A}_{Z}\mathrm{X}\right)\right|$ amu, i.e. ***<u>the atomic mass of any element is equal to the value of the molar mass expressed in amu</u>***.

From Ptable



$m_p = 1.007277$ amu
$m_n = 1.008665$ amu
$m_e = 0.0005486$ amu

Chemistry – Periodic Table of *Elements*

Group ► / Period ▼	1	2		3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
1	1 H																		2 He
2	3 Li	4 Be												5 B	6 C	7 N	8 O	9 F	10 Ne
3	11 Na	12 Mg												13 Al	14 Si	15 P	16 S	17 Cl	18 Ar
4	19 K	20 Ca		21 Sc	22 Ti	23 V	24 Cr	25 Mn	26 Fe	27 Co	28 Ni	29 Cu	30 Zn	31 Ga	32 Ge	33 As	34 Se	35 Br	36 Kr
5	37 Rb	38 Sr		39 Y	40 Zr	41 Nb	42 Mo	43 Tc	44 Ru	45 Rh	46 Pd	47 Ag	48 Cd	49 In	50 Sn	51 Sb	52 Te	53 I	54 Xe
6	55 Cs	56 Ba	La to Yb	71 Lu	72 Hf	73 Ta	74 W	75 Re	76 Os	77 Ir	78 Pt	79 Au	80 Hg	81 Tl	82 Pb	83 Bi	84 Po	85 At	86 Rn
7	87 Fr	88 Ra	Ac to No	103 Lr	104 Rf	105 Db	106 Sg	107 Bh	108 Hs	109 Mt	110 Ds	111 Rg	112 Cn	113 Nh	114 Fl	115 Mc	116 Lv	117 Ts	118 Og
	s-block (incl. He)		f-block	d-block										p-block (excl. He)					

Nonmetals (above the dashed line) / *Metals* (below)

Transition metals (sometimes excl. group 12)

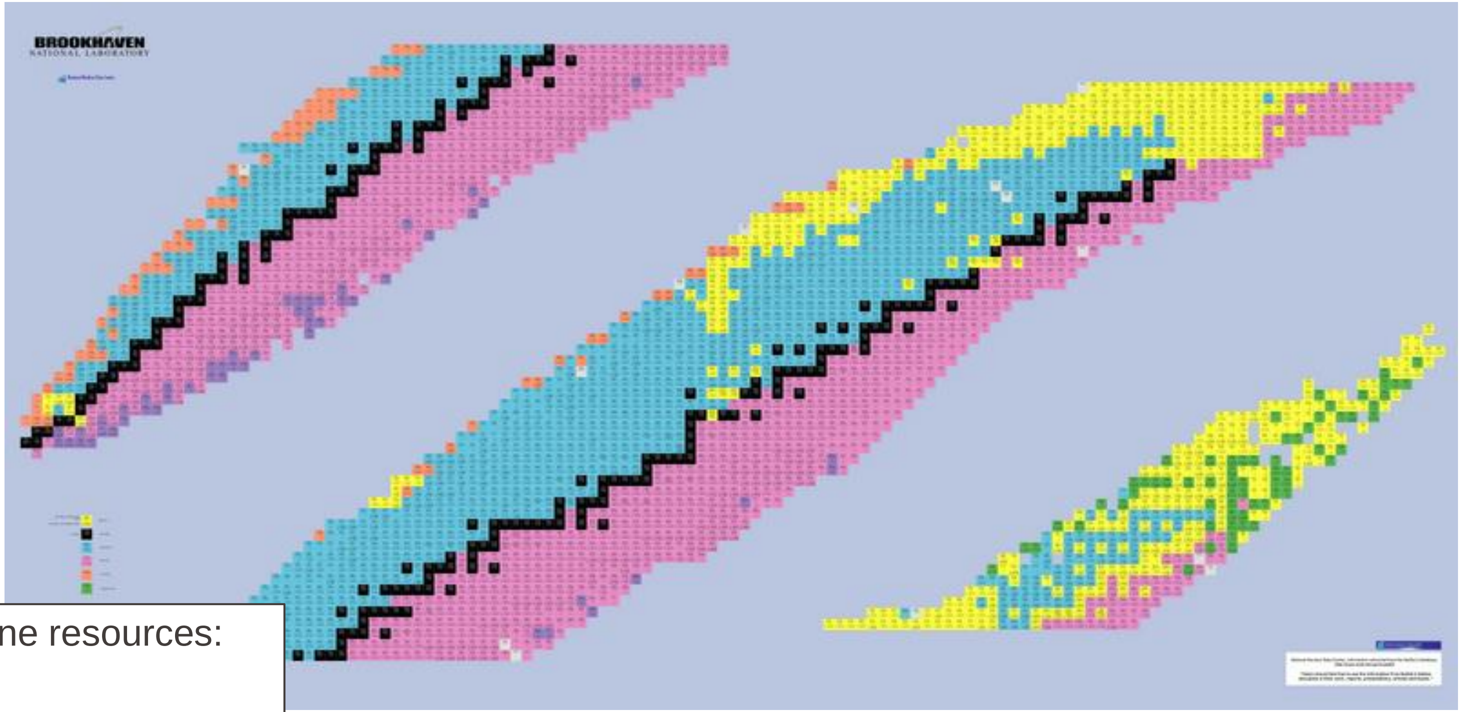
Some elements near the dashed staircase are sometimes called *metalloids*

Noble gases (group 18)

Lanthanides	57 La	58 Ce	59 Pr	60 Nd	61 Pm	62 Sm	63 Eu	64 Gd	65 Tb	66 Dy	67 Ho	68 Er	69 Tm	70 Yb
Actinides	89 Ac	90 Th	91 Pa	92 U	93 Np	94 Pu	95 Am	96 Cm	97 Bk	98 Cf	99 Es	100 Fm	101 Md	102 No

Nuclear Physics – Chart of *Nuclides*

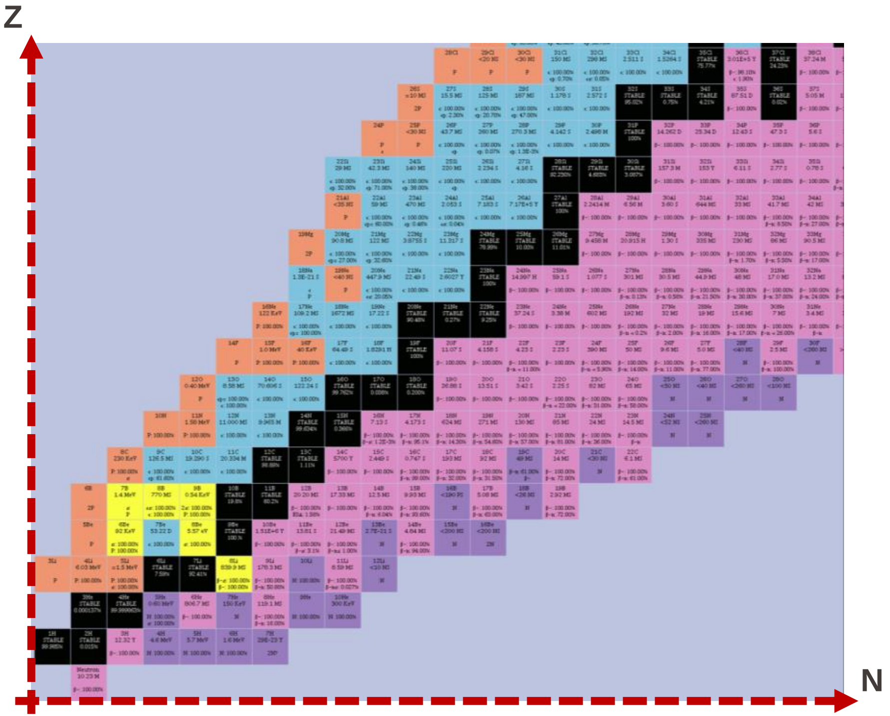
Wikipedia – Public Domain



Some online resources:

- IAEA
- KAERI

Nuclides & Isotopes



- **Nuclides**: Nuclei with specific Z protons and A nucleons (protons + neutrons)
- **Isotopes**: Nuclides of the same element (same Z) but different number of neutrons (different A)
- Some isotopes occur naturally, others are produced in reactors or particle accelerators.

- **Examples of isotopic abundance**:
  - ^{1}H (99.985%), ^{2}H (0.015%)
  - ^{234}U (0.006%), ^{235}U (0.72%), ^{238}U (99.27%)
  - ^{6}Li (7.6%), ^{7}Li (92.4%)
- **NOTE - Hydrogen isotopes names:**
  - ^{2}H → Deuterium or D
  - ^{3}H (radioactive) → Tritium or T

Exercise – Notation

- Complete isotope name/symbol, then provide the number of protons and neutrons in each isotope:

1. $^{3}_{2}?$
2. $^{6}_{3}?$ and $^{7}_{3}?$
3. $^{90}_{40}?$
4. $^{137}_{55}?$
5. $^{238}_{92}?$ and $^{235}_{92}?$

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Exercise – Notation

- Complete isotope name/symbol, then provide the number of protons and neutrons in each isotope:

1. $^{3}_{2}\mathrm{He}$
2. $^{6}_{3}\mathrm{Li}$ and $^{7}_{3}\mathrm{Li}$
3. $^{90}_{40}\mathrm{Zr}$
4. $^{137}_{55}\mathrm{Cs}$
5. $^{238}_{92}\mathrm{U}$ and $^{235}_{92}\mathrm{U}$

Exercise – How Many Atoms in a Nuclear Fuel Pellet?

- A UO_2 fuel pellet used in nuclear reactors is typically a cylinder with:
  - Diameter = 9 mm
  - Height = 10 mm
  - Density = 10.5 g/cm^3

- **Questions:**
  - How many UO_2 moles are in one pellet?
    How many U atoms?
  - Calculate the total mass of fuel in a reactor assuming 300 pellets per rod and 50k rod per reactor.

Exercise – How Many Atoms in a Nuclear Fuel Pellet?

- Fuel pellet volume: $\mathrm{V} = \pi \cdot 4.5^2 \cdot 10 = 636.17\, mm^3 = 0.636\, cm^3$
- Mass of pellet: $\mathrm{m} = \mathrm{V} \cdot \rho = 0.636 \cdot 10.5 = 6.678\, g$
- Molar mass of UO_2: $M_{UO_2} = 238 + 2 \cdot 16 = 270 \frac{g}{mol}$
- Number of atoms: $N = \frac{6.678}{270} 6.02 \cdot 10^{23} = 1.49 \cdot 10^{22} atoms$

- Fuel mass in the reactor: $m_{reactor} = 6.678 \cdot 300 \cdot 50000 \approx 100$ tons

Atomic vs Nuclear Physics - Energy Levels & Electron Volts

- 1 **electron volt (eV)** is the energy gained by an electron when accelerated through a potential difference of 1 volt.

$$1\ \mathrm{eV} = 1.602 \times 10^{-19}\ \mathrm{J}$$

- In chemistry & atomic physics, reactions involve the **rearrangement of electrons in atomic shells**.
- Typical scale: 1-100 eV (order of ionization energies).
- Example, combustion of methane: $\mathrm{CH_4} + 2\mathrm{O_2} \rightarrow \mathrm{CO_2} + 2\mathrm{H_2 0} + \sim 8\ \mathrm{eV}$

- In nuclear physics energy transitions involve changes **within the nucleus** (protons and neutrons).
- Much higher energies: keV to MeV (1,000 - 1,000,000 eV).
- Examples: nuclear fission (U-235): ~200 MeV

Exercise – How Much Fuel Do You Need in a Lifetime

- **Goal**: Estimate the fuel mass required to power an individual's lifetime electricity consumption if the energy is extracted solely from: coal, natural gas, or fission (U-235)

- **Assumptions**:
  - Annual electricity consumption per household (4-persons): 5000 kWh
  - Average lifetime: 80 years
  - Efficiencies:
    - Coal power plant: 35%
    - Gas power plant: 50%
    - Nuclear reactor (U-235 fission): ~33% efficiency
  - Fuel Energy Content:
    - Coal: 24 MJ/kg
    - Natural gas (CH_4): 36 MJ/m^3
    - U-235 (Fission Energy): 200 MeV/fission

Exercise – How Much Fuel Do You Need in a Lifetime

- Annual electricity consumption per person: 5000/4 = 1250 kWh
- **Coal:**
  - Electricity produced per person: 1250/0.35 ≈ 3571.4 kWh → 3571.4 * 3600 ≈ 12857 MJ
  - Fuel per person: 12857 / 24 = 535.71 kg per year
  - Fuel per person for 80 years: 42.8 tons
- **Gas:**
  - Electricity produced per person: 1250/0.50 ≈ 2500 kWh → 2500 * 3600 ≈ 9000 MJ
  - Fuel per person: 9000 / 36 = 250 m^3 per year
  - Fuel per person for 80 years: 20000 m^3
- **Uranium:**
  - Electricity produced per person: 1250/0.33 ≈ 3787.88 kWh → 3787.88 * 3600 ≈ 13636 MJ
  - Energy per fission: 200E+6 eV * 1.6E-19 J/eV = 320E-19 MJ/fission
  - Energy per kg: (1 kg / (0.235 kg/mol) * 6.02E+23) * 320E-19 MJ/fission = 81’974’468 MJ/kg
  - Fuel per person: 13636 / 81’974’468 = 0.166 g per year
  - Fuel per person for 80 years: 13.28 g

Exercise – How Much Fuel for 1GW for a day?

- **Goal**: Estimate the amount of fuel required to generate 1 GW-day (GWd), i.e. 1 gigawatt (GW) of electricity for one day (24h).

- **Assumptions**: Same as previous exercise.

- **Questions**:
  - How much coal (tons) is burned per day?
    - How many coal train cars does this require? (Assume 100 tons per train car)
  - How much natural gas (m^3) is burned per day?
  - How much U-235 (kg) is needed per day?
    How much U (238+235) if we consider a realistic typical ***enrichment*** of 5%?

Exercise – How Much Fuel for 1GW for a day?

- **Goal**: Estimate the amount of fuel required to generate 1 GW-day (GWd), i.e. 1 gigawatt (GW) of electricity for one day (24h).

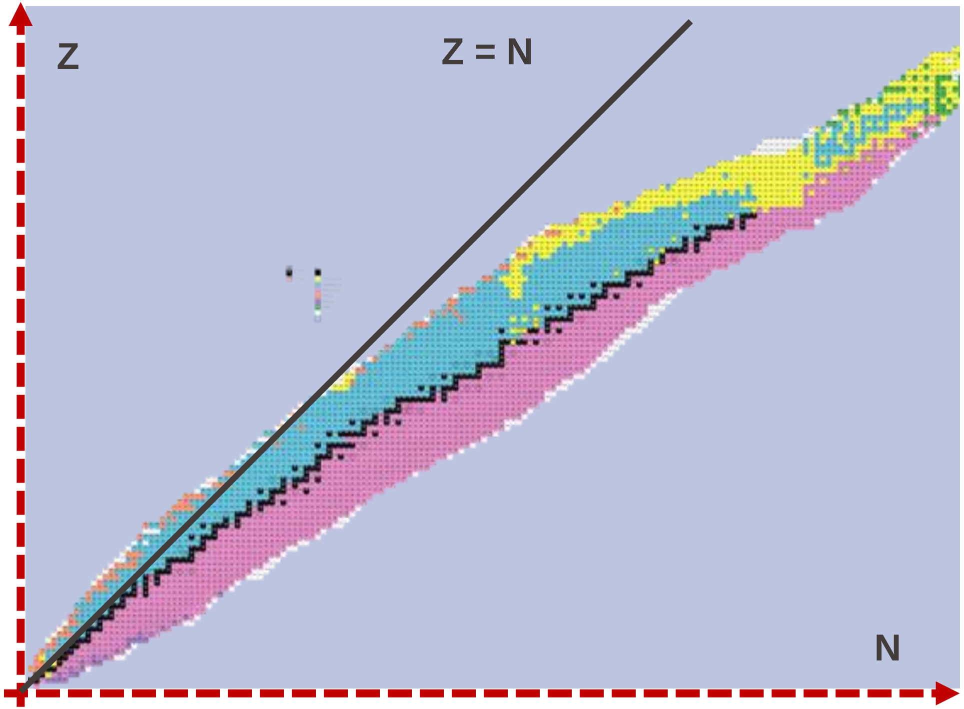
- Energy per day in MJ: 1000 MW * 24 * 3600 = 86’400’000 MJ

- **Coal (consider efficiency of plant)**:
  - Fuel mass: (86’400’000 / 0.35) / 24 = 10’286 tons → 102 train cars

- **Gas (consider efficiency of plant)**: :
  - Fuel volume: (86’400’000 / 0.5) / 36 = 4’800’000 m^3

- **U235 (consider efficiency of plant)**: :
  - U235 mass: (86’400’000 / 0.33) / 81’974’468 = 3.18 kg
  - U238 mass: 3.18 /0.05 = 63.6 kg

Radioactive Decay & Nuclide Stability



Some nuclides are unstable and undergo **radioactive decay:**

- Spontaneous process
- Release of energy to become more stable and emission of particles/radiation (e.g. α, β, or γ).

“Stability line” in Z vs. N plot (black squares):

- **Light elements** (Z<20): Stability occurs when $N \approx Z$ (equal protons and neutrons).
- **Heavy elements** (Z>20): Stability requires $N > Z$ to compensate for increasing Coulomb repulsion between protons (more neutrons strengthen the strong nuclear force, balancing repulsive forces).

Radioactivity Decay – Fundamental Law

- The probability of decay per unit time λ is constant: $-\frac{dN(t)}{N(t)\cdot dt} = \lambda \longrightarrow -\frac{dN(t)}{dt} = \lambda N(t) = A(t)$

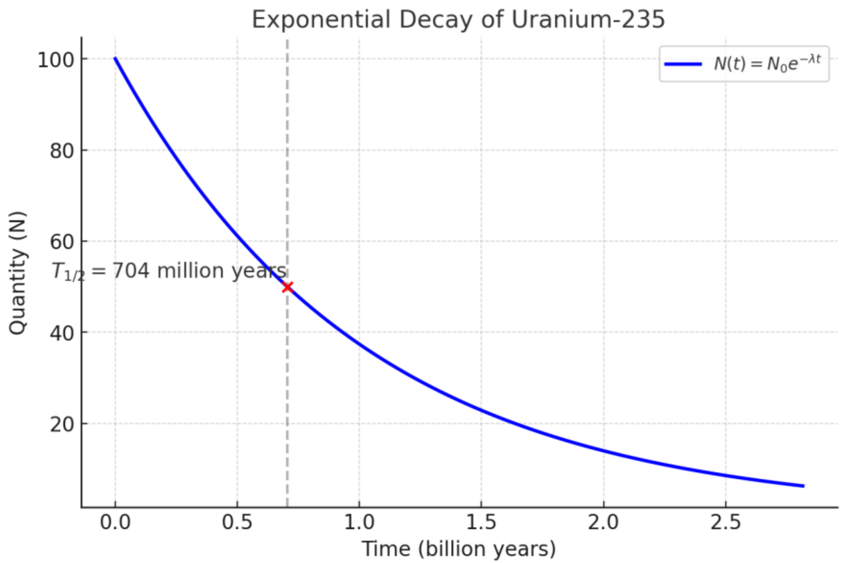
- **Activity A(t)** measured in Becquerel **(1 Bq = 1 decay/s)**

- Integrating the equation gives:

$$N(t) = N(0)\cdot e^{-\lambda t} \qquad A(t) = A(0)\cdot e^{-\lambda t}$$

- **Half-Life $T_{1/2}$** - Time for the activity or number of nuclei to reduce by half:

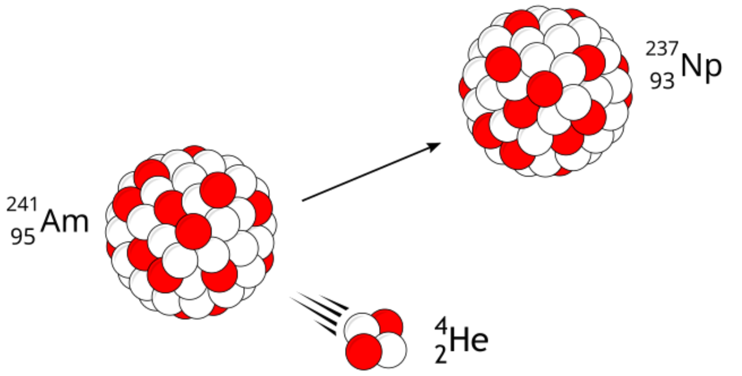
$$\frac{N(T_{1/2})}{N(0)} = \frac{1}{2} = e^{-\lambda\cdot T_{1/2}} \longrightarrow T_{1/2} = \frac{\ln 2}{\lambda} \cong \frac{0.693}{\lambda}$$



Radioactive Decay Modes – α Decay

- Emission of a stable $^4_2\mathrm{He}$ nucleus also called ***alpha particle*** → as a result, **Z decreases by 2, A decreases by 4.**
- Occurs in **heavy, unstable nuclei** to reduce Coulomb repulsion (too many protons)
- **Energy release**: high, typically in the MeV range (~4-9 MeV).
- **Penetration**: it can be stopped by paper or skin…
- **Safety: *very harmful if inhaled or ingested!***

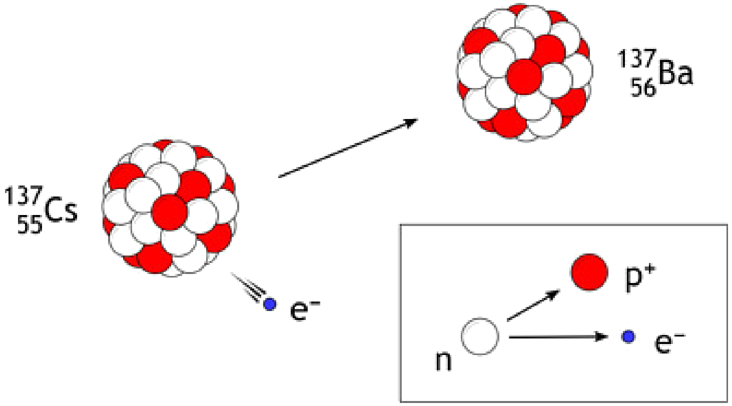
Example: $^{241}_{95}Am \rightarrow ^{237}_{93}Np + \alpha$



Radioactive Decay Modes – β^- Decay

- Occurs in **neutron-rich nuclei**, converting a neutron into a proton + electron or ***β*− particle** (emitted together with an antineutrino $\bar{\nu}_e$)
- Atomic number Z **increases by 1 (new element is formed);** mass number A **remains unchanged**.

Example: $^{137}_{55}Cs \rightarrow {}^{137}_{56}Ba + \beta^- + \bar{\nu}_e$



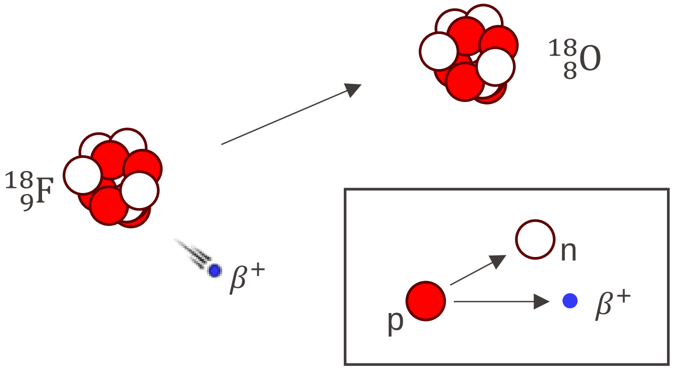
- **Energy release:** in the range of a few keV to ~1 MeV.
- **Penetration:** higher than α particles; stopped by a few mm of plastic or glass.
- **Safety**: can cause skin burns; internal exposure is dangerous!

Radioactive Decay Modes – β^+ Decay

- Occurs in **proton-rich nuclei**, converting a proton into a neutron + positron or β^+ particle (emitted together with a neutrino ν_e).

- Atomic number Z **decreases by 1 (new element is formed);** mass number A **remains unchanged.**

Example: $^{18}_{9}F \rightarrow {}^{18}_{8}O + \beta^+ + \nu_e$



- **Energy release:** in the range of a few keV to ~2 MeV

- **Penetration:** Similar to β^- particles, but annihilates with an electron upon contact

- **Safety**: positron annihilation produces gamma radiation (511 keV photons), which can penetrate further than β

Exercise – Complete the decay

1. $^{210}_{84}\text{Po} \rightarrow \; ? + {}^{4}_{2}\text{He}$

2. $^{131}_{53}\text{I} \rightarrow \; ? + {}^{0}_{-1}\beta + \overline{\nu_e}$

3. $^{40}_{19}\text{K} \rightarrow \; ? + {}^{0}_{1}\beta + \nu_e$

4. $? \rightarrow {}^{140}_{55}\text{Cs} + {}^{109}_{43}\text{Tc} + 3\,{}^{1}_{0}\text{n}$

Exercise – Complete the decay

1. $^{210}_{84}\mathrm{Po} \rightarrow {}^{206}_{82}\mathrm{Pb} + {}^{4}_{2}\mathrm{He}$

2. $^{131}_{53}\mathrm{I} \rightarrow {}^{131}_{54}\mathrm{Xe} + {}^{0}_{-1}\beta + \overline{\nu}_e$

3. $^{40}_{19}\mathrm{K} \rightarrow {}^{40}_{18}\mathrm{Ar} + {}^{0}_{1}\beta + \nu_e$

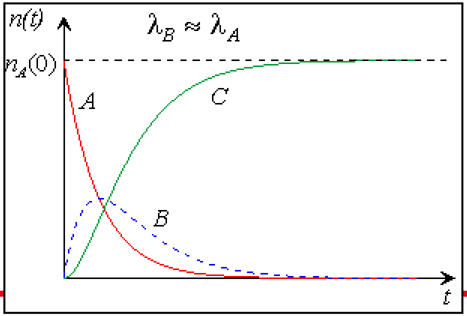
4. $^{252}_{98}\mathrm{Cf} \rightarrow {}^{140}_{55}\mathrm{Cs} + {}^{109}_{43}\mathrm{Tc} + 3\,{}^{1}_{0}\mathrm{n}$

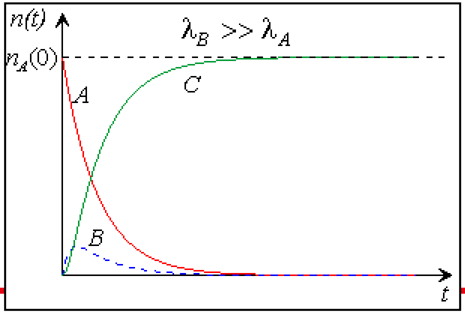
Decay Chains

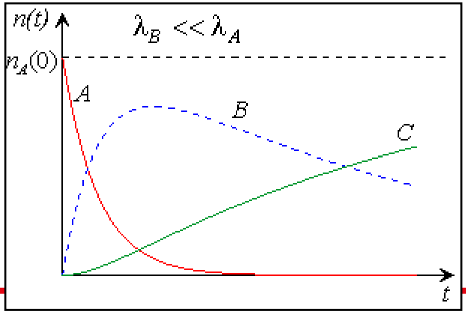
- The decay of a nucleus can lead to a nucleus that is also radioactive leading to a chain.
- It applies to natural radioactivity (e.g. $^{232}_{90}\mathrm{Th}$) but also to fission products.
- In general, we can build a system

$$\begin{cases} \frac{dN_1(t)}{dt} = -\lambda_1 N_1(t) \\ \frac{dN_2(t)}{dt} = -\lambda_2 N_2(t) + \lambda_1 N_1(t) \\ \frac{dN_3(t)}{dt} = -\lambda_3 N_3(t) + \lambda_2 N_2(t) \\ \text{etc.} \end{cases}$$

- For a simple case (A→B→C) with C stable, there are 3 main cases:





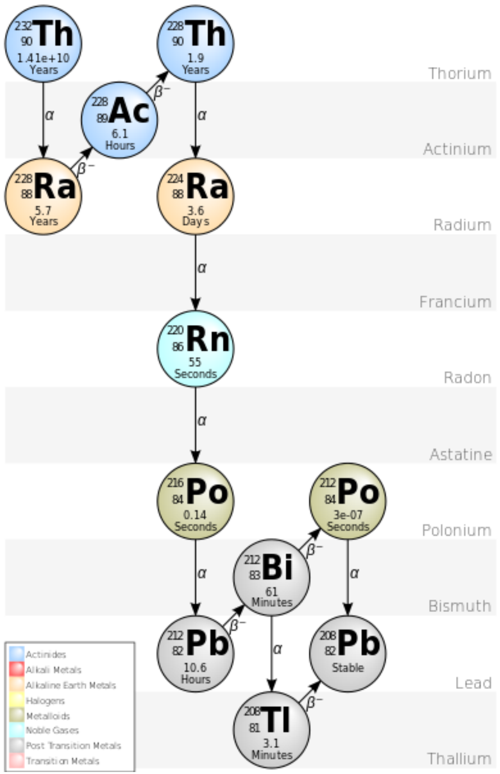


Radioactive or secular equilibrium

- Occurs when the half-life of the parent nucleus is significantly longer than that of all its daughter nuclides (or $\lambda_1 \ll \lambda_2$)
- Time to equilibrium will be short relative to the parent nucleus's half-life, but long compared to the half-lives of daughter nuclides.
- At equilibrium, the rate of decay of each nuclide (dN/dt) in ≈ zero so: $\lambda_1 N_1 \approx \lambda_2 N_2 \approx \lambda_3 N_3$ etc.

Consequences:

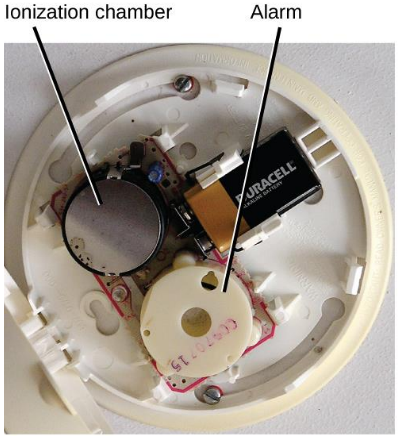
- Activities of all nuclides in the chain become equal at equilibrium…
- …but the nuclide concentration is different $\frac{N_i}{N_1} = \frac{\lambda_1}{\lambda_i} = \frac{T_i}{T_1}$
- The shorter the half-life, the lower the concentration of the nuclide in the natural state.

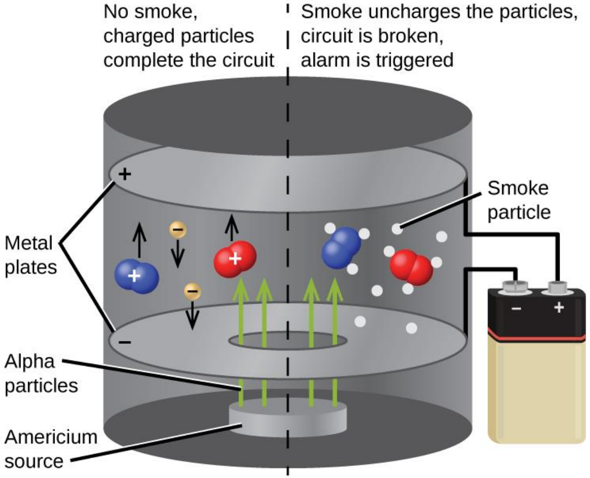


How Do Smoke Detectors Work?

Many household smoke detectors use $^{241}_{95}\mathrm{Am}$!

- A small $^{241}_{95}\mathrm{Am}$ source is placed in a ionization chamber between two electrically charged plates.
- With no smoke: α particles ionize air molecules, creating a small electric current between plates.
- When smoke enters: smoke particles absorb ions, reducing the current, triggering the alarm.





No safety concerns:

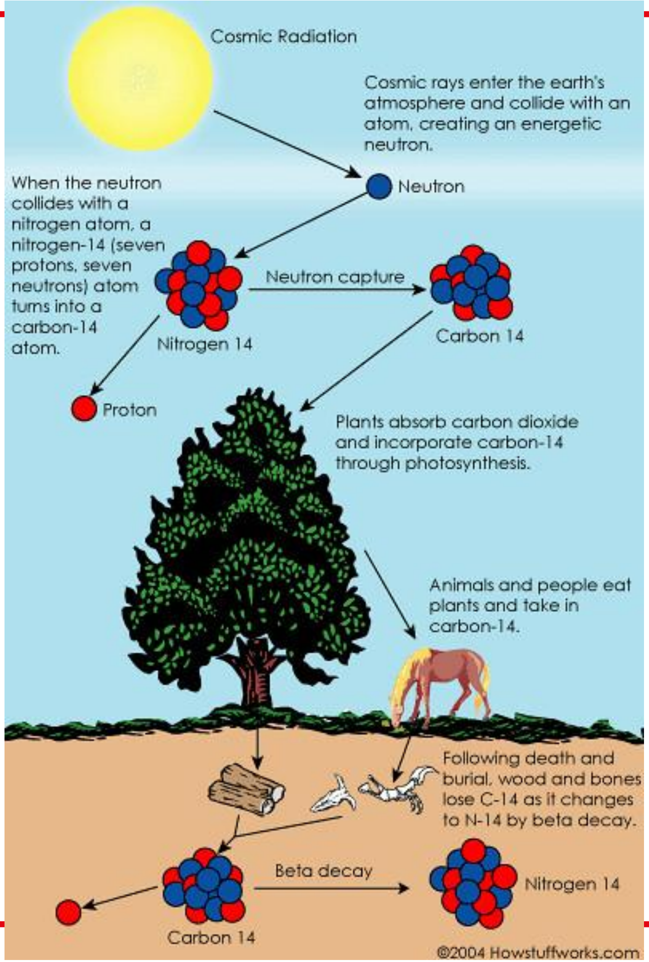
- α particles have low penetration → cannot escape the detector casing.
- Very small amounts (~0.3 μg per detector) are used, posing no radiation risk in normal use.

Carbon Dating – Determining the Age of Ancient Artifacts

- ${}^{14}_{6}\mathrm{C}$ is naturally found in the atmosphere.
- Living organisms absorb ${}^{14}_{6}\mathrm{C}$ through CO_2 exchange, maintaining a ***constant ratio of*** $\boldsymbol{{}^{14}_{6}C}$ ***to*** $\boldsymbol{{}^{12}_{6}C}$.
- When an organism dies, it stops absorbing carbon, and ${}^{14}_{6}\mathrm{C}$ begins to decay with $T_{1/2}$ of 5’730 years:

$${}^{14}_{6}\mathrm{C} \rightarrow {}^{14}_{7}\mathrm{N} + \beta^{-} + \overline{\nu_e}$$

- The remaining ${}^{14}_{6}\mathrm{C}$ fraction in a sample indicates its age! (Carbon dating is effective for objects up to ~50,000)



PET scans and radio-isotopes

PET scans use β^+ decay to create detailed 3D images of metabolic processes inside the body.

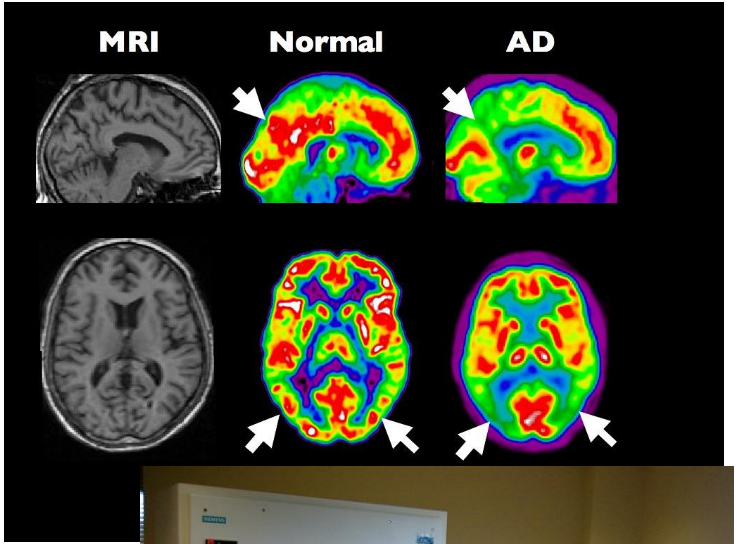
- A radioactive tracer (e.g., ${}^{18}_{9}F$) is injected into the patient.
- The tracer undergoes β^+ decay, emitting a positron.
- The positron annihilates with an electron, producing two 511 keV gamma photons.
- PET scanners detect these gamma rays to form an image.

Production of PET Tracers:

- PET tracers like ${}^{18}_{9}F$ are produced in a **cyclotron**, a particle accelerator that bombards a target material with protons to induce nuclear reactions

$$ {}^{18}_{8}O + p \rightarrow {}^{18}_{9}F + n $$

- PET tracers have short half-lives (e.g., ${}^{18}_{9}F$: 110 min) → they must be produced close to hospitals.



Exercise – Smoke Detectors & Carbon Dating

1. Calculate the activity of a new smoke detector
   - Assume 0.3 µg of ${}^{241}_{95}\mathrm{Am}$ at the start.
   - Consider half life of 432.6y
2. How much ${}^{241}_{95}\mathrm{Am}$ is left after ten-years?

3. Archaeologists find a wooden artifact and want to determine its age using carbon-14 dating. The wood sample has 25% of the original ${}^{14}_{6}C$ activity remaining. Given that $T_{1/2} = 5730\ y$, estimate the age of the sample.

Exercise – Smoke Detectors & Carbon Dating

1. Calculate the activity of a new smoke detector
   - Number of atoms in 0.3 µg of ${}^{241}_{95}\mathrm{Am}$: $N_{Am} = \frac{0.3\cdot 10^{-6}}{241}\cdot 6.02\cdot 10^{23} = 749.3\cdot 10^{12}$ atoms
   - Decay constant: $\lambda_{Am} = \frac{\ln(2)}{432.6\cdot 365\cdot 24\cdot 3600} = 5.08\cdot 10^{-11}\ s$
   - **Activity:** $\mathbf{A = \lambda_{Am}\cdot N_{Am} = 38\ kBq}$

2. How much ${}^{241}_{95}\mathrm{Am}$ is left after ten-years?
   - Mass of americium decays with half life of 432.6 years → $\mathrm{m(10y)} = m_0 e^{-\frac{\ln(2)}{432.6}10} = \mathbf{0.295\mu g}$

3. Archaeologists find a wooden artifact and want to determine its age using carbon-14 dating. The wood sample has 25% of the original ${}^{14}_{6}\mathrm{C}$ activity remaining. Given that $\mathrm{T_{1/2} = 5730\ y}$, estimate the age of the sample.
   - $\mathrm{N_{C14}(t) = N_{C14,0}\cdot \exp\left(-\frac{\ln(2)}{5730}t\right) \rightarrow \frac{N_{C14}(t)}{N_{C14,0}} = 0.25 = \exp\left(-\frac{\ln(2)}{5730}t\right)}$
   - $\mathrm{\ln(0.25) = \ln\left(\frac{1}{4}\right) = -\ln(4) = \left(-\frac{\ln(2)}{5730}t\right) \rightarrow \mathbf{t = 5730\frac{\ln(4)}{\ln(2)} = 11460\ years}}$

Exercise – Production of a Radio-Isotope

- A medical facility requires a daily supply of 5 GBq of ${}^{18}_{9}F$ at the time of delivery. The production process involves bombarding an enriched ${}^{18}_{8}O$ target with protons in a cyclotron that is **1hr away**.

- **Estimate how long the cyclotron must irradiate the target to produce the required activity.**

- **Assumptions**:
  - The half-life of ${}^{18}_{9}F$ is 109.7 min.
  - The production rate in the cyclotron is constant and equal to 1E+10 atoms/sec
  - The decay loss during transport should be considered.

Exercise – Production of a Radio-Isotope

- The decay law with an additional constant production term R is:

$$\frac{dN}{dt} = R - \lambda N$$

- We integrate this equation by multiplying both sides with the integrating factor $e^{\lambda t}$

$$\mathbf{A(t) = R \cdot (1 - e^{-\lambda t})}$$

- We need 5 GBq of ${}^{18}_{9}F$ after delivery. Thus, considering the decay that takes place during 1hr transport, the activity at the exit from the cyclotron should be:

$$A(1hr) = 5\ \text{GBq} = A_{exit} e^{-\frac{\ln(2)}{1.828\ hr} \cdot 1\ hr} \rightarrow \mathbf{A_{exit} = 7.305\ GBq}$$

- Using the equation derived before to find the t_{irr} with a production rate of 1E+10 at/s:

$$A(t_{irr}) = A_{exit} = 7.305\ \text{GBq} = 1E{+}10\left(1 - e^{-\frac{ln(2)}{1.828\ hr} \cdot t_{irr}}\right)$$

$$\mathbf{t_{irr} = 3.45\ \mathit{hr}}$$