ME-459 - Formula Sheet

Heat pump systems

General definitions and relations:

$$E = KE + PE + U = m\left(\frac{w^2}{2} + gz + u\right)$$

$$dS = \left.\frac{\delta Q}{T}\right|_{rev}$$

$$H = U + PV$$

$$K = (H - H_0) + T_0(S - S_0) + KE + PE$$

$$Ex = (E - E_0) + p_0(V - V_0) - T_0(S - S_0)$$

$$dh = vdP + Tds$$

$$Tds = du + Pdv$$

$$dh|_p = c_p dT$$

$$du|_v = c_v dT$$

$$x = \frac{s_x - s'}{s'' - s'} = \frac{h_x - h'}{h'' - h'}$$

$$h_{0x} = h_x + \frac{c_x^2}{2}$$

$$M_x = c_x/a_x$$

$$\eta_{t-is} = \frac{\dot{W}^-_{real}}{\dot{W}^-_{is}} = \frac{h_{0,in} - h_{0,out}}{h_{0,in} - h_{0,out-is}}$$

$$\eta_{k-is} = \frac{\dot{W}^+_{is}}{\dot{W}^+_{real}} = \frac{h_{0,out-is} - h_{0,in}}{h_{0,out} - h_{0,in}}$$

$$y = y_1 + \frac{(x - x_1)}{(x_2 - x_1)}(y_2 - y_1)$$

Relations valid for a perfect gas:

$$du = c_v dT$$

$$dh = c_p dT$$

$$c_P = c_v + R$$

$$Pv = RT$$

$$PV = mRT$$

$$\left(\frac{T_2}{T_1}\right) = \left(\frac{P_2}{P_1}\right)^{\frac{\gamma-1}{\gamma}} = \left(\frac{v_1}{v_2}\right)^{\gamma-1}$$

$$\gamma = \frac{c_p}{c_v}$$

$$\Delta s = c_p\, ln\left(\frac{T_{final}}{T_{initial}}\right) - R\, ln\left(\frac{P_{final}}{P_{initial}}\right)$$

$$a_x = \sqrt{\gamma R T_x}$$

Thermodynamic balance equations for closed systems:

$$\delta W = -pdV$$
$$\frac{dE}{dt} = \dot{W} + \dot{Q}$$
$$dU = \delta Q + \delta W$$
$$\frac{dS}{dt} = \frac{\dot{Q}}{T} + \dot{\sigma}$$
$$\oint \frac{\delta Q}{T} = -\sigma$$
$$\frac{dEx}{dt} = \left(1 - \frac{T_0}{T}\right)\dot{Q} + \dot{W} + p_0\frac{dV}{dt} - T_0\dot{\sigma}$$

Thermodynamic balance equations for open systems:

$$\frac{dm_{cv}}{dt} = \sum_{in}\dot{m}_{in} - \sum_{out}\dot{m}_{out}$$
$$\frac{dE_{cv}}{dt} = \dot{W}_{cv} + \dot{Q}_{cv} + \sum_{in}\dot{m}_{in}\left(h + \frac{w^2}{2} + gz\right)_{in} - \sum_{out}\dot{m}_{out}\left(h + \frac{w^2}{2} + gz\right)_{out}$$
$$\frac{dS_{cv}}{dt} = \sum_j \frac{\dot{Q}_j}{T_j} + \sum_{in}\dot{m}_{in}s_{in} - \sum_{out}\dot{m}_{out}s_{out} + \dot{\sigma}_{cv}$$
$$\frac{dEx_{cv}}{dt} = \sum_j\left(1 - \frac{T_0}{T_j}\right)\dot{Q}_j + \dot{W}_{cv} + p_0\frac{dV_{cv}}{dt} + \sum_{in}\dot{m}_{in}k_{in} - \sum_{out}\dot{m}_{out}k_{out} - T_0\dot{\sigma}_{cv}$$

Power cycles:

$$\eta_{th} = \frac{\dot{W}^-}{\dot{Q}^+_{Hot}} = 1 - \frac{\dot{Q}^-_{Cold}}{\dot{Q}^+_{Hot}}$$
$$\eta_{Carnot} = 1 - \frac{T_{cold}}{T_{hot}}$$
$$\eta_{ex} = \frac{\eta_{th}}{\eta_{Carnot}}$$

Heat pump cycles:

$$COP_h = \frac{q_h^-}{e^+} = \frac{1}{\Theta_h}(1 - \frac{l}{e^+})$$
$$\Theta_h = 1 - \frac{T_a}{T_h}$$
$$COP_f = \frac{q_f^+}{e^+} = -\frac{1}{\Theta_f}(1 - \frac{l}{e^+})$$

$$\Theta_f = 1 - \frac{T_a}{T_f}$$

$$COP = \frac{q_L}{w_c}$$

$$\eta_{ex} = 1 - \frac{l}{e^+}$$

$$e^+ = q_h^- - q_f^+$$

$$e_{qh}^- = (1 - \frac{T_a}{T_h})q_h^-$$

$$e_{qf}^- = -(1 - \frac{T_a}{T_f})q_f^+$$

$$\eta = \frac{e_{qf}^-}{e^+}$$

$$\eta = \frac{e_{qh}^-}{e^+}$$

Heat pump cycles: Exercise-Based Formula (Week 5 and 6)

$$\epsilon \dot{E}_x^+ = \dot{m}_x \Delta h_x$$

$$\dot{Q}_x^+(power - heat) = \sum_{n=1}(\dot{m}_n * (h_{n\,out} - h_{n\,in})) + \sum_{m=1}(\epsilon\dot{E}_m^- - \epsilon\dot{E}_m^+)$$

$$\dot{Q}_x^-(energetic\,losses\,(power - heat\,losses)) = \sum_{n=1}(\dot{m}_n * (h_{n\,in} - h_{n\,out})) + \sum_{m=1}(\epsilon\dot{E}_m^+ - \epsilon\dot{E}_m^-)$$

$$\dot{Q}_x^- = (1 - \epsilon_x) * \dot{E}_x^+$$

$$\epsilon = \frac{\dot{Q}_{given}}{\sum_{n=1}\dot{E}_n^+}$$

$$\dot{E}_{qx}^-(exergy\,heat) = -(1 - \frac{T_a}{T_x}) * \dot{Q}_x^+$$

$$\dot{E}_{qx}^-(exergy\,heat\,given) = (1 - \frac{T_a}{T_x}) * \dot{Q}_x^-$$

$$\dot{E}_{x\,total}^+(exergy\,power) = \sum_{n=1}\dot{E}_n^+ + \sum_{m=1}\dot{E}_{qm}^+$$

$$k_x = h_x - T_A S_x$$

$$k_{a-b} = h_b - h_a - T_A C_p ln(\frac{T_b}{T_a})$$

$$\dot{L}_x(exergetic\,losses) = \sum_{n=1}(\dot{m}_n * (k_{n\,in} - k_{n\,out})) + \sum_{m=1}(\dot{E}_m^+ - \dot{E}_m^-)$$

$$\eta = \frac{\dot{E}_{qx}^-}{\dot{E}_{x\,total}^+}$$

Volumetric compressors:

$$\dot{V}_s = \frac{\dot{m}}{\rho_{in}}$$

$$\dot{V}_{st} = V_{st} * f_c$$

$$\eta_V = \frac{V_s}{V_{st}} = \eta_{V1} \cdot \eta_{V2} \cdot \eta_{V3}$$

$$\eta_{V1} = \frac{p_1'}{p_1}$$

$$\eta_{V2} = 1 - \frac{V_0}{V_{st}} \left[\Pi^{\frac{1}{\kappa}} - 1 \right]$$

$$\eta_{is} = \eta_{is-max} \cdot \eta_{th}$$

$$\eta_{th} = \frac{\kappa \left[\Pi^{\frac{\kappa - 1}{\kappa}} - 1 \right]}{V_R^{\kappa-1} - \kappa + (\kappa - 1) \cdot \frac{\Pi}{V_R}}$$

$$\kappa = \frac{c_p}{c_v}$$

$$\Pi = \frac{p_2}{p_1}$$

$$e_{is} = C_p (T_{2is} - T_1)$$

$$P_{el} = \frac{e_{is}}{\eta_{is}} \cdot \dot{m}$$

Turbomachinery:

$$n_s = \frac{\omega \cdot \dot{V}^{0.5}}{e_{is}^{0.75}}$$

$$d_s = \frac{d_4 \cdot e_{is}^{0.25}}{\dot{V}^{0.5}}$$

$$\eta_{c-is} = c_p T_{01} \frac{\left(\frac{p_{02}}{p_{01}} \right)^{(\gamma-1)/\gamma} - 1}{U_2 c_{\theta 2} - U_1 c_{\theta 1}}$$

$$e^+ = \Delta w = U_2 c_{\theta 2} - U_1 c_{\theta 1} = h_{02} - h_{01}$$

$$\Omega = \frac{rpm}{60} 2\pi$$

$$\eta_D = \frac{h_{2s} - h_1}{h_2 - h_1} = \frac{c_1^2 - c_{2s}^2}{c_1^2 - c_2^2} = \frac{T_{2s}/T_1 - 1}{T_2/T_1 - 1}$$

$$\eta_p = \frac{dh_{is}}{dh} = \frac{vdp}{c_p dT}$$

$$\dot{m} = \rho v A$$

$$r_t = U_2/\Omega$$

$$\dot{W}_{act} = \frac{\dot{W}_c}{\eta_m} = \frac{\dot{m}\Delta w}{\eta_m}$$

$$c = \left(c_\theta^2 + c_r^2\right)^{1/2}$$

$$c_r = \frac{\dot{m}}{\rho 2\pi r_t b}$$

$$\tan\beta = \frac{V_r}{V_\theta}$$

$$\dot{W}_{shaft} = \dot{m}\omega\left[r_2 V_{2\theta} - r_1 V_{1\theta}\right]$$

Heat transfer:

$$\dot{Q} = -kA\nabla T = \frac{\Delta T}{R_{th-cond}}$$

$$R_{th-cond} = \frac{L}{kA}$$

$$R_{th-cond-cyl} = \frac{ln(\frac{r_2}{r_1})}{2\pi Lk}$$

$$\dot{Q} = hA(T_s - T_\infty) = \frac{\Delta T}{R_{th-conv}}$$

$$R_{th-cond} = \frac{1}{hA}$$

$$R_{th-cond} = \frac{1}{h2\pi rL}$$

$$Re = \frac{vx}{\nu}$$

$$Nu = \frac{hx}{k_f}$$

$$Pr = \frac{\nu}{\alpha}$$

$$\alpha = \frac{k}{\rho c_p}$$

$$Gr = \frac{g\beta\Delta T x^3}{\nu^2}$$

$$\dot{Q} = A\epsilon\sigma T^4$$

$$\sigma = 5.67\cdot 10^{-8}[\frac{W}{m^2K^4}]$$

$$1 = \rho + \tau + \alpha$$

$$\nu = \frac{\mu}{\rho}$$

$$\dot{Q} = UA\Delta T_{lm}$$

$$\Delta T_{lm} = \frac{T_{h2} - T_{c1} - T_{h1} + T_{c2}}{ln\frac{T_{h2}-T_{c1}}{T_{h1}-T_{c2}}}$$

$$UA = \frac{1}{R_{total}} = \sum \frac{1}{R}$$

Ejectors:

$$\left(\frac{\dot{m}_s}{\dot{m}_p}\right)_{\text{ideal}} = \frac{h_p - h_{(p+1)s}}{h_{(s+1)s} - h_s}$$

$$\eta = \frac{(\dot{m}_s/\dot{m}_p)_{\text{actual}}}{(\dot{m}_s/\dot{m}_p)_{\text{ideal}}}$$

$$h_{discharge} = \frac{\dot{m}_s h_s + \dot{m}_p h_p}{\dot{m}_s + \dot{m}_p}$$