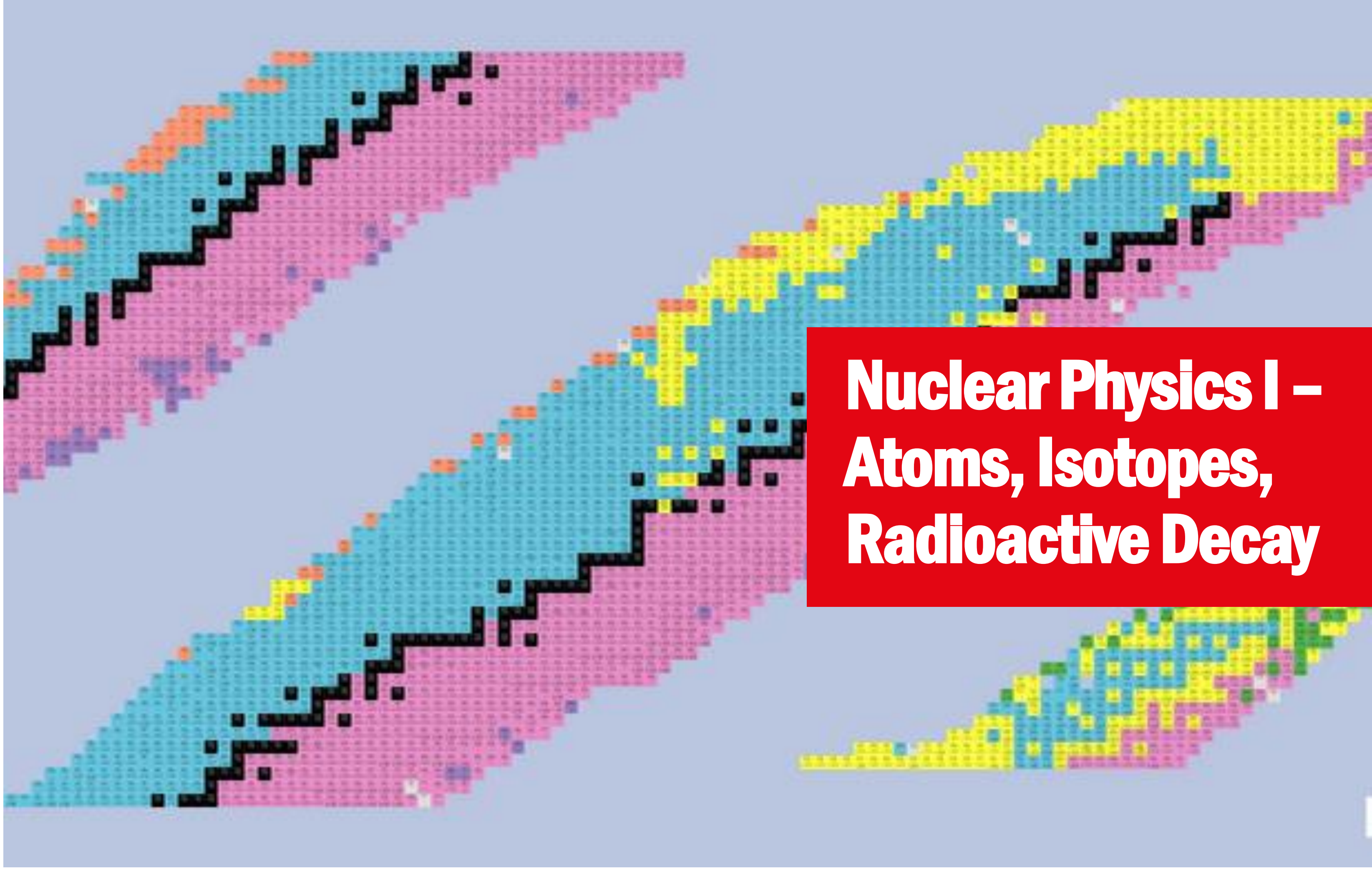
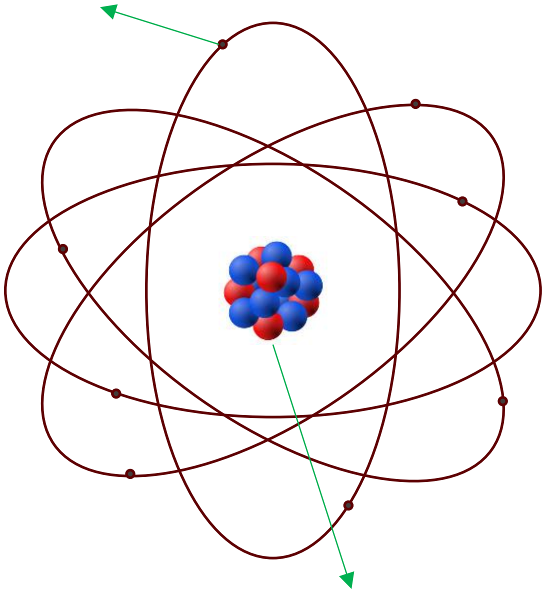
Nuclear Physics I – Atoms, Isotopes, Radioactive Decay



Fundamental Components of the Atom



The Figure is not accurate and not to scale

Basic atomic structure:

- **Electrons (e)**
  - negative charge (q_e ~ -1.6·10^{-19} C)
  - very light (m_e ~ 9.11·10^{-31} kg)
- **Protons (p)**
  - positive charge ($q_p = +|q_e|$)
  - m_p ~ 1800 times larger than m_e (m_p ~ 1.67 ·10^{-27} kg)
- **Neutrons (n)**
  - no charge
  - m_n slightly greater than m_p

Relative sizes – "Classical Dimensions“:

- Nucleus ~ 1 femtometer (1 fm = 10^{-15} m or 10^{-12} cm)
- Atom ~ 0.1 nm or 1 Ångström (1 Å = 10^{-10} meters or 10^{-8} cm)

Reminder on Notation

$$^{A}_{Z}X \rightarrow {}^{16}_{8}O \text{ or } {}^{16}O$$

Z or Atomic Number:

- The number of protons in a nucleus, which also equals the number of electrons in a neutral atom.
- Determines the identity of the element and its position in the periodic table.
- Sometimes omitted when it is clear from the element symbol X.

A or Mass Number:

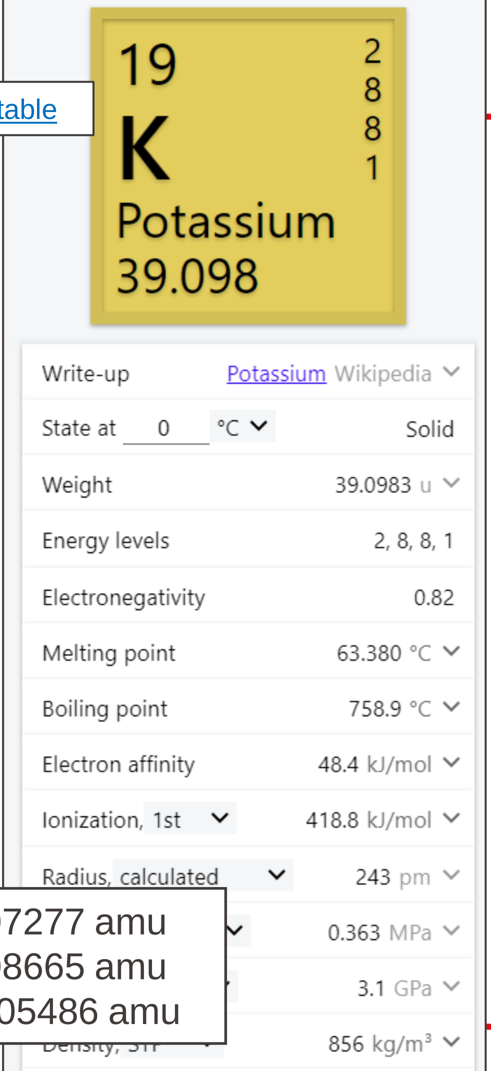
- The total number of nucleons (protons p + neutrons n) in a nucleus.
- Thus, a nucleus is made of Z protons and (A-Z) neutrons.

Reminder on Atomic and Molar Mass

- **Mole:** amount of substance that contains the same number of entities (atoms, molecules) as there are atoms in 12 grams of ${}^{12}_{6}\mathrm{C}$
- This number is the **Avogrado Number** $N_A = 6.023 \times 10^{23}$
- **Molar mass:** mass in g of 1 mole of an element $M({}^{A}_{Z}X)$. By definition, $M({}^{12}_{6}C)$ is set to 12g.

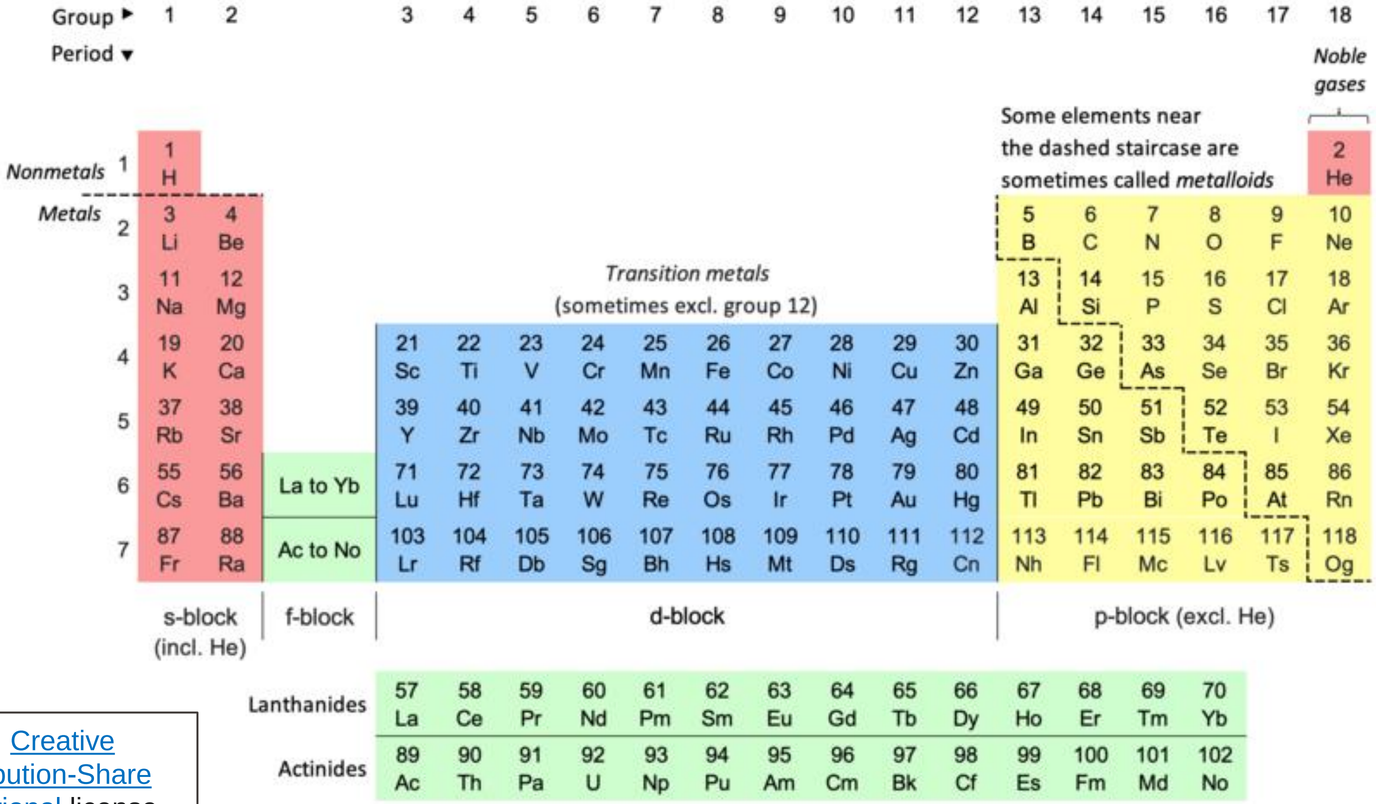
- **Atomic mass unit (amu):** Defined as 1/12 of the weight of a ${}^{12}_{6}\mathrm{C}$ atom: 1 amu $= \frac{M({}^{12}_{6}C)}{12 \cdot N_A} = 1.66 \times 10^{-27}$ kg
- Thus, the **atomic mass** of one ${}^{12}_{6}\mathrm{C}$ atom is exactly $m({}^{12}_{6}C) = 12.00$ amu.
- It follows that $m({}^{A}_{Z}X) = |M({}^{A}_{Z}X)|$ amu, i.e. ***the atomic mass of any element is equal to the value of the molar mass expressed in amu***.

From Ptable



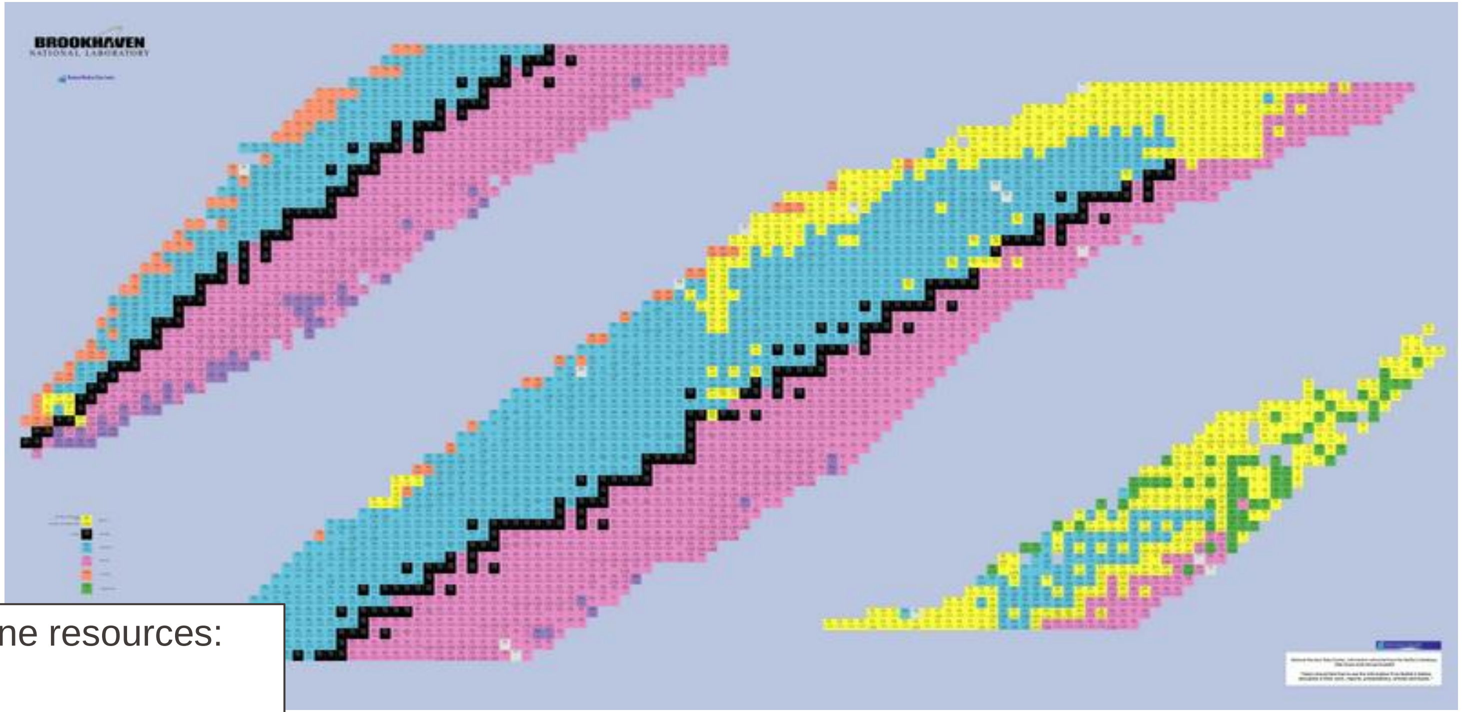
$m_p = 1.007277$ amu
$m_n = 1.008665$ amu
$m_e = 0.0005486$ amu

Chemistry – Periodic Table of *Elements*



Nuclear Physics – Chart of *Nuclides*

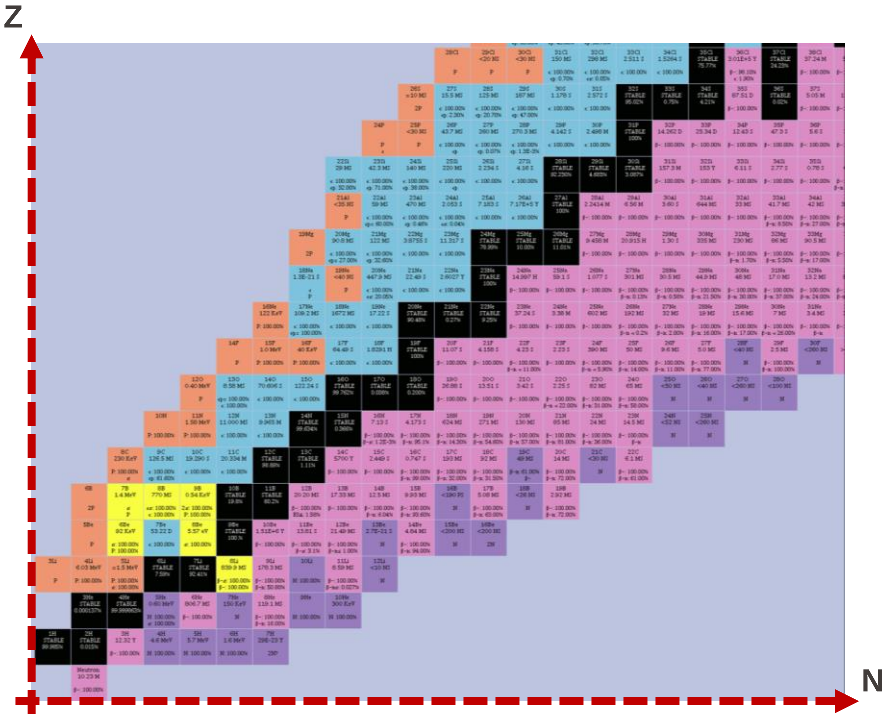
Wikipedia – Public Domain



Some online resources:

- IAEA
- KAERI

Nuclides & Isotopes



- **Nuclides**: Nuclei with specific Z protons and A nucleons (protons + neutrons)
- **Isotopes**: Nuclides of the same element (same Z) but different number of neutrons (different A)
- Some isotopes occur naturally, others are produced in reactors or particle accelerators.

- **Examples of isotopic abundance**:
  - ^{1}H (99.985%), ^{2}H (0.015%)
  - ^{234}U (0.006%), ^{235}U (0.72%), ^{238}U (99.27%)
  - ^{6}Li (7.6%), ^{7}Li (92.4%)
- **NOTE - Hydrogen isotopes names:**
  - ^{2}H → Deuterium or D
  - ^{3}H (radioactive) → Tritium or T

Exercise – Notation

- Complete isotope name/symbol, then provide the number of protons and neutrons in each isotope:

1. ${}^{3}_{2}?$
2. ${}^{6}_{3}?$ and ${}^{7}_{3}?$
3. ${}^{90}_{40}?$
4. ${}^{137}_{55}?$
5. ${}^{238}_{92}?$ and ${}^{235}_{92}?$

Exercise – How Many Atoms in a Nuclear Fuel Pellet?

- A UO_2 fuel pellet used in nuclear reactors is typically a cylinder with:
  - Diameter = 9 mm
  - Height = 10 mm
  - Density = 10.5 g/cm^3

- **Questions:**
  - How many UO_2 moles are in one pellet?
    How many U atoms?
  - Calculate the total mass of fuel in a reactor assuming 300 pellets per rod and 50k rod per reactor.

Atomic vs Nuclear Physics - Energy Levels & Electron Volts

- 1 **electron volt (eV)** is the energy gained by an electron when accelerated through a potential difference of 1 volt.

$$1\ \mathrm{eV} = 1.602 \times 10^{-19}\ \mathrm{J}$$

- In chemistry & atomic physics, reactions involve the **rearrangement of electrons in atomic shells**.
- Typical scale: 1-100 eV (order of ionization energies).
- Example, combustion of methane: $CH_4 + 2O_2 \rightarrow CO_2 + 2H_20 + {\sim}8\ \mathrm{eV}$

- In nuclear physics energy transitions involve changes **within the nucleus** (protons and neutrons).
- Much higher energies: keV to MeV (1,000 - 1,000,000 eV).
- Examples: nuclear fission (U-235): ~200 MeV

Exercise – How Much Fuel Do You Need in a Lifetime

- **Goal**: Estimate the fuel mass required to power an individual's lifetime electricity consumption if the energy is extracted solely from: coal, natural gas, or fission (U-235)

- **Assumptions**:
  - Annual electricity consumption per household (4-persons): 5000 kWh
  - Average lifetime: 80 years
  - Efficiencies:
    - Coal power plant: 35%
    - Gas power plant: 50%
    - Nuclear reactor (U-235 fission): ~33% efficiency
  - Fuel Energy Content:
    - Coal: 24 MJ/kg
    - Natural gas (CH_4): 36 MJ/m^3
    - U-235 (Fission Energy): 200 MeV/fission

Exercise – How Much Fuel for 1GW for a day?

- **Goal**: Estimate the amount of fuel required to generate 1 GW-day (GWd), i.e. 1 gigawatt (GW) of electricity for one day (24h).

- **Assumptions**: Same as previous exercise.

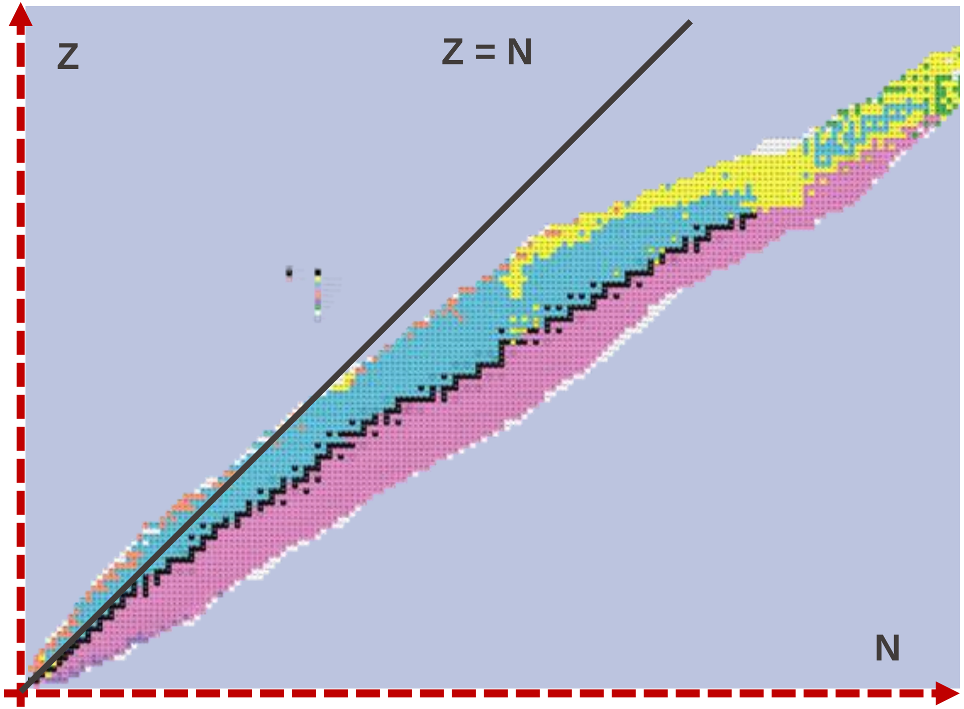
- **Questions**:

  - How much coal (tons) is burned per day?
    - How many coal train cars does this require? (Assume 100 tons per train car)

  - How much natural gas (m³) is burned per day?

  - How much U-235 (kg) is needed per day?
    How much U (238+235) if we consider a realistic typical ***enrichment*** of 5%?

Radioactive Decay & Nuclide Stability



Some nuclides are unstable and undergo **radioactive decay:**

- Spontaneous process
- Release of energy to become more stable and emission of particles/radiation (e.g. α, β, or γ).

“Stability line” in Z vs. N plot (black squares):

- **Light elements** (Z<20): Stability occurs when $N \approx Z$ (equal protons and neutrons).
- **Heavy elements** (Z>20): Stability requires $N > Z$ to compensate for increasing Coulomb repulsion between protons (more neutrons strengthen the strong nuclear force, balancing repulsive forces).

Radioactivity Decay – Fundamental Law

- The probability of decay per unit time λ is constant: $-\frac{dN(t)}{N(t)\cdot dt} = \lambda \longrightarrow -\frac{dN(t)}{dt} = \lambda N(t) = A(t)$

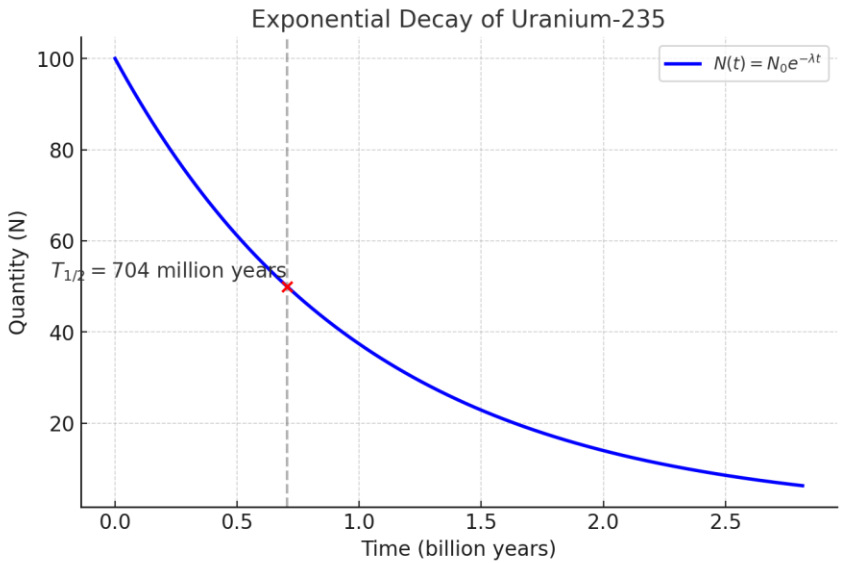
- **Activity A(t)** measured in Becquerel **(1 Bq = 1 decay/s)**

- Integrating the equation gives:

$$N(t) = N(0)\cdot e^{-\lambda t} \qquad A(t) = A(0)\cdot e^{-\lambda t}$$

- **Half-Life $T_{1/2}$** - Time for the activity or number of nuclei to reduce by half:

$$\frac{N(T_{1/2})}{N(0)} = \frac{1}{2} = e^{-\lambda\cdot T_{1/2}} \longrightarrow T_{1/2} = \frac{\ln 2}{\lambda} \cong \frac{0.693}{\lambda}$$

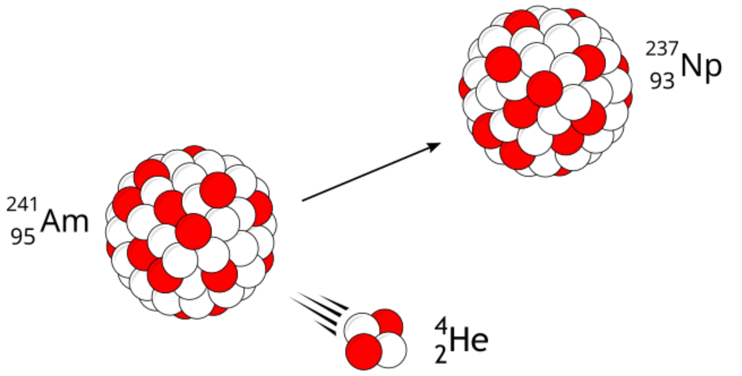


Radioactive Decay Modes – α Decay

- Emission of a stable $^{4}_{2}\mathrm{He}$ nucleus also called ***alpha particle*** → as a result, **Z decreases by 2, A decreases by 4.**
- Occurs in **heavy, unstable nuclei** to reduce Coulomb repulsion (too many protons)

Example: $^{241}_{95}Am \rightarrow ^{237}_{93}Np + \alpha$

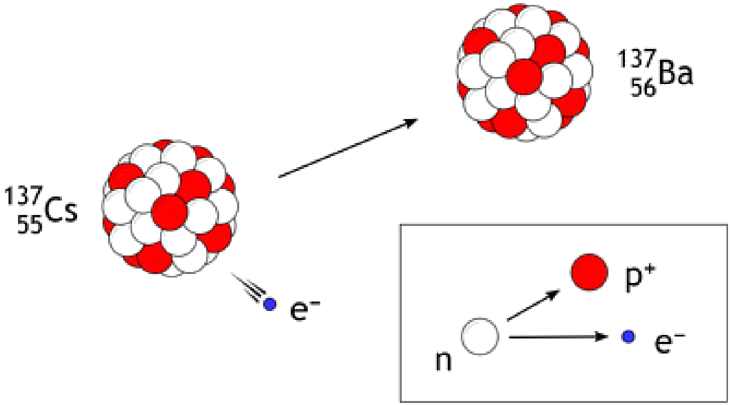
- **Energy release**: high, typically in the MeV range (~4-9 MeV).
- **Penetration**: it can be stopped by paper or skin…
- **Safety:** ***very harmful if inhaled or ingested!***



Radioactive Decay Modes – β^- Decay

- Occurs in **neutron-rich nuclei**, converting a neutron into a proton + electron or ***β*− particle** (emitted together with an antineutrino $\bar{\nu}_e$)
- Atomic number Z **increases by 1 (new element is formed);** mass number A **remains unchanged**.

Example: $^{137}_{55}Cs \rightarrow ^{137}_{56}Ba + \beta^- + \bar{\nu}_e$



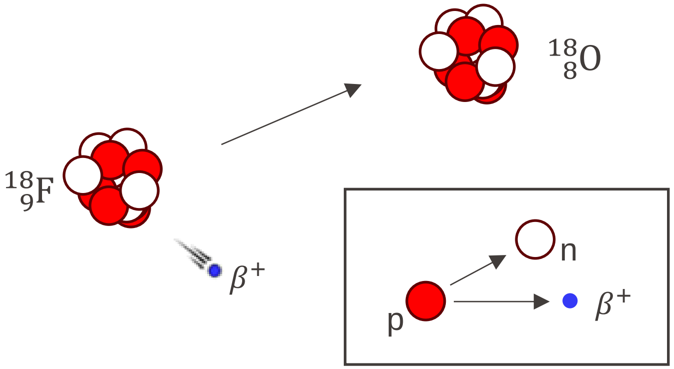
- **Energy release:** in the range of a few keV to ~1 MeV.
- **Penetration:** higher than α particles; stopped by a few mm of plastic or glass.
- **Safety**: can cause skin burns; internal exposure is dangerous!

Radioactive Decay Modes – β^+ Decay

- Occurs in **proton-rich nuclei**, converting a proton into a neutron + positron or β^+ particle (emitted together with a neutrino ν_e).

- Atomic number Z **decreases by 1 (new element is formed);** mass number A **remains unchanged.**

Example: $^{18}_{9}F \rightarrow {}^{18}_{8}O + \beta^+ + \nu_e$



- **Energy release:** in the range of a few keV to ~2 MeV

- **Penetration:** Similar to β^- particles, but annihilates with an electron upon contact

- **Safety**: positron annihilation produces gamma radiation (511 keV photons), which can penetrate further than β

Exercise – Complete the decay

1. $^{210}_{84}\mathrm{Po} \rightarrow\ ? + {}^{4}_{2}\mathrm{He}$

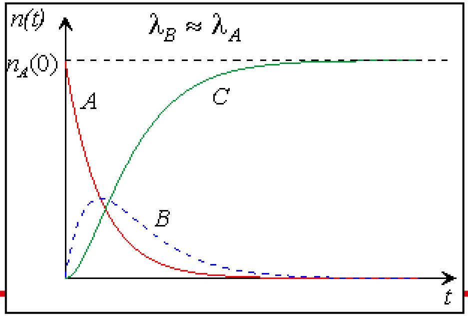
2. $^{131}_{53}\mathrm{I} \rightarrow\ ? + {}^{0}_{-1}\beta + \overline{\nu_e}$

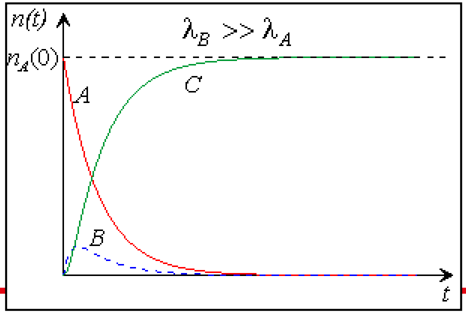
3. $^{40}_{19}\mathrm{K} \rightarrow\ ? + {}^{0}_{1}\beta + \nu_e$

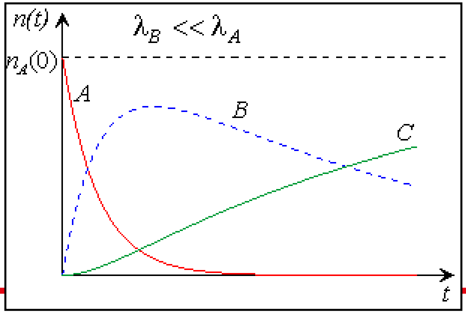
4. $? \rightarrow {}^{140}_{55}\mathrm{Cs} + {}^{109}_{43}\mathrm{Tc} + 3\,{}^{1}_{0}\mathrm{n}$

Decay Chains

- The decay of a nucleus can lead to a nucleus that is also radioactive leading to a chain.
- It applies to natural radioactivity (e.g. $^{232}_{90}\mathrm{Th}$) but also to fission products.
- In general, we can build a system $$\begin{cases} \frac{dN_1(t)}{dt} = -\lambda_1 N_1(t) \\ \frac{dN_2(t)}{dt} = -\lambda_2 N_2(t) + \lambda_1 N_1(t) \\ \frac{dN_3(t)}{dt} = -\lambda_3 N_3(t) + \lambda_2 N_2(t) \\ \text{etc.} \end{cases}$$
- For a simple case (A→B→C) with C stable, there are 3 main cases:





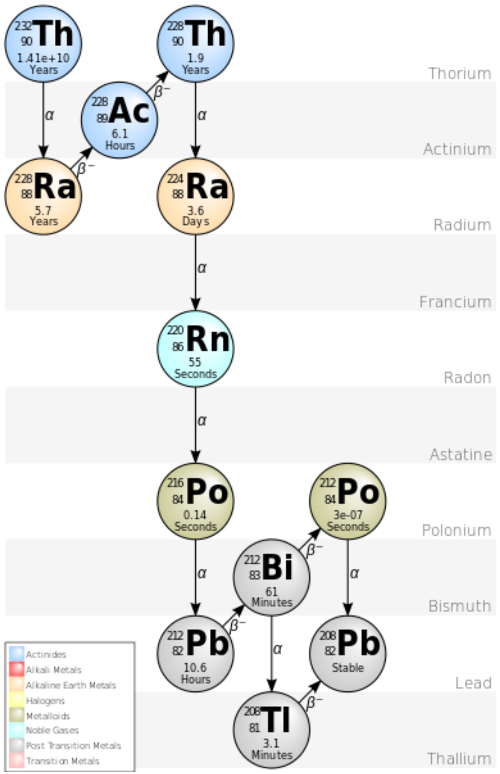


Radioactive or secular equilibrium

- Occurs when the half-life of the parent nucleus is significantly longer than that of all its daughter nuclides (or $\lambda_1 \ll \lambda_2$)
- Time to equilibrium will be short relative to the parent nucleus's half-life, but long compared to the half-lives of daughter nuclides.
- At equilibrium, the rate of decay of each nuclide (dN/dt) in ≈ zero so: $\lambda_1 N_1 \approx \lambda_2 N_2 \approx \lambda_3 N_3$ etc.

Consequences:

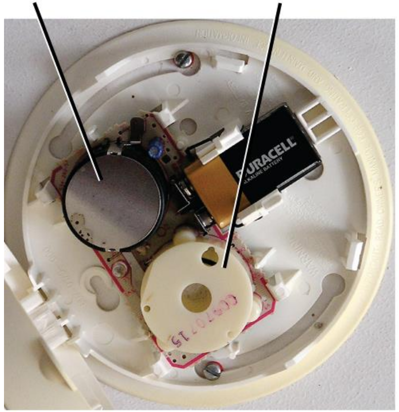
- Activities of all nuclides in the chain become equal at equilibrium…
- …but the nuclide concentration is different $\frac{N_i}{N_1} = \frac{\lambda_1}{\lambda_i} = \frac{T_i}{T_1}$
- The shorter the half-life, the lower the concentration of the nuclide in the natural state.

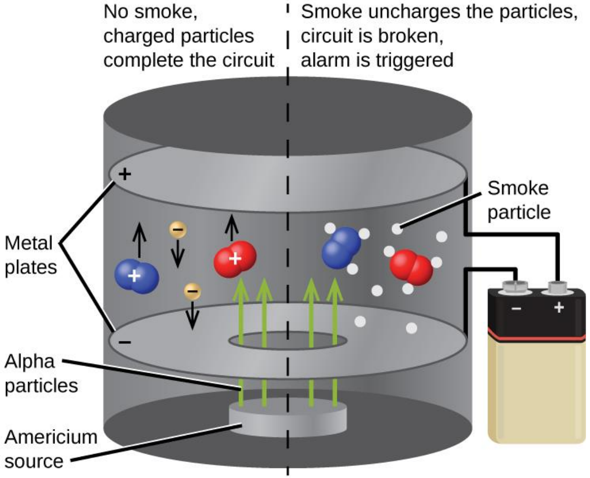


How Do Smoke Detectors Work?

Many household smoke detectors use $^{241}_{95}\mathrm{Am}$!

- A small $^{241}_{95}\mathrm{Am}$ source is placed in a ionization chamber between two electrically charged plates.
- With no smoke: α particles ionize air molecules, creating a small electric current between plates.
- When smoke enters: smoke particles absorb ions, reducing the current, triggering the alarm.





No safety concerns:

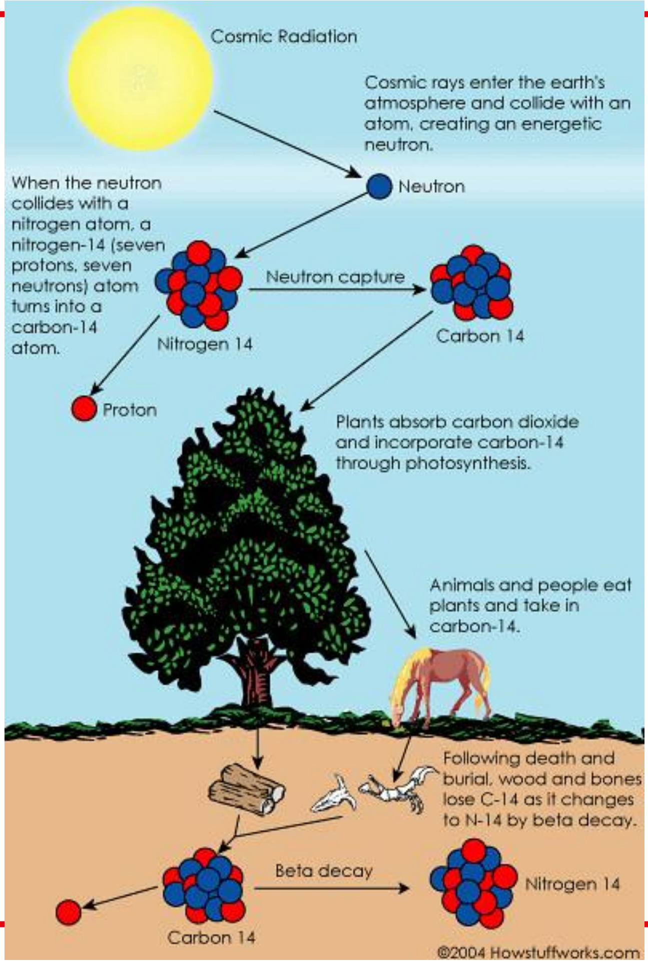
- α particles have low penetration → cannot escape the detector casing.
- Very small amounts (~0.3 µg per detector) are used, posing no radiation risk in normal use.

Carbon Dating – Determining the Age of Ancient Artifacts

- ${}^{14}_{6}\mathrm{C}$ is naturally found in the atmosphere.
- Living organisms absorb ${}^{14}_{6}\mathrm{C}$ through CO_2 exchange, maintaining a ***constant ratio of*** $\boldsymbol{{}^{14}_{6}C}$ ***to*** $\boldsymbol{{}^{12}_{6}C}$.
- When an organism dies, it stops absorbing carbon, and ${}^{14}_{6}\mathrm{C}$ begins to decay with $T_{1/2}$ of 5'730 years:

$$ {}^{14}_{6}\mathrm{C} \rightarrow {}^{14}_{7}\mathrm{N} + \beta^{-} + \overline{\nu_e} $$

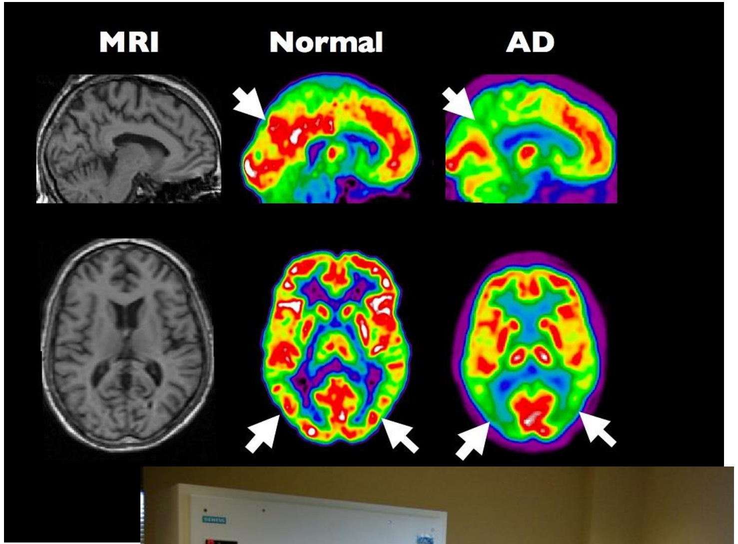
- The remaining ${}^{14}_{6}\mathrm{C}$ fraction in a sample indicates its age! (Carbon dating is effective for objects up to ~50,000)



PET scans and radio-isotopes

PET scans use β^+ decay to create detailed 3D images of metabolic processes inside the body.

- A radioactive tracer (e.g., ${}^{18}_{9}F$) is injected into the patient.
- The tracer undergoes β^+ decay, emitting a positron.
- The positron annihilates with an electron, producing two 511 keV gamma photons.
- PET scanners detect these gamma rays to form an image.



Production of PET Tracers:

- PET tracers like ${}^{18}_{9}F$ are produced in a **cyclotron**, a particle accelerator that bombards a target material with protons to induce nuclear reactions

$${}^{18}_{8}O + p \rightarrow {}^{18}_{9}F + n$$

- PET tracers have short half-lives (e.g., ${}^{18}_{9}F$: 110 min) → they must be produced close to hospitals.

Exercise – Smoke Detectors & Carbon Dating

1. Calculate the activity of a new smoke detector
   - Assume 0.3 µg of $^{241}_{95}\mathrm{Am}$ at the start.
   - Consider half life of 432.6y
2. How much $^{241}_{95}\mathrm{Am}$ is left after ten-years?

3. Archaeologists find a wooden artifact and want to determine its age using carbon-14 dating. The wood sample has 25% of the original $^{14}_{6}C$ activity remaining. Given that $T_{1/2} = 5730\ y$, estimate the age of the sample.

Exercise – Production of a Radio-Isotope

- A medical facility requires a daily supply of 5 GBq of $^{18}_{9}F$ at the time of delivery. The production process involves bombarding an enriched $^{18}_{8}O$ target with protons in a cyclotron that is **1hr away**.

- **Estimate how long the cyclotron must irradiate the target to produce the required activity.**

- **Assumptions**:
  - The half-life of $^{18}_{9}F$ is 109.7 min.
  - The production rate in the cyclotron is constant and equal to 1E+10 atoms/hr
  - The decay loss during transport should be considered.