Heat Pump Systems

Summary W9

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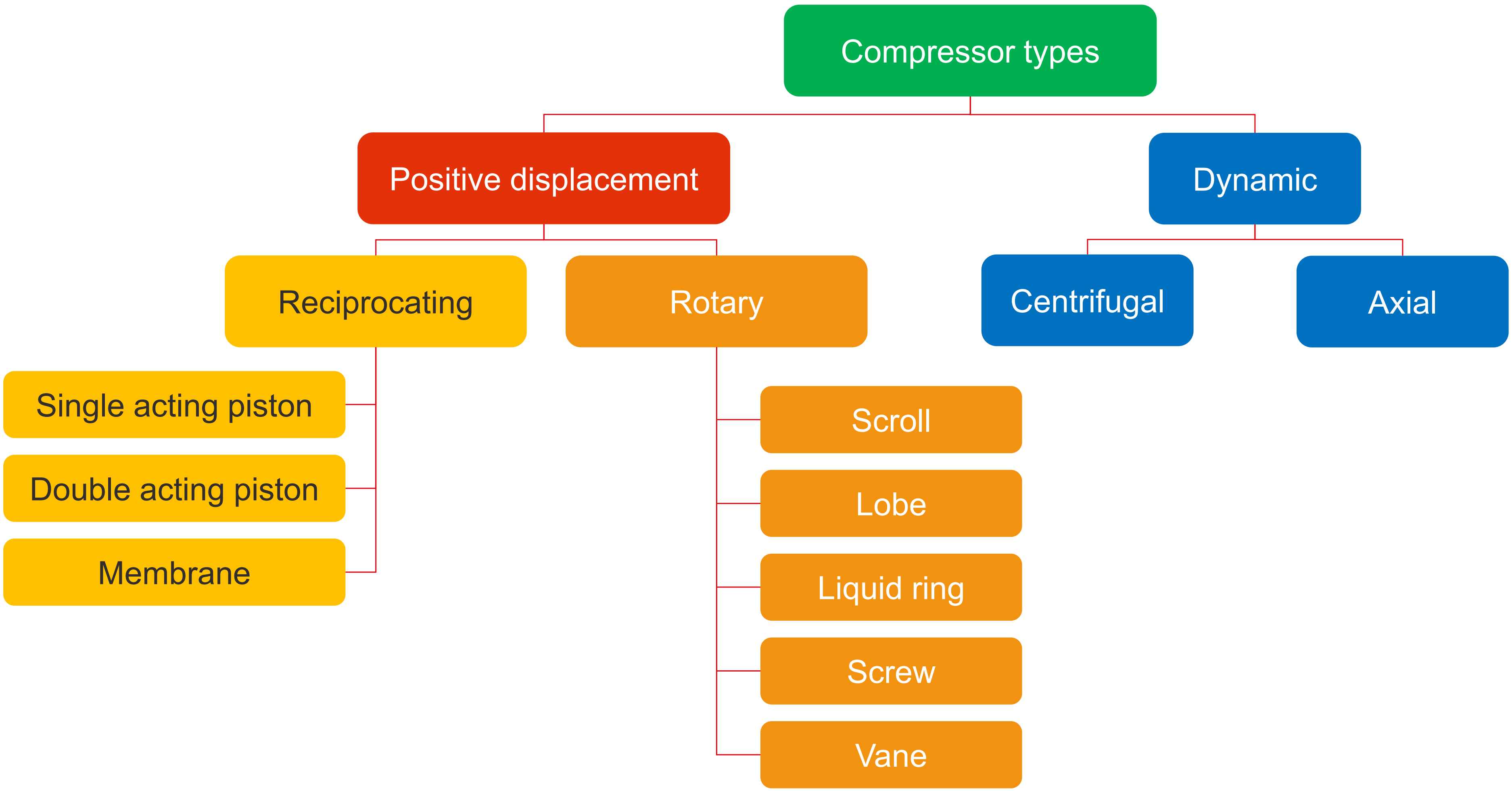
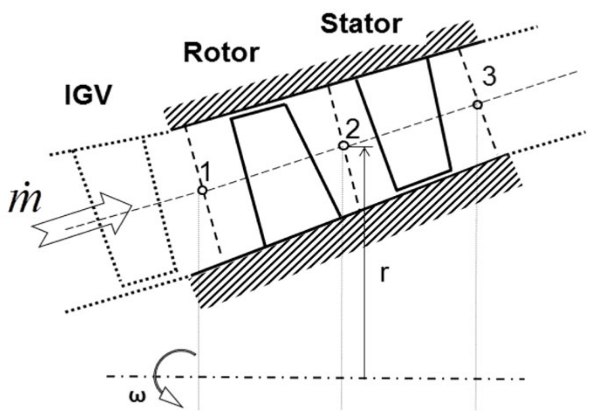
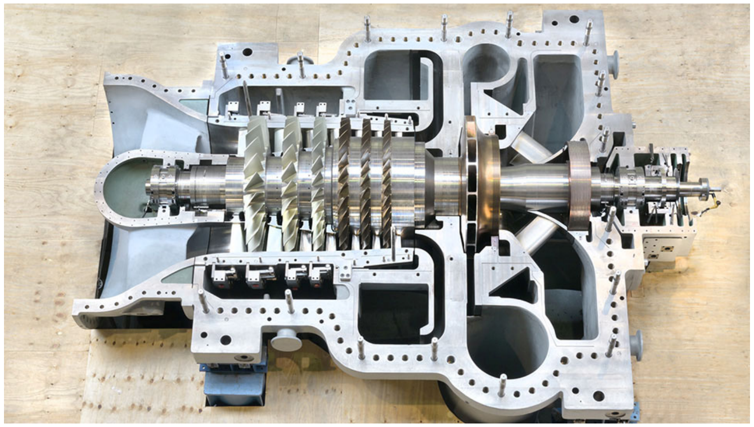
Compressor Classification



Working Principle

- Composed of bladed rotating part (rotor) exchanging work with fluid and stationary bladed part to direct flow (stator)
  - Compressor rotor blade row adds energy to fluid by increasing its swirl and kinetic energy
  - Stator blade row converts kinetic energy into pressure rise
- Stage (rotor & stator) is smallest functional entity of turbocompressor





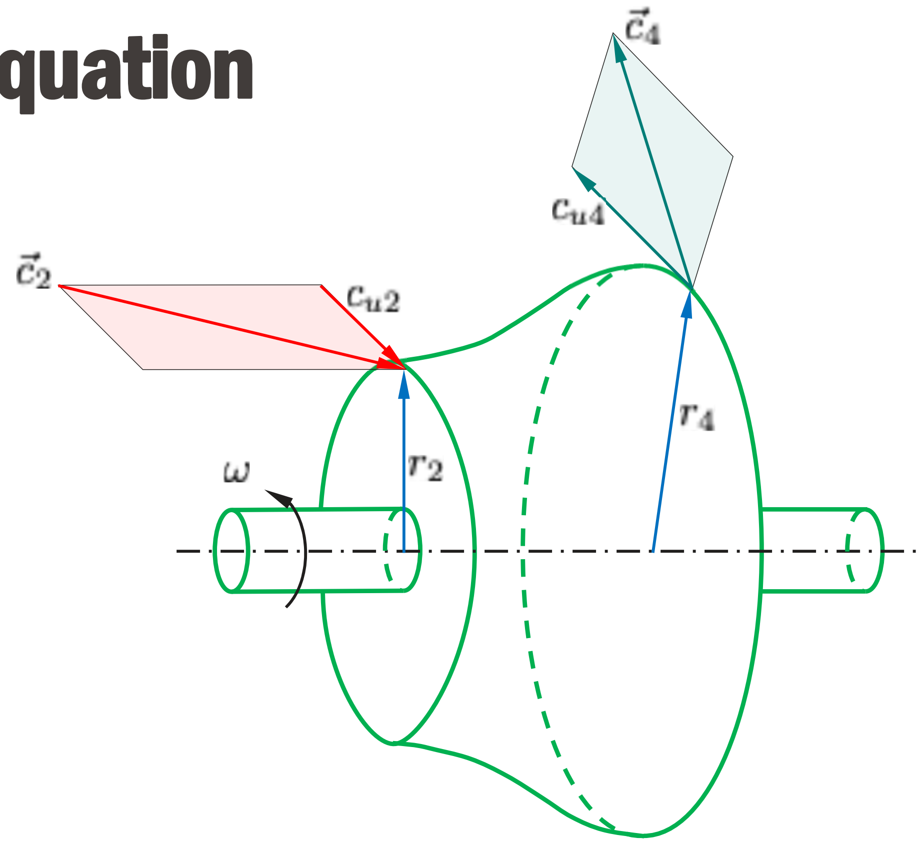
https://www.man-es.com/process-industry/products/compressors/axial

Euler Turbomachinery Equation

- Euler turbomachinery equation

$$e^{+} = \omega\left(r_4 c_{4u} - r_2 c_{2u}\right)$$

$$e^{+} = u_4 c_{4u} - u_2 c_{2u}$$



- Universally applicable
- Determines work from changes between mean conditions at inlet and outlet
- No knowledge on inner workings required

Insights from Euler Equation

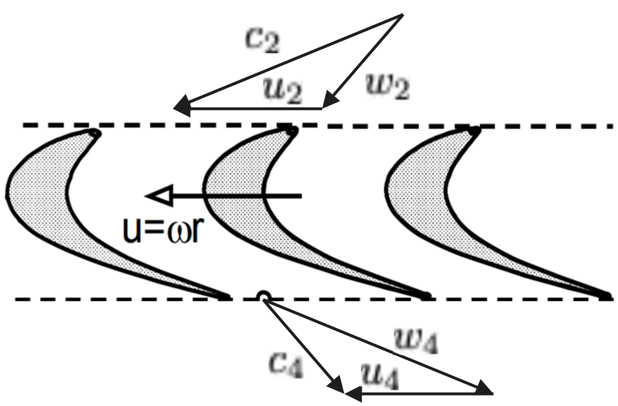
- Work is determined by the change in circumferential component of absolute velocity → change in swirl

$$e^{+} = u_4 c_{4u} - u_2 c_{2u}$$

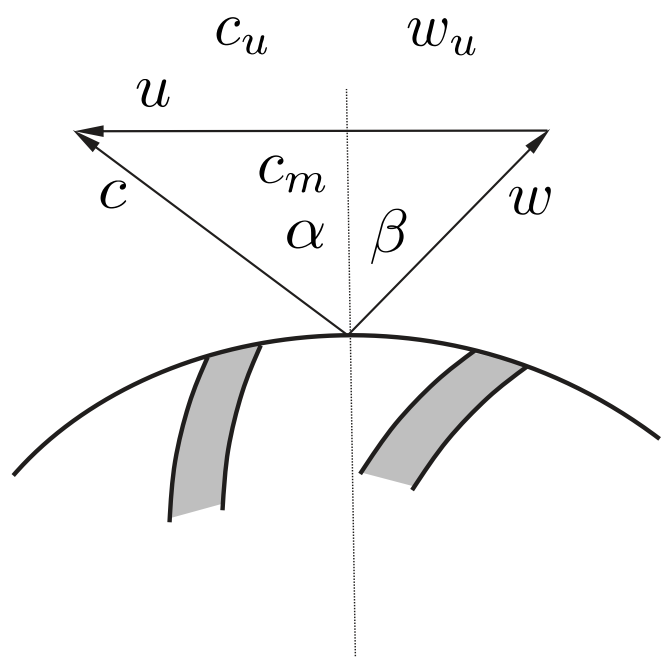
- Change in swirl and flow guidance is achieved by sufficiently closely spaced blades

$$c_u = f\left(u, w\right)$$

- Mastery of velocity magnitude and direction is key in turbomachinery design



Velocity Triangles



- Velocity triangle connects relative to absolute measurements

$$\vec{c} = \vec{w} + \vec{u}$$

- Trigonometry yields

$$u c_u = \frac{1}{2}\left(c^2 + u^2 - w^2\right)$$

$$e^+ = u_4 c_{4u} - u_2 c_{2u} = \frac{1}{2}\left[\left(c_4^2 - c_2^2\right) + \left(w_2^2 - w_4^2\right) + \left(u_4^2 - u_2^2\right)\right]$$

Link Between Euler and Thermodynamics

- Assume a radial outflow, no inlet pre-swirl

$$e^+ = c_{u4}u_4 - c_{u2}u_2 = u_4^2$$

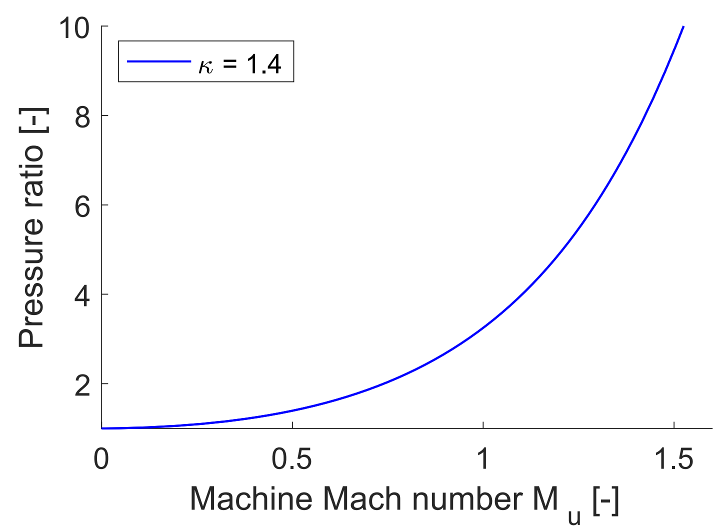
$$e^+ = u_4^2 = \frac{\kappa}{\kappa-1} R T_{01} \left[\Pi^{\frac{\kappa-1}{\kappa}} - 1 \right]$$

$$\Pi = \frac{P_{out}}{P_{in}}$$

$$\Pi = \left[\underbrace{\frac{u_4^2}{\kappa R T_{01}}} (\kappa - 1) \right]^{\frac{\kappa}{\kappa-1}}$$

$$\frac{u_4^2}{a_{01}^2} = \mathrm{Mu}^2$$

$$\Pi = \left[\mathrm{Mu}^2 (\kappa - 1) \right]^{\frac{\kappa}{\kappa-1}}$$



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Heat Pump Systems

Thermodynamics in Turbocompressors

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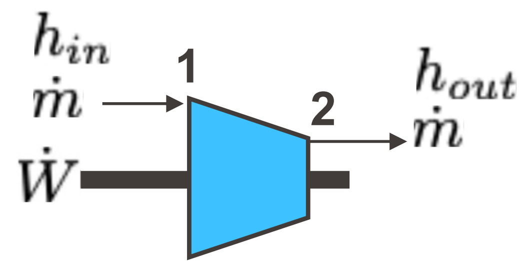
Total and Static Conditions



- Energy balance for open system

$$\dot{W}^+ + \dot{Q}^+ = \dot{m}\left[(h_2 - h_1) + 1/2\left(c_2^2 - c_1^2\right) + g\left(z_2 - z_1\right)\right]$$

- Neglecting gravity, assuming adiabatic operation

$$\frac{\dot{W}^+}{\dot{m}} = \underbrace{\left(h_2 + \frac{1}{2}c_2^2\right)}_{h_{02}} - \underbrace{\left(h_1 + \frac{1}{2}c_1^2\right)}_{h_{01}}$$

- Total enthalpy → fictive thermodynamic state variable

$$h_0 = h + \frac{c^2}{2}$$

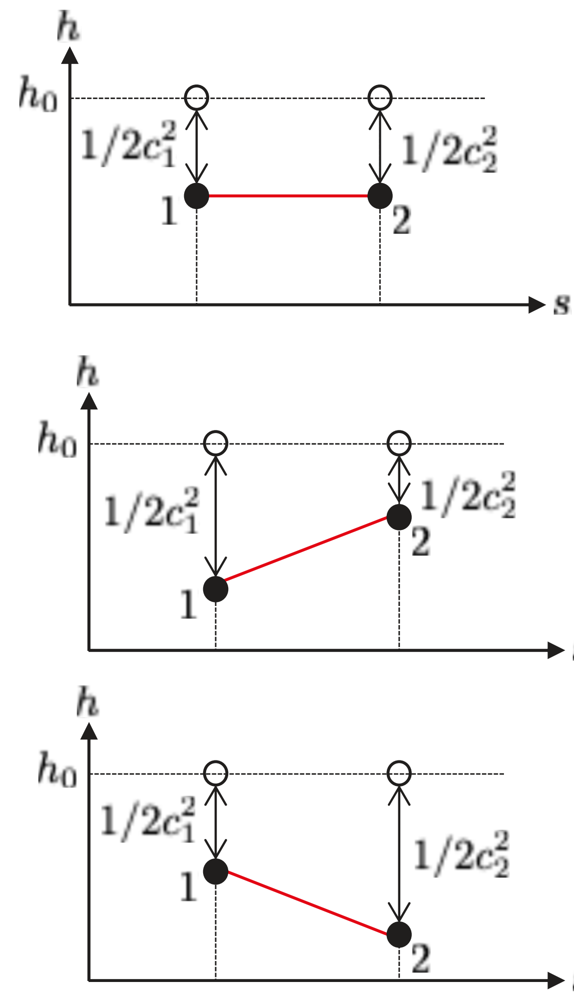
Specific Cases

- Adiabatic work process $w^+ = \Delta h_0$
  - Compressor with no heat transfer
- Diabatic work process $w^+ + q^+ = \Delta h_0$
  - Cooled/heated compressor
- Adiabatic flow process $0 = \Delta h_0$
  - Non-rotating components such as inducer, diffuser,…
- Diabatic flow process $q^+ = \Delta h_0$
  - Stationary components such as HEX

Typical Adiabatic Flow Processes

$$0 = \Delta h_0$$

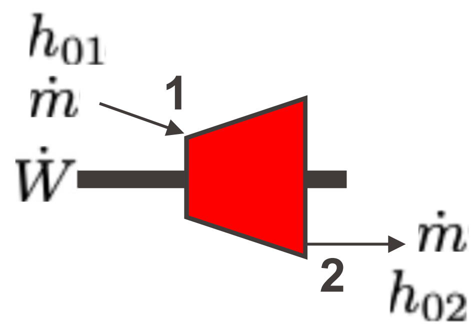
- Constant area pipe flow
- Diffuser
- Nozzle

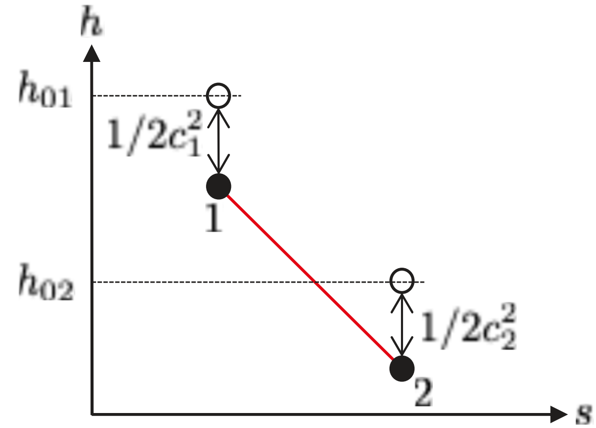


Typical Adiabatic Work Processes

- Turbine

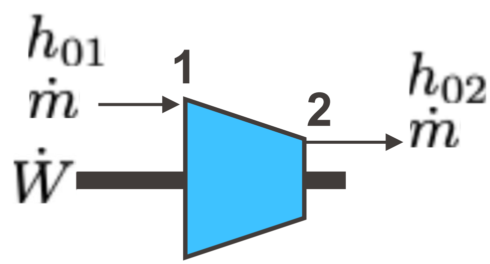
$$w^+ = h_{02} - h_{01} < 0$$

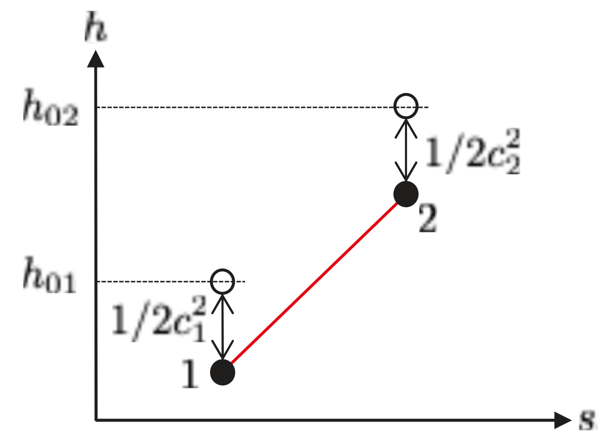




- Compressor

$$w^+ = h_{02} - h_{01} > 0$$





Definition of Rothalpy

- Coupling Euler equation with thermodynamic states ($\dot{q} = 0$)

$$e^+ = u_4 c_{4u} - u_2 c_{2u} = h_{04} - h_{02} = \left(h_4 + \frac{c_4^2}{2}\right) - \left(h_2 + \frac{c_2^2}{2}\right)$$

- Rearranging…

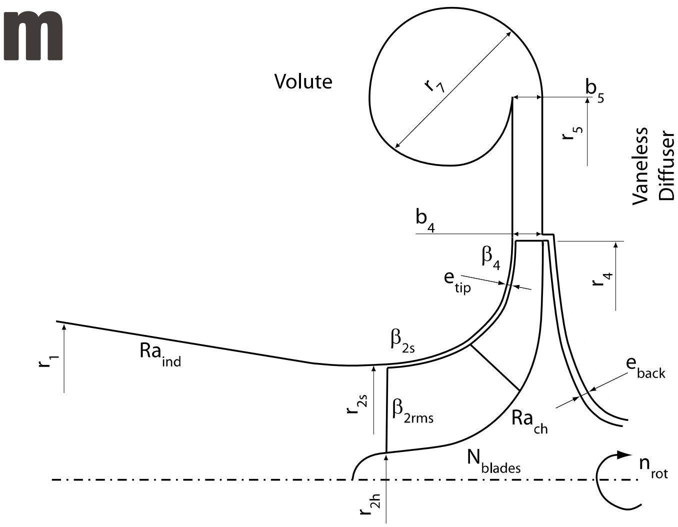
$$\underbrace{h_4 + \frac{c_4^2}{2} - u_4 c_{4u}}_{I_4} = \underbrace{h_2 + \frac{c_2^2}{2} - u_2 c_{2u}}_{I_2}$$

Rothalpy

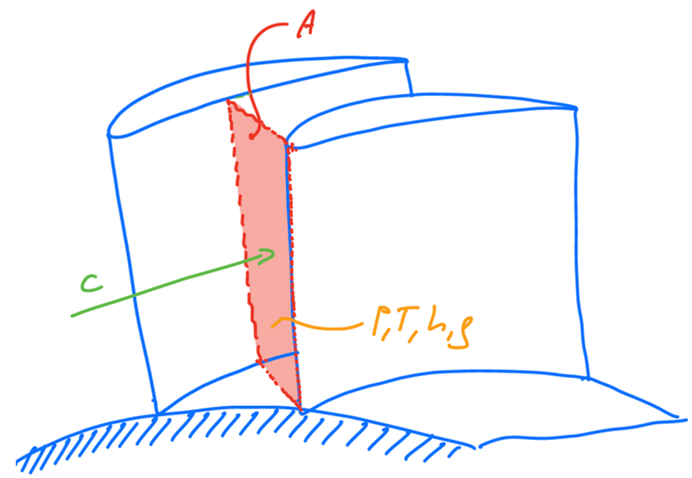
- Coupling with velocity triangles

$$\underbrace{h_4 + \frac{w_4^2}{2}}_{h_{0,rel,4}} - \frac{u_4^2}{2} = \underbrace{h_2 + \frac{w_2^2}{2}}_{h_{0,rel,2}} - \frac{u_2^2}{2}$$

Turbocompressor in h-s-Diagram

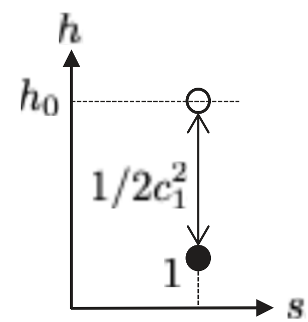


Mass Flow and Flow Velocity

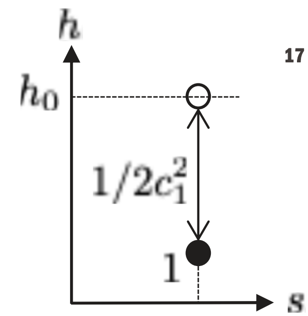


- So far looked at velocity triangles and link to thermodynamics
- Often, only total conditions, area and mass flow are known
- Mass flow and velocity linked through static property, which depend on velocity for given total condition
- How to translate mass flow into flow velocity?

$$\dot{m} = \rho c A \qquad \rho = f\left(P, T\right)$$



Dimensionless Mass Flow Equation I



- Perfect gas assumption

$$h_0 = h + 1/2c^2 \qquad h = c_p T$$

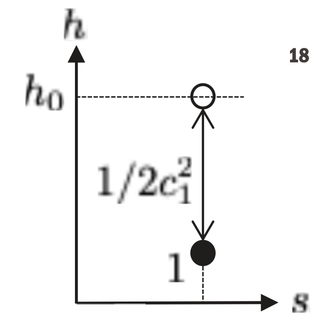
$$T_0 = T + \frac{c^2}{2c_p} \quad \text{use } c_p - c_v = R \quad \kappa = c_p/c_v \quad a = \sqrt{\kappa R T} \quad M = c/a$$

$$\frac{T_0}{T} = 1 + \frac{1}{2}c^2\frac{\kappa - 1}{\kappa R T} = 1 + \frac{\kappa - 1}{2}M^2$$

$$\frac{P_0}{P} = \left(\frac{T_0}{T}\right)^{\kappa/(\kappa-1)} = \left(1 + \frac{\kappa - 1}{2}M^2\right)^{\kappa/(\kappa-1)}$$

- Ratio of total to static states only function of Mach number

Dimensionless Mass Flow Equation II

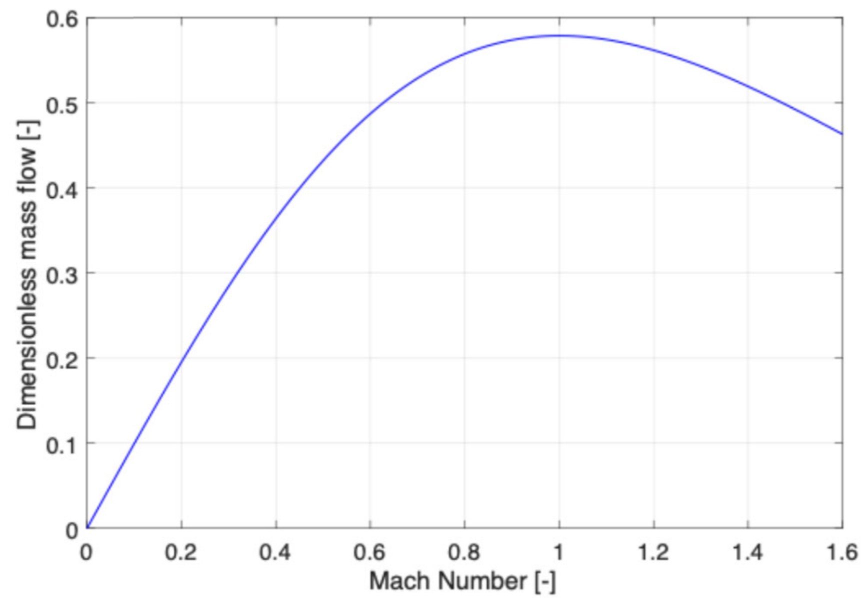


$$\dot{m} = \rho c A \quad \text{use} \quad a = \sqrt{\kappa R T} \quad M = c/a \Rightarrow \frac{\dot{m}}{A} = \frac{P}{RT} M \sqrt{\kappa R T}$$

Replace static with total states

$$\frac{\dot{m}\sqrt{RT_0/\kappa}}{AP_0} = M\left(1 + \frac{\kappa - 1}{2} M^2\right)^{-\frac{\kappa+1}{2(\kappa-1)}}$$

- Knowledge of mass-flow, area A, and total conditions (P_0, T_0) allows calculating Mach number
- Requires numerical approach



Summary

- Turbomachinery applications span across wide range of power and applications
- All turbomachinery governed by same principle → Euler equation
- Mastering velocity angles is key to induce change in swirl → blades
- Link between Euler equation and thermodynamics yields pressure ratio as a function of machine Mach number
- Velocity across area for given mass-flow and total conditions needs to be found iteratively

Heat Pump Systems

Centrifugal Compressors

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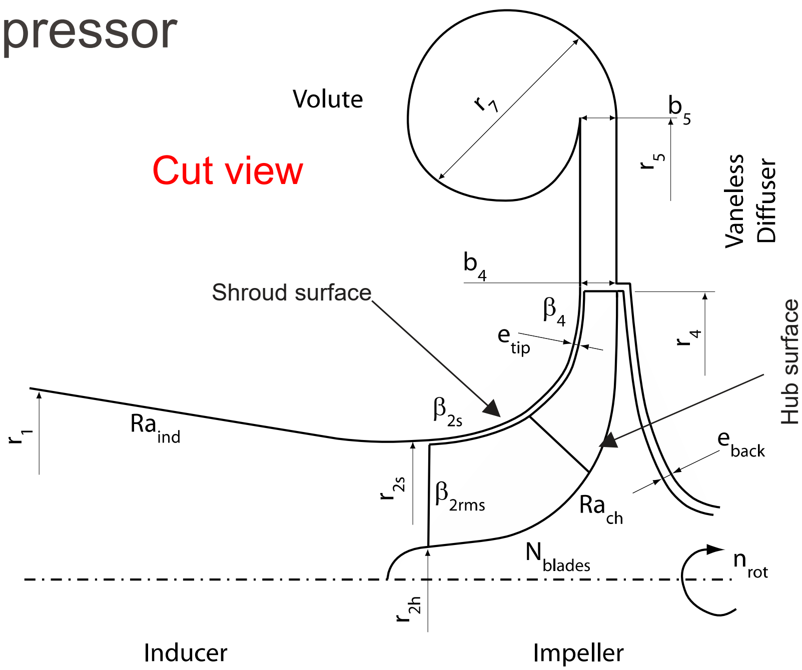
Radial Compressor Stage

- Stage is smallest functional entity of turbocompressor
- The inducer accelerates fluid into the compressor
- Rotor transfers energy from shaft to fluid
- Stator (diffuser) converts kinetic energy out of impeller into pressure increase
- Volute collects flow at discharge

Cut view



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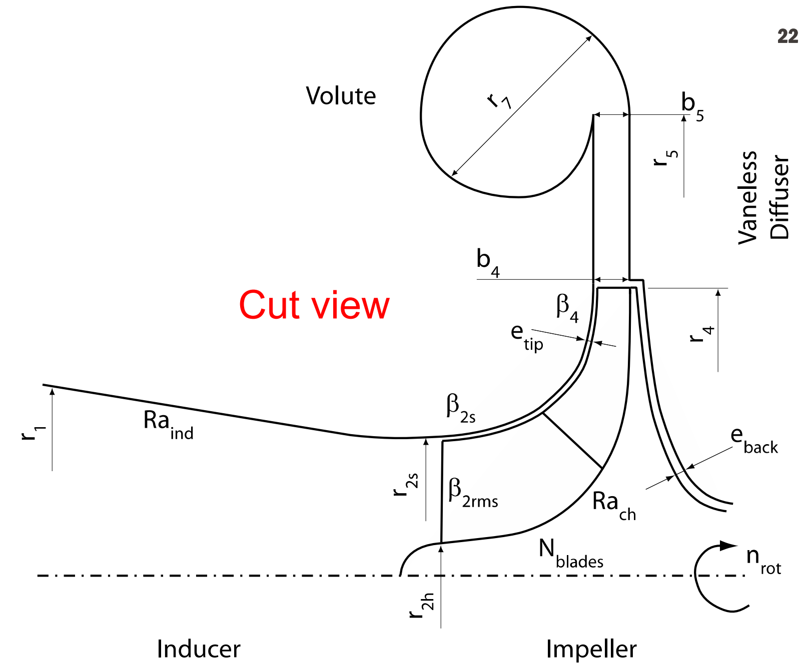
Radial Compressor Geomery

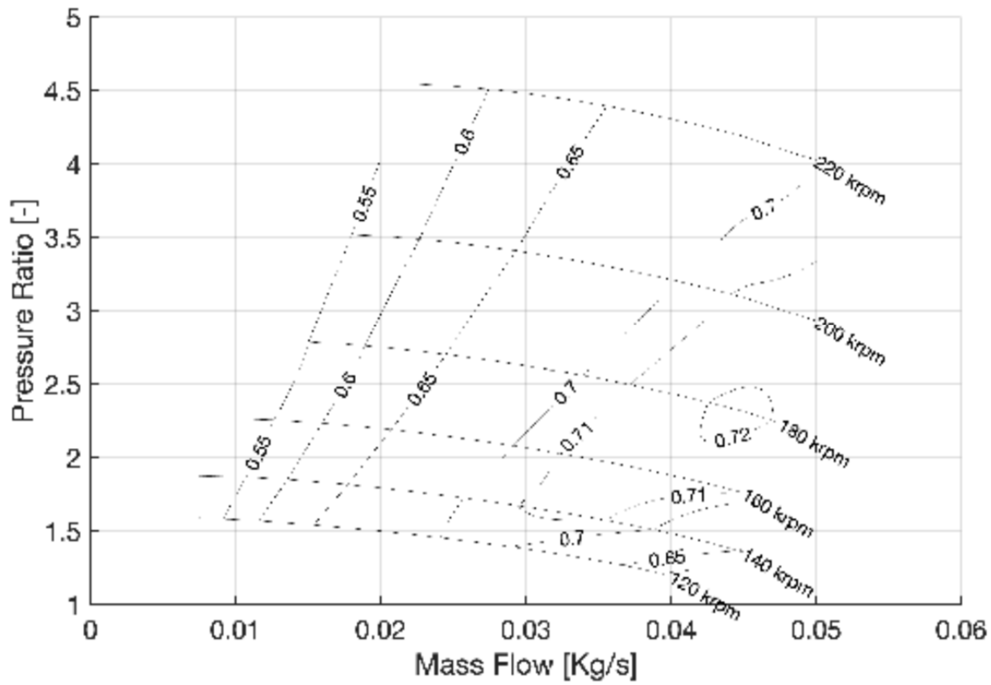
- Typical geometric features
  - Impeller tip width ratio
    $$0.02 < b_4/r_4 < 0.2$$
  - Relative impeller tip clearance
    $$e_{tip}/b_4 < 0.05$$
  - Number of blades
    $$12 < N_{blades} < 32$$
  - Axial length
    $$L/r_4 \approx 0.35$$
  - Inlet diameter ratio
    $$0.3 < r_{2s}/r_4 < 0.7$$
  - Blade angles
    $$\beta_{2s} = -60^\circ \qquad \beta_4 \approx -40^\circ$$

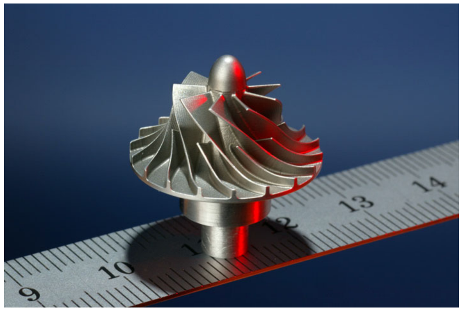
Cut view



Typical Compressor Map

- Map limited towards lower mass flows by surge and towards higher mass by choke





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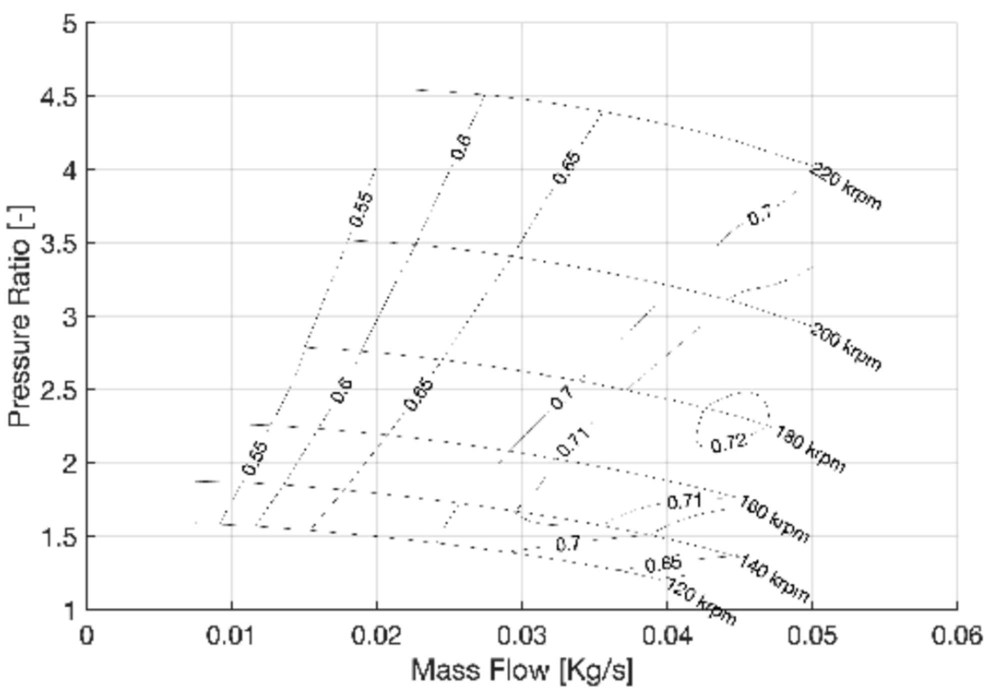
Comments on Compressor Map

- Mass flow or rotational speed changes lead to a change in velocity triangles
- Energy transfer and enthalpy distribution in stage changes
- Velocity triangles influence losses → entropy distribution → dissipation and efficiency change with operating point
- Change in pressure ratio is result of changed enthalpy and losses

Velocity Triangles Along Iso-Speedline

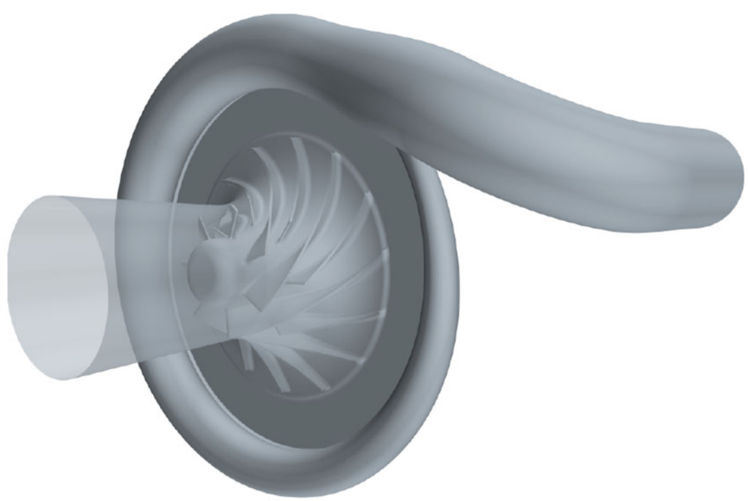


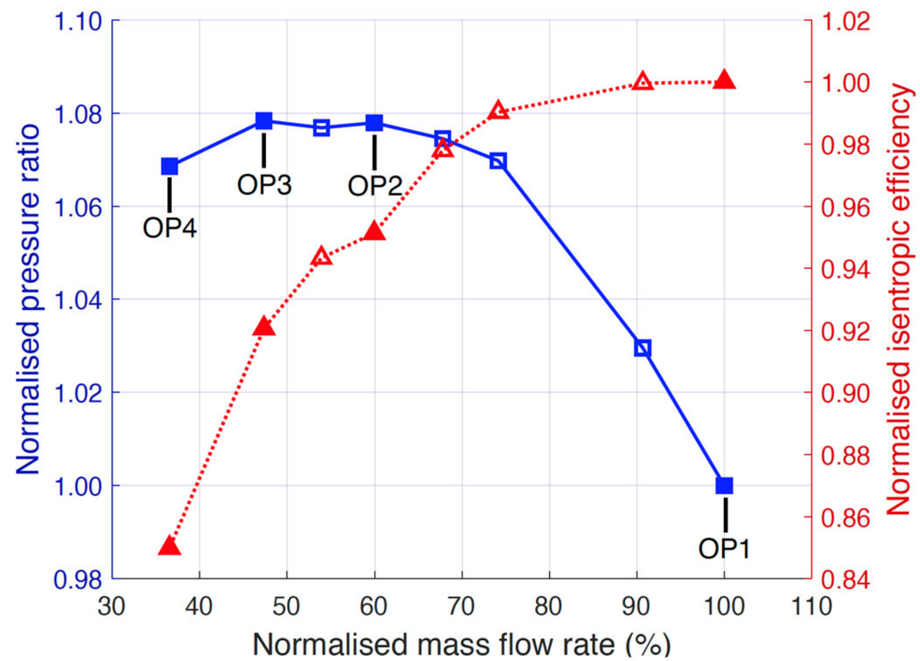
Compressor at Choke

- Choke defines upper mass flow for each speed line
  - Reducing back pressure increases velocity until it reaches the speed of sound in smallest passage
  - Further reduction in back pressure yields no increase in mass flow
  - Speed line is vertical after choking has occurred
  - Choking can occur at inlet, impeller, or diffuser
  - Higher speeds shift choke to higher mass flows until inlet is choked
  - Throat usually near impeller inlet or diffuser inlet

Compressor at Surge I

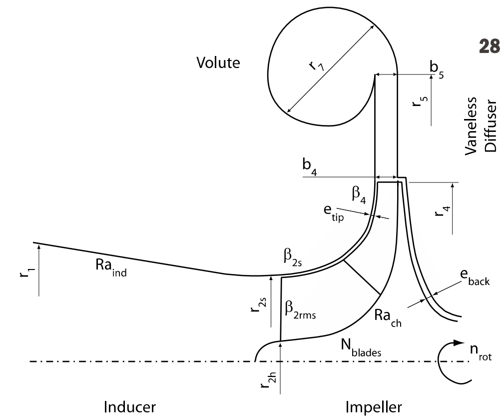
- Radial compressor with vaneless diffuser
  - 100krpm rotor speed, 0.1kg/s mass flow, PR = 2
  - URANS simulation
  - Fuel cell application



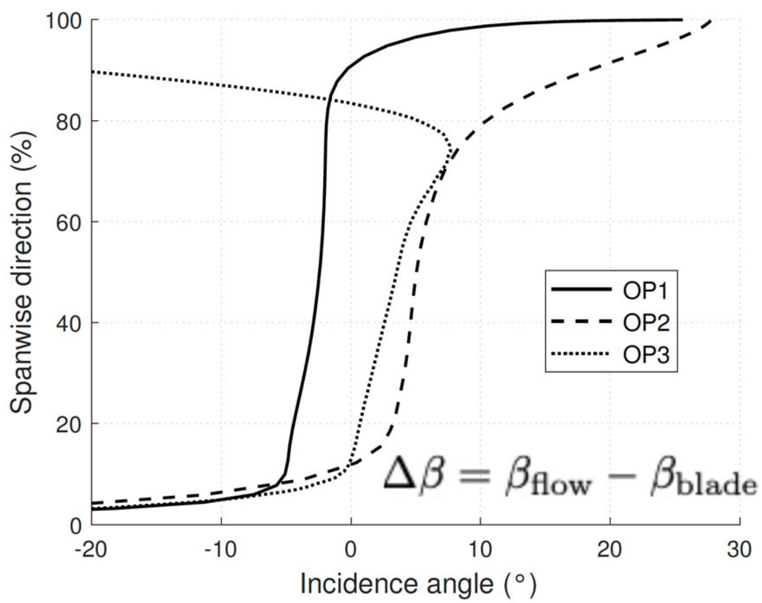


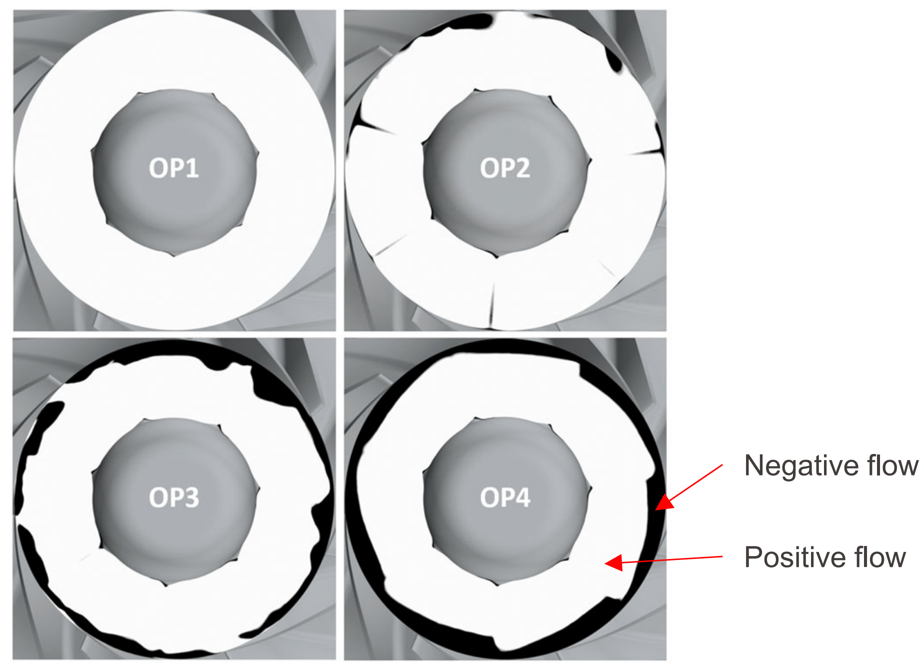
Flete, X., et al. (2024). Analysis and Prediction of the Stability Limit for Centrifugal Compressors with Vaneless Diffusers. *International Journal of Turbomachinery, Propulsion and Power*, *9*(3), 29

Compressor at Surge II



- Mass-flow reduction increases incidence at compressor inlet → flow detaches → inlet recirculation → blockage → reattachment





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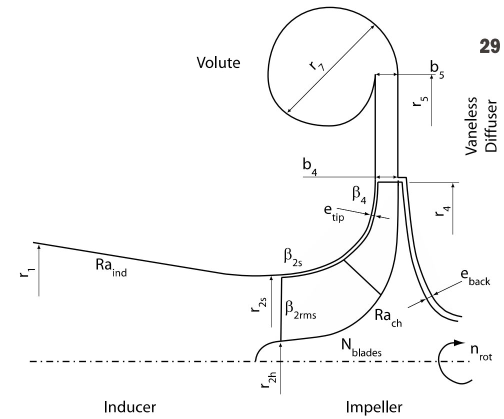
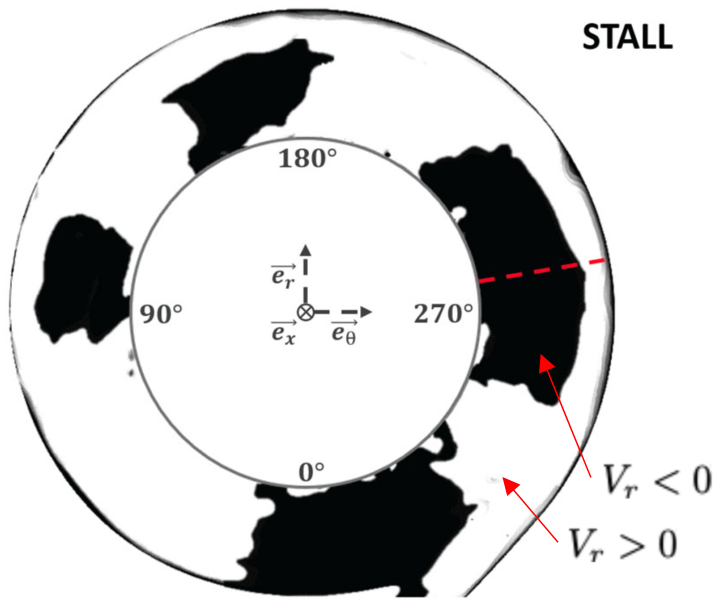
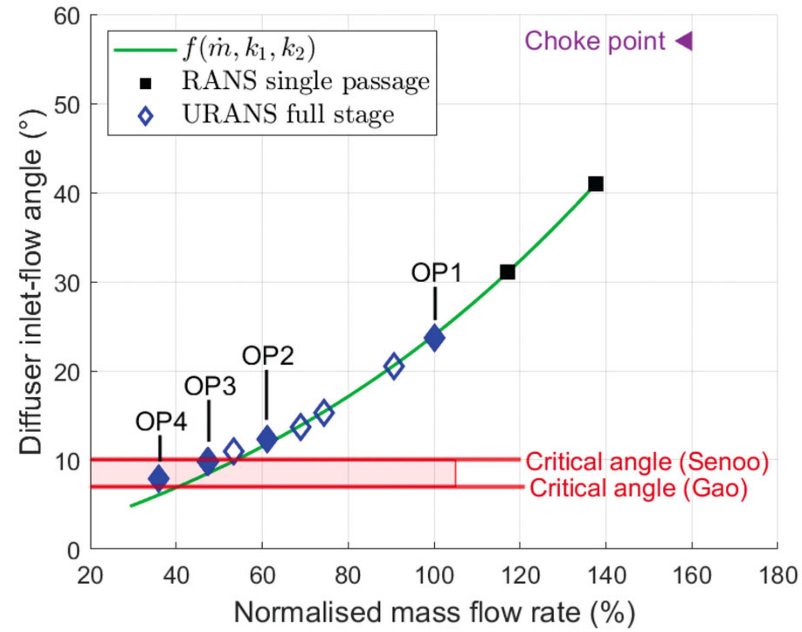
Compressor at Surge III

- Vaneless diffuser stall after inlet recirculation stabilizes
- Diffuser stall characterized by critical flow angle







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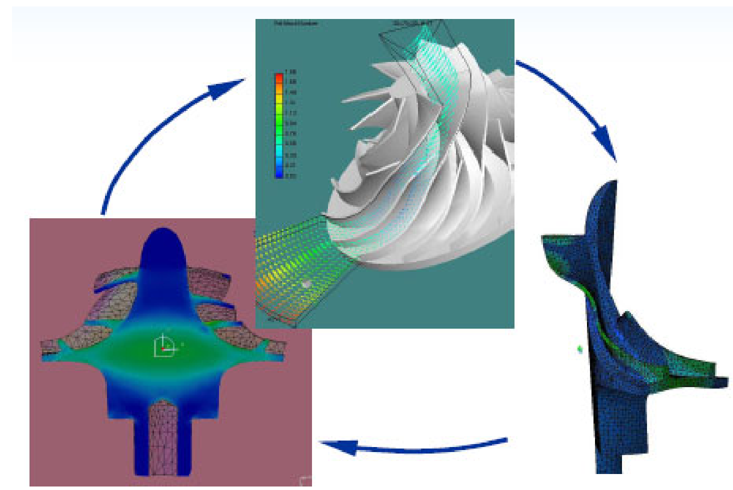
Compressor at Surge IV

- Surge line marks the low flow limit of region of stable operation
  - Caused by increase in loading as the mass flow is reduced (lower mass flow → increased incidence → increased loading → separation)
  - Depends on compressor itself & system configuration

- Instability can have several forms
  - Rotating Stall: separation in blade rows which jumps from one blade to next → mass flow nearly constant with small high-frequency pressure fluctuations
  - Mild Surge: Pulsations in mass flow and pressure without backflow
  - Deep Surge: Strong periodic backflow through compressor with large pressure and mass flow variations → should be avoided

Design Process of Turbomachinery

- Design of turbocompressor is interdisciplinary task
  - Aerodynamics
  - Mechanical stress
  - Dynamics
  - Thermal



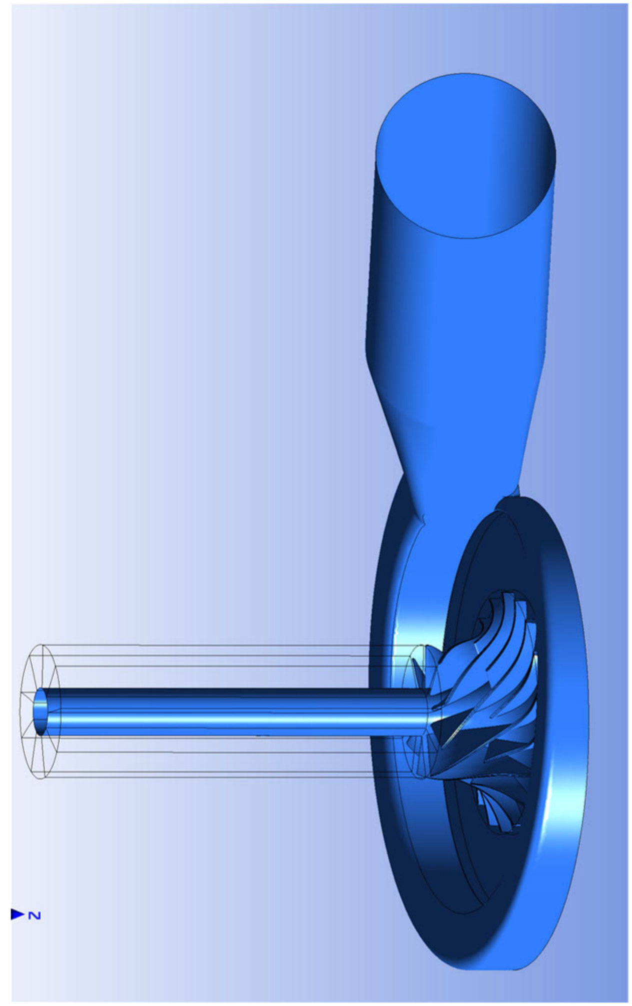
- Flow in turbomachinery is highly complex

EPFL

3D – CFD I

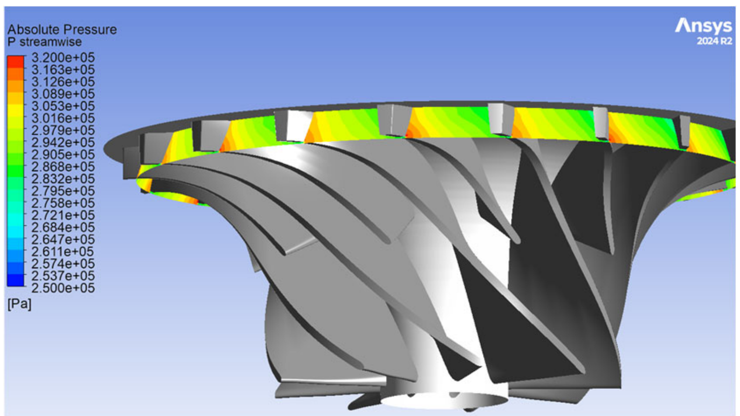
- Mesh of 30 million nodes
  → 15h computational time for one operating point → computational resources

- Not well suited for design

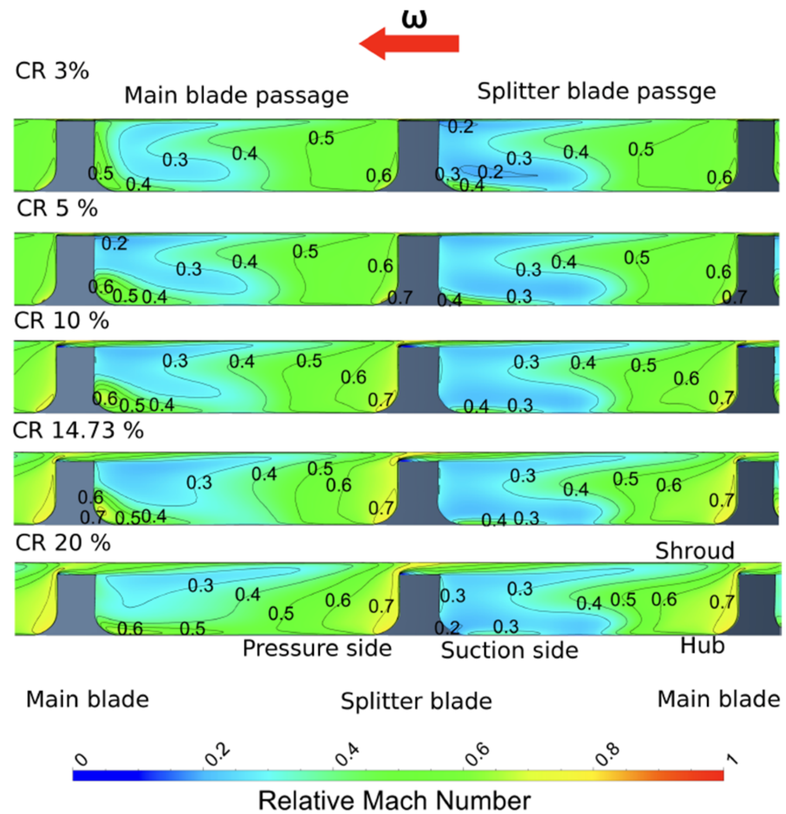


3D – CFD II

- Great for visualizing detailed flow features and analysis



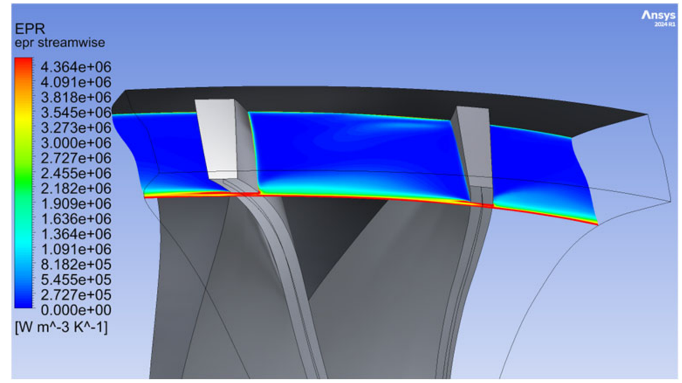
Pressure on pressure side larger than on suction side



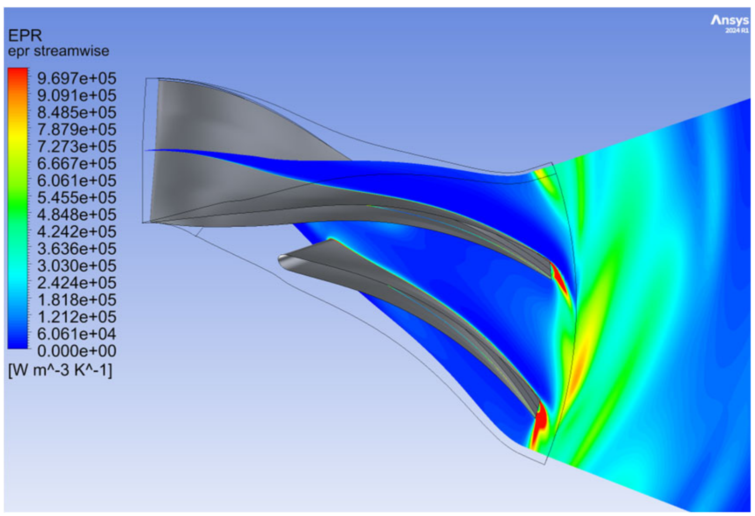
Accumulation of low momentum fluid close to suction side → jet-wake pattern → mixing losses into diffuser

3D – CFD III

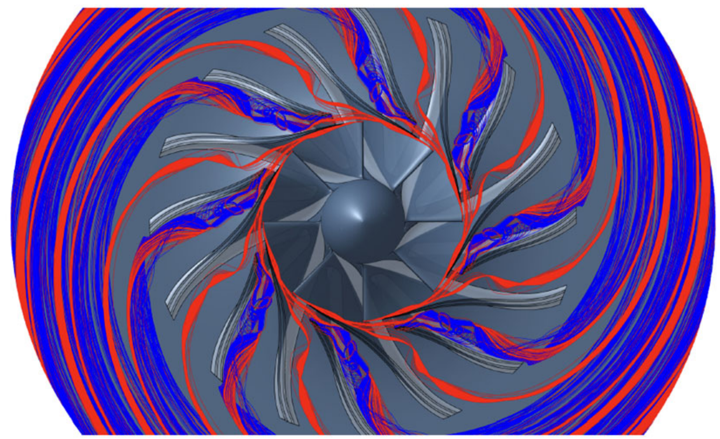
- Great for visualizing detailed flow features and analysis



Entropy production rate highest in tip gap and on surfaces → skin friction



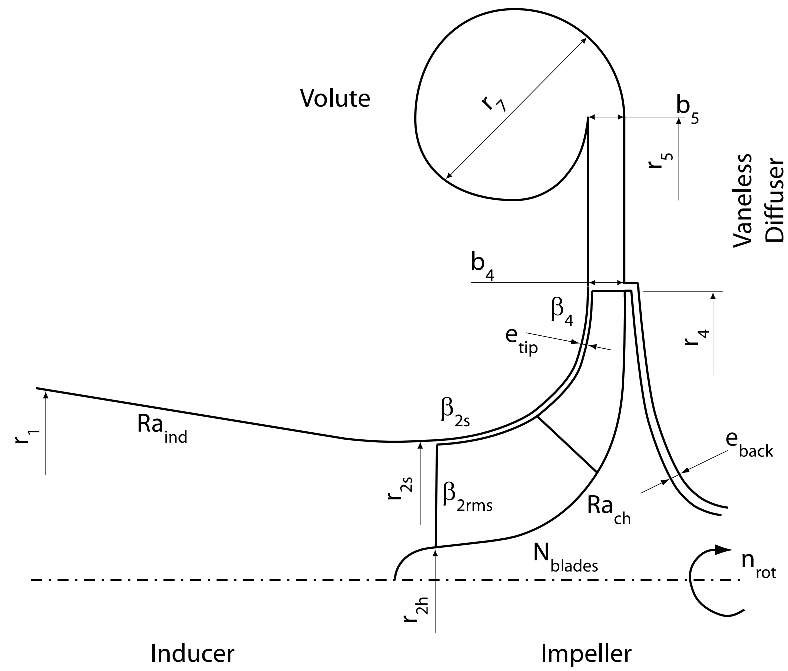
Trailing edge mixing into diffuser



Trajectories of main (red streamlines) and splitter blade (blue streamlines) tip leakage vortices

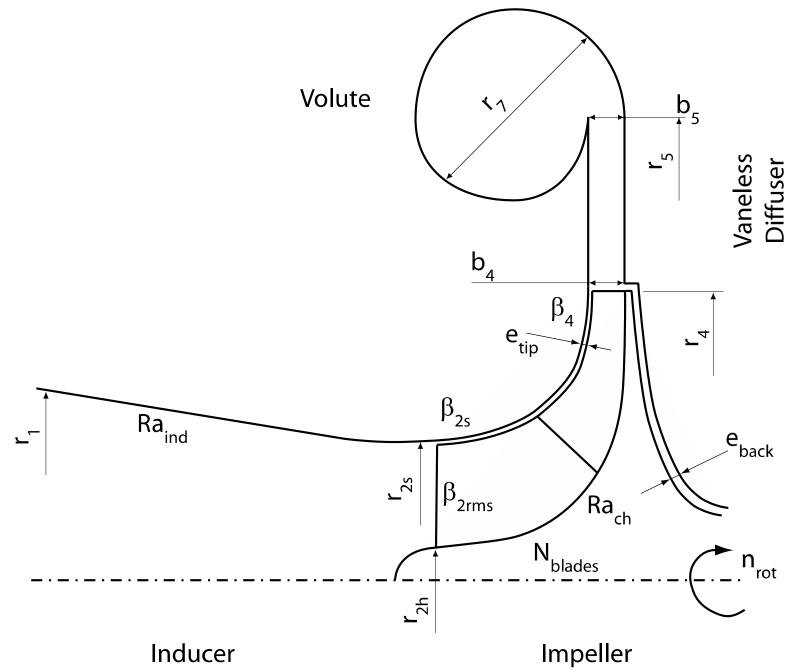
1D – Meanline Model

- Modeling along flow direction, component by component
- Based on velocity triangles, mass- & energy conservation, h-s-diagram
- Representation of losses through empirical correlations



1D – Meanline Model

- Proven Empirical Loss Correlations
  - Skin Friction } Inducer
  - Incidence
  - Skin Friction
  - Diffusion
  - Clearance } Impeller
  - Disk friction
  - Recirculation
  - Skin Friction
  - Trailing edge mixing } Diffuser



1D – Meanline Model

- Further effects
  - Flow does not follow blade angle → slip → causes a decrease in work input

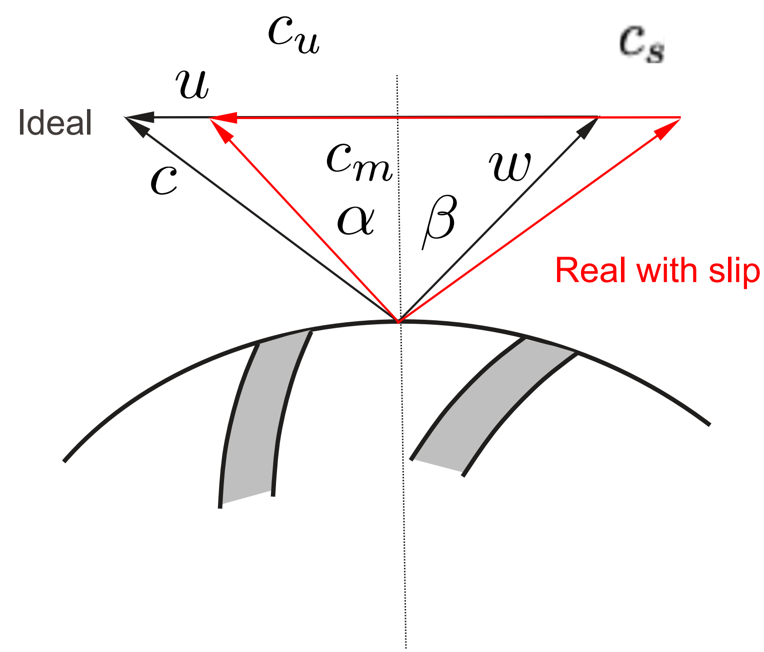
$$\frac{c_s}{u_4} = \frac{\sqrt{cos\beta_{4_{bl}}}}{N_{bl}^{0.7}}$$

  - Aerodynamic blockage

$$B_i = A_{\mathrm{eff}}/A_{\mathrm{geom}}$$

  - Surge in vaneless diffuser

$$\alpha_{crit} = f\left(M_4, b_4/r_4, r_5/r_4\right)$$

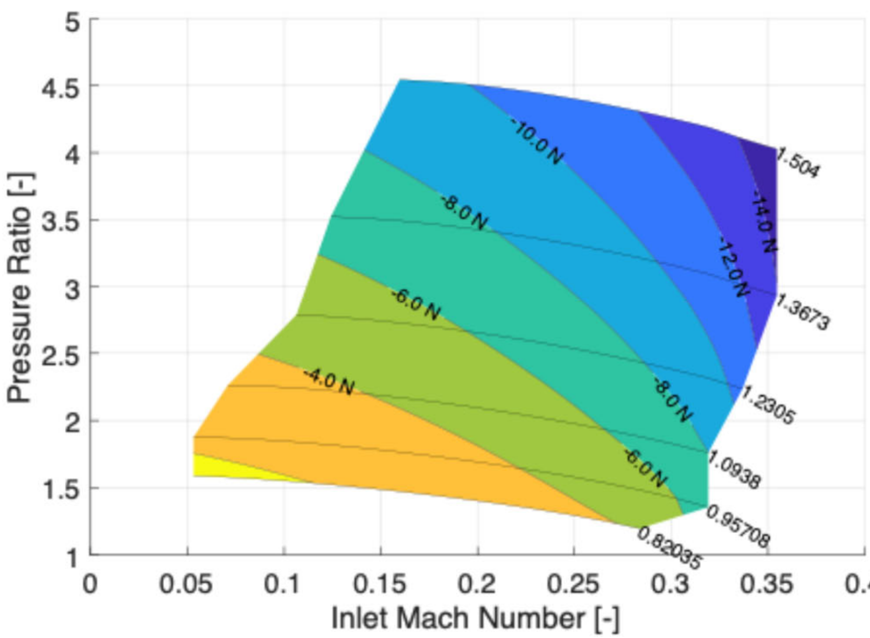


1D – Meanline Model

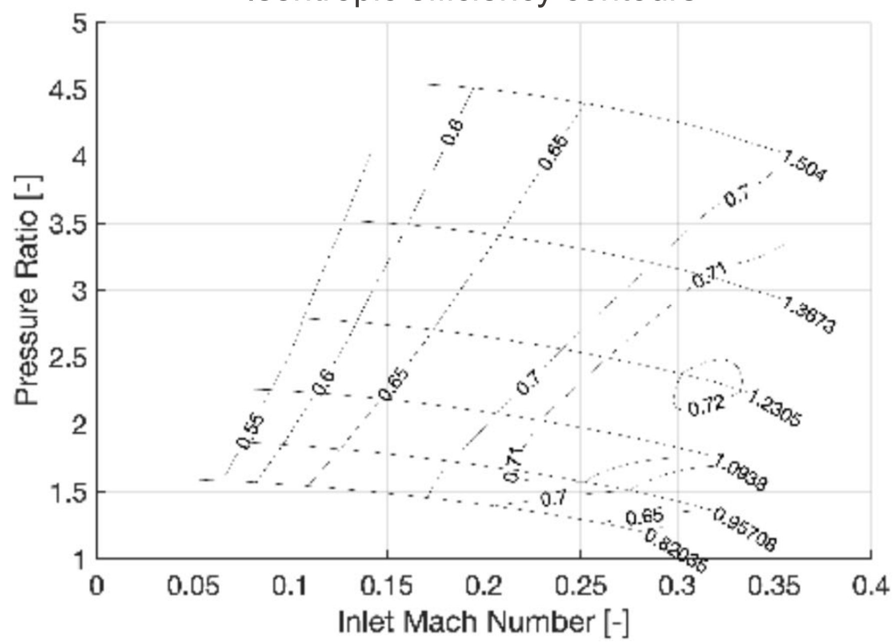
$$[PR, \eta_{is}, \text{surge}, \text{choke}] = f\left(\dot{m}, N_{rot}, P_{01}, T_{01}, \text{Fluid}, \text{Geom}\right)$$

- Fast compressor map prediction → great for design
- No capturing of 3D flow patterns

Thrust force contours

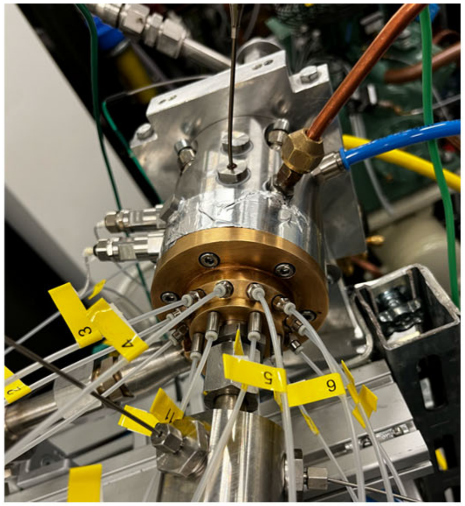


Isentropic efficiency contours

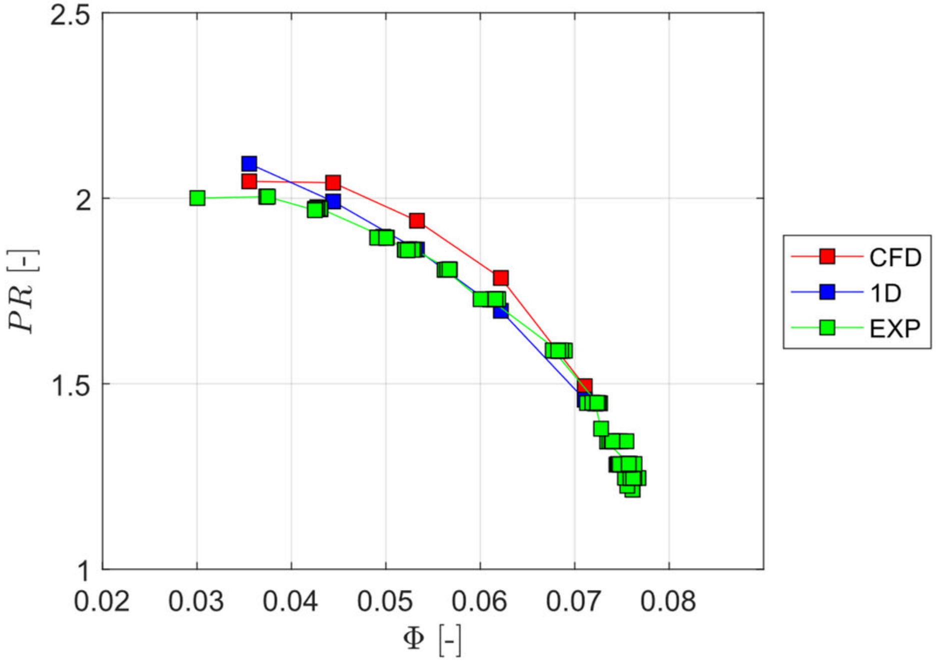


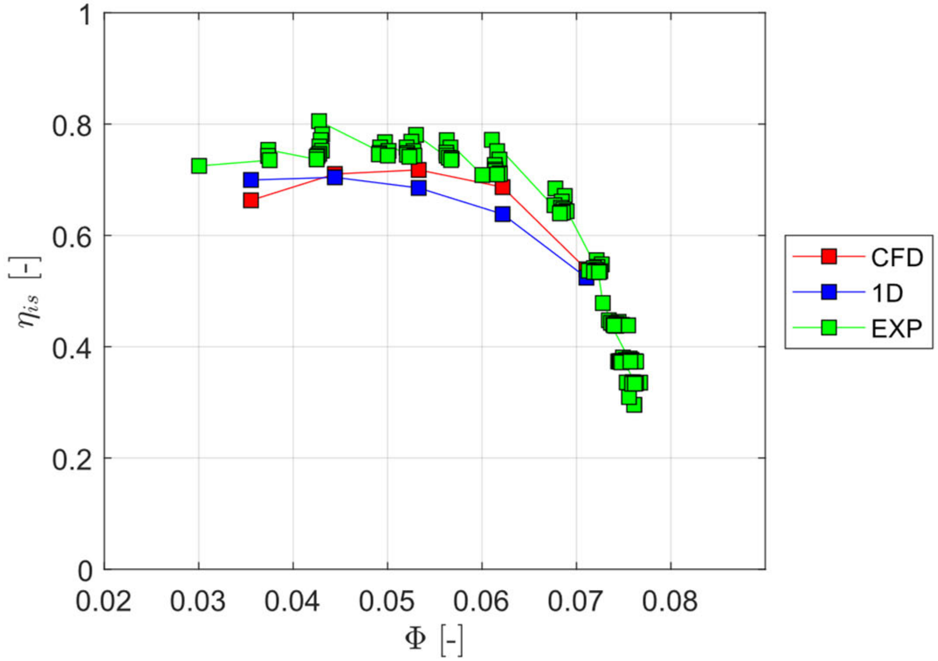
EPFL

Comparison CFD – Meanline Model



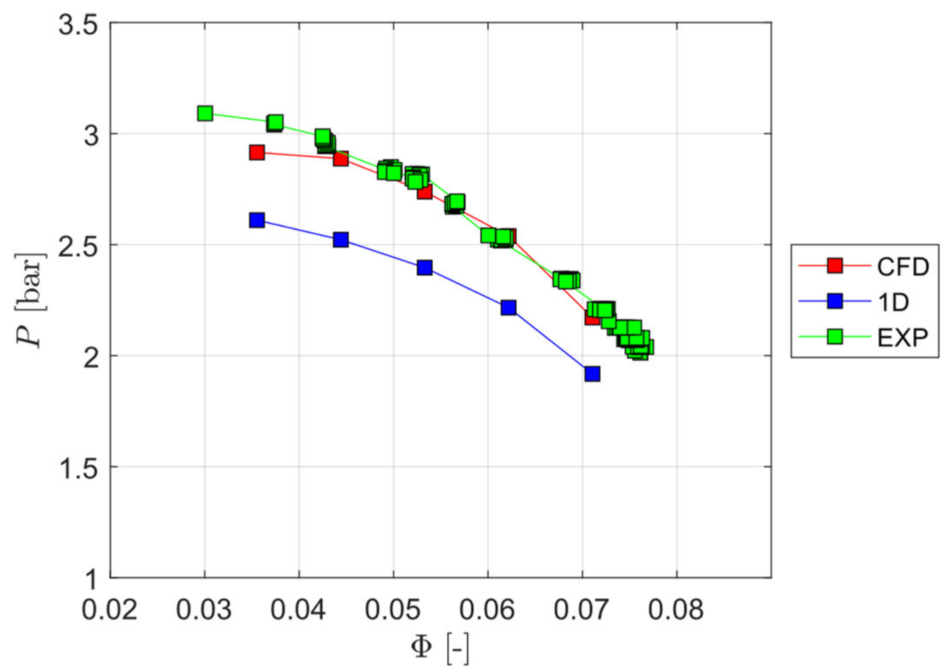
- Excellent overall (out-in) agreement between experimental data, CFD, and meanline-model



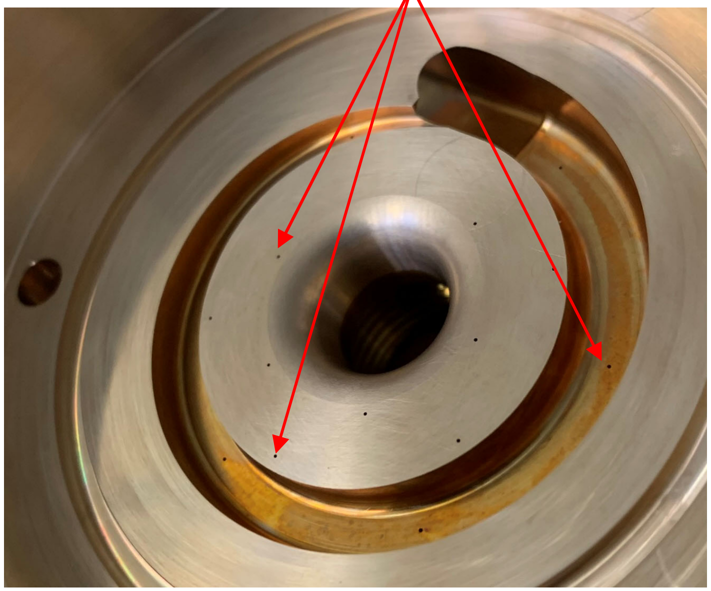


Comparison CFD – Meanline Model

- Inconsistency between meanline model, CFD & experiments at component level Impeller pressure ratio

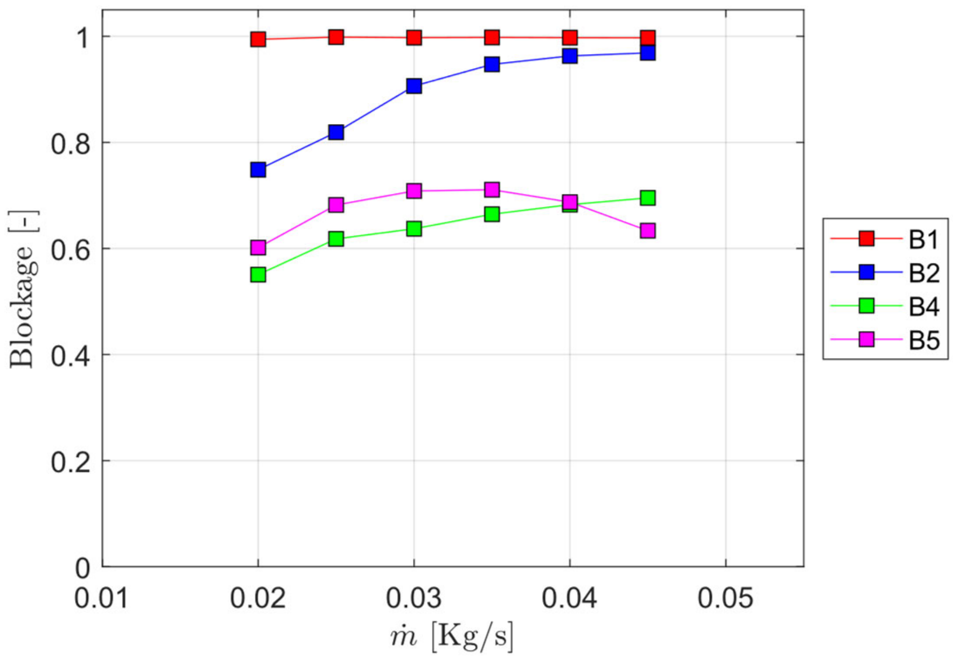


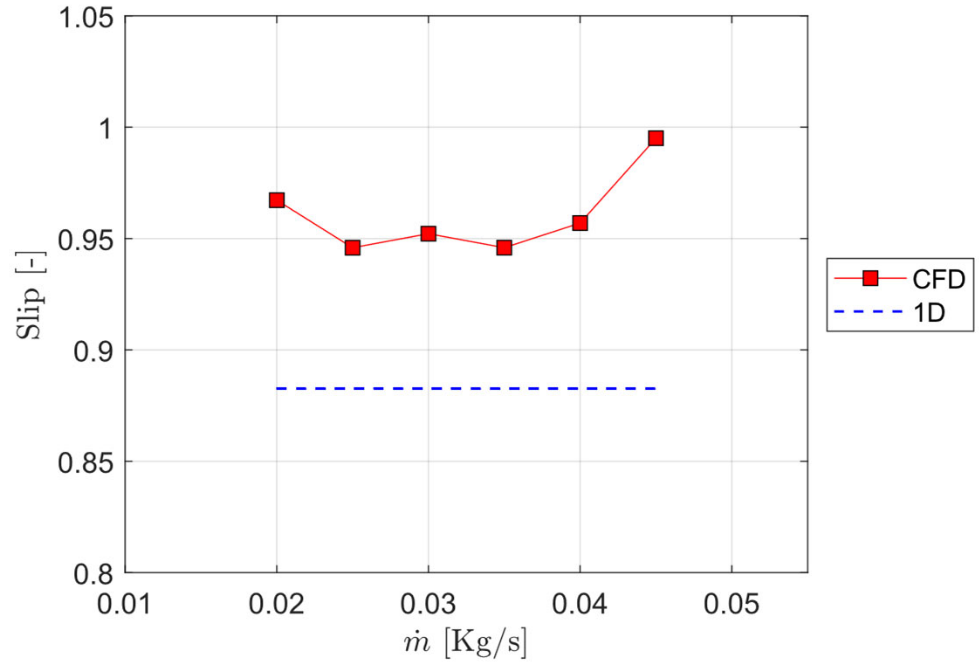
Pressure measurements



Meanline Model Calibration

- Correction of aerodynamic blockage factors, slip, and losses based on CFD data

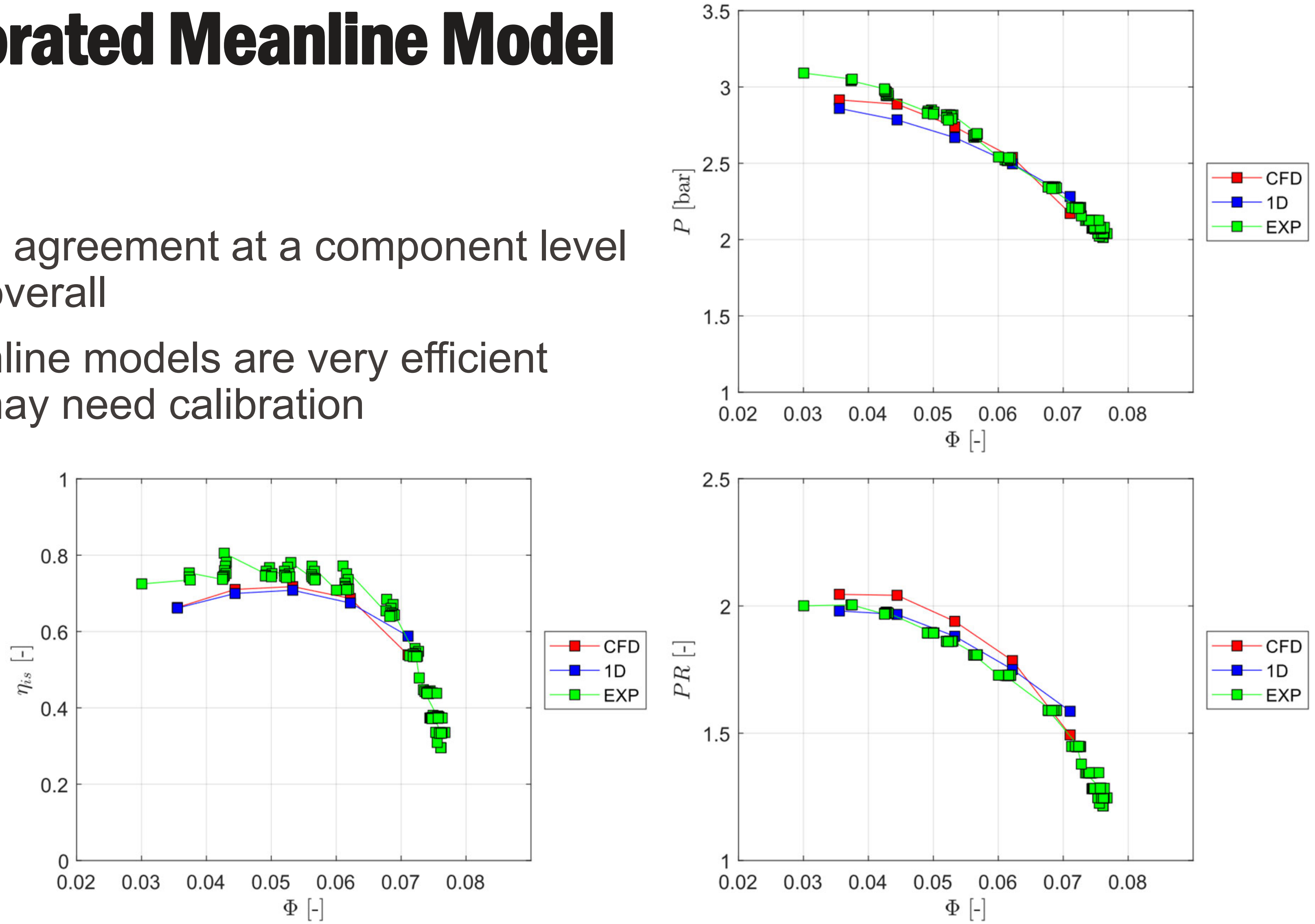




EPFL

Calibrated Meanline Model

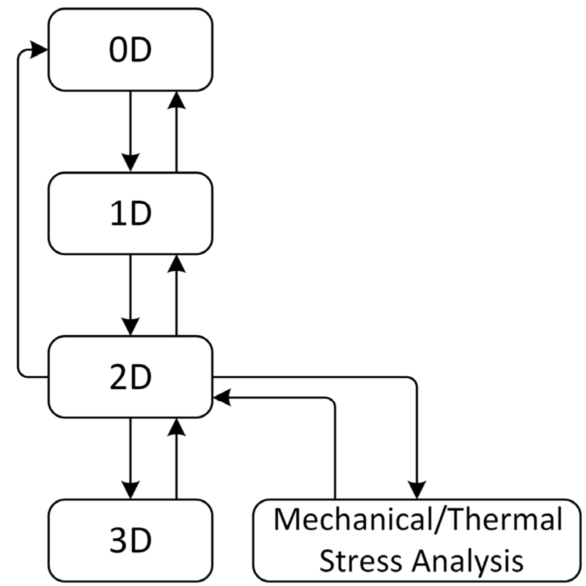
- Good agreement at a component level and overall
- Meanline models are very efficient but may need calibration



Design Process of Turbomachinery

- Iterative combination of increasingly complex models
  - 0D: Overall dimensions – based on empirical performance maps
  - 1D: Details of inlet and exhaust areas → meanline model
  - 2D: Definition of blade geometry
  - 3D: Assessment of detailed flow patterns

- Accurate starting point (0D) key for efficient design process
  - Requires design maps



Dimensional Analysis

- Parameters influencing compressor performance

$$[PR, \eta_{is}, \text{surge}, \text{choke}] = f\left(\dot{m}, N_{rot}, P_{01}, T_{01}, \text{Fluid}, \text{Geom}\right)$$

- Using dimensional analysis

$$[\lambda, \eta_{is}] = f\left(\phi_{01}, M_{u4}, Re, \text{Fluid}, \text{Geom}\right)$$

$$M_{u4} = \frac{u_4}{a_{01}}$$ Machine Mach number

$$\phi_{01} = \frac{\dot{m}}{\rho_{01} D_4^2 u_4^2}$$ Flow coefficient

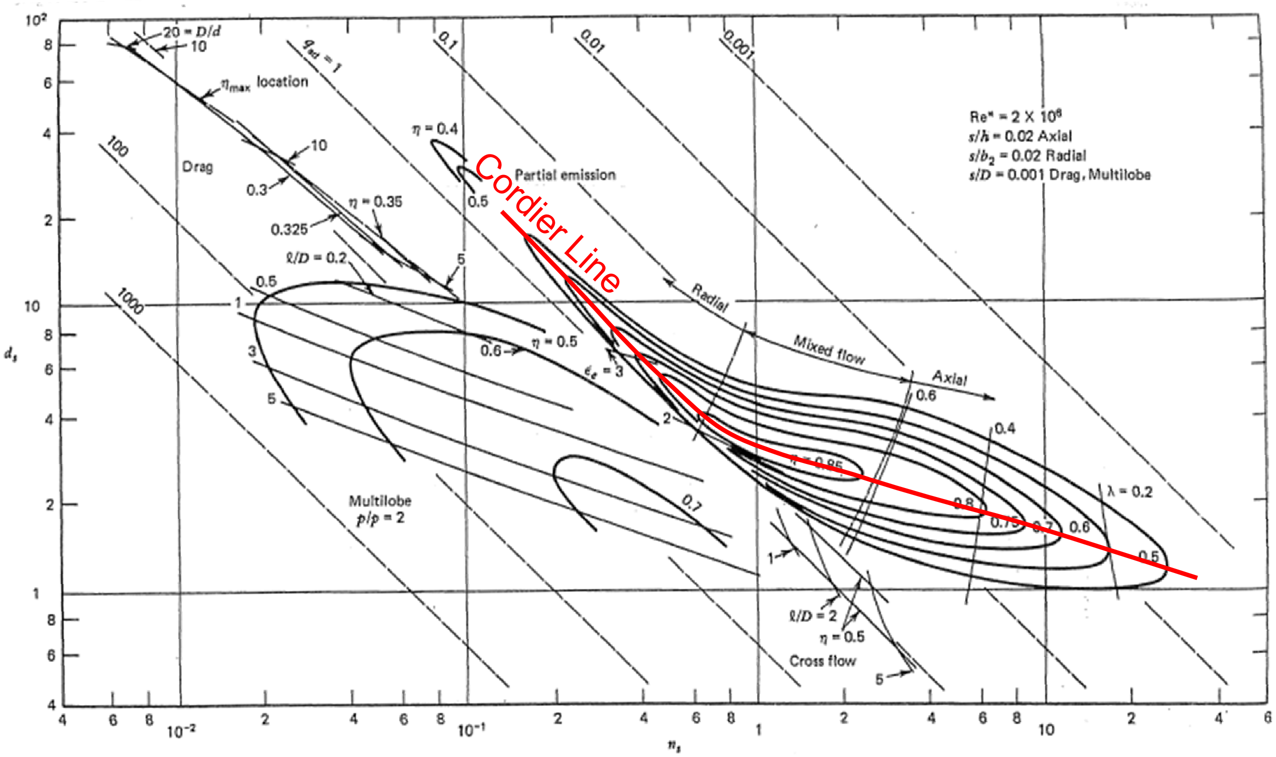
$$\lambda = \frac{h_{04} - h_{01}}{u_4^2}$$ Work input coefficient

Compressor Design Maps

- Specific speed and diameter alternative set of dimensionless parameters

$$n_s = \omega \frac{\dot{V}_{01}^{0.5}}{\Delta h_{is}^{0.75}}$$

$$d_s = d_4 \frac{\Delta h_{is}^{0.25}}{\dot{V}_{01}^{0.5}}$$

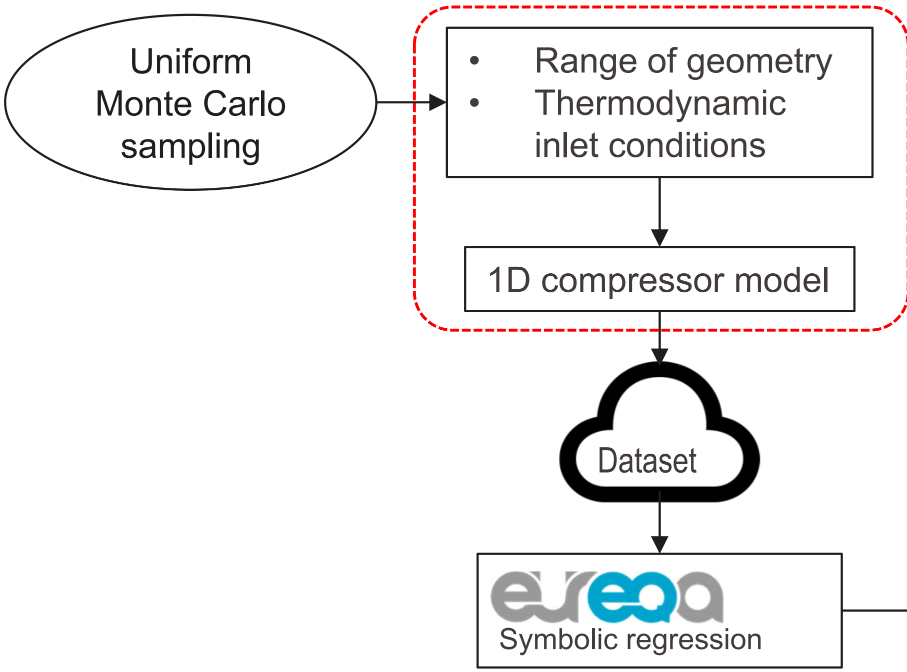


O. Balje, Turbomachines: A guide to design, selection and theory, 1981, Wiley

EPFL

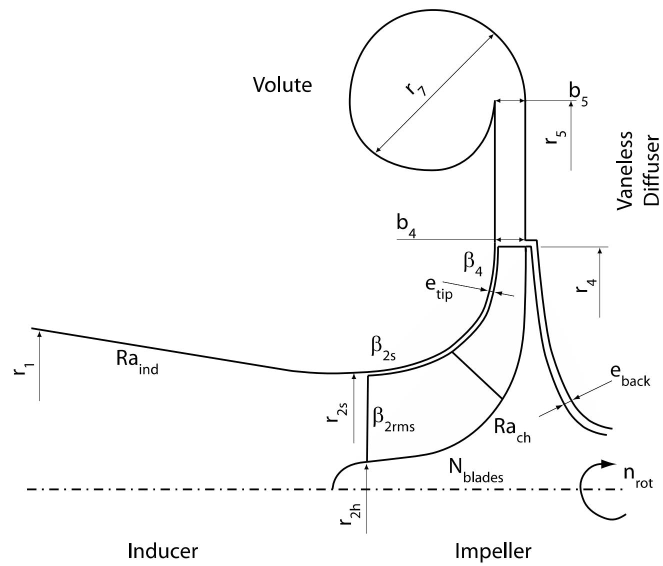
Updated Compressor Design Maps

- Experimentally validated tool used to generate surrogate model



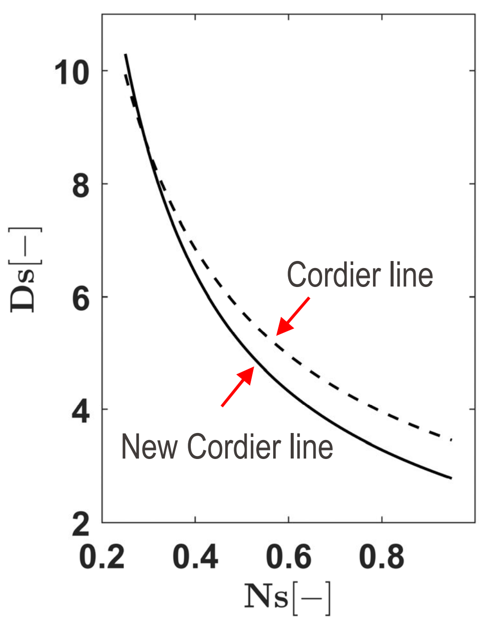
https://www.nutonian.com/products/eureqa/

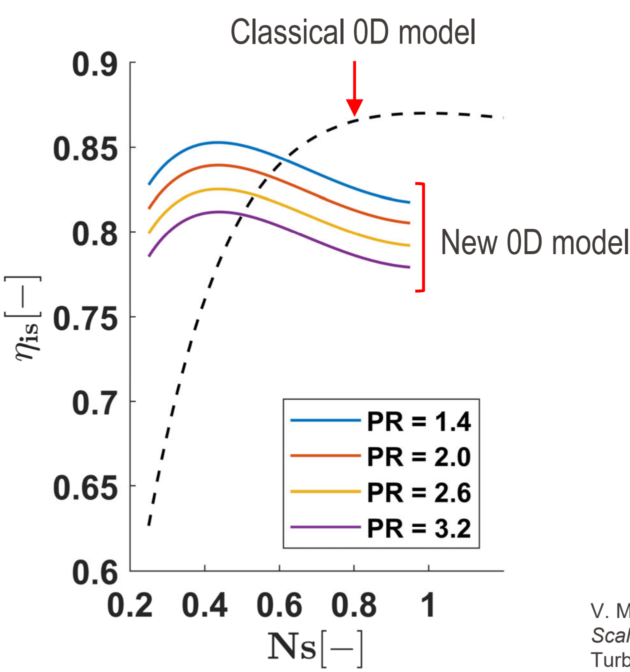
$$\eta_{is,0D} = f(Ns, Ds, PR, \overline{b_4}, \overline{r_{2s}})$$

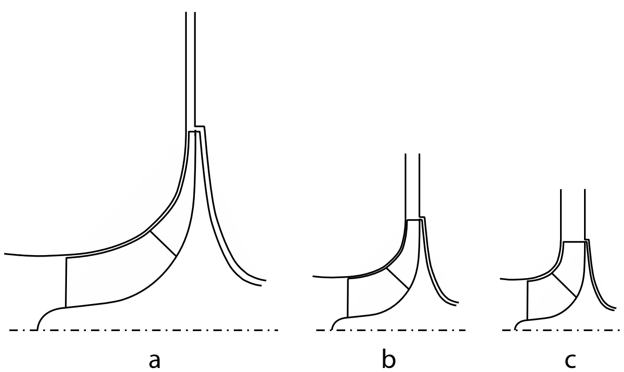


Updated Design Maps

- Updated model for reduced scale compressors deviates from literature
- New model includes additional design variables → more information



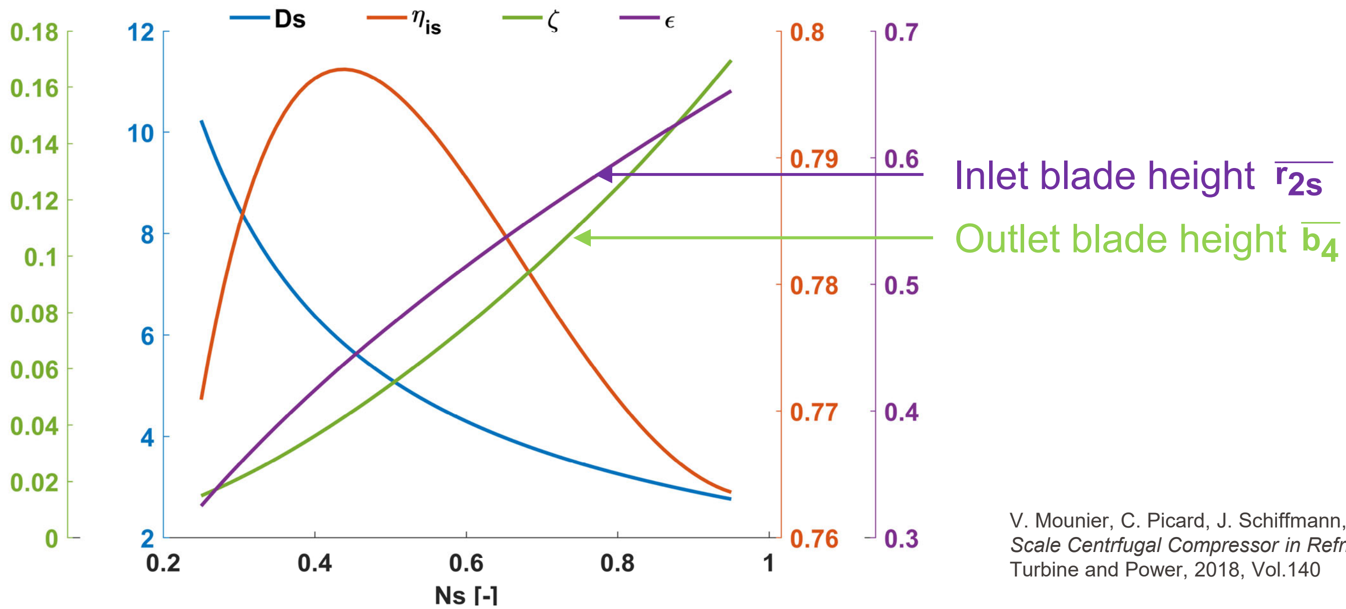




V. Mounier, C. Picard, J. Schiffmann, *Data-Driven Predesign Tool for Small-Scale Centrfugal Compressor in Refrigeration*, Journal for Engineering of Gas Turbine and Power, 2018, Vol.140

Impact of Updated Design Maps

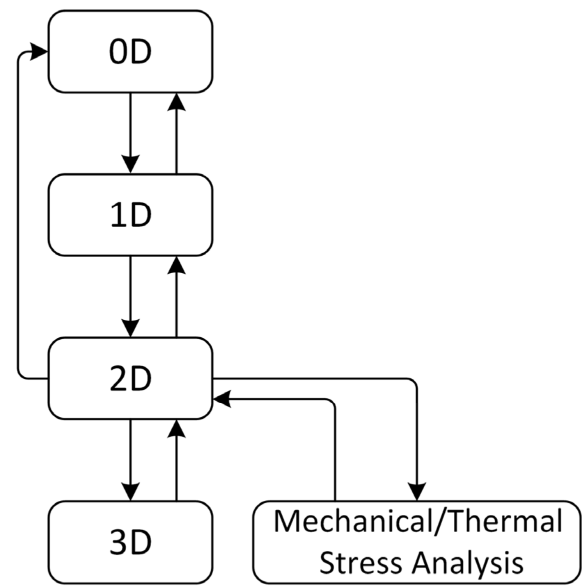
- Compared to classical tools new model yields more design information
- Data driven tool improves starting point and bypasses 1D design loop
- New tool is 1'500 x faster than meanline model with similar accuracy



V. Mounier, C. Picard, J. Schiffmann, *Data-Driven Predesign Tool for Small-Scale Centrfugal Compressor in Refrigeration*, Journal for Engineering of Gas Turbine and Power, 2018, Vol.140

Turbocompressor Design

- Design of radial compressor is iterative process
- Involves aerodynamic, mechanical, and thermal aspects
- Design starts with “first guess” and evolves with engaging increasingly complex models
- Research efforts
  - Improve meanline models
  - Increase speed of high-fidelity models
  - Surge prediction
  - Behavior of turbomachinery at small scale



Outlook W11

- On Small-Scale Turbocompressors for Heat Pumps

Exercises W10

- Comprehension questions
- Centrifugal compressor analysis
- Flow through a turbine runner