

# Heat Pump Systems

Summary W2

Prof. J. Schiffmann

# Open System Balances

- Energy balance:

$$\frac{dE_{cv}}{dt} = \dot{W}_{cv} + \dot{Q}_{cv} + \sum \dot{m}_{in} \left( h + \frac{w^2}{2} + gz \right)_{in} - \sum \dot{m}_{out} \left( h + \frac{w^2}{2} + gz \right)_{out}$$

Net convected power

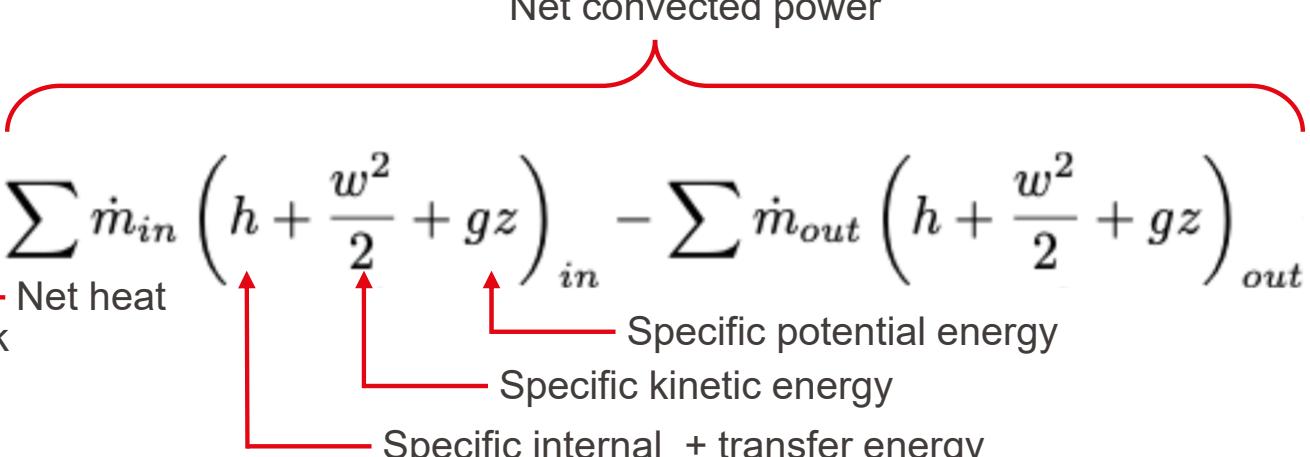
Net work

Net heat

Specific potential energy

Specific kinetic energy

Specific internal + transfer energy



- Mass balance:

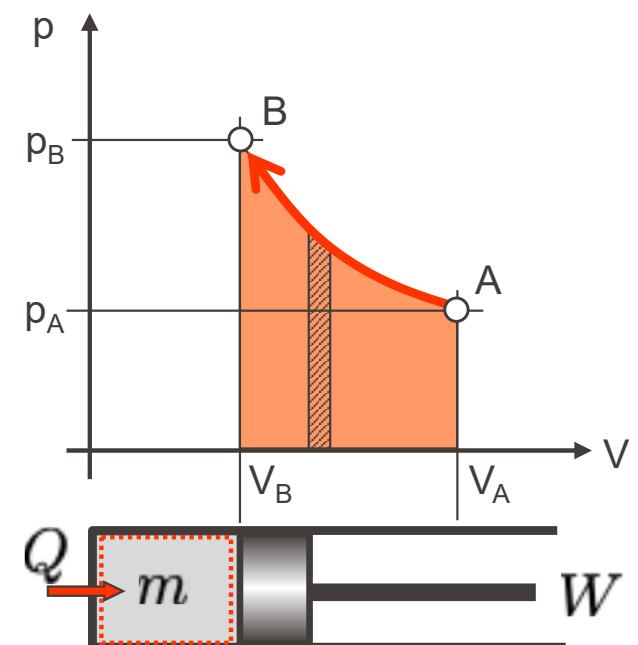
$$\frac{dm_{cv}}{dt} = \sum \dot{m}_{in} - \sum \dot{m}_{out}$$

# Work

- Consider closed system where piston moves from A to B

$$W = \int_A^B \delta W = - \int_{x_A}^{x_B} pAdx = - \int_{V_A}^{V_B} pdV$$

- Work depends on evolution of p vs. V  
→ work depends on process details
- Surface under transformation line in pV-diagram represents work
- Work is no thermodynamic state property



- Conduction: Fourier's law

$$\dot{q} = \frac{\dot{Q}}{A} = -\lambda \frac{dT}{dx}$$

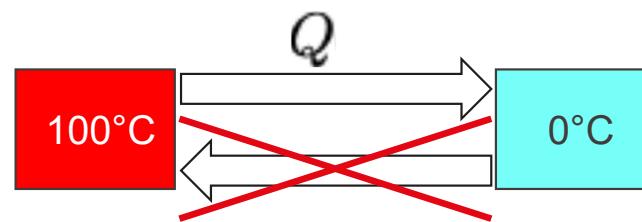
- Radiation: Boltzmann-equation  $\dot{q} = \epsilon \sigma T^4$

- Convection: Newton's law

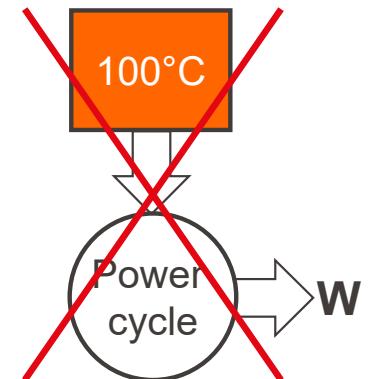
$$\dot{q} = \frac{\dot{Q}}{A} = \alpha (T_{Wall} - T_{Fluid})$$

# Formulations of 2<sup>nd</sup> Law

- By Clausius (1854): There is no change of state whose only result is the transfer of heat from a body at a lower temperature to a body at a higher temperature



- Kelvin-Planck (1848/1926): It is impossible to construct a device which, operating in a cycle, will produce no other effect than the extraction of heat from a reservoir and the performance of equivalent amount of work



# Carnot Principles

- The thermal efficiency of an irreversible power cycle is always lower than that of a reversible cycle between the same thermal reservoirs
- All reversible power cycles between the same thermal reservoirs have the same thermal efficiency
- The efficiency of a reversible machine is independent of the process, the components, and the working fluid

# Thermal Efficiency

- Through Kelvins definition, thermal efficiency of reversible cycle expressed as:

$$\eta_{th-rev.} = 1 - \frac{T_{cold}}{T_{hot}} = \eta_c$$

- Carnot cycle is one famous reversible power cycle
- Thermal efficiency of reversible cycle is called Carnot-efficiency

?

# Heat Pump Systems

Thermodynamics Crash Course  
Entropy

# Formulations of 2<sup>nd</sup> Law

- There is no change of state whose only result is the transfer of heat from a body at a lower temperature to a body at a higher temperature (Clausius)
- It is impossible to construct a device which, operating in a cycle, will produce no other effect than the extraction of heat from a reservoir and the performance of equivalent amount of work (Kelvin-Planck)
- Formulations are qualitative

# Alternative Formulation of 2<sup>nd</sup> Law

- Mathematical formulation of 2<sup>nd</sup> law possible via Clausius inequality for any cycle process

$$\oint \frac{\delta Q}{T} \leq 0$$

Heat across system boundary

Temperature of heat transfer across system boundary

- Valid for any cycle process
- Reversible if integral = 0
- Irreversible if integral < 0
- 2<sup>nd</sup> law forbids processes with integral > 0

# Example: Carnot Cycle

- Carnot efficiency

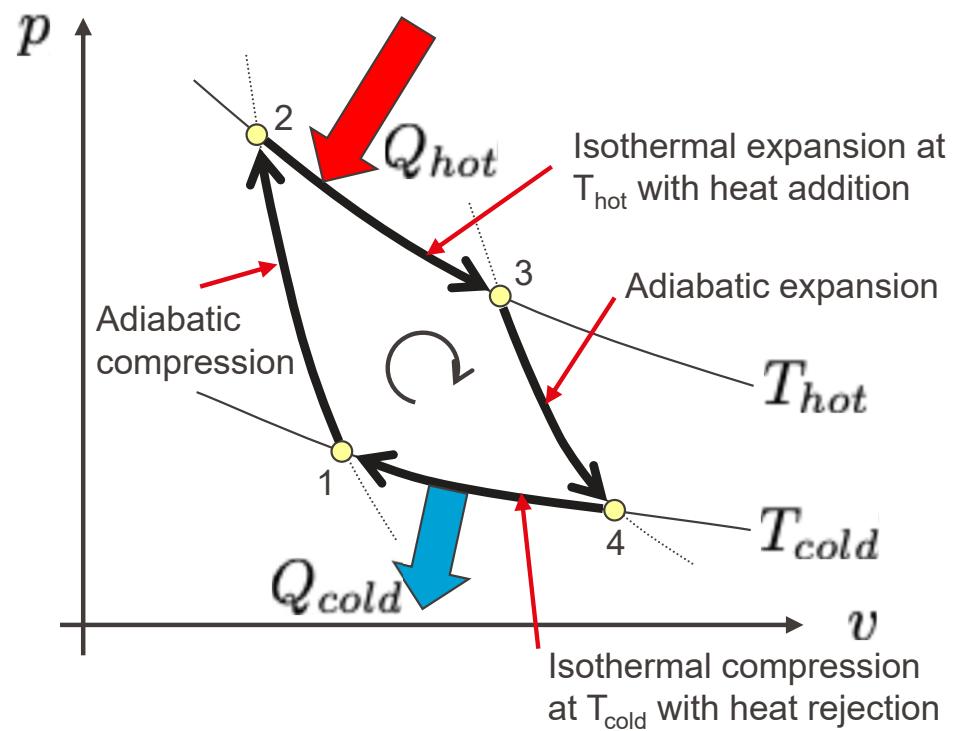
$$\eta_c = 1 - \frac{Q_{cold}^-}{Q_{hot}^+} = 1 - \frac{T_{cold}}{T_{hot}}$$

$$\frac{Q_{cold}^-}{Q_{hot}^+} = \frac{T_{cold}}{T_{hot}}$$

- Clausius inequality  $\rightarrow$  equality

$$\oint \frac{\delta Q}{T} = \frac{Q_{hot}^+}{T_{hot}} - \frac{Q_{cold}^-}{T_{cold}}$$

$$\oint \frac{\delta Q}{T} = 0$$



# Alternative Formulation of 2<sup>nd</sup> Law

- Writing Clausius inequality as equation

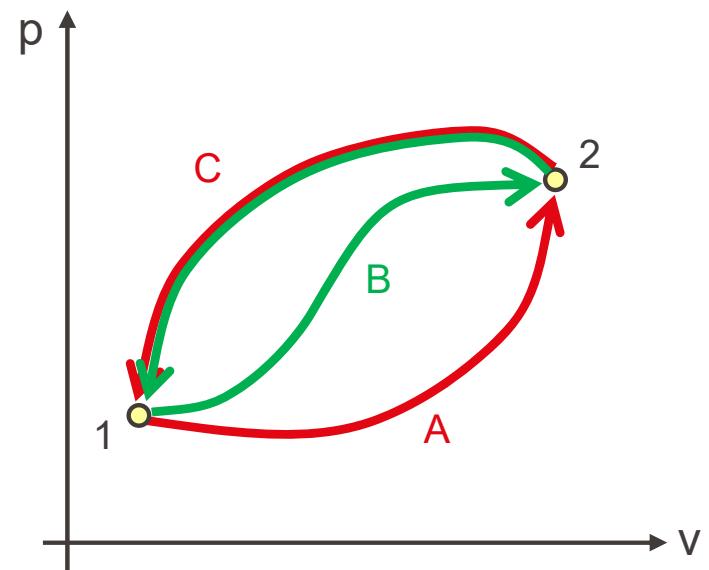
$$\oint \frac{\delta Q}{T} = -\sigma$$

- 2<sup>nd</sup> law now contained in  $\sigma$ 
  - $\sigma = 0$   $\rightarrow$  reversible process
  - $\sigma > 0$   $\rightarrow$  irreversible process
  - $\sigma < 0$   $\rightarrow$  Not possible (2<sup>nd</sup> law)

- Consider two reversible cycles, AC & BC, going through states 1 & 2
- Since reversible  $r_{AC} = r_{BC} = 0$

$$\int_1^2 \frac{\delta Q}{T} \Big|_A + \int_2^1 \frac{\delta Q}{T} \Big|_C = 0$$

$$\int_1^2 \frac{\delta Q}{T} \Big|_B + \int_2^1 \frac{\delta Q}{T} \Big|_C = 0$$



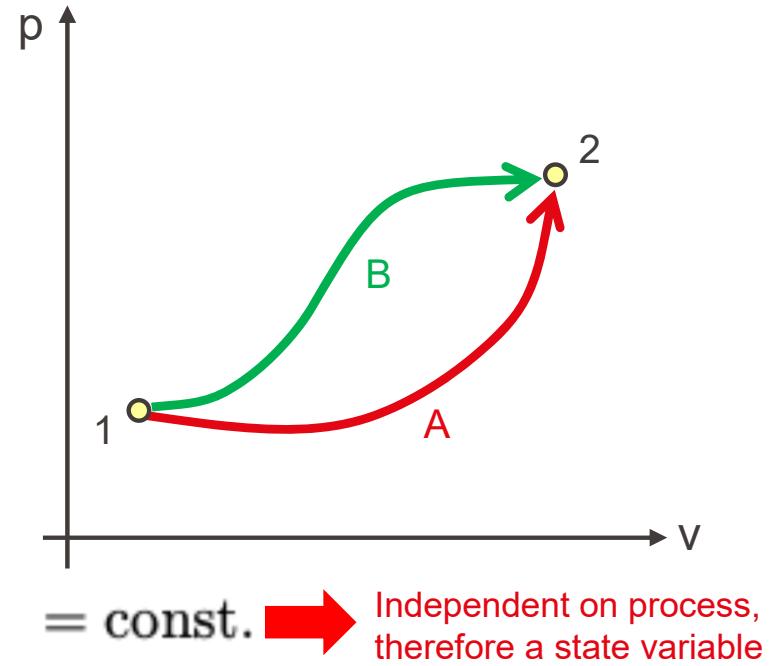
- Consider two reversible cycles, AC & BC, going through states 1 & 2
- Since reversible  $r_{AC}$  and  $r_{BC} = 0$

$$\int_1^2 \frac{\delta Q}{T} \Big|_A + \int_2^1 \frac{\delta Q}{T} \Big|_C = 0$$

$$\int_1^2 \frac{\delta Q}{T} \Big|_B + \int_2^1 \frac{\delta Q}{T} \Big|_C = 0$$

- It follows

$$\int_1^2 \frac{\delta Q}{T} \Big|_A = \int_1^2 \frac{\delta Q}{T} \Big|_B \quad \rightarrow \quad \int_1^2 \frac{\delta Q}{T} \Big|_{rev} = \text{const.}$$



# Entropy Definition

- New state variable entropy (S) defined through Clausius

$$S_2 - S_1 = \int_1^2 \frac{\delta Q}{T} \Big|_{rev} \quad \rightarrow \quad dS = \frac{\delta Q}{T} \Big|_{rev} \quad \rightarrow \quad TdS = \delta Q \Big|_{rev}$$

- Entropy is a state property  $\rightarrow$  Knowledge of two other state properties defines also entropy of state

# Entropy in Irreversible Processes?

- Consider cycle with reversible and irreversible process
  - Clausius leads to

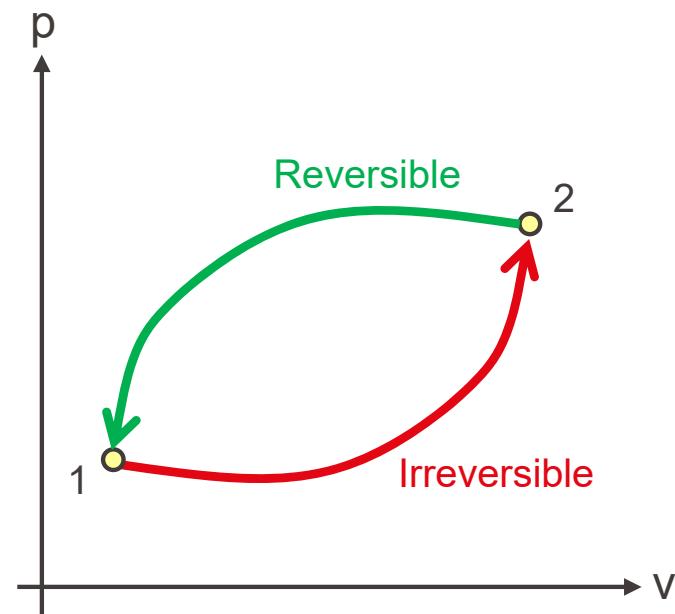
$$\int_1^2 \frac{\delta Q}{T} + \int_2^1 \frac{\delta Q}{T} \Big|_{rev} = -\sigma$$

- Definition of entropy change

$$S_1 - S_2 = \int_2^1 \frac{\delta Q}{T} \Big|_{rev}$$

- Leads to

$$S_2 - S_1 = \int_1^2 \frac{\delta Q}{T} + \sigma$$



# Entropy Balance

$$S_2 - S_1 = \int_1^2 \frac{\delta Q}{T} + \sigma$$

- Entropy change between two states results from
  - Entropy transfer due to heat transfer → dependent on process and independent from work
  - Entropy production through irreversibility → dependent on process, always  $> 0$  due to 2<sup>nd</sup> law!

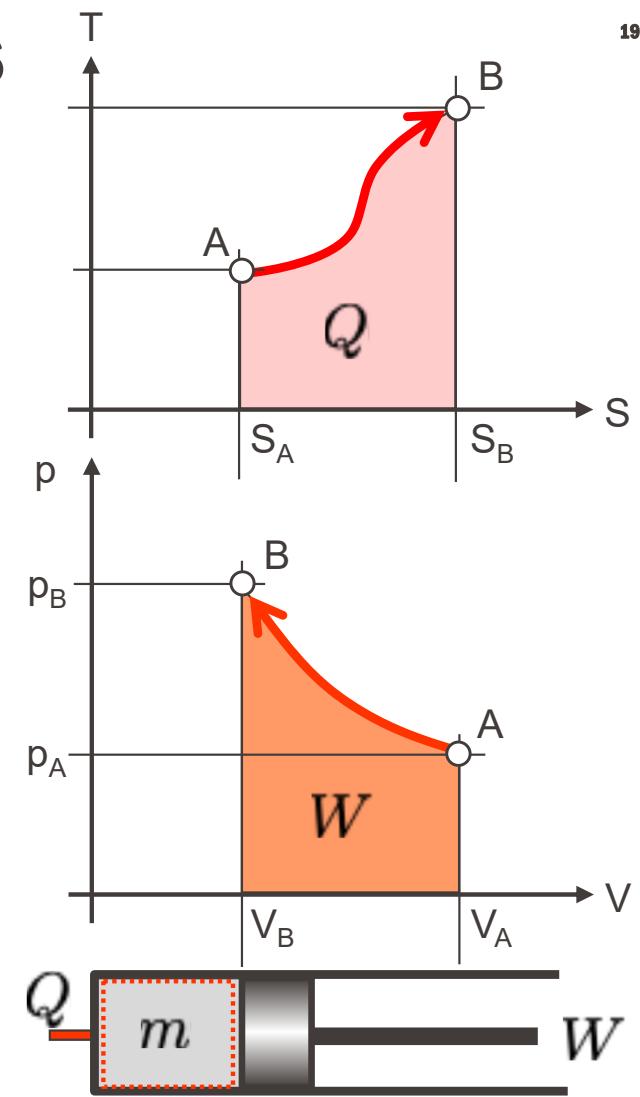
# Entropy Balance in Closed Systems

- Consider reversible closed system
- Integral in PV-diagram corresponds to work

$$W = - \int_1^2 p dV$$

- Integral in TS-diagram corresponds to heat

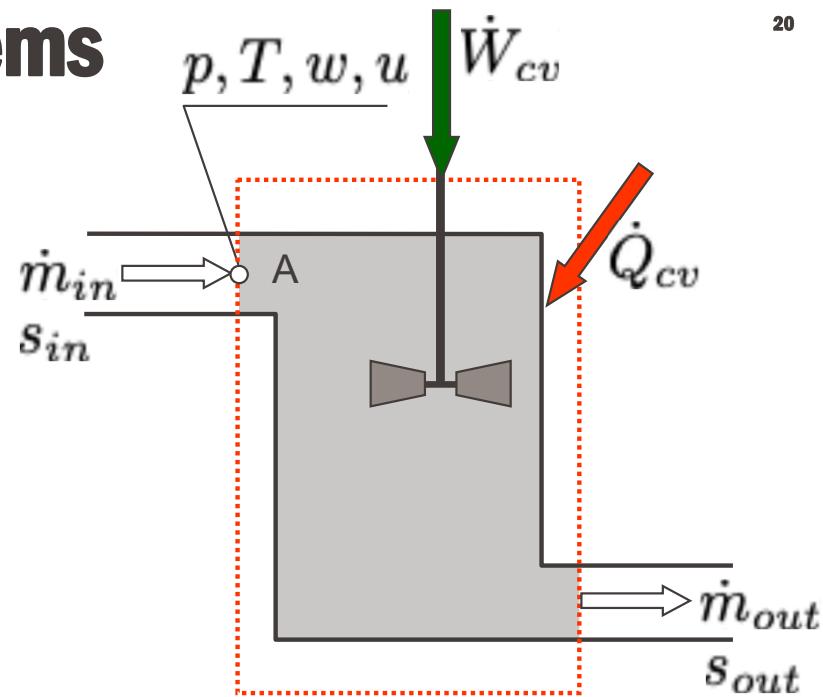
$$Q_{rev} = \int_1^2 T dS$$



# Entropy Balance in Open Systems

- Entropy can also be convected across system through mass fluxes
- Entropy balance for open system

$$\frac{dS_{cv}}{dt} = \sum_j \frac{\dot{Q}_j}{T_j} + \sum_{in} \dot{ms}|_{in} - \sum_{out} \dot{ms}|_{out} + \dot{\sigma}_{cv}$$



- 1<sup>st</sup> law
  - No energy produced only transformed
  - Equivalence of work and heat
- 2<sup>nd</sup> law
  - No conservation of entropy, entropy can be produced
  - Entropy transfer associated with heat transfer → can be positive or negative
  - In open systems entropy is convected across system boundary through mass fluxes
  - Irreversibility produces entropy
  - Entropy production (irreversibility) corresponds to lost work
  - Change of entropy in closed system is result of heat transfer and dissipation

# Heat Pump Systems

Thermodynamics Crash Course  
Isentropic Processes & Efficiency

Prof. J. Schiffmann

# Isentropic Processes

- Definition

$$\Delta S = \int_1^2 \frac{\delta Q}{T} + \sigma = 0$$

- Often system is considered adiabatic  $\rightarrow \delta Q = 0 \rightarrow \sigma = 0$
- Under such assumption, isentropic transformation is perfect process
- Isentropic, adiabatic process can be used as reference to assess performance of real machines

# Example: Turbine

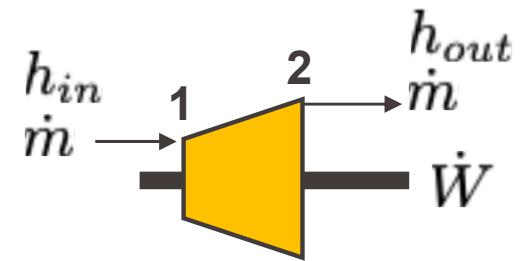
- Machine used to expand gas to recover work
- Mass flow established through pressure difference
- Work recovered through change of momentum
- Assumptions
  - Adiabatic and stationary operation
  - Negligible change in kinetic and potential energy



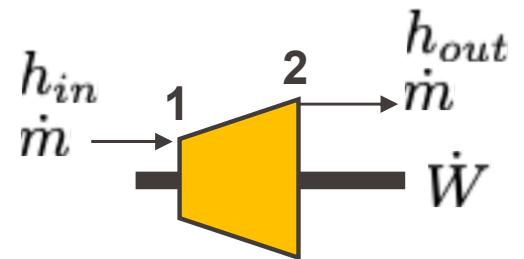
# Example: Turbine (p<sub>1</sub> to p<sub>2</sub>)

- Mass balance

$$\frac{dm_{cv}}{dt} = \sum \dot{m}_{in} - \sum \dot{m}_{out} = 0 \quad \rightarrow \quad \dot{m}_{in} = \dot{m}_{out} = \dot{m}$$



# Example: Turbine (p<sub>1</sub> to p<sub>2</sub>)



- Mass balance

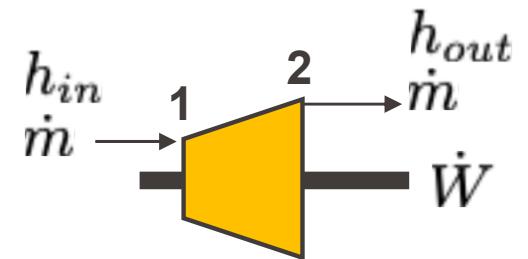
$$\frac{dm_{cv}}{dt} = \sum \dot{m}_{in} - \sum \dot{m}_{out} = 0 \quad \rightarrow \quad \dot{m}_{in} = \dot{m}_{out} = \dot{m}$$

- Energy balance

$$\frac{dE_{cv}}{dt} = \dot{W}_{cv} + \dot{Q}_{cv} + \sum \dot{m}_{in} \left( h + \frac{w^2}{2} + gz \right)_{in} - \sum \dot{m}_{out} \left( h + \frac{w^2}{2} + gz \right)_{out}$$

$$\dot{W}_{cv} = \dot{m} (h_{in} - h_{out})$$

# Example: Turbine (p<sub>1</sub> to p<sub>2</sub>)



- Mass balance

$$\frac{dm_{cv}}{dt} = \sum \dot{m}_{in} - \sum \dot{m}_{out} = 0 \quad \rightarrow \quad \dot{m}_{in} = \dot{m}_{out} = \dot{m}$$

- Energy balance

$$\frac{dE_{cv}}{dt} = \dot{W}_{cv} + \dot{Q}_{cv} + \sum \dot{m}_{in} \left( h + \frac{w^2}{2} + gz \right)_{in} - \sum \dot{m}_{out} \left( h + \frac{w^2}{2} + gz \right)_{out}$$

$$\dot{W}_{cv} = \dot{m} (h_{in} - h_{out})$$

- Entropy balance

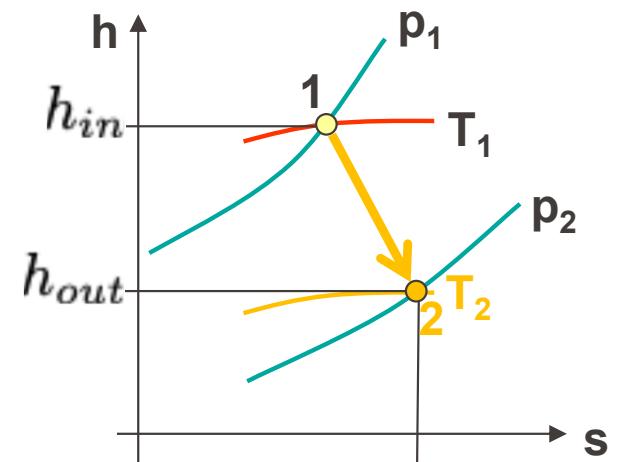
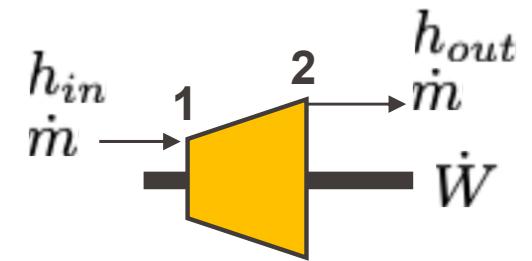
$$\frac{dS_{cv}}{dt} = \sum_j \frac{\dot{Q}_j}{T_j} + \sum_i \dot{ms}|_i - \sum_o \dot{ms}|_o + \dot{\sigma}_{cv} \quad \rightarrow \quad s_{in} - s_{out} = \frac{\dot{\sigma}_{cv}}{\dot{m}}$$

# Example: Turbine (p<sub>1</sub> to p<sub>2</sub>)

- Real process (entropy rise due to irreversibility)

$$\dot{W}_{cv}^- = \dot{m} (h_{in} - h_{out})$$

$$s_{in} - s_{out} = \frac{\dot{\sigma}_{cv}}{\dot{m}}$$



# Example: Turbine (p<sub>1</sub> to p<sub>2</sub>)

- Real process (entropy rise due to irreversibility)

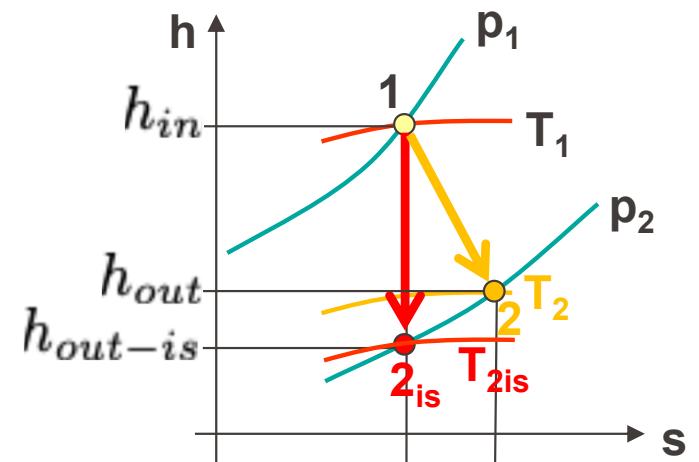
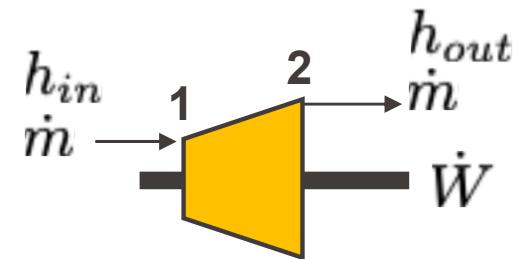
$$\dot{W}_{real}^- = \dot{m} (h_{in} - h_{out})$$

$$s_{in} - s_{out} = \frac{\dot{\sigma}_{cv}}{\dot{m}}$$

- Isentropic process

$$\dot{W}_{is}^- = \dot{m} (h_{in} - h_{out-is})$$

$$s_{in} - s_{out} = \frac{\dot{\sigma}_{cv}}{\dot{m}} = 0$$



# Example: Turbine (p<sub>1</sub> to p<sub>2</sub>)

- Real process (entropy rise due to irreversibility)

$$\dot{W}_{real}^- = \dot{m} (h_{in} - h_{out})$$

$$s_{in} - s_{out} = \frac{\dot{\sigma}_{cv}}{\dot{m}}$$

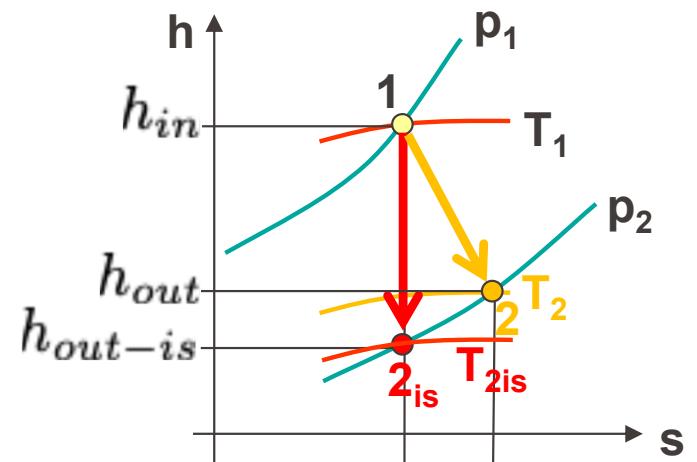
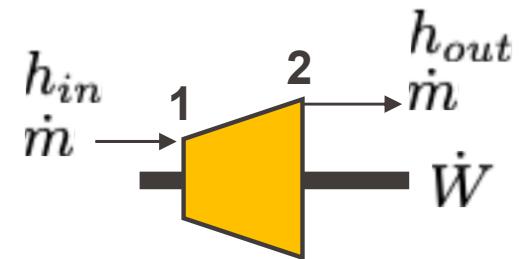
- Isentropic process

$$\dot{W}_{is}^- = \dot{m} (h_{in} - h_{out-is})$$

$$s_{in} - s_{out} = \frac{\dot{\sigma}_{cv}}{\dot{m}} = 0$$

- Isentropic turbine efficiency

$$\eta_{t-is} = \frac{\dot{W}_{real}^-}{\dot{W}_{is}^-} = \frac{h_{in} - h_{out}}{h_{in} - h_{out-is}}$$



# Example: Compressor ( $p_1$ to $p_2$ )

- Real process (entropy rise due to irreversibility)

$$\dot{W}_{real}^+ = \dot{m} (h_{out} - h_{in})$$

$$s_{out} - s_{in} = \frac{\dot{\sigma}_{cv}}{\dot{m}}$$

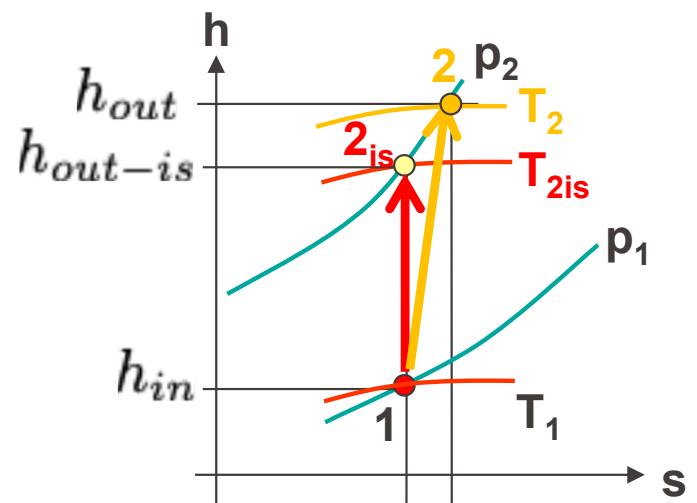
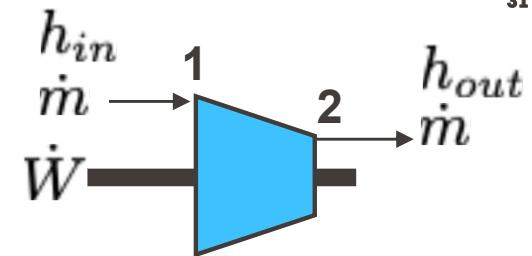
- Isentropic process

$$\dot{W}_{is}^+ = \dot{m} (h_{out-is} - h_{in})$$

$$s_{out} - s_{in} = \frac{\dot{\sigma}_{cv}}{\dot{m}} = 0$$

- Isentropic compressor efficiency

$$\eta_{k-is} = \frac{\dot{W}_{is}^+}{\dot{W}_{real}^+} = \frac{h_{out-is} - h_{in}}{h_{out} - h_{in}}$$



# Heat Pump Systems

Thermodynamics Crash Course  
Exergy

Prof. J. Schiffmann

- Thermodynamic analysis governed by two laws
- First law
  - Conservation of energy
  - Energy can be transformed into different and equivalent forms
- Second law
  - Dictates direction of natural energy transfer
  - Requires cycles to work between two thermal reservoirs
  - Limits maximum efficiency of thermal machines
  - Allows distinction between perfect and real processes

# Issue with 1<sup>st</sup> Law Efficiency Definition

- Classical approach to assess performance is through 1<sup>st</sup> law efficiency

$$\eta_{th} = \frac{\text{yield}}{\text{investment}} = \frac{W^-}{Q^+} = 1 - \frac{Q^-}{Q^+}$$

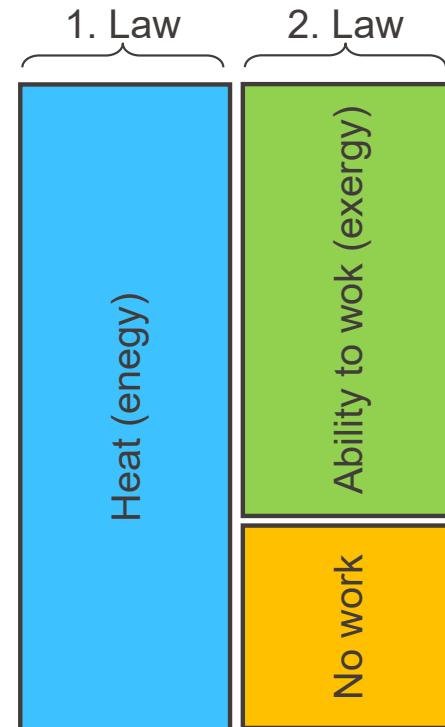
- For heat pump this definition leads to efficiency  $> 1$
- In case of combined heat and power plants, yield is heat and work, efficiency close to 1
- 1<sup>st</sup> law efficiency definition makes no distinction between quality of energy  $\rightarrow$  inadequate to measure degree of thermodynamic perfection

# Quality of Energy

- Second law gives indication regarding quality of energy
- Heat delivered at certain temperature can only partially be transformed into work

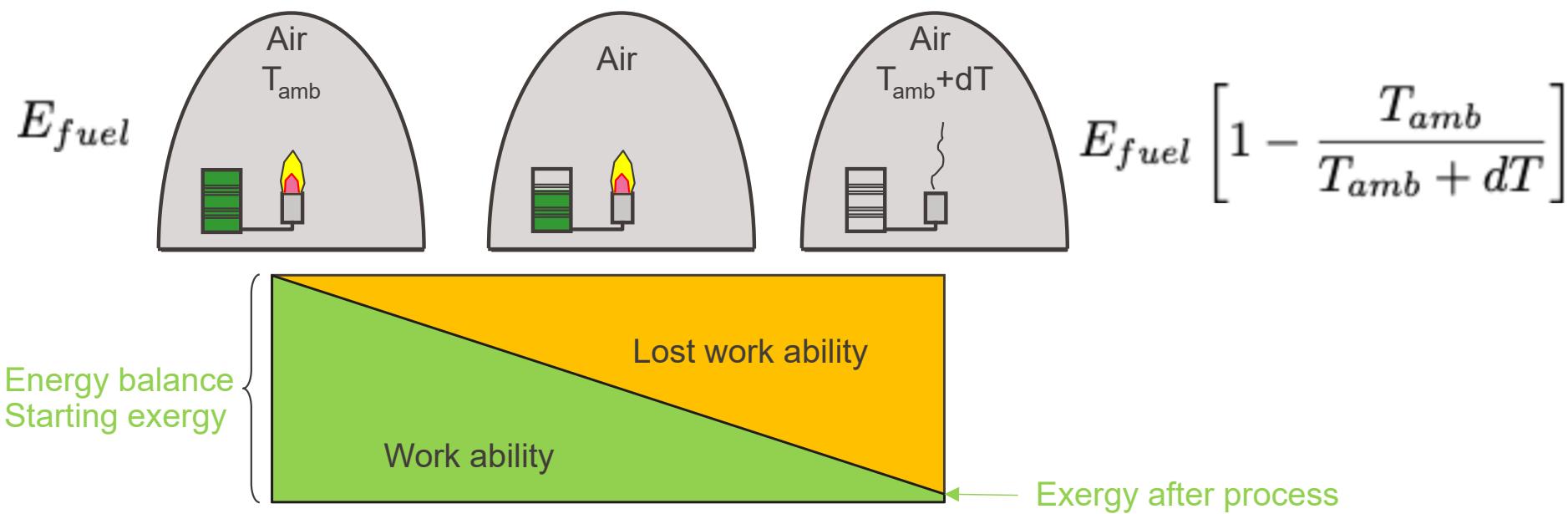
$$\eta_c = 1 - \frac{Q_{cold}^-}{Q_{hot}^+} = 1 - \frac{T_{cold}}{T_{hot}}$$

- Work has higher quality than heat, but 1<sup>st</sup> law does not differentiate



# Quality of Energy: Illustration

- Consider isolated system with fuel container
- Chemical work potential of fuel initially vs. lukewarm air at end



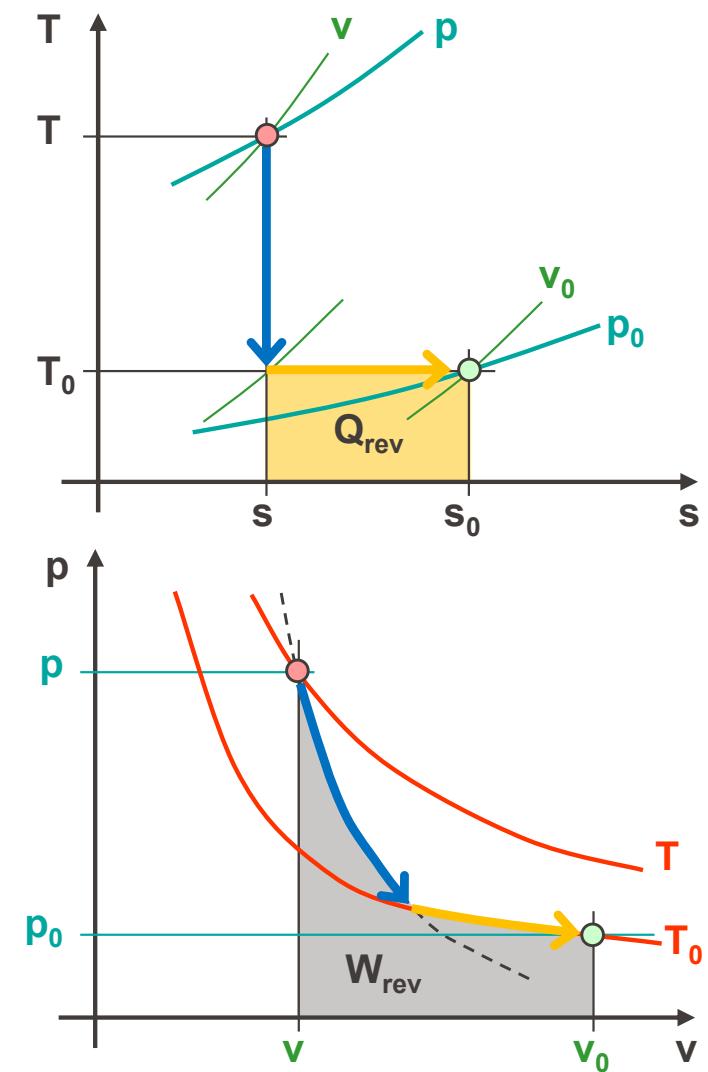
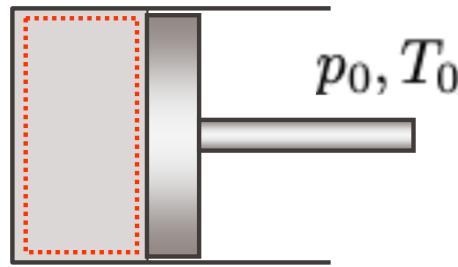
# Concept of Exergy

- Idea is to indicate maximum work of a system relative to ambient conditions (pressure & temperature)
- All system that has a thermodynamic “tension” relative to ambient has ability to transform it into work via reversible process → exergy
- Exergy is a thermodynamic state variable
- Such approach ensures simultaneous satisfaction of 1<sup>st</sup> & 2<sup>nd</sup> law

# Concept of Exergy

- Consider pressurized vessel at rest
- Idea is to indicate maximum work relative to ambient ( $p_0$  &  $T_0$ )
- Process involves adiabatic reversible expansion until  $T_0$ , followed by isothermal expansion until  $p_0$

$m, p, T, S, U, V, KE, PE$



# Concept of Exergy

- Change in energy

$$E_0 - E = Q_{rev} + W_{rev}$$

where

$$Q_{rev} = T_0 (S_0 - S)$$

$$E_0 - E = U_0 - U - KE - PE$$

- Reversible work delivered by system

$$W_{rev} = U_0 - U - T_0 (S_0 - S) - KE - PE$$

$$W_{use} = U_0 - U - T_0 (S_0 - S) - KE - PE + p_0 \underbrace{(V_0 - V)}_{\text{Reduction from reversible work due to ambient displacement work}}$$

# Concept of Exergy

- Exergy in closed system corresponds to useful work (positive sign)

$$Ex = -W_{use}$$

$$Ex = (U - U_0) + p_0 (V - V_0) - T_0 (S - S_0) + KE + PE$$

$$Ex = (E - E_0) + p_0 (V - V_0) - T_0 (S - S_0)$$

- Exergy is thermodynamic state
- Between two states

$$Ex_2 - Ex_1 = (E_2 - E_1) + p_0 (V_2 - V_1) - T_0 (S_2 - S_1)$$

# Exergy Balance in Closed Systems

- Energy and entropy balance

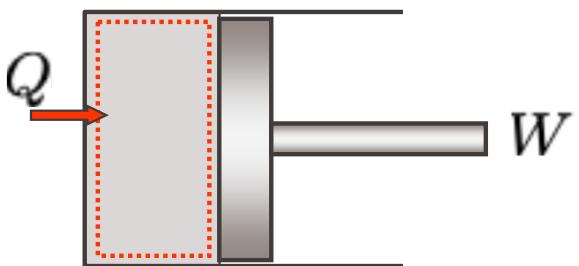
$$E_2 - E_1 = \int_1^2 \delta Q + W_{12}$$

$$S_2 - S_1 = \int_1^2 \frac{\delta Q}{T} + \sigma$$

- Change of exergy

$$Ex_2 - Ex_1 = (E_2 - E_1) + p_0 (V_2 - V_1) - T_0 (S_2 - S_1)$$

$$Ex_2 - Ex_1 = \int_1^2 \left(1 - \frac{T_0}{T}\right) \delta Q + [W_{12} + p_0 (V_2 - V_1)] - T_0 \sigma$$



# Exergy Balance in Closed Systems

$$Ex_2 - Ex_1 = \underbrace{\int_1^2 \left(1 - \frac{T_0}{T}\right) \delta Q}_{\text{State variables}} + \underbrace{[W_{12} + p_0 (V_2 - V_1)]}_{\text{Process dependent}} - \underbrace{T_0 \sigma}_{\text{Process dependent}}$$

# Exergy Balance in Closed Systems

$$Ex_2 - Ex_1 = \underbrace{\int_1^2 \left(1 - \frac{T_0}{T}\right) \delta Q}_{\text{Process independent}} + \underbrace{[W_{12} + p_0 (V_2 - V_1)]}_{\text{Process dependent}} - \underbrace{T_0 \sigma}_{\text{Process independent}}$$

- Exergy transfer through heat
- Heat energy reduced by Carnot efficiency
- Best process to transform heat into useful work

# Exergy Balance in Closed Systems

$$Ex_2 - Ex_1 = \underbrace{\int_1^2 \left(1 - \frac{T_0}{T}\right) \delta Q}_{\substack{\text{State variables} \\ \text{Process independent}}} + \underbrace{[W_{12} + p_0 (V_2 - V_1)]}_{\substack{\text{Process dependent} \\ \text{Exergy transfer through work}}} - \underbrace{T_0 \sigma}_{\substack{\text{Reduced by ambient displacement work}}}$$

- Exergy transfer through heat
- Heat energy reduced by Carnot efficiency
- Best process to transform heat into useful work

- Exergy transfer through work
- Reduced by ambient displacement work

# Exergy Balance in Closed Systems

$$Ex_2 - Ex_1 = \int_1^2 \left(1 - \frac{T_0}{T}\right) \delta Q + [W_{12} + p_0 (V_2 - V_1)] - T_0 \sigma$$

State variables  
Process independent

Process dependent

- Exergy transfer through heat
- Heat energy reduced by Carnot efficiency
- Best process to transform heat into useful work
- Exergy transfer through work
- Reduced by ambient displacement work
- Exergy losses due to process irreversibility

# Exergy Balance in Open Systems

- Transfer of exergy through convection across system boundary
- Definition of co-enthalpy in analogous way as for closed system and with definition of enthalpy

$$K = (H - H_0) - T_0 (S - S_0) + KE + PE$$

- Co-enthalpy is thermodynamic state
- Between two states

$$k_2 - k_1 = (h_2 - h_1) - T_0 (s_2 - s_1) + \frac{w_2^2 - w_1^2}{2} + g (z_2 - z_1)$$

# Exergy Balance in Open Systems

- Transfer of exergy through convection across system boundary
- Definition of co-enthalpy in analogous way as for closed system and with definition of enthalpy

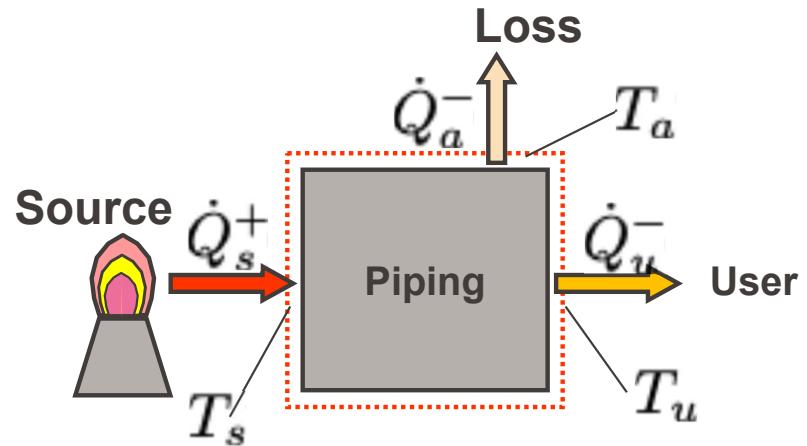
$$\frac{dEx_{cv}}{dt} = \underbrace{\sum_j \int \left(1 - \frac{T_0}{T_j}\right) \delta \dot{Q}}_{\text{Balance of heat exergy}} + \underbrace{\dot{W}_{cv} + p_0 \frac{dV_{cv}}{dt}}_{\text{Balance of work}} + \underbrace{\sum_i \dot{m}_i k_i -}_{\text{Balance of co-enthalpy}} + \underbrace{\sum_o \dot{m}_o k_o -}_{\text{Exergy losses}} \underbrace{T_0 \dot{\sigma}_{cv}}_{\dot{L}}$$

# Example: Piping System in Fuel Boiler

- Consider fuel boiler system
- Stationary system, no work
- Energy balance

$$\frac{dE_{cv}}{dt} = \dot{W}_{cv} + \dot{Q}_{cv} + \sum \dot{m}_{in} \left( h + \frac{w^2}{2} + gz \right)_{in} - \sum \dot{m}_{out} \left( h + \frac{w^2}{2} + gz \right)_{out}$$

$$\dot{Q}_s^+ - \dot{Q}_u^- - \dot{Q}_a^- = 0$$



# Example: Piping System in Fuel Boiler

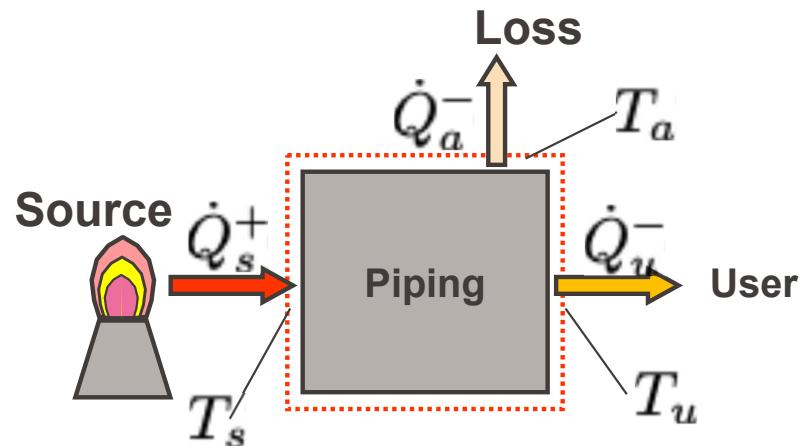
- Consider fuel boiler system
- Stationary system, no work
- Energy balance

$$\frac{dE_{cv}}{dt} = \dot{W}_{cv} + \dot{Q}_{cv} + \sum \dot{m}_{in} \left( h + \frac{w^2}{2} + gz \right)_{in} - \sum \dot{m}_{out} \left( h + \frac{w^2}{2} + gz \right)_{out}$$

$$\dot{Q}_s^+ - \dot{Q}_u^- - \dot{Q}_a^- = 0$$

- Energy efficiency

$$\eta = \frac{\dot{Q}_u^-}{\dot{Q}_s^+} \approx 1$$

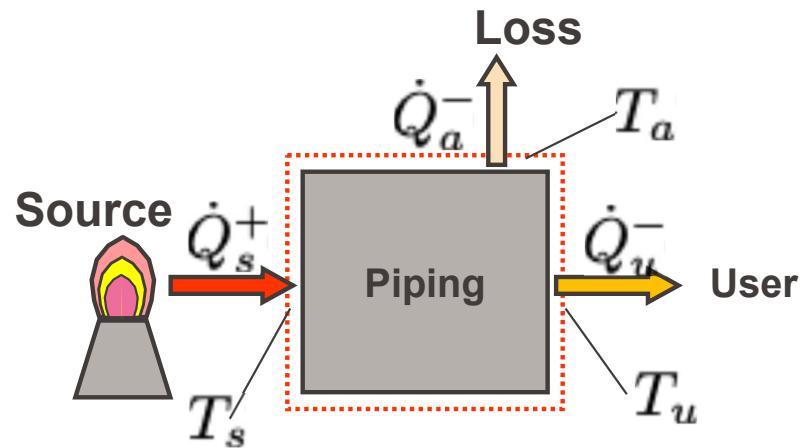


# Example: Piping System in Fuel Boiler

- Consider fuel boiler system
- Stationary system, no work
- **Exergy** balance

$$\frac{dEx_{cv}}{dt} = \sum_j \int \left(1 - \frac{T_0}{T_j}\right) \delta \dot{Q} + \dot{W}_{cv} + p_0 \frac{dV_{cv}}{dt} + \sum_i \dot{m}_i k_i - + \sum_o \dot{m}_o k_o - T_0 \dot{\sigma}_{cv}$$

$$\left(1 - \frac{T_0}{T_s}\right) \dot{Q}_s^+ - \left(1 - \frac{T_0}{T_u}\right) \dot{Q}_u^- - \left(1 - \frac{T_0}{T_a}\right) \dot{Q}_a^- = \dot{L}$$



# Example: Piping System in Fuel Boiler

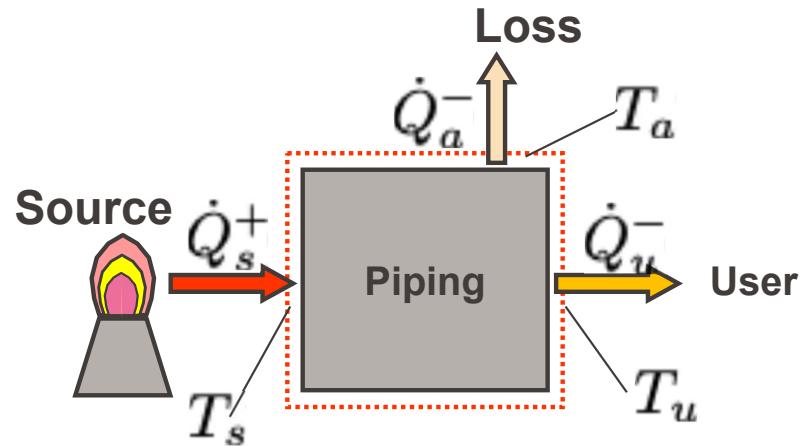
- Consider fuel boiler system
- Stationary system, no work

- **Exergy** balance

$$\left(1 - \frac{T_0}{T_s}\right) \dot{Q}_s^+ - \left(1 - \frac{T_0}{T_u}\right) \dot{Q}_u^- - \left(1 - \frac{T_0}{T_a}\right) \dot{Q}_a^- = \dot{L}$$

- Exergy efficiency

$$\eta_{ex} = \frac{\left(1 - \frac{T_0}{T_u}\right) \dot{Q}_u^-}{\left(1 - \frac{T_0}{T_s}\right) \dot{Q}_s^+}$$



# Example: Piping System in Fuel Boiler

- Consider fuel boiler system
- Stationary system, no work

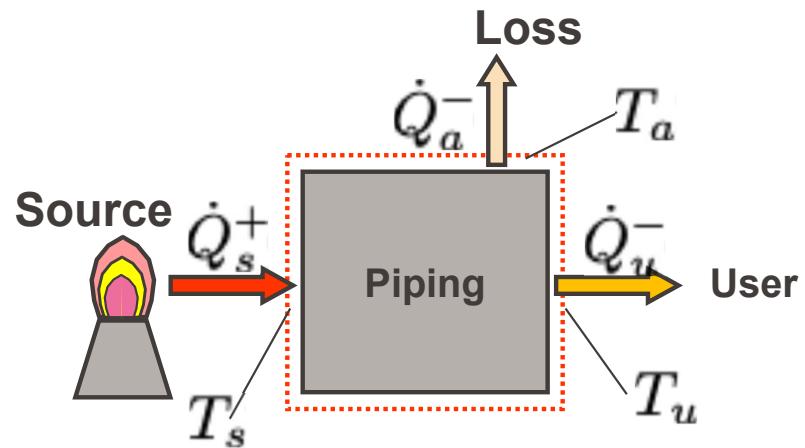
- **Exergy** balance

$$\left(1 - \frac{T_0}{T_s}\right) \dot{Q}_s^+ - \left(1 - \frac{T_0}{T_u}\right) \dot{Q}_u^- - \left(1 - \frac{T_0}{T_a}\right) \dot{Q}_a^- = \dot{L}$$

- Exergy efficiency

$$\eta_{ex} = \frac{\left(1 - \frac{T_0}{T_u}\right) \dot{Q}_u^-}{\left(1 - \frac{T_0}{T_s}\right) \dot{Q}_s^+} = \frac{\left(1 - \frac{T_0}{T_u}\right)}{\left(1 - \frac{T_0}{T_s}\right)} \eta \ll 1$$

- 90% of heat exergy received by source is lost
- Due primarily to the temperature de-evaluation of high temperature source to low temperature in piping system



# Example: Power Cycle

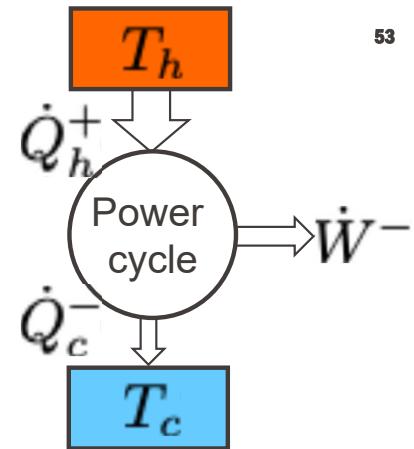
- Consider power cycle
- Stationary system
- **Energy** balance

$$\frac{dE_{cv}}{dt} = \dot{W}_{cv} + \dot{Q}_{cv} + \sum \dot{m}_{in} \left( h + \frac{w^2}{2} + gz \right)_{in} - \sum \dot{m}_{out} \left( h + \frac{w^2}{2} + gz \right)_{out}$$

$$\dot{Q}_h^+ - \dot{Q}_c^- - \dot{W}^- = 0$$

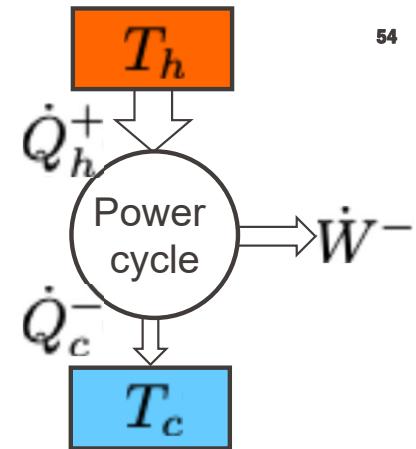
- **Energy** efficiency

$$\eta_{th} = \frac{\dot{W}^-}{\dot{Q}_h^+}$$



# Example: Power Cycle

- Consider power cycle
- Stationary system, no work
- **Exergy** balance



$$\frac{dEx_{cv}}{dt} = \sum_j \int \left(1 - \frac{T_0}{T_j}\right) \delta \dot{Q} + \dot{W}_{cv} + p_0 \frac{dV_{cv}}{dt} + \sum_i \dot{m}_i k_i - + \sum_o \dot{m}_o k_o - T_0 \dot{\sigma}_{cv}$$

$$\left(1 - \frac{T_0}{T_h}\right) \dot{Q}_h^+ - \left(1 - \frac{T_0}{T_c}\right) \dot{Q}_c^- - \dot{W}^- = \dot{L}$$

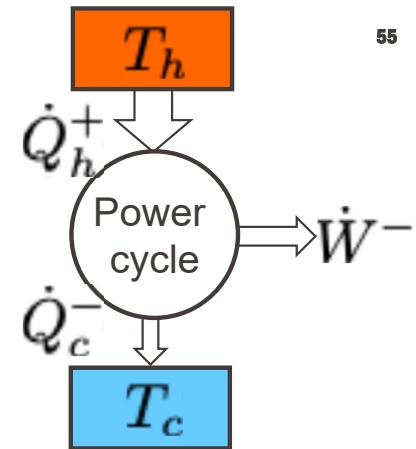
# Example: Power Cycle

- Consider power cycle
- Stationary system, no work
- **Exergy** balance

$$\left(1 - \frac{T_0}{T_h}\right) \dot{Q}_h^+ - \left(1 - \frac{T_0}{T_c}\right) \dot{Q}_c^- - \dot{W}^- = \dot{L}$$

- **Exergy** efficiency

$$\eta_{ex} = \frac{\dot{W}^-}{\left(1 - \frac{T_0}{T_h}\right) \dot{Q}_h^+} = \frac{\eta_{th}}{\left(1 - \frac{T_0}{T_h}\right)} = \frac{\eta_{th}}{\eta_c}$$



# Conclusion

- Opposed to energy balance, exergy analysis combines 1<sup>st</sup> and 2<sup>nd</sup> law into a new thermodynamic state
- Exergy analysis automatically merges energy balance with feasibility limits imposed by 2<sup>nd</sup> law
- Energy efficiency does not consider quality of energy → may lead to spurious values
- Exergy efficiency suggested to be more sound approach to assess quality of thermodynamic system

- Introduction to heat pumps
- Thermodynamic analysis of heat pumps (energy & exergy)
- Technical challenges and limitations
- Main components of heat pumps

- Theory questions
- Entropy and exergy analysis of a power plant
- Entropy and exergy analysis of a turbine
- Entropy and exergy analysis of a compressor
- Cold room of a refrigeration plant