Heat Pump Systems

Summary W8

Prof. J. Schiffmann

École
polytechnique
fédérale
de Lausanne

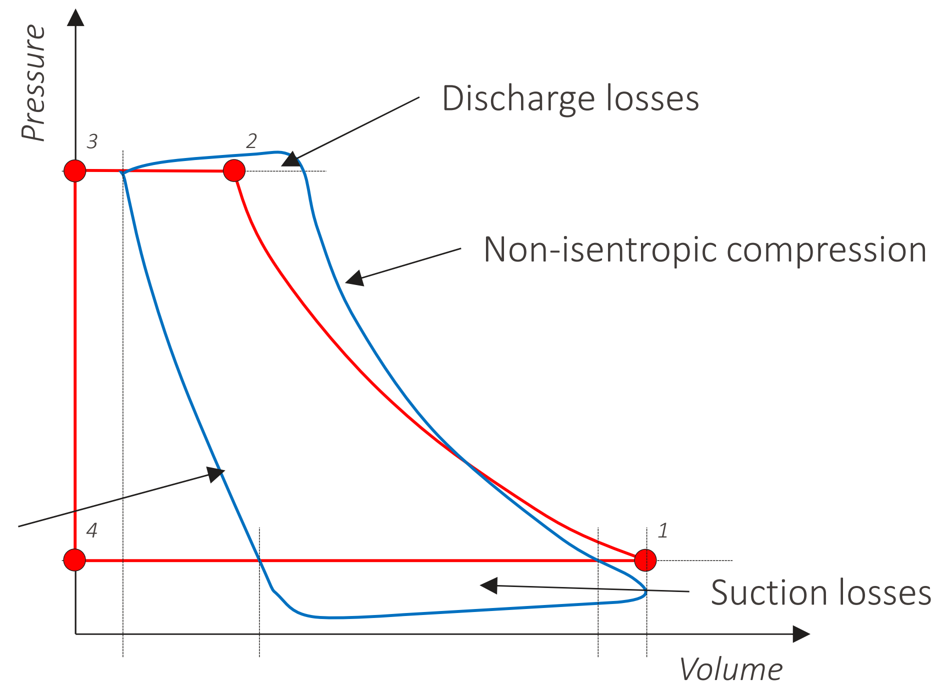
Real Compression Process

- Real compression deviates from ideal process
- Characterized by an isentropic and a volumetric efficiency

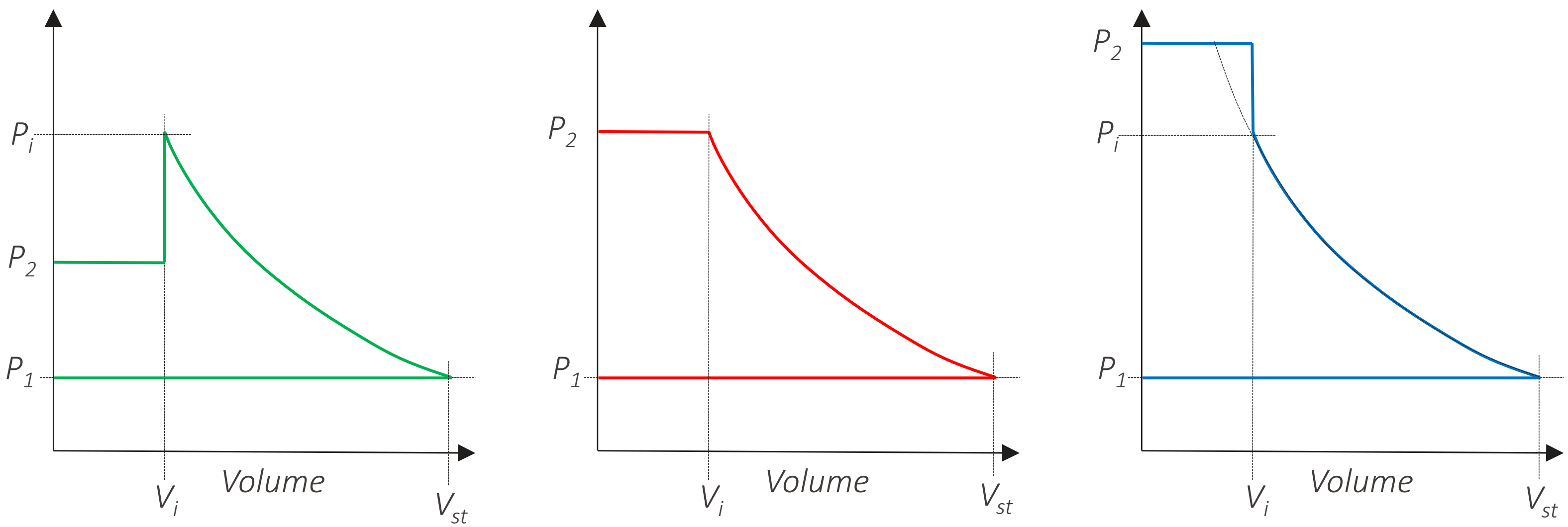
$$\eta_v = \frac{\dot{V}_s}{\dot{V}_{th}} = \eta_{v1}\eta_{v2}\eta_{v3}$$

Suction losses (η_{v1}), Clearance volume expansion (η_{v2}), Internal leakage (η_{v3})

$$\eta_{is} = \eta_{el}\eta_m\eta_{is-int}$$



Over- & Under-Compression in Fixed Volume Ratio Compressors



$$\eta_{th} = \frac{\kappa\left[\Pi^{\frac{\kappa-1}{\kappa}} - 1\right]}{V_R^{\kappa-1} - \kappa + (\kappa-1)\,\frac{\Pi}{V_R}}$$

Comparison to Measured Data

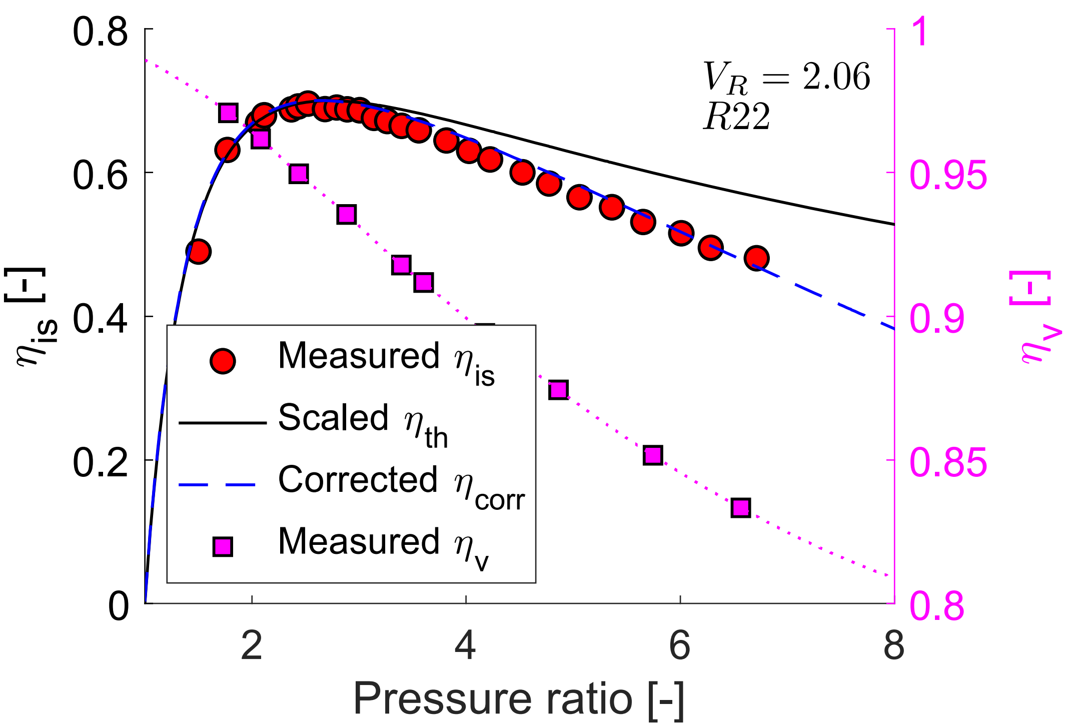
- Experimental scroll compressor data with $\eta_{is-max} = 0.71$

- Scaled theoretical efficiency overpredicted at high Π

- Correction to account for effect of leakage on isentropic efficiency

$$\eta_{is} = \eta_{is-max}\eta_{th} - c\left(1 - \eta_v\right)\Pi$$

$$c = 0.07$$

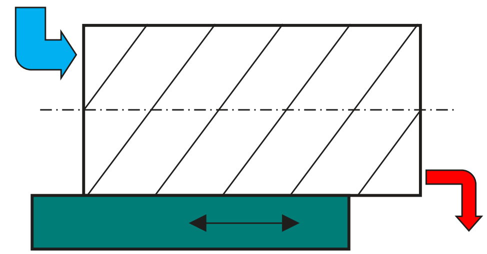


Varying Compressor Operating Conditions

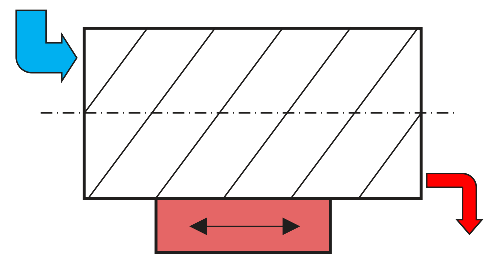
- Heat pumps need to be able to operate under varying conditions
- Changing temperature levels in evaporator and condenser imply varying pressure ratios across compressor
- Changing heating and cooling power at constant temperature levels implies changing mass-flow / volumetric flow delivered by compressor
- Ideal positive displacement compressor should be able to change installed volume ratio and volume flow

Flexibility with Screw Compressors

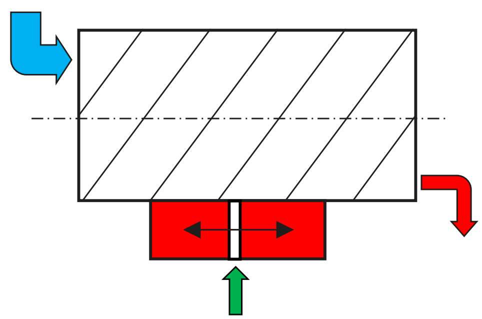
- Introduction of slide vane allows adaptation of:
  - Installed volume ratio



  - Suction volume

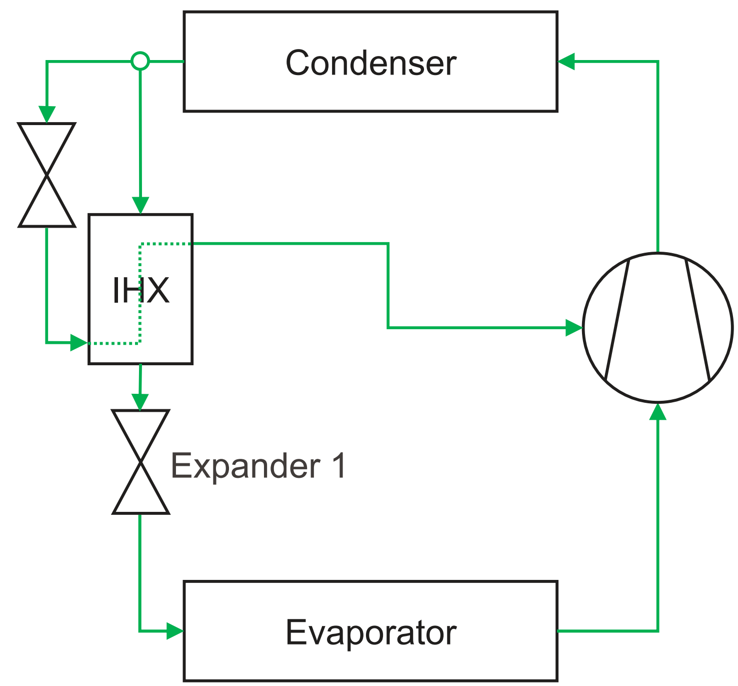


  - Intermediate injection



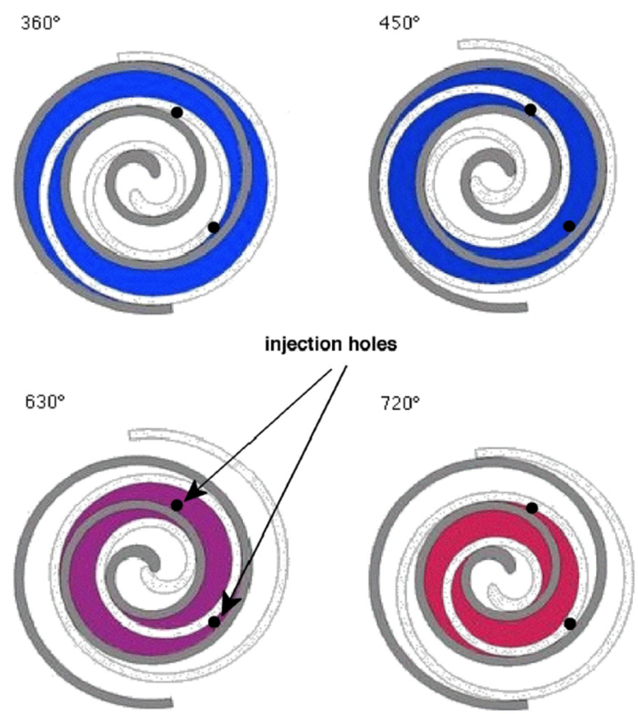
Intermediate Injection

- Use single stage compressor with intermediate injection port to avoid investment of two compressors
- Intermediate injection of cold liquid cools down compressor working chamber and reduces exhaust temperature
- Cycle approaches two stage cycle performance with less cost



Injected Scroll Compressor

- Scroll compressors can also be designed with economizer ports
- Injection ports located in fixed scroll
- Injection occurs into closed compression chambers after compression initiated



?

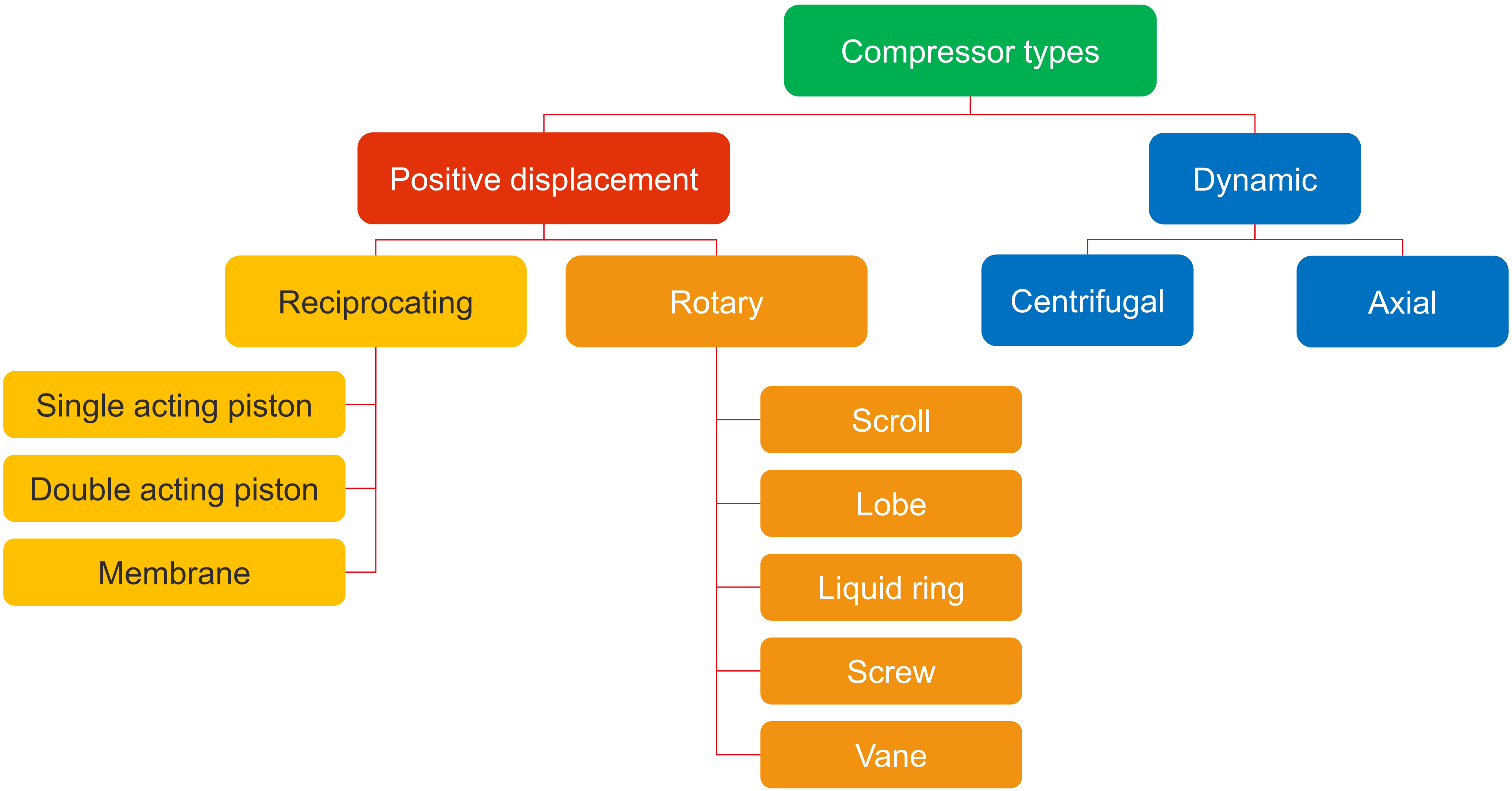
Heat Pump Systems

Introduction to Turbocompressors

Prof. J. Schiffmann

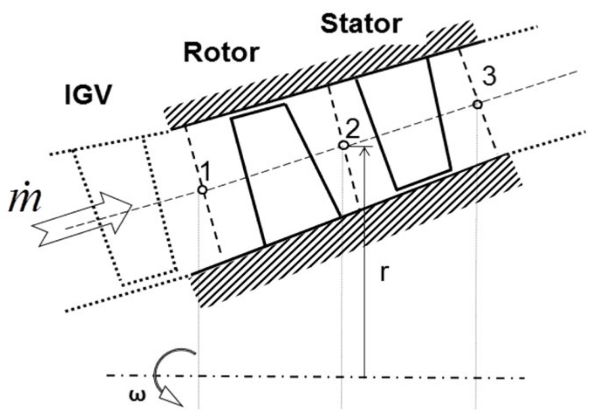
École
polytechnique
fédérale
de Lausanne

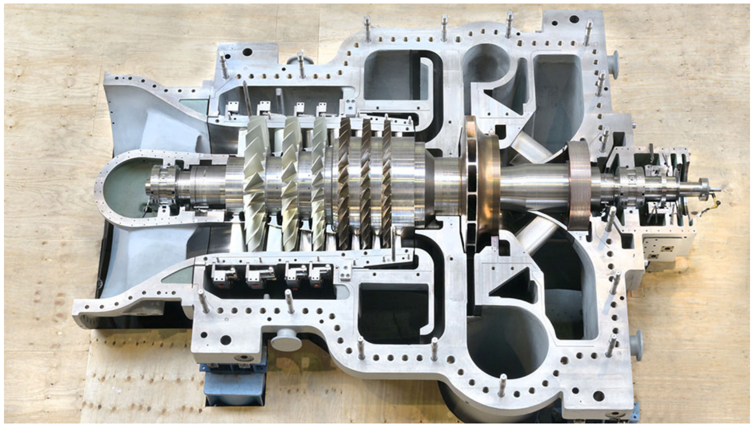
Compressor Classification



Working Principle

- Composed of bladed rotating part (rotor) exchanging work with fluid and stationary bladed part to direct flow (stator)
  - Compressor rotor blade row adds energy to fluid by increasing its swirl and kinetic energy
  - Stator blade row converts kinetic energy into pressure rise
- Stage (rotor & stator) is smallest functional entity of turbocompressor





https://www.man-es.com/process-industry/products/compressors/axial

Wide Range of Applications

- Same underlying physical phenomena used across wide range of applications and power levels

www.ebmpapst.com

www.edf.fr

www.bjreview.com

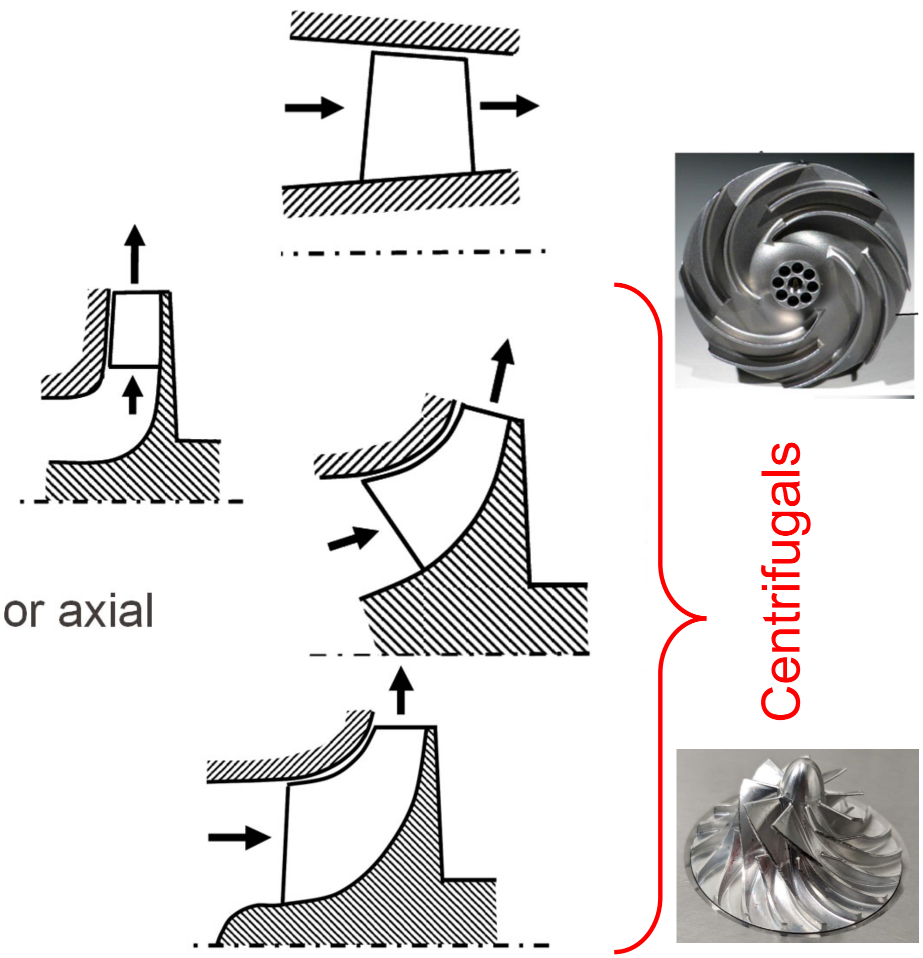
Cooling fan < Ø25mm, 0.5W

Steam turbine, 1.77GW

Francis turbine, 1GW

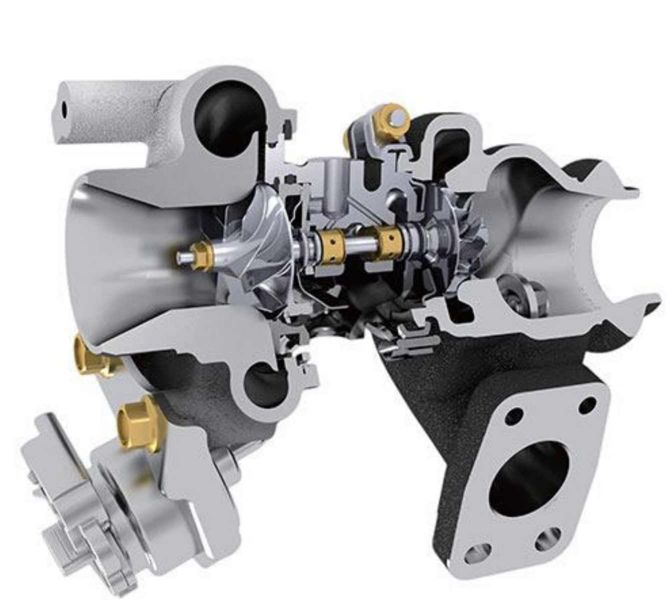
Classification Along Flow Direction

- Axial
  - Flow parallel to axis of rotation
- Radial
  - Main flow perpendicular to axis
- Diagonal
  - Leading or trailing edge neither radial nor axial
- Mixed
  - Leading edge axial, trailing radial



Applications

- Turbochargers

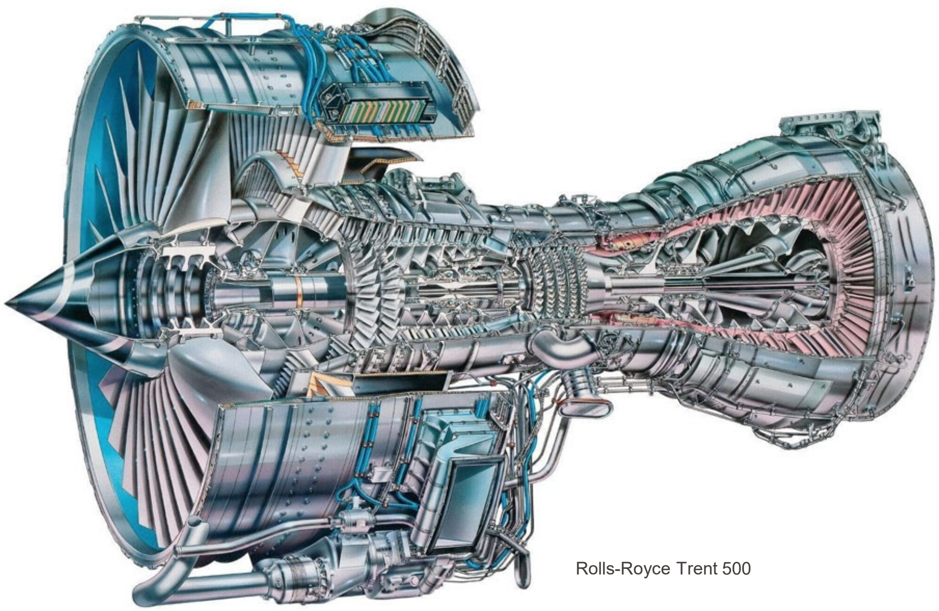


https://www.mhi.com

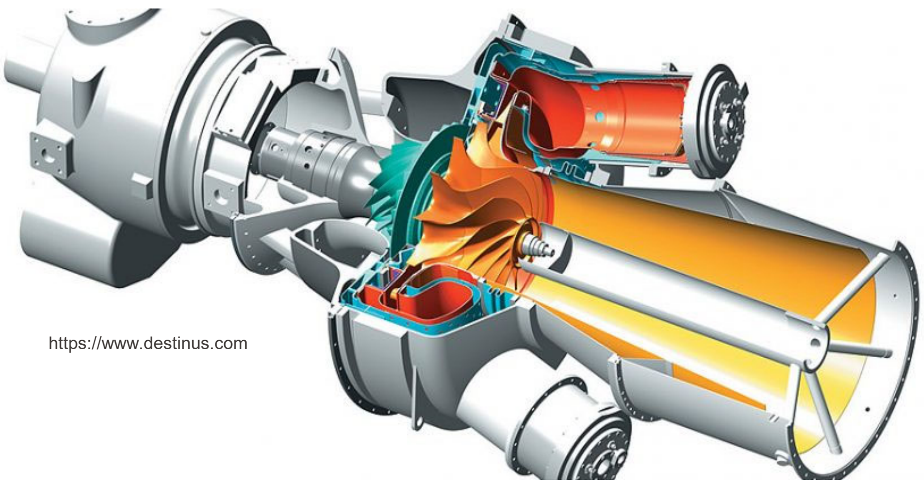
www.abb.com

Applications

- Aircraft engines / gas turbine engines



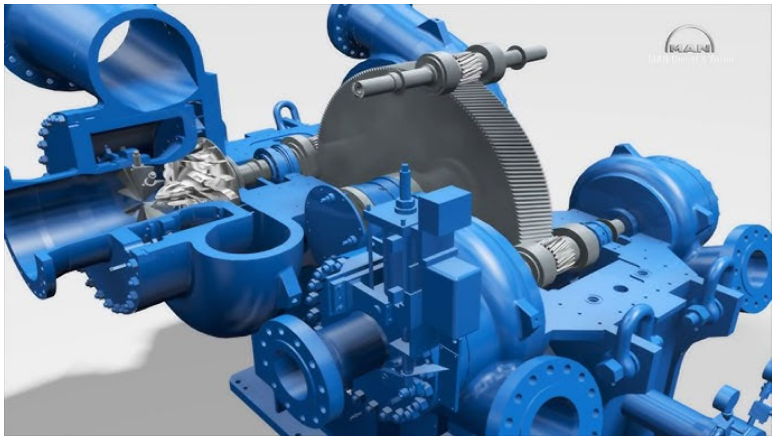
Rolls-Royce Trent 500



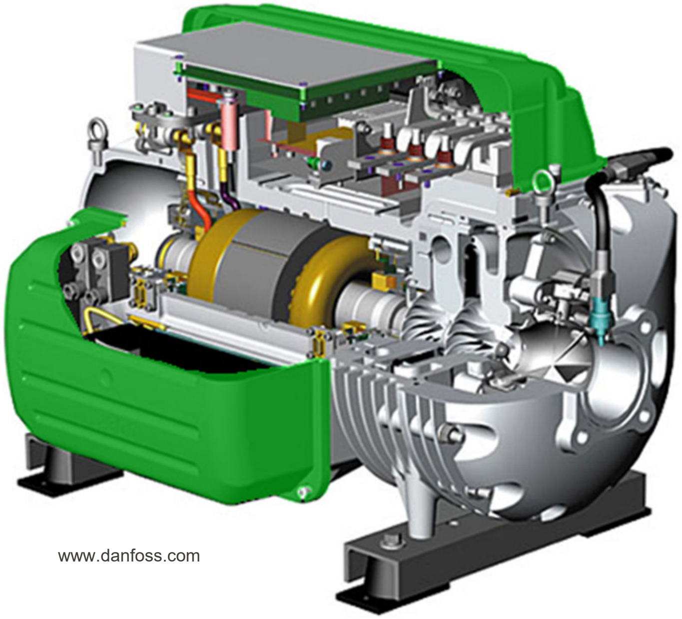
https://www.destinus.com

Applications

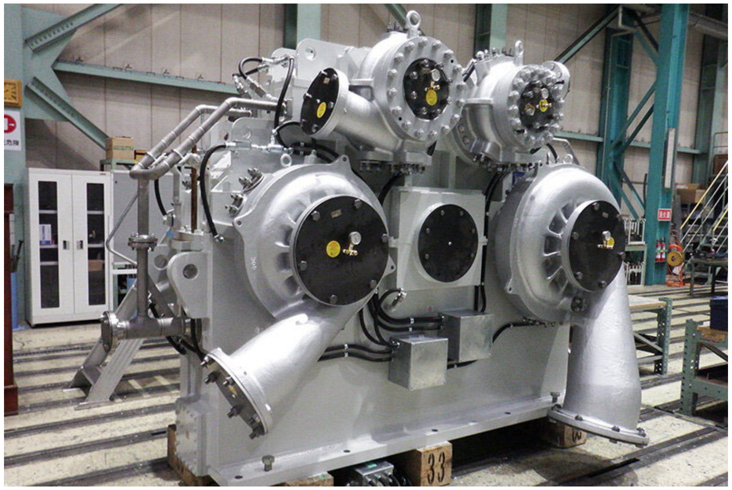
- Refrigeration compressors



www.man.com

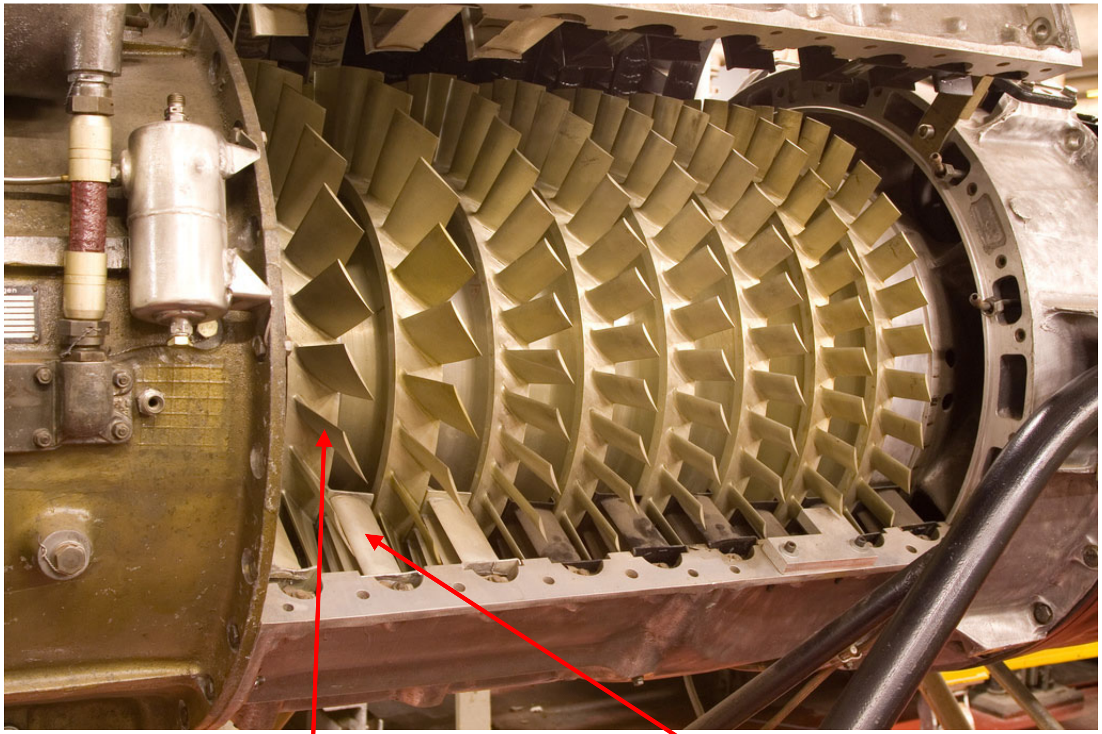


www.danfoss.com



www.kobelco.com

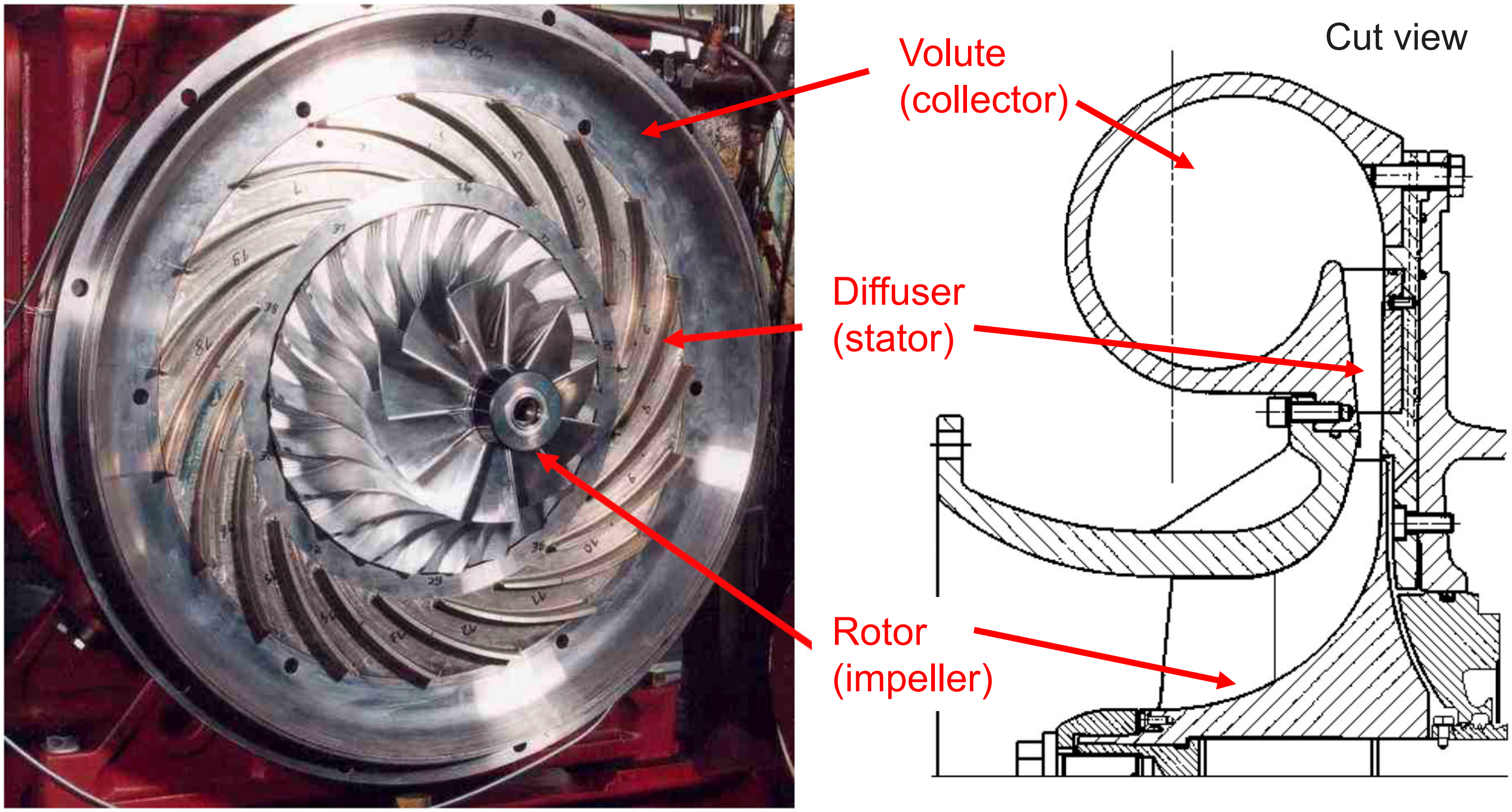
Axial Turbocompressor Stage



Stator blades

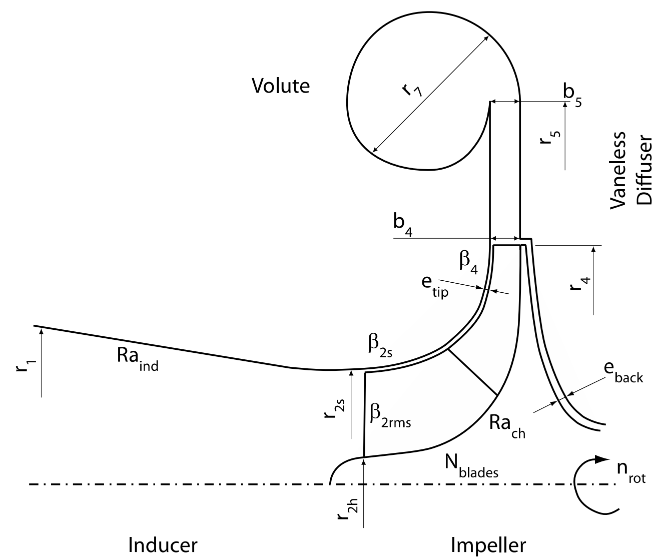
Rotor blades

Radial Turbocompressor Stage



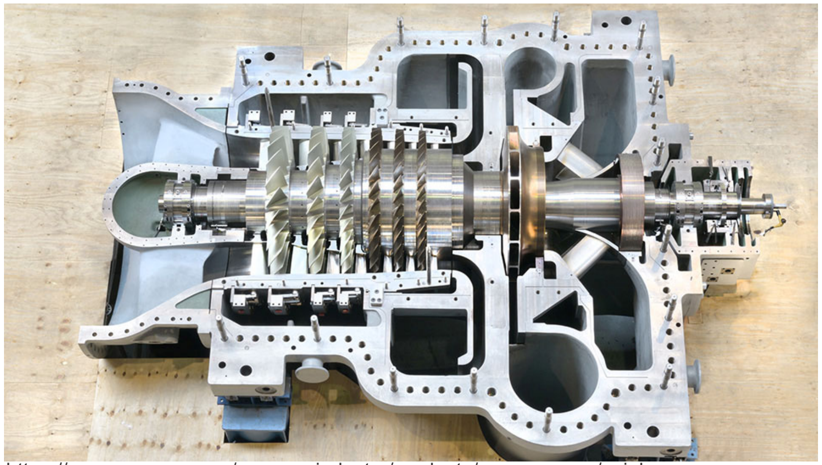
Compressor Stage

- Stage (rotor & stator) is smallest functional entity of turbocompressor
- Inducer accelerates fluid into compressor
- Rotor transfers energy from shaft to fluid
- Stator (diffuser) converts kinetic energy out of impeller into pressure increase
- Volute/collector collects flow at discharge



Multistage Compressor

- Multiple stages (axial and radial) can be used used when required pressure ratio cannot be achieved with one stage

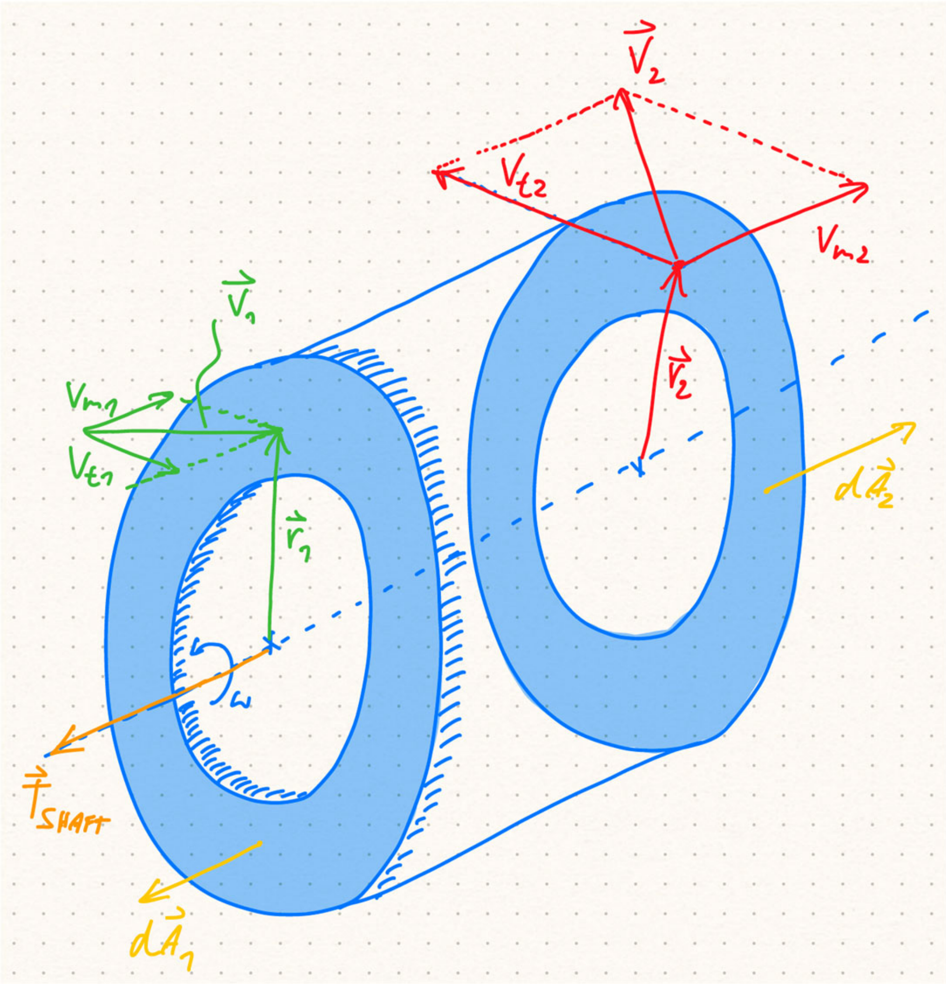


https://www.man-es.com/process-industry/products/compressors/axial

www.man-es.com

Principle of Turbomachinery

- Characterized by
  - Rotating body
  - Control volume
  - Control surface
  - Inlet flow
  - Discharge flow
  - Torque



Torque on Fluid Control Volume I

- Total torque and angular momentum

$$\vec{T} = \frac{d\vec{H}}{dt} \qquad \vec{H} = \int_M \vec{r} \times \vec{V} dm$$

- Total torque acting on system (friction, gravity, torque)

$$\vec{T} = \vec{r} \times \vec{F}_s + \int_M \vec{r} \times \vec{g} dm + \vec{T}_{\text{shaft}}$$

- Rate of change of angular momentum in system

$$\frac{d\vec{H}}{dt} = \frac{\delta}{\delta t} \int_{CV} \vec{r} \times \vec{V} \rho dV + \int_{CS} \vec{r} \times \vec{V} \rho \vec{V} d\vec{A}$$

Torque on Fluid Control Volume II

- Total torque and angular momentum

$$\underbrace{\vec{r} \times \vec{F}_s + \int_M \vec{r} \times \vec{g} dm + \vec{T}_{\text{shaft}}}_{\vec{T}} = \underbrace{\frac{\delta}{\delta t} \int_{CV} \vec{r} \times \vec{V} \rho dV + \int_{CS} \vec{r} \times \vec{V} \rho \vec{V} d\vec{A}}_{d\vec{H}/dt}$$

- Assume no surface forces on system, stationary conditions, no body forces

$$\vec{T} = \int_{CS} \vec{r} \times \vec{V} \rho \vec{V} d\vec{A}$$

Torque on Fluid Control Volume Applied to Turbomachinery

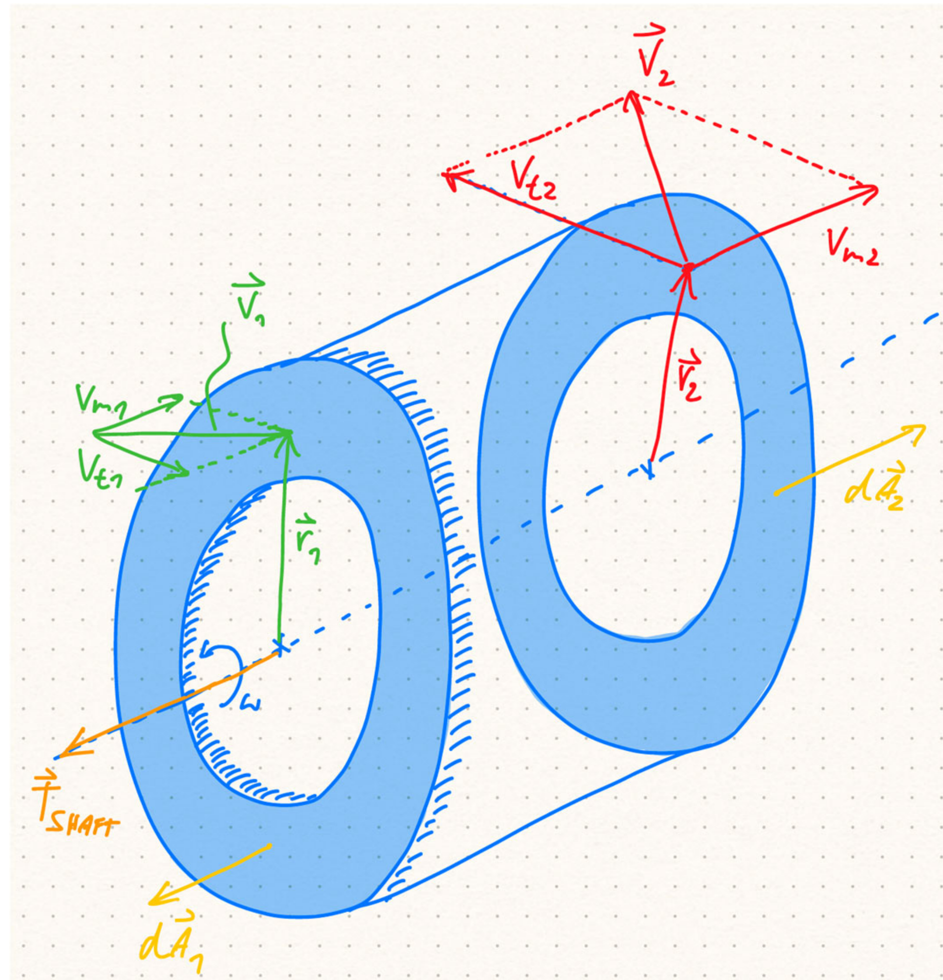
- Total torque on system

$$\vec{T} = \int_{CS} \vec{r} \times \vec{V} \rho \vec{V} d\vec{A}$$

- Applied to turbomachinery, assuming homogenous flow

$$T_{\text{shaft}} = (r_2 V_{t2} - r_1 V_{t1})\, \dot{m}$$

- Change in swirl requires/generates torque



Euler Turbomachinery Equation

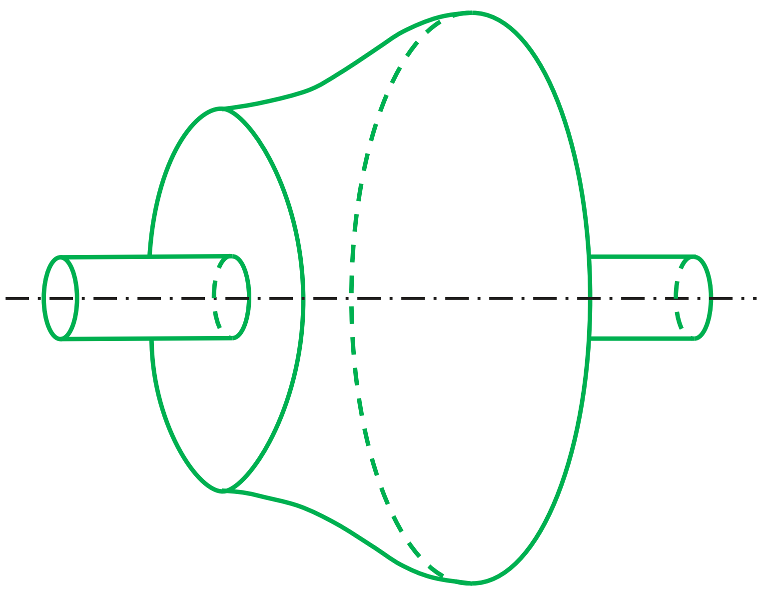
- Change of angular momentum about compressor axis between discharge and inlet requires torque

$$T = \dot{m}\left(r_4 c_{4u} - r_2 c_{2u}\right)$$

- Specific shaft work

$$e^{+} = \omega\left(r_4 c_{4u} - r_2 c_{2u}\right)$$

$$e^{+} = u_4 c_{4u} - u_2 c_{2u}$$



Euler Turbomachinery Equation

- Specific shaft work

$$e^+ = \omega\left(r_4 c_{4u} - r_2 c_{2u}\right)$$

$$e^+ = u_4 c_{4u} - u_2 c_{2u}$$

- Euler turbomachinery equation is:
  - Universally applicable
  - Determines work from changes between mean conditions at inlet and outlet
  - No knowledge on inner workings required

Insights from Euler Equation

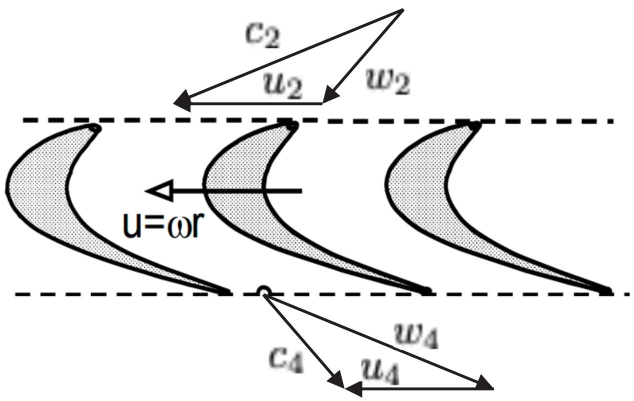
- Work is determined by the change in circumferential component of absolute velocity → change in swirl

$$e^{+} = u_4 c_{4u} - u_2 c_{2u}$$

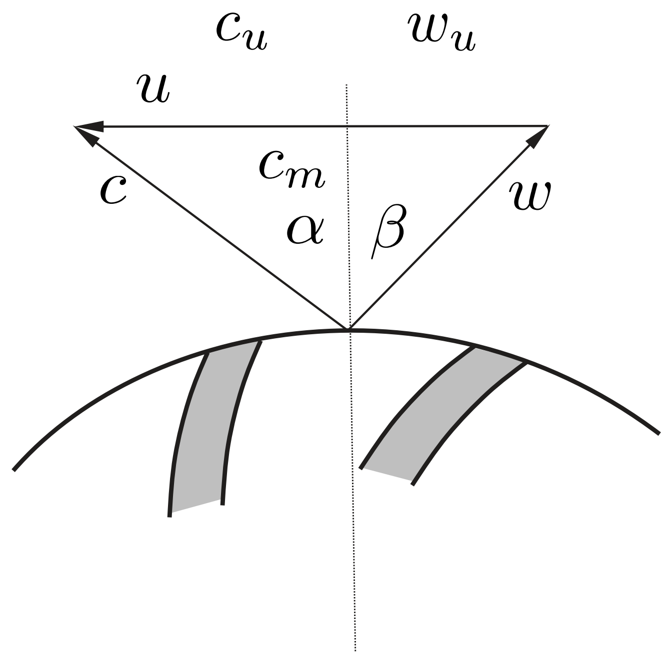
- Change in swirl and flow guidance is achieved by sufficiently closely spaced blades

$$c_u = f\,(u, w)$$

- Mastery of velocity magnitude and direction is key in turbomachinery design



Velocity Triangles



- Velocity triangle connects relative to absolute measurements

$$\vec{c} = \vec{w} + \vec{u}$$

- Trigonometry yields

$$uc_u = \frac{1}{2}\left(c^2 + u^2 - w^2\right)$$

$$e^+ = u_4 c_{4u} - u_2 c_{2u} = \frac{1}{2}\left[\left(c_4^2 - c_2^2\right) + \left(w_2^2 - w_4^2\right) + \left(u_4^2 - u_2^2\right)\right]$$

Energy Transformation in Turbomachinery

$$e^{+} = u_4 c_{4u} - u_2 c_{2u} = \frac{1}{2}\left[\left(c_4^2 - c_2^2\right) + \left(w_2^2 - w_4^2\right) + \left(u_4^2 - u_2^2\right)\right]$$

- Velocity triangles yields alternative form of Euler equation which identifies work provided by
  - Change in kinetic energy of absolute flow through rotor $\frac{1}{2}\left(c_4^2 - c_2^2\right)$
    - Positive in compressors
    - Negative in turbines
  - Change in kinetic energy of relative flow in rotor $\frac{1}{2}\left(w_2^2 - w_4^2\right)$
    - Limited diffusion in compressor
    - Unlimited acceleration in turbines
  - Centrifugal effect $\frac{1}{2}\left(u_4^2 - u_2^2\right)$

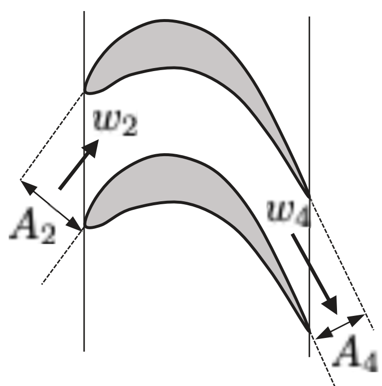
Turbine vs. Compressor

$$e^{+} = \frac{1}{2}\left[\left(c_4^2 - c_2^2\right) + \left(w_2^2 - w_4^2\right) + \left(u_4^2 - u_2^2\right)\right]$$

- Turbine
  - Blades accelerate relative flow → nozzle-like

$$e^{+} < 0$$

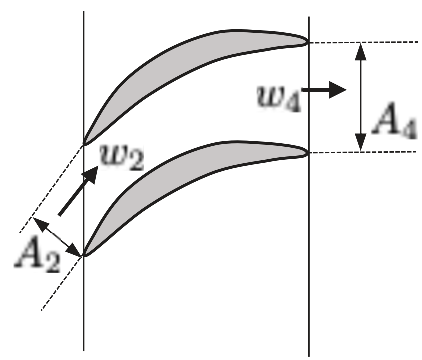
$$c_4 < c_2 \qquad w_4 > w_2$$



- Compressor
  - Blades decelerate relative flow → diffuser

$$e^{+} > 0$$

$$c_4 > c_2 \qquad w_4 < w_2$$



Axial vs. Radial Compressor

$$e^{+} = u_4 c_{4u} - u_2 c_{2u}$$

- Axial machine

$$c_{2u} = 0 \qquad w_4/w_2 = 0.7 \qquad u_4 \approx u_2$$

- Radial machine

$$c_{2u} = 0 \qquad w_4/w_2 = 0.7 \qquad u_4 \approx 2u_2$$

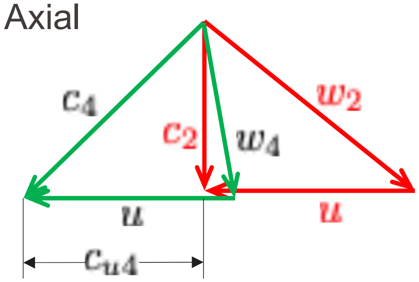
- Comparison

$$w_2|_{radial} = w_2|_{axial}$$

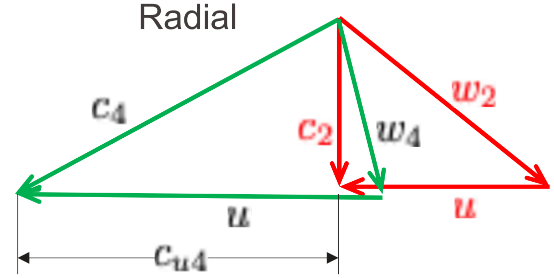
$$c_{4u}|_{radial} > c_{4u}|_{axial}$$

$$u_4 c_{4u}|_{radial} \gg u_4 c_{4u}|_{axial}$$

Axial

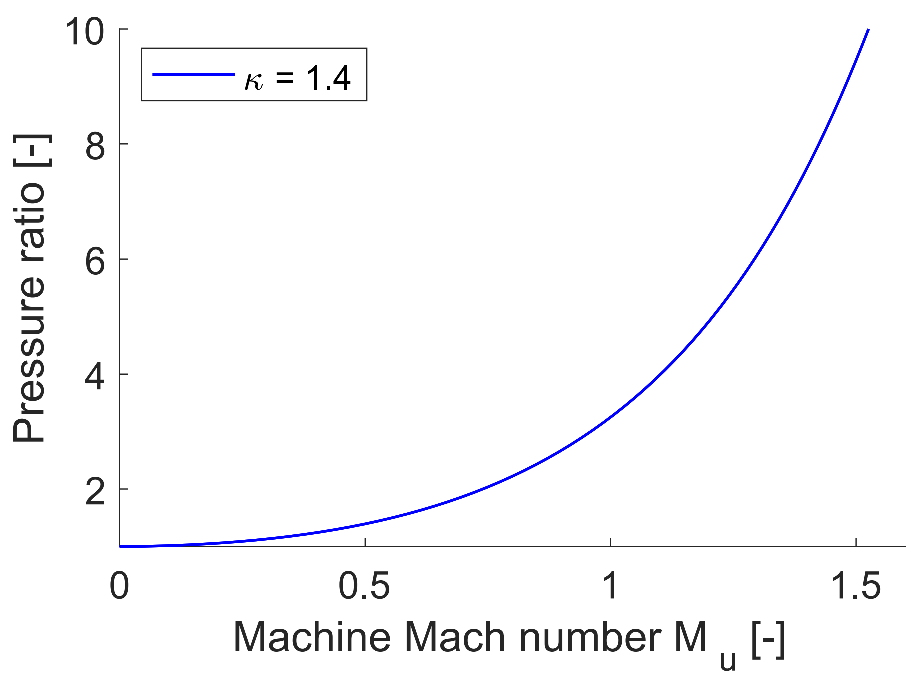


Radial

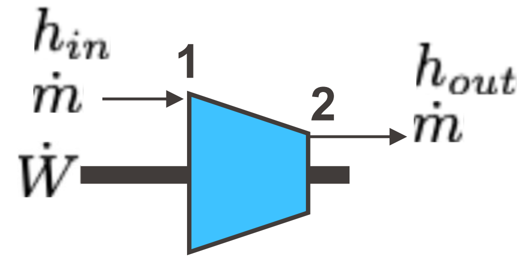


- Nealy 50% of work due to centrifugal effect
- Radial compressor stage can replace 3-4 axial stages

Link Between Euler and Thermodynamics



Total and Static Conditions



- Energy balance for open system

$$\dot{W}^{+} + \dot{Q}^{+} = \dot{m}\left[(h_2 - h_1) + 1/2\left(c_2^2 - c_1^2\right) + g\left(z_2 - z_1\right)\right]$$

- Neglecting gravity, assuming adiabatic operation

$$\frac{\dot{W}^{+}}{\dot{m}} = \underbrace{\left(h_2 + \frac{1}{2}c_2^2\right)}_{h_{02}} - \underbrace{\left(h_1 + \frac{1}{2}c_1^2\right)}_{h_{01}}$$

- Total enthalpy → fictive thermodynamic state variable

$$h_0 = h + \frac{c^2}{2}$$

EPFL

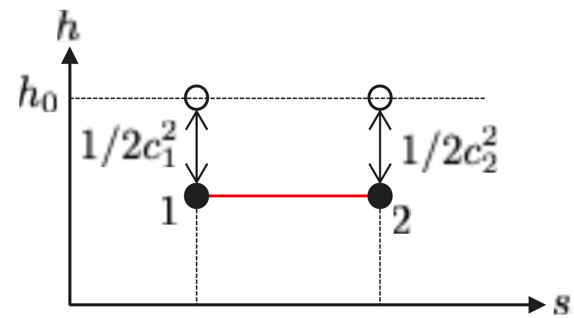
Specific Cases

- Adiabatic work process $w^+ = \Delta h_0$
  - Compressor with no heat transfer
- Diabatic work process $w^+ + q^+ = \Delta h_0$
  - Cooled/heated compressor
- Adiabatic flow process $0 = \Delta h_0$
  - Non-rotating components such as inducer, diffuser,…
- Diabatic flow process $q^+ = \Delta h_0$
  - Stationary components such as HEX

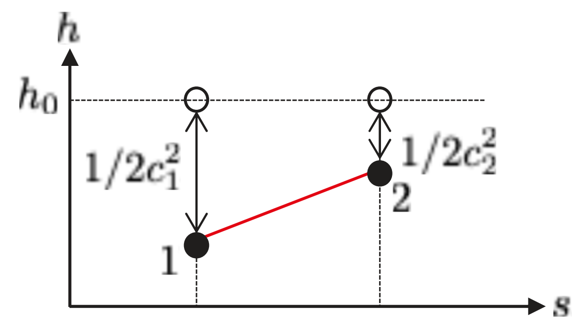
Typical Adiabatic Flow Processes

$$0 = \Delta h_0$$

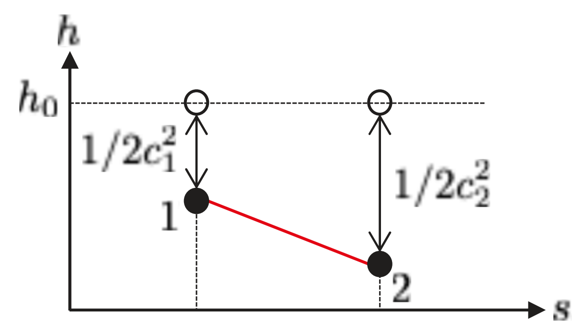
- Constant area pipe flow



- Diffuser



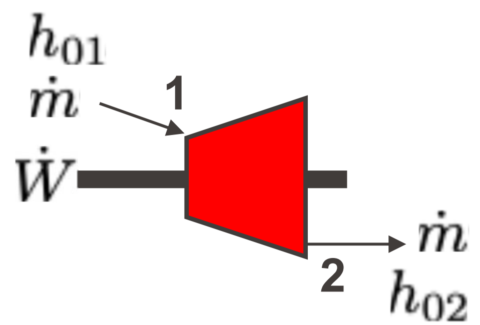
- Nozzle

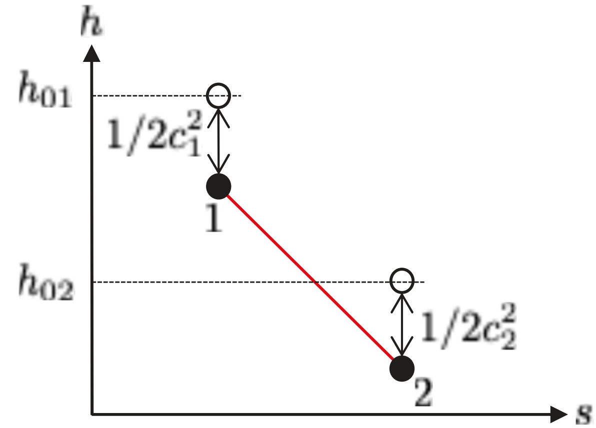


Typical Adiabatic Work Processes

- Turbine

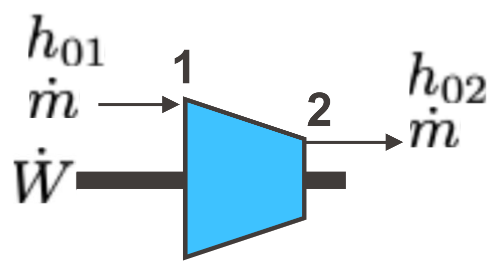
$$w^+ = h_{02} - h_{01} < 0$$

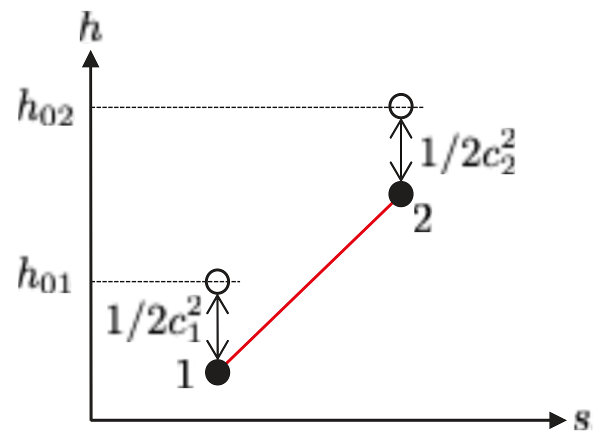




- Compressor

$$w^+ = h_{02} - h_{01} > 0$$





EPFL

Definition of Rothalpy

- Coupling Euler equation with thermodynamic states ($\dot{q} = 0$)

$$e^{+} = u_4 c_{4u} - u_2 c_{2u} = h_{04} - h_{02} = \left(h_4 + \frac{c_4^2}{2}\right) - \left(h_2 + \frac{c_2^2}{2}\right)$$

- Rearranging…

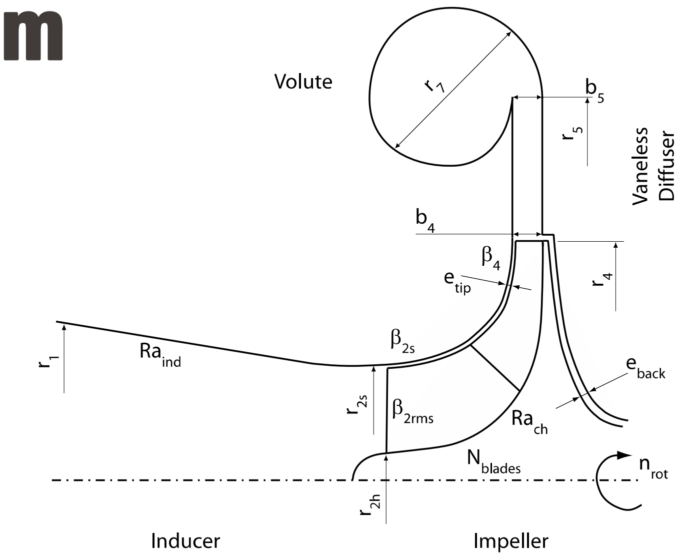
$$\underbrace{h_4 + \frac{c_4^2}{2} - u_4 c_{4u}}_{I_4} = \underbrace{h_2 + \frac{c_2^2}{2} - u_2 c_{2u}}_{I_2}$$

I_2 ← Rothalpy

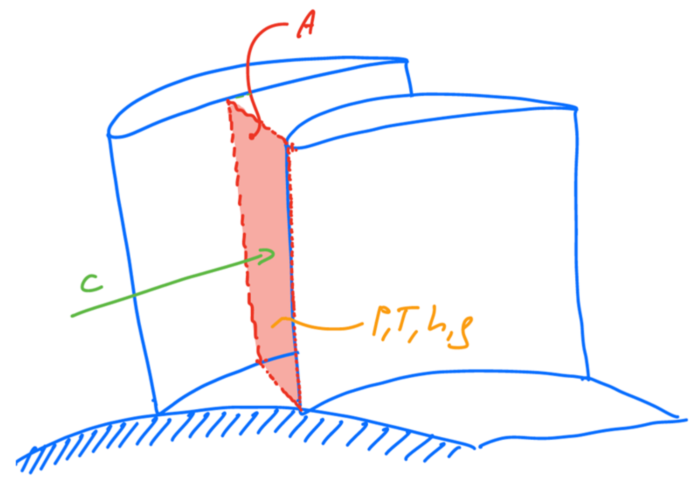
- Coupling with velocity triangles

$$\underbrace{h_4 + \frac{w_4^2}{2}}_{h_{0,rel,4}} - \frac{u_4^2}{2} = \underbrace{h_2 + \frac{w_2^2}{2}}_{h_{0,rel,2}} - \frac{u_2^2}{2}$$

Turbocompressor in h-s-Diagram

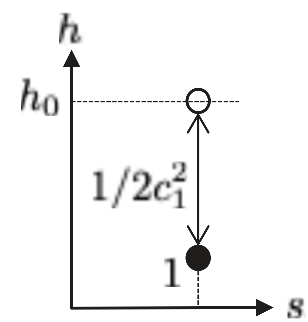


Mass Flow and Flow Velocity

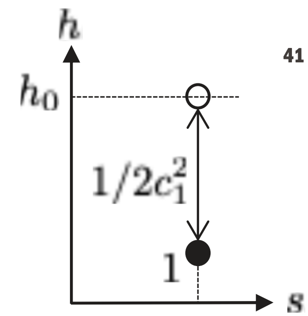


- So far looked at velocity triangles and link to thermodynamics
- Often, only total conditions, area and mass flow are known
- Mass flow and velocity linked through static property, which depend on velocity for given total condition
- How to translate mass flow into flow velocity?

$$\dot{m} = \rho c A \qquad \rho = f(P, T)$$



Dimensionless Mass Flow Equation I



- Perfect gas assumption

$$h_0 = h + 1/2c^2 \qquad h = c_p T$$

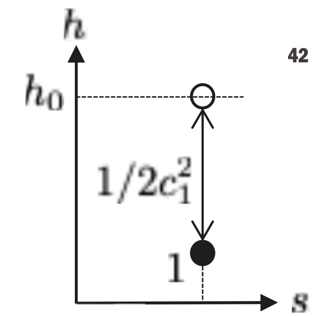
$$T_0 = T + \frac{c^2}{2c_p} \quad \text{use } c_p - c_v = R \quad \kappa = c_p/c_v \quad a = \sqrt{\kappa R T} \quad M = c/a$$

$$\frac{T_0}{T} = 1 + \frac{1}{2}c^2 \frac{\kappa - 1}{\kappa R T} = 1 + \frac{\kappa - 1}{2} M^2$$

$$\frac{P_0}{P} = \left(\frac{T_0}{T}\right)^{\kappa/(\kappa-1)} = \left(1 + \frac{\kappa - 1}{2} M^2\right)^{\kappa/(\kappa-1)}$$

- Ratio of total to static states only function of Mach number

Dimensionless Mass Flow Equation II



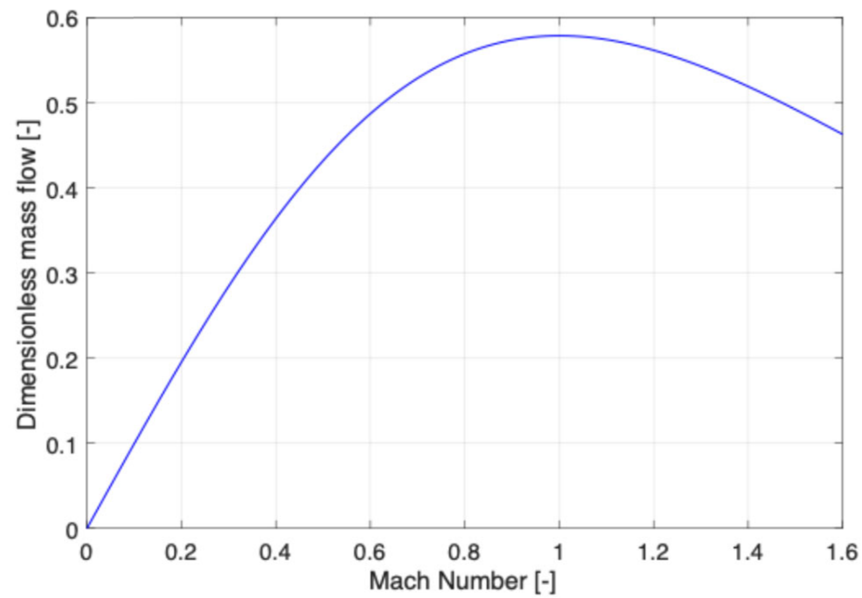
$$\dot{m} = \rho c A \quad \text{use} \quad a = \sqrt{\kappa R T} \quad M = c/a \quad \Rightarrow \quad \frac{\dot{m}}{A} = \frac{P}{RT} M \sqrt{\kappa R T}$$

Replace static with total states

$$\frac{\dot{m}\sqrt{RT_0/\kappa}}{AP_0} = M\left(1 + \frac{\kappa - 1}{2} M^2\right)^{-\frac{\kappa+1}{2(\kappa-1)}}$$

- Knowledge of mass-flow, area A, and total conditions (P_0, T_0) allows calculating Mach number

- Requires numerical approach



Summary

- Turbomachinery applications span across wide range of power and applications
- All turbomachinery governed by same principle → Euler equation
- Mastering velocity triangles is key to induce change in swirl
- Link between Euler equation and thermodynamics yields pressure ratio, representation of losses
- Velocity across area for given mass-flow and total conditions needs to be found iteratively

Exercises W9

- Comprehension questions
- A diffuser problem
- Axial flow air compressor