

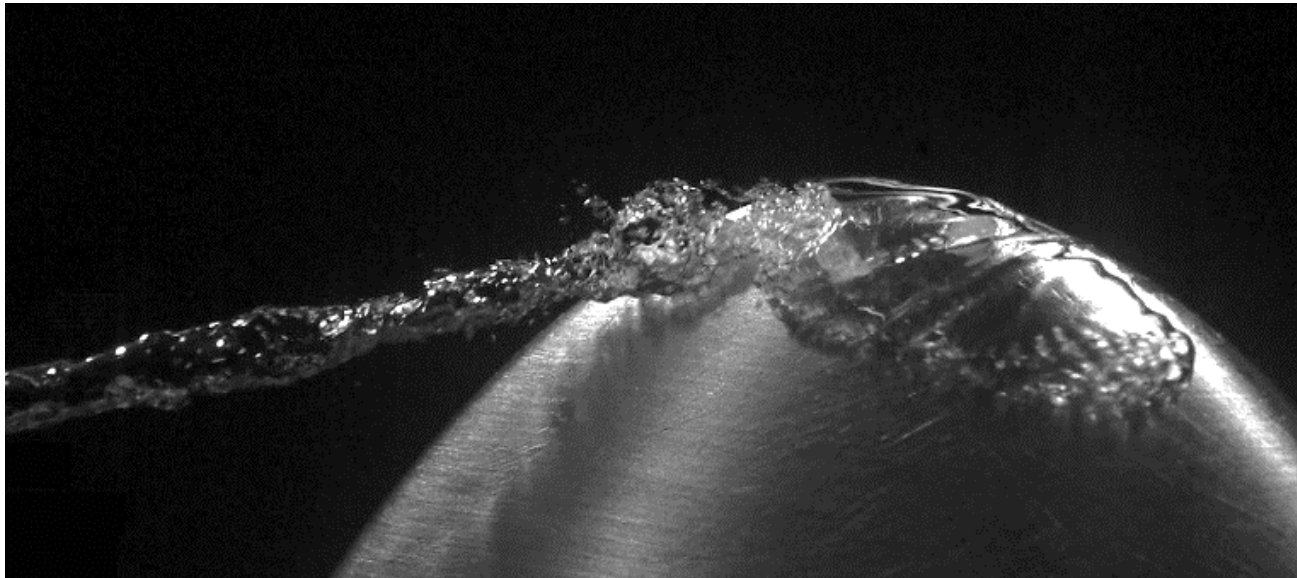
ÉCOLE POLYTECHNIQUE FÉDÉRALE DE LAUSANNE

SGM - 6th & 8th Semester, Fall 2022

CAVITATION AND INTERFACE PHENOMENA

Chapter 5: Vortex Cavitation

5.2: Effect of Gas Content on Tip Vortex Cavitation



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Velocity and pressure fields in a vortex

Lamb-Oseen vortex model:

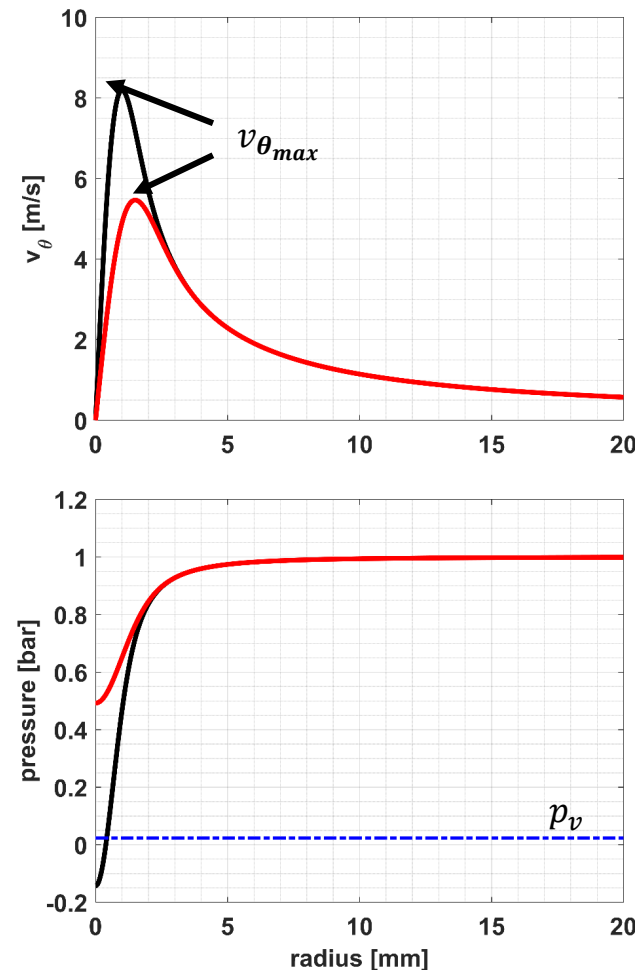
$$v_{\theta}(r) = \frac{\Gamma}{2\pi r} \left(1 - e^{-1.256(r/r_c)^2} \right)$$

- r_c is the **viscous core radius** (location of $u_{\theta_{max}}$).
- Γ is the vortex strength or circulation.

Let's try an example. Consider a vortex with $\Gamma = 0.07 \frac{m^2}{s}$

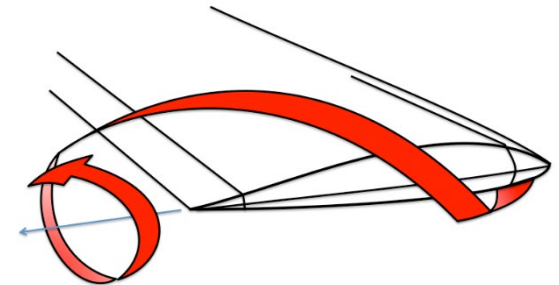
- What is the effect of the viscous core?
 - Velocity profile for $r_c = 1$ and 1.5 mm
 - Significant influence on $v_{\theta_{max}}$ and p_{min}

- Pressure profile $\left(\frac{\partial p}{\partial r} = \frac{\rho v_{\theta}^2(r)}{r} \right)$

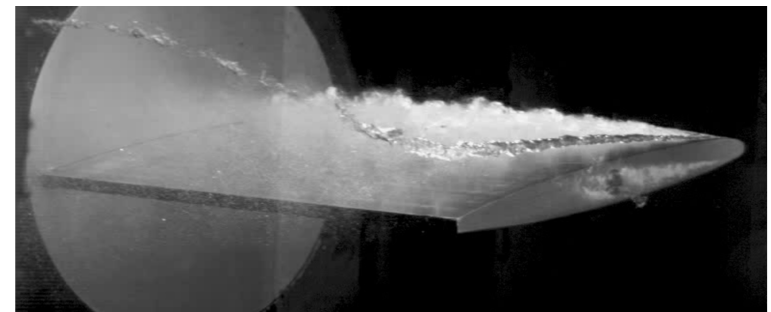


Tip vortex flow and cavitation

- **Finite-span lifting bodies** generate tip vortices
- **Low static pressures** develop in the vortex core
 - Tip vortices are very prone to cavitation
 - Usually the **first** type of cavitation that incepts



- Occurrence of Tip Vortex Cavitation (TVC) may lead to **severe erosion, noise emission, and structural vibrations**
- Very sensitive to gas content



Visualization of TVC on NACA-0009 hydrofoil

What is gas content?

- Micro bubbles → Required for cavitation inception

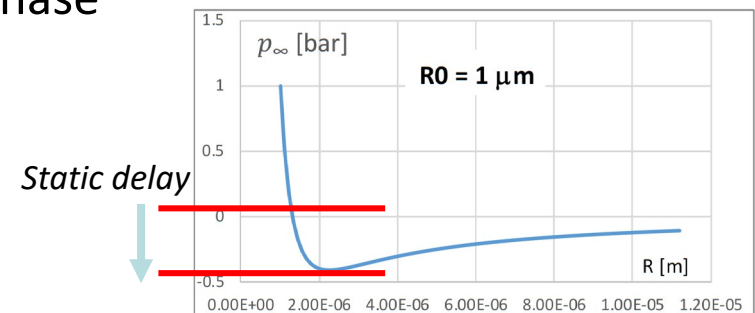
- Critical pressure and radius:

$$R_c = \sqrt{\frac{3p_{g0}R_0^3}{2S}} \quad \text{and} \quad p_c = p_v - \frac{4S}{3R_c}$$



- Dissolved gases → molecular Oxygen, Nitrogen, etc.

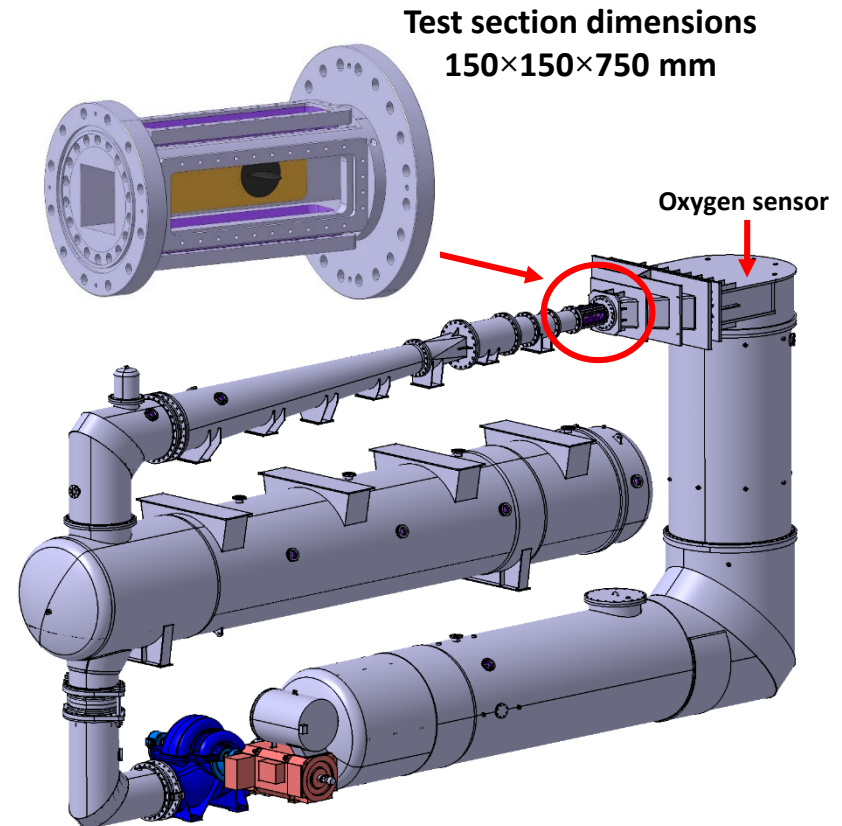
- Diffusion into the vaporous phase



Experimental Setup

High-speed cavitation tunnel

- Maximum velocity → **50 m/s**
- Maximum pressure → **16 bars**
- Velocity measurements
 - Stereo-PIV & LDV
- Various sensors
 - Oxygen saturation level
 - Lift and drag forces
- Flow visualization
 - Still photography
 - High-speed visualization



Experimental Setup

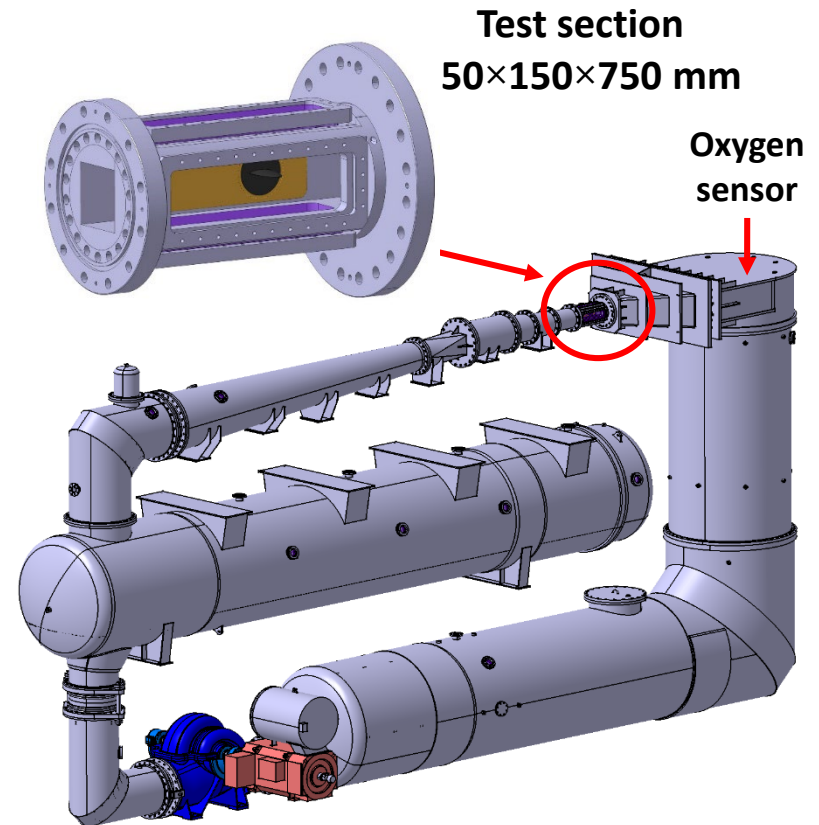
High-speed cavitation tunnel

- Maximum velocity $\rightarrow$ 50 m/s
- Maximum pressure $\rightarrow$ 16 bars
- Control of gas content level:

Super-cavitation
Degassing



Free-surface flow
Engassing



Experimental Setup

– Cross-section:

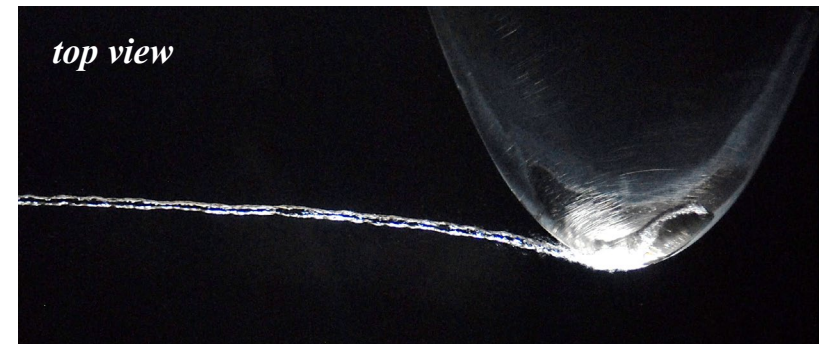
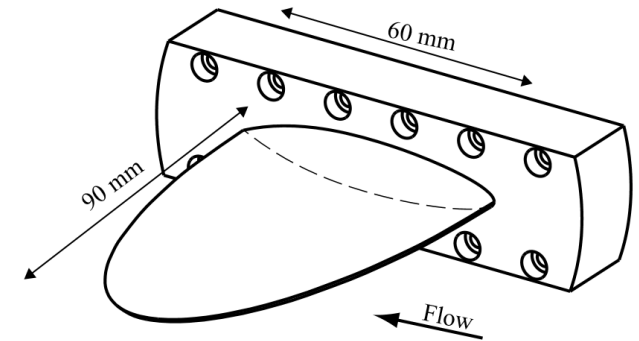
- NACA 16-020
- Symmetrical

– Planform:

- Elliptical

– Main feature:

- Well-defined tip vortex

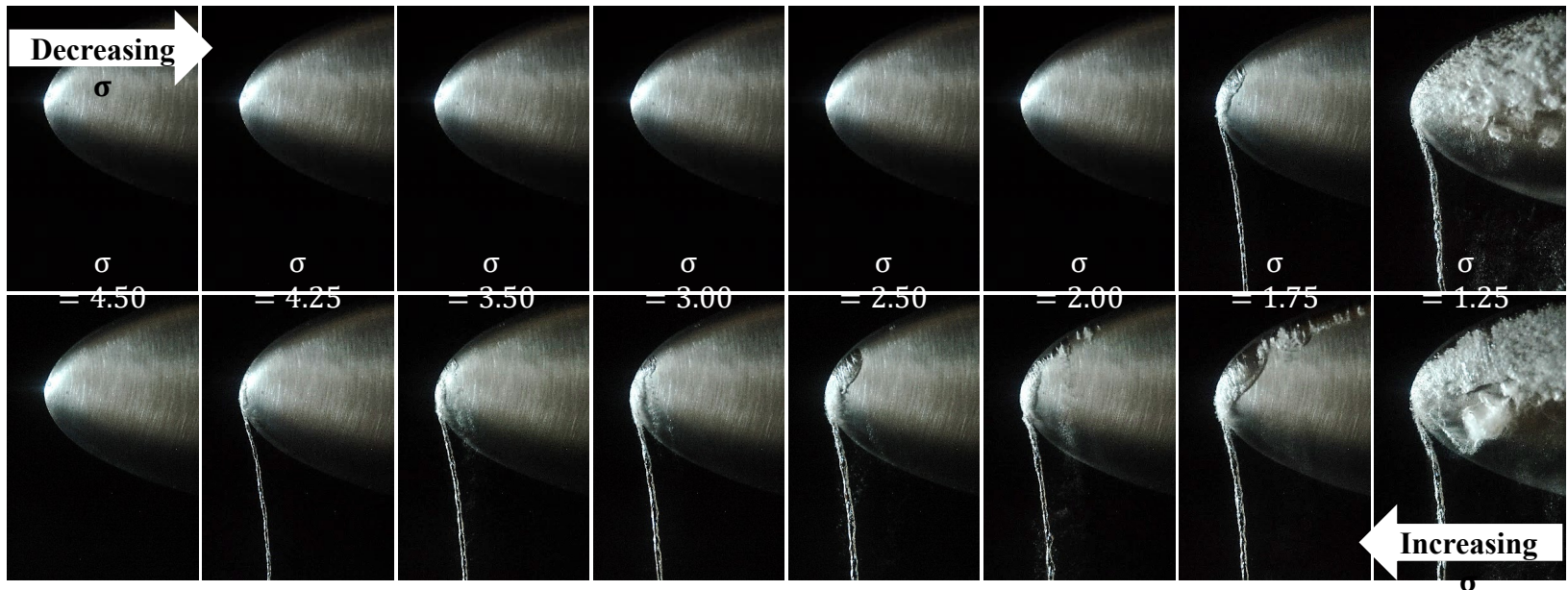


TVC Hysteresis

Hysteresis in TVC inception and desinence

Large inception-desinence thresholds observed for specific flow conditions

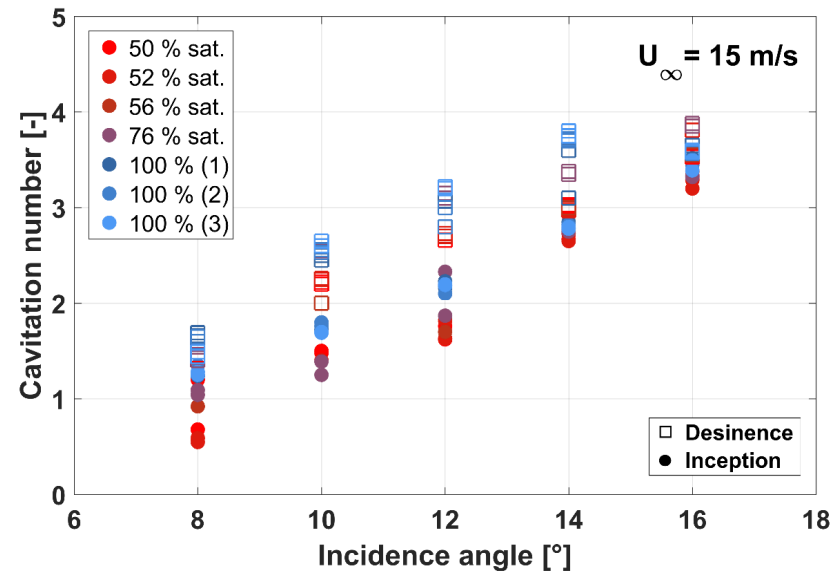
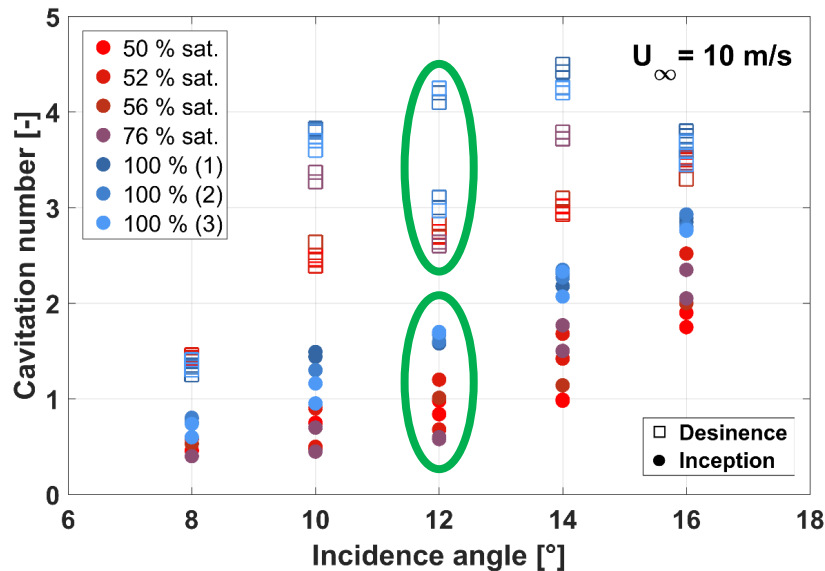
$$U_{\infty} = 10 \text{ m/s}, \alpha = 12^{\circ}$$



Ali Amini, Martino Reclari, Takeshi Sano, Masamichi Iino, Matthieu Dreyer, and Mohamed Farhat. "On the physical mechanism of tip vortex cavitation hysteresis." *Experiments in Fluids* 60, no. 7 (2019): 118.

TVC inception/desinence tests

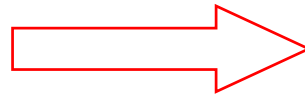
- σ_i depends on gas content $\rightarrow$ more dissolved $O_2 \rightarrow$ higher concentrations of large nuclei
- σ_d depends on gas content $\rightarrow p_d$ is not necessarily equal to p_v and is usually higher
- The extent of the hysteresis depends on the bulk flow parameters.



TVC hysteresis

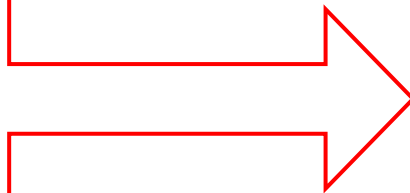
- The hysteresis depends on

- The dissolved gas content



- Freestream velocity

- Angle of attack

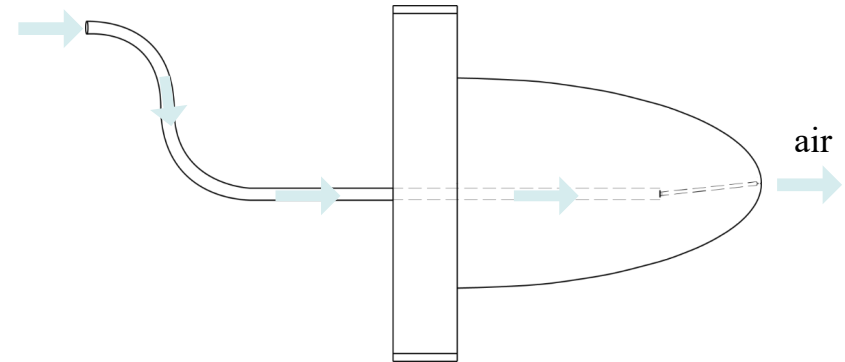
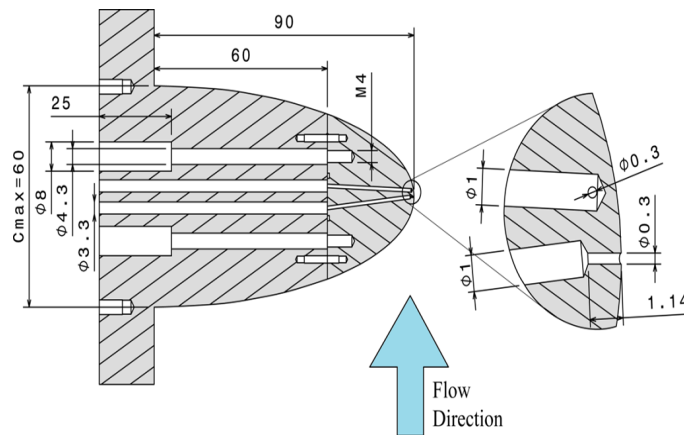


- Mass diffusion into TVC
 - Gaseous cavitation

- Boundary layer characteristics
 - Wake flow patterns

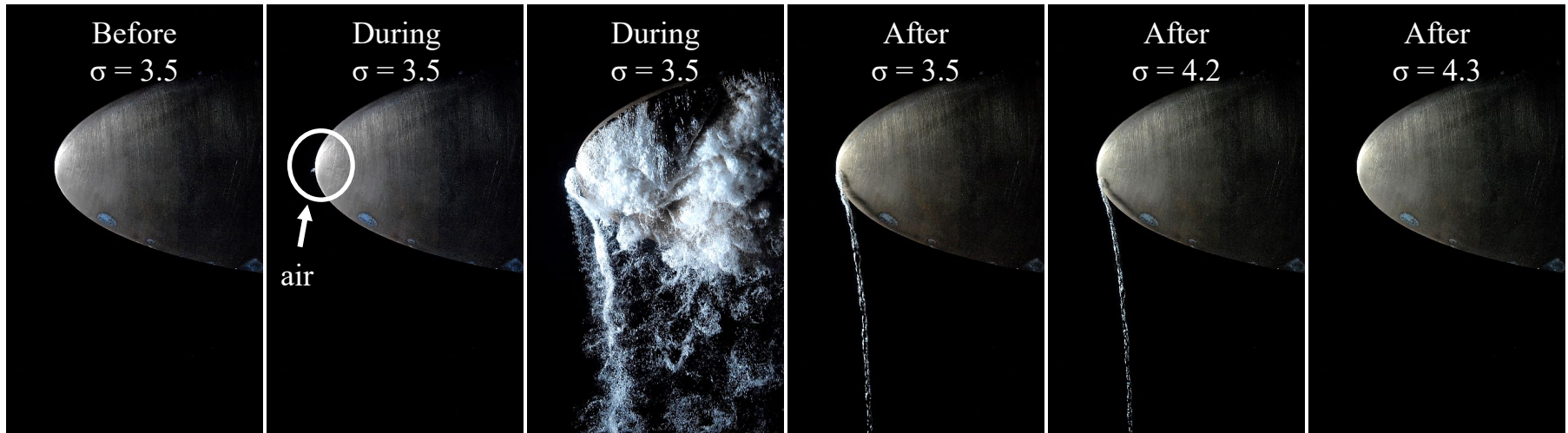
Transient air injection

- A specific hydrofoil equipped with a 0.3-mm ejection channel at the tip
- Flow conditions: $U_{\infty} = 10 \text{ m/s}$, $\alpha = 12^\circ$
 - Initial saturation level $\sim 98 \%$
 - At injection: $\sigma_{initial} = 3.5$ (No cavitation)



Transient air injection

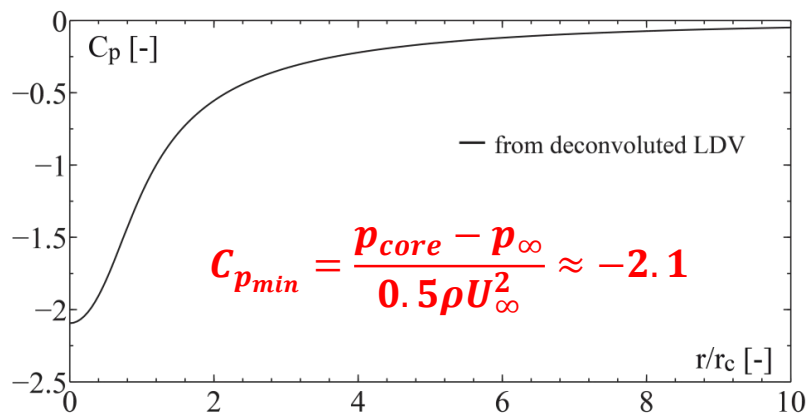
- Flow conditions: $U_\infty = 10 \text{ m/s}$, $\alpha = 12^\circ$, $\sigma_{initial} = 3.5$



Transient air injection

- What is the pressure in the vortex core?

➤ LDV measurements have revealed that for $U_\infty = 10 \text{ m/s}$ and $\alpha = 12^\circ$:



Calculation of $C_{p_{min}}$ by Dr. M. Dreyer in his PhD thesis (EPFL)

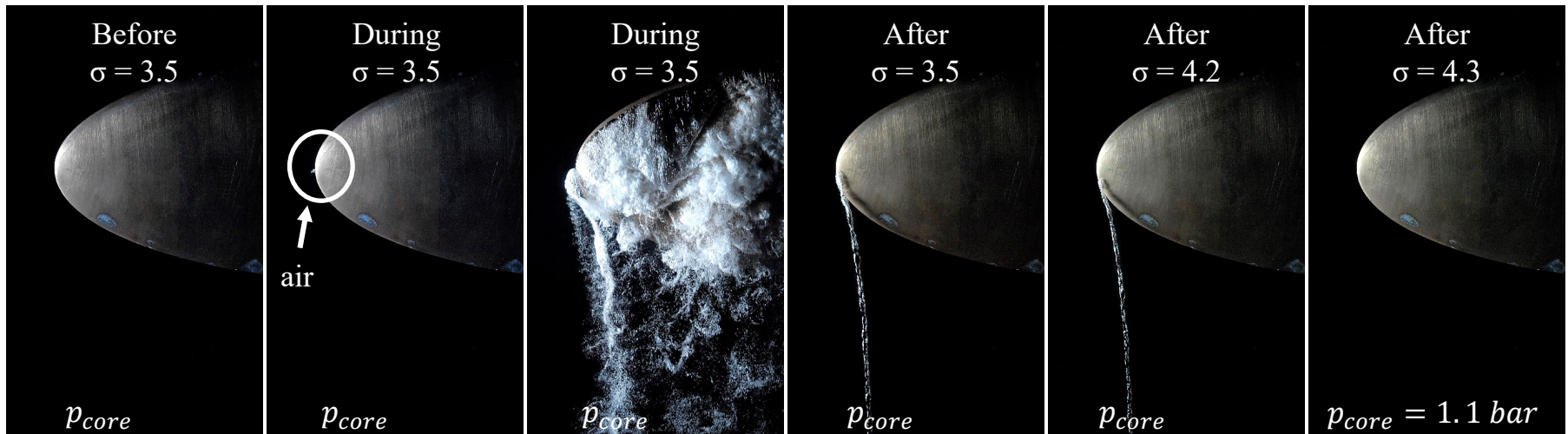
$C_{p_{min}}$ is equal to -2.1

From the definitions of cavitation number and pressure coefficient ($p_v \approx 0$):

$$p_{core} = \frac{1}{2} \rho U_\infty^2 (\sigma + C_{p_{core}})$$

Transient air injection

- Flow conditions: $U_{\infty} = 10 \text{ m/s}$, $\alpha = 12^\circ$, $\sigma_{initial} = 3.5$
- Pressure coefficient at the core is measured: $C_{p_{min}} \approx -2.1$

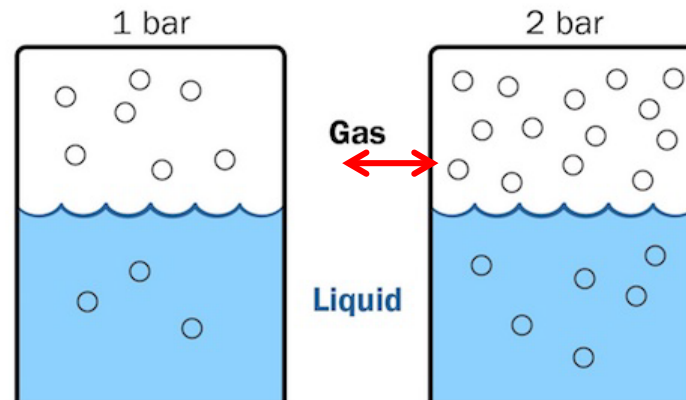


TVC disappears at $\approx 1 \text{ bar}$, which is the initial saturation pressure (p_{sat_i}) of water

Transient air injection

Henry's law: solubility of a gaseous species in a liquid is proportional to its partial pressure

$$c = \mathcal{H} p_g$$



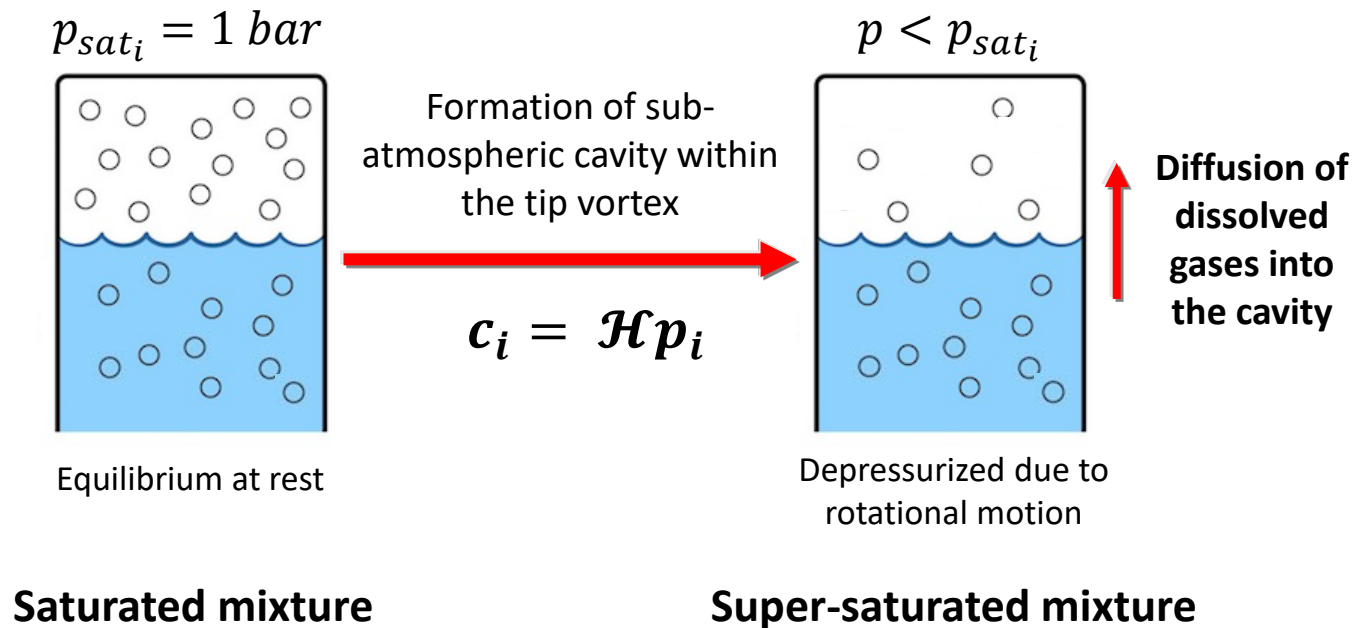
c : concentration of the i species in the liquid phase $[\frac{mol}{m^3}]$

$\mathcal{H}$: Henry's law constant $[\frac{mol}{m^3 \cdot atm}]$ (varies for different substances and with temperature)

p_g : partial pressure of the i species in the gas phase $[atm]$

Transient air injection

What happens for a cavitating tip vortex?



Transient air injection

□ Once a cavity forms within the tip vortex:

- Water flowing around it is exposed to a gaseous medium with a pressure (p_{core}) that deviates from the initial saturation condition (p_{sat_i})
- **While** there is **no change** in the amount of dissolved gases.

□ Therefore in our case:

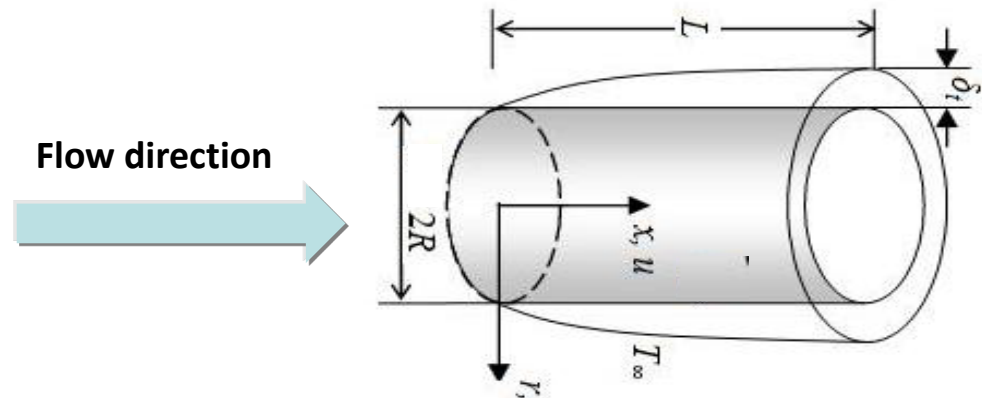
- For $\sigma > 2.1 \rightarrow p_{min} > p_v$
 - No more vaporization occurs.
- For $\sigma < 4.2 \rightarrow p_{min} < p_{atm} = p_{sat_i}$
 - Water flowing along the cavity is **supersaturated**. As a result, the dissolved air diffuses into the cavity and forms a gaseous TVC.

Modelling the outgassing process

- A concentration boundary layer develops along the *cylindrical* cavity:

$$\frac{\partial c}{\partial t} + v_r \frac{\partial c}{\partial r} + \frac{v_\theta}{r} \frac{\partial c}{\partial \theta} + v_z \frac{\partial c}{\partial z} = \frac{1}{r} \frac{\partial}{\partial r} \left(D r \frac{\partial c}{\partial r} \right) + \frac{1}{r^2} \frac{\partial}{\partial \theta} \left(D \frac{\partial c}{\partial \theta} \right) + \frac{\partial}{\partial z} \left(D \frac{\partial c}{\partial z} \right)$$

- c : concentration of the species
- D : mass diffusion coefficient

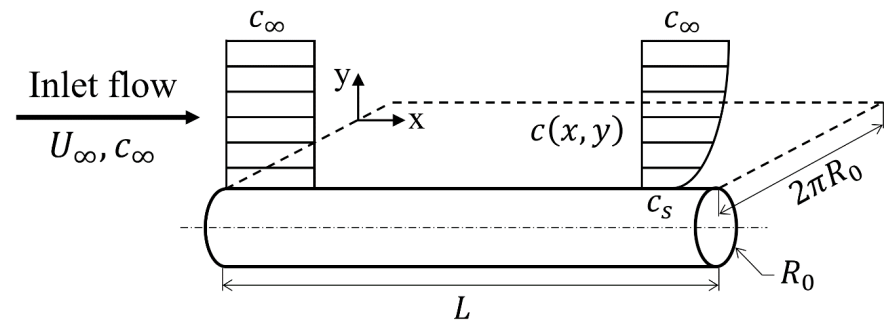


Development of a concentration boundary layer similar to a thermal boundary layer

Modelling the outgassing process

- **Outgassing** → A convective diffusion that occurs along a hollow cylinder
- We employ the ‘**wrap-around**’ technique → Cylindrical to planar coordinates
- Two-dimensional Cartesian species transfer equation:

$$\frac{\partial c}{\partial t} + u \frac{\partial c}{\partial x} + v \frac{\partial c}{\partial y} = D \left(\frac{\partial^2 c}{\partial x^2} + \frac{\partial^2 c}{\partial y^2} \right)$$



Modelling the outgassing process

□ Assumptions:

- Constant velocity field: $\mathbf{u} = U_\infty, \mathbf{v} = 0$
- Steady state
- Large Peclet numbers ($Pe = \frac{U_\infty L}{D}$)
 - Diffusion in x is negligible compared to diffusion in y
- A concentration boundary layer develops along the TVC interface
 - The thickness (δ) scales with $\sqrt{Dx/U_\infty}$

$$\frac{\partial c}{\partial t} + u \frac{\partial c}{\partial x} + v \frac{\partial c}{\partial y} = D \left(\frac{\partial^2 c}{\partial x^2} + \frac{\partial^2 c}{\partial y^2} \right)$$



$$\frac{\partial c}{\partial x} = \frac{D}{U_\infty} \frac{\partial^2 c}{\partial y^2}$$

$$BC's \rightarrow \begin{cases} x = 0 & \rightarrow c(0, y) = c_\infty \\ y = 0 & \rightarrow c(x, 0) = c_s \\ y \rightarrow \infty & \rightarrow c(x, \infty) = c_\infty \end{cases}$$

Modelling the outgassing process

- Similarity solution in the form of

$$f(\eta) = \frac{c(\eta) - c_s}{c_\infty - c_s}$$

where $\eta = y/\delta = y\sqrt{U_\infty/Dx}$ is the similarity variable and $f(\eta)$ is the normalized concentration of the dissolved gases.

$$\frac{\partial c}{\partial x} = \frac{D}{U_\infty} \frac{\partial^2 c}{\partial y^2}$$



$$f'' + 0.5f' = 0$$

$$\begin{cases} \eta = 0 & \rightarrow f(0) = 0 \\ \eta \rightarrow \infty & \rightarrow f(\infty) \rightarrow 1 \end{cases}$$

Modelling the outgassing process

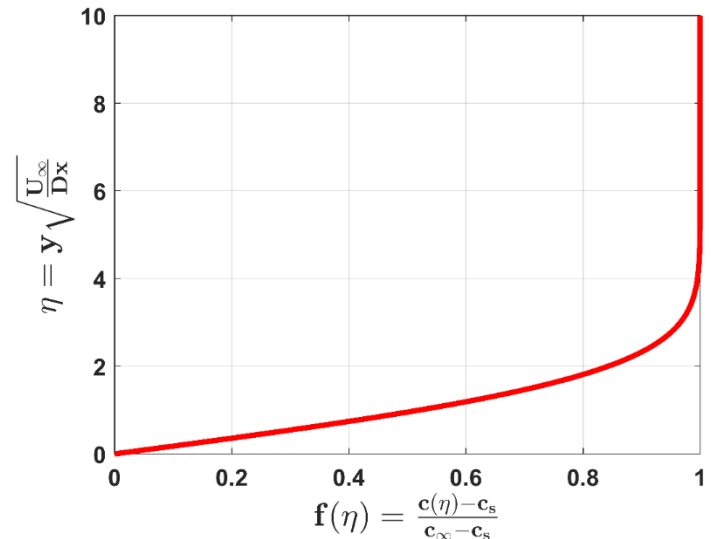
- This equation has an analytical solution:
- Local mass flux into TVC can be calculated by *Fick's second law*:

$$m_y''(x) = -D \left. \frac{\partial c}{\partial y} \right|_{y=0}$$

- Wrapping the solution around the TVC with radius R_0 and taking the integral along the whole length L , the total mass transfer rate in [mol/s] is found to be

$$\dot{M}_{tot} = 4R_0(c_\infty - c_s)\sqrt{\pi U_\infty DL}$$

$$f(\eta) = \text{erf}(\eta/2)$$



Modelling the outgassing process

- Variation of air concentration with time and the characteristic time of the system (from 100% down to 36.8%):

$$\frac{dc}{dt} = -\frac{\dot{M}_{tot}}{V_{tot}} \Rightarrow c(t) = (c_{\infty} - c_s)e^{(-t/\tau)} + c_s$$

$$\tau = \frac{V_{tot}}{4R_0\sqrt{\pi U_{\infty}DL}}$$

- With typical values of:
 - $R_0 = 1 \text{ mm}$, $U_{\infty} = 10 \text{ m/s}$, $L = 1 \text{ m}$
 - $D = 7 \times 10^{-5} \text{ m}^2/\text{s}$ (calculated based on a mixing length model from Parkin & Ravindra (1991)).
 - $\sigma = 2.5$ and $C_{pmin} = -2.1$

Number of volume change per second is 4.25 and:

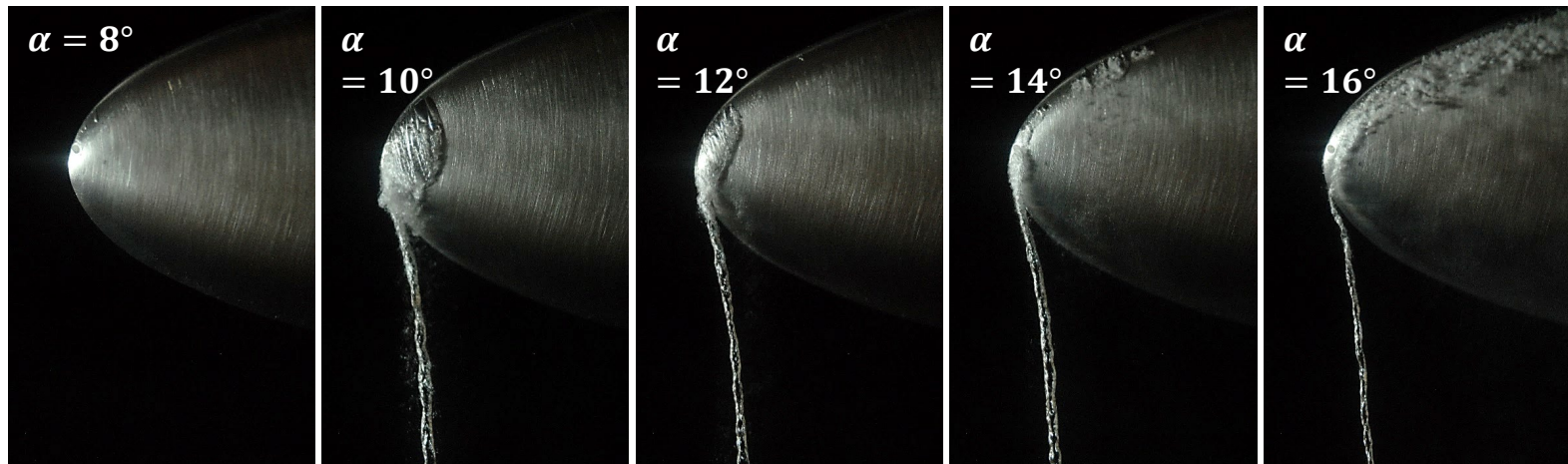
$$\frac{\dot{V}_{diff}}{\dot{V}_{conv}} = 0.43$$

→ $\tau = 6.8 \text{ days} !!!$

Effect of Flow Structures on Outgassing Process

Effect of **incidence angle** on TVC Hysteresis

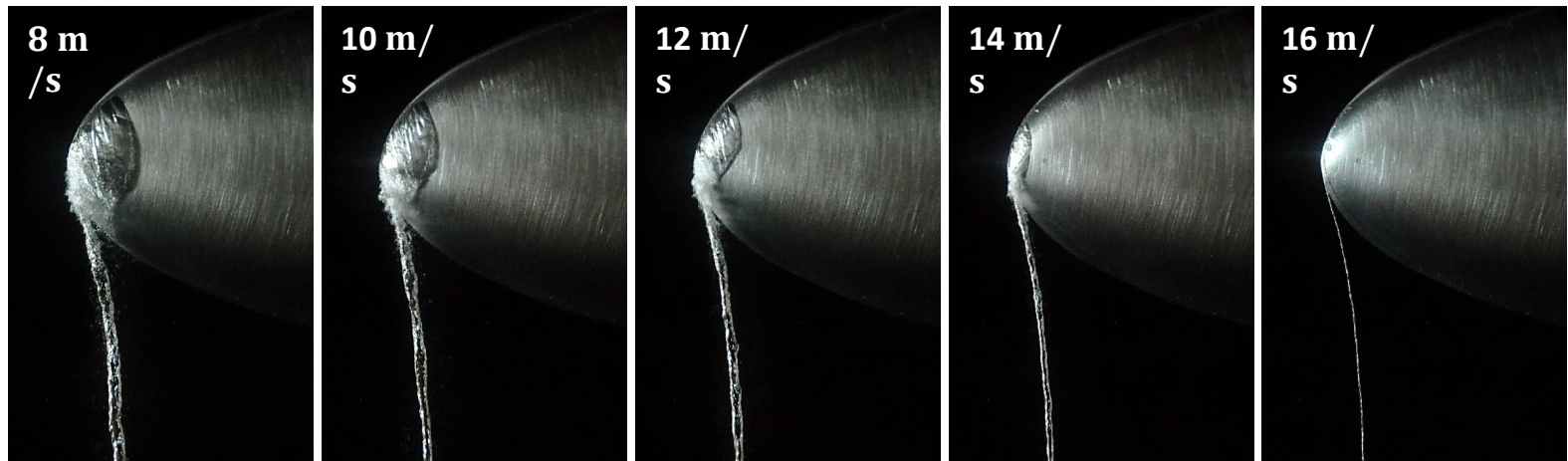
- Flow conditions: $U_{\infty} = 10 \text{ m/s}$, $\sigma = 2.5$



It is observed that the shape and aspect of the cavity at the tip of the hydrofoil changes drastically as the incidence angle is changed.

Effect of **Reynolds number** on TVC Hysteresis

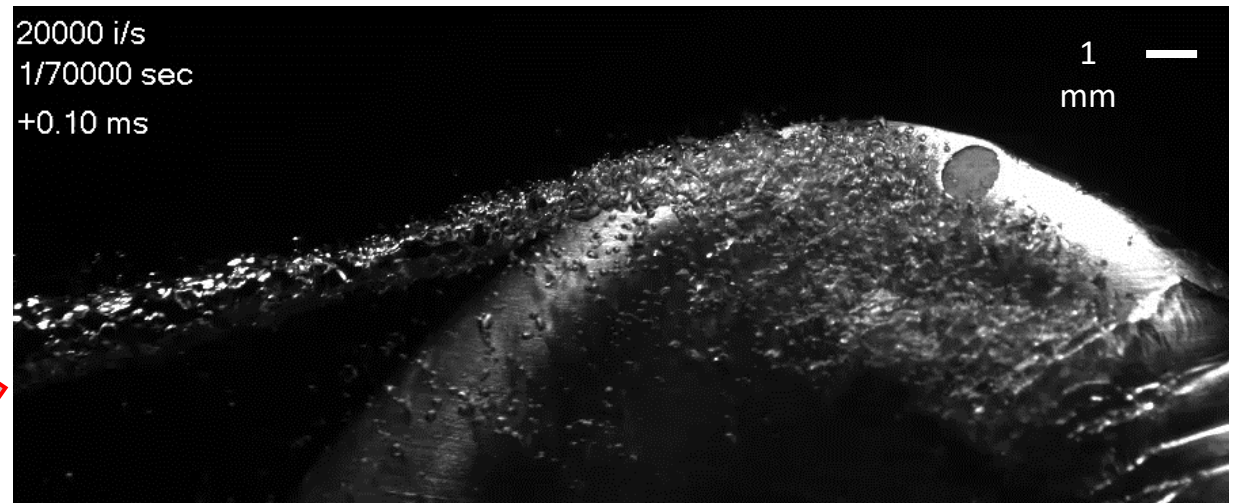
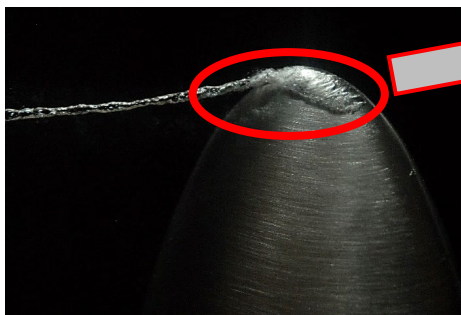
- Flow conditions: $\alpha = 10^\circ$, $\sigma = 2.4$



It is likely that the gas diffusion is enhanced when a laminar separation bubble forms at the leading edge near the tip

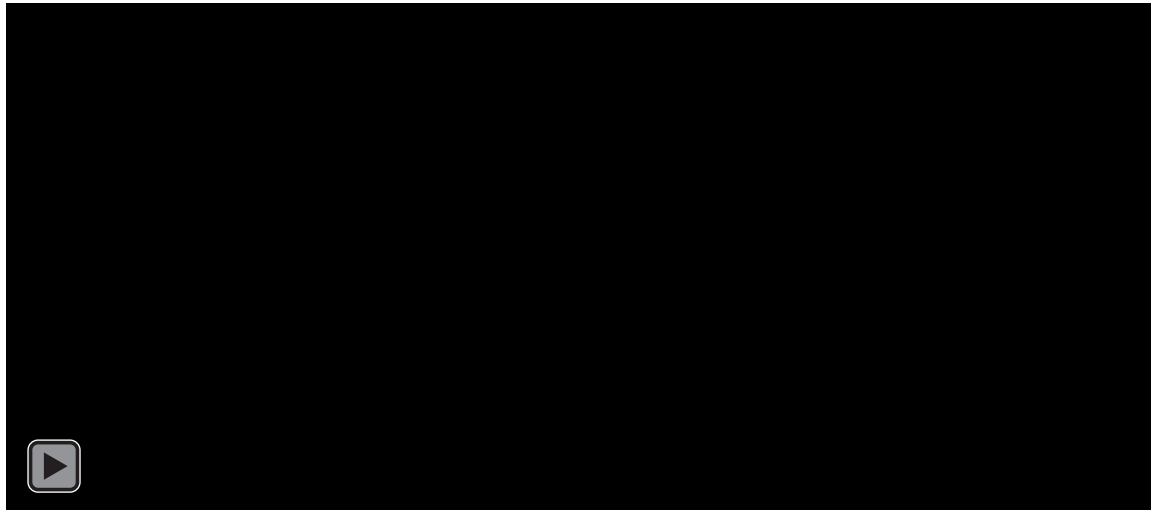
Laminar separation bubble

- Flow conditions: $\alpha = 12^\circ$, $U_\infty = 5.3 \text{ m/s}$, $\sigma = 6.38$



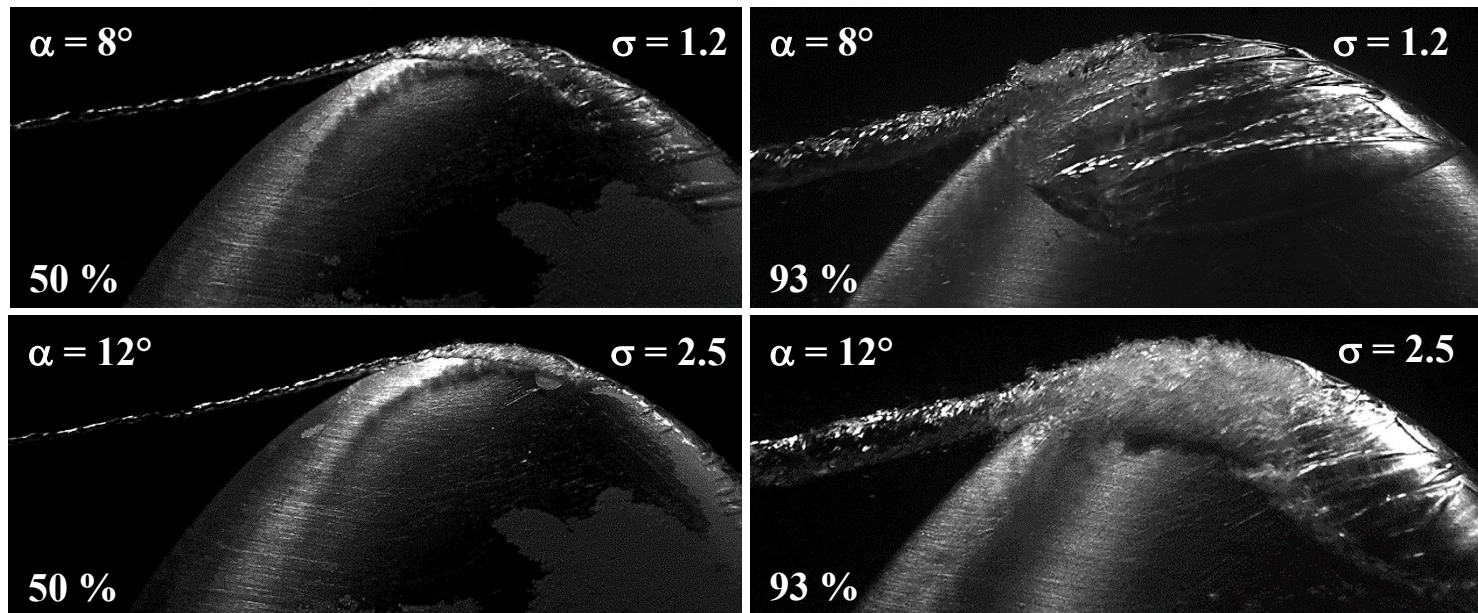
Laminar separation bubble

- Flow conditions: $\alpha = 12^\circ$, $U_\infty = 5.5 \text{ m/s}$, $\sigma = 6.0$



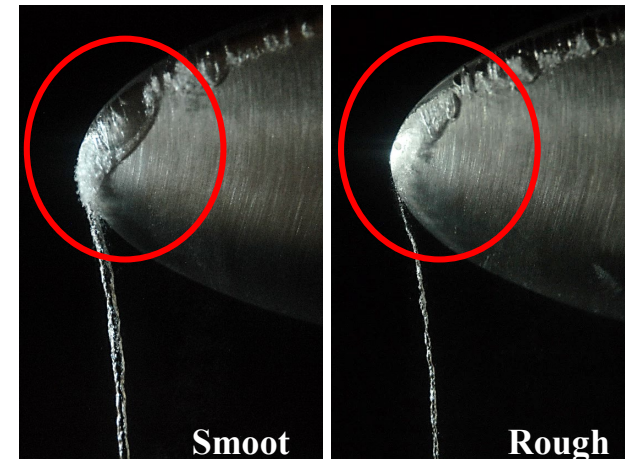
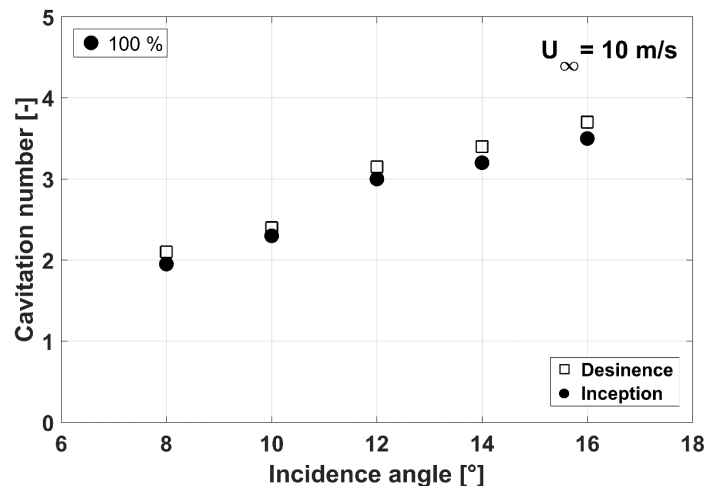
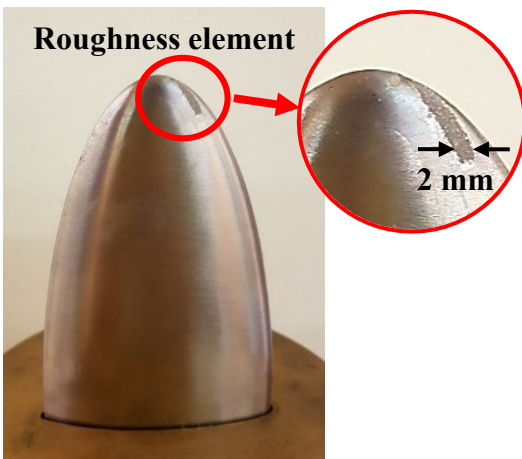
Effect of gas content on TVC

Gas diffusion occurs **spontaneously** for all the cavitating vortices, although it is the **nature of the BL separation** that defines its effectiveness in retarding the desinence.



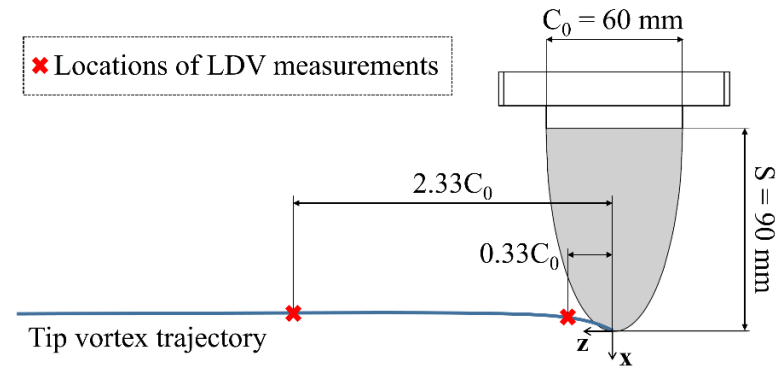
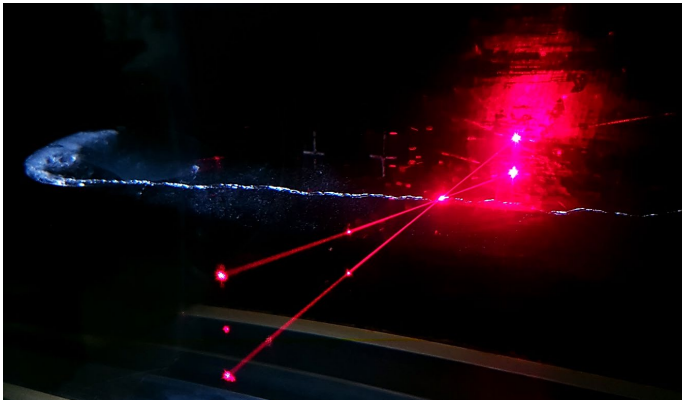
Let's Destroy the Shelter Zone!

- **Roughness** (125- μm -diameter sand) is added to only the **last 10 %** of the span.
- The hysteresis is totally suppressed due to turbulence.



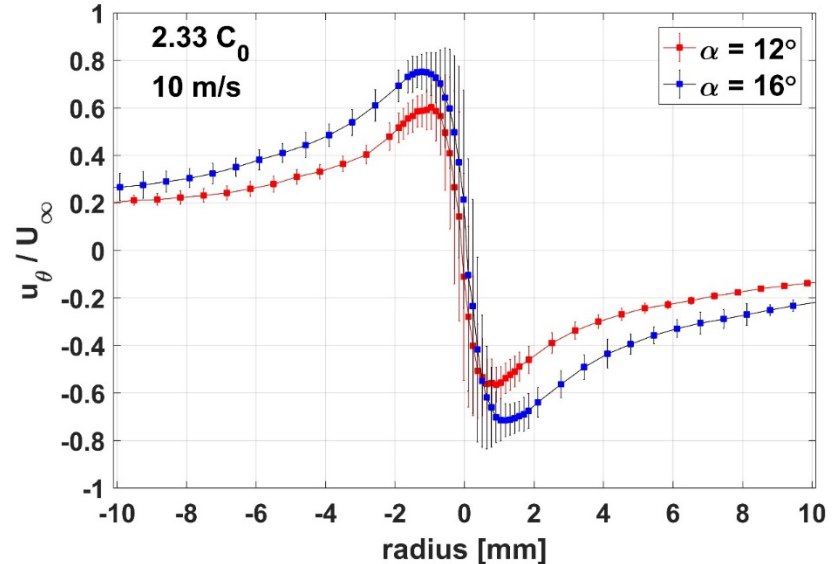
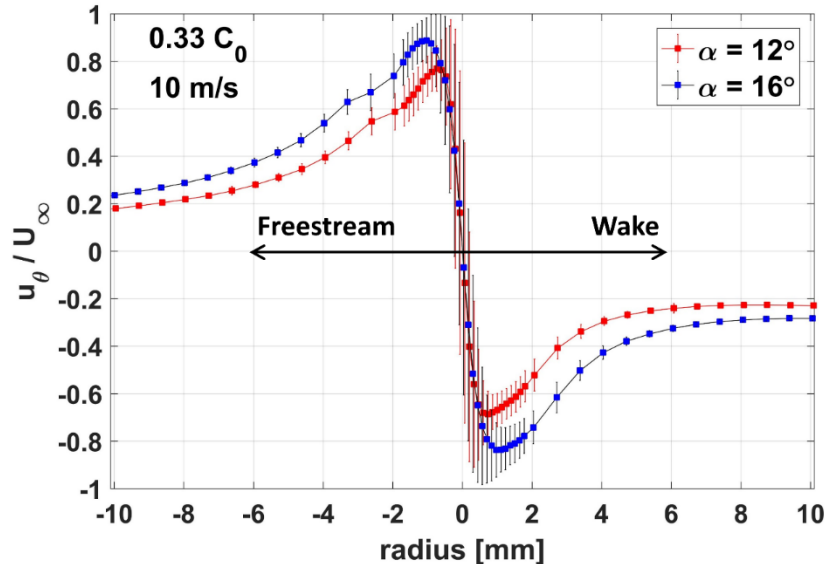
Velocity measurements in the wake flow

- LDV measurements of the tangential and axial velocity components



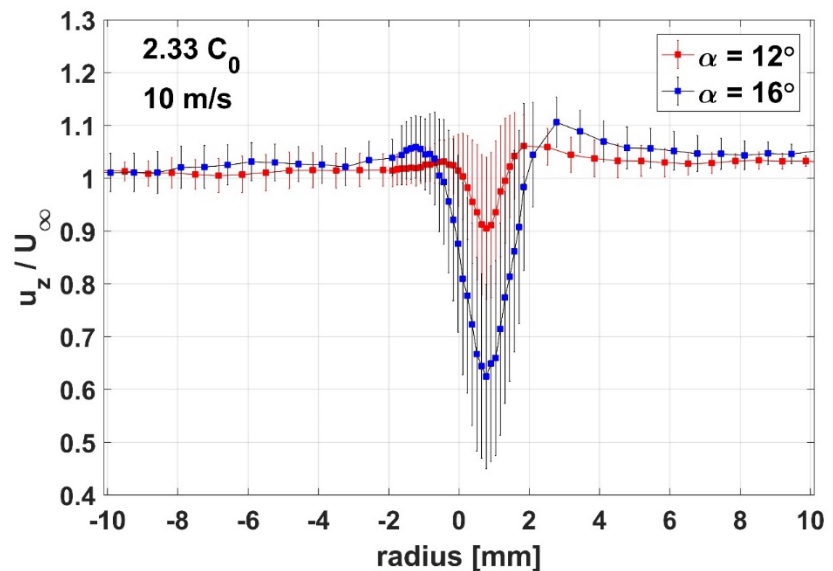
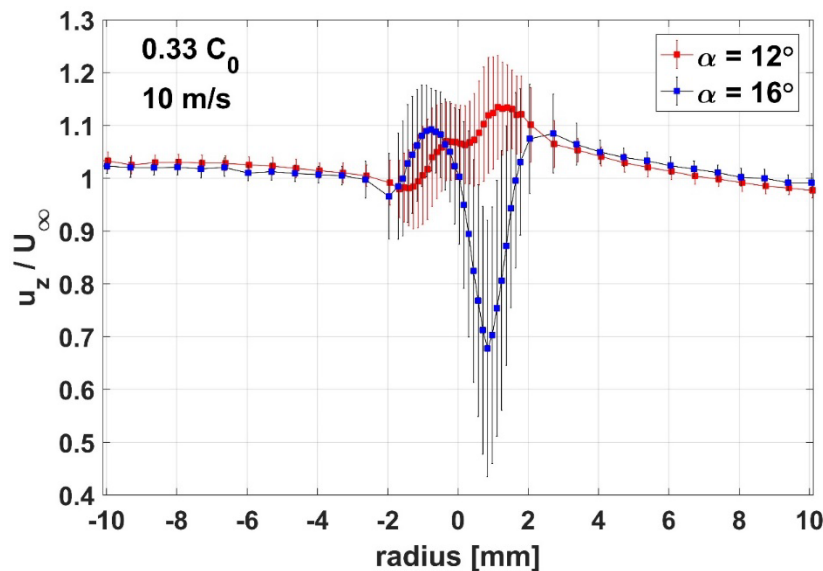
Velocity measurements in the wake flow

- LDV measurements of the **tangential** velocity component:
 - Evolution of tip vortex velocity profiles with the downstream distance from the tip
 - Similar patterns are observed for both angles of attack

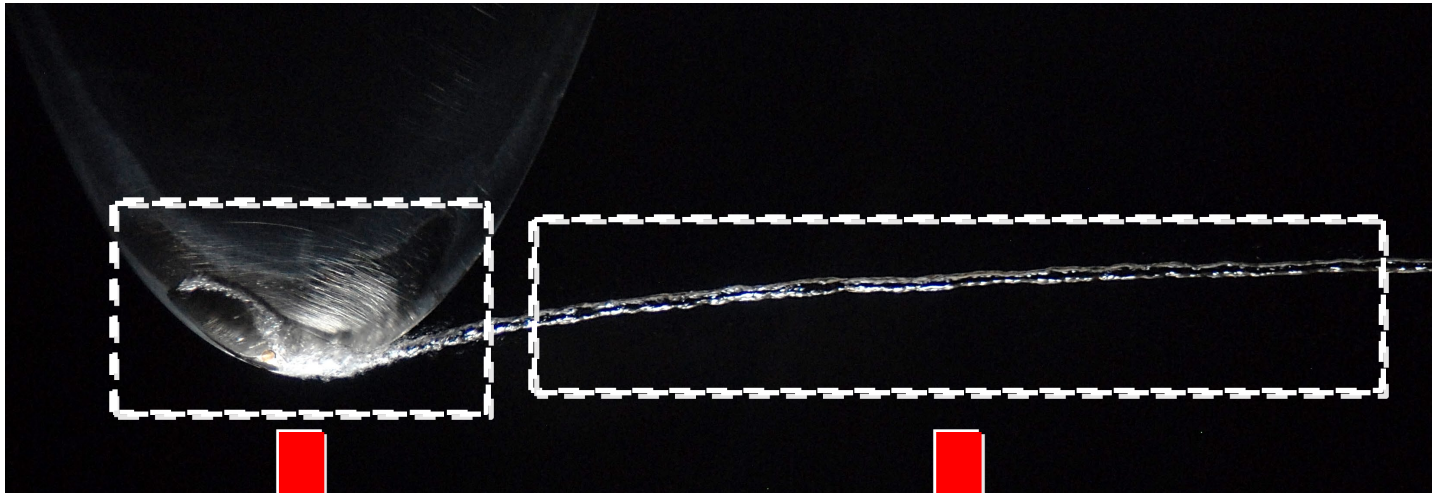


Velocity measurements in the wake flow

- LDV measurements of the **axial** velocity component:
 - Evolution of tip vortex velocity profiles with the downstream distance from the tip
 - A jet-like velocity profile for $\alpha = 12^\circ$, while a wake-like one occurs for $\alpha = 16^\circ$.



Conclusion



- Recirculation area
- Roll-up process
- Shelter for TVC

- Air diffusion into the cavity
- Development of concentration BL
- Jet / wake-like behavior