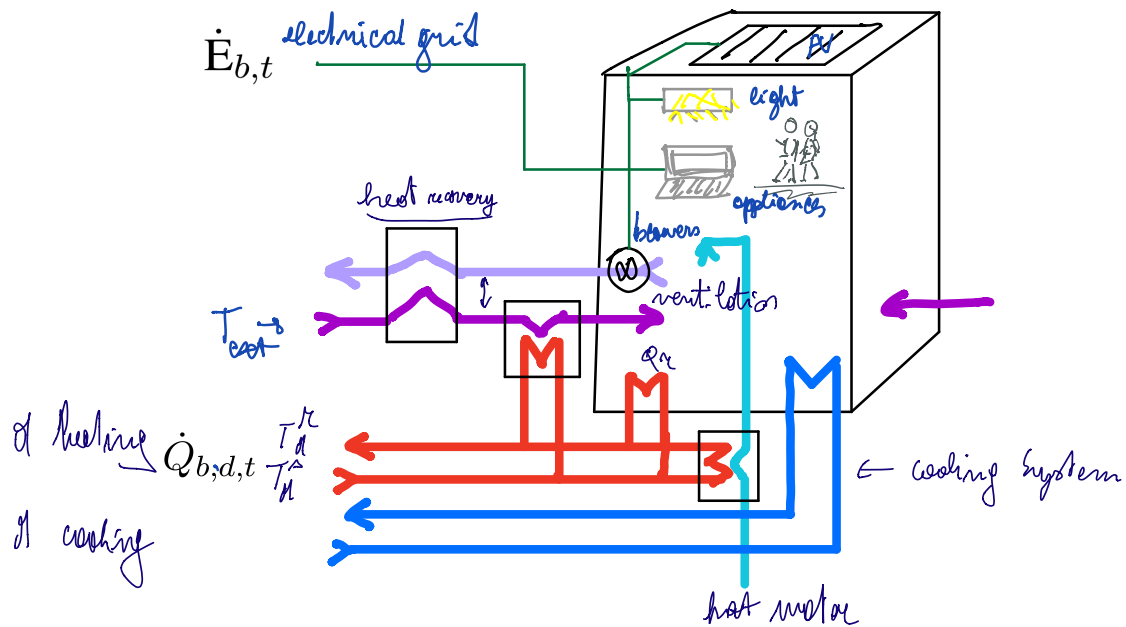


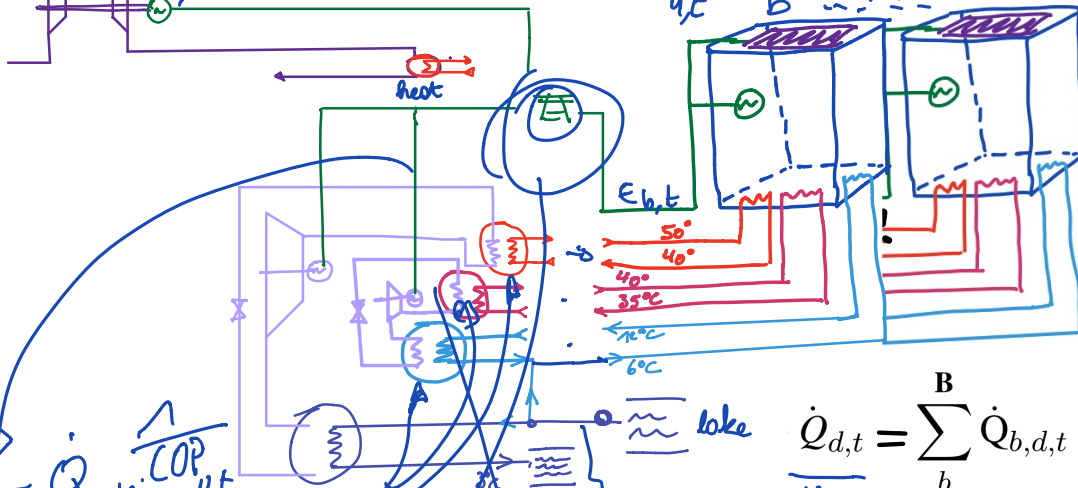
BUILDING b



refer to architect \rightarrow building \Rightarrow ??? motor
 refer heating system \rightarrow measures

$$\dot{m}_{FUEL} \cdot LHV \cdot \eta_{GT} = E_{GT,t} - E_{GT,t}$$

$$\dot{Q}_{d,t} = \sum_b \dot{Q}_{b,d,t}$$



$$\dot{E}_{u,t} = \dot{Q}_{u,t} \cdot \frac{1}{COP_{u,t}}$$

$$\dot{Q}_{d,t} = \sum_b \dot{Q}_{b,d,t}$$

$$COP_{u,t} = \frac{\dot{Q}_{u,t}}{E_{u,t}}$$

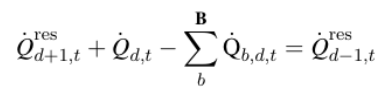
$$\sum_u \dot{q}_{u,d} - \dot{Q}_{d,t} = 0$$

$$\dot{Q}_{d,t} = \dot{m}_{d,t} c_{p,d} (T_{d,t}^s - T_{d,t}^r)$$

supply return

$$Q_{u,t} = \text{heat pump low } T = f_{u,t} \cdot \dot{Q}_{u,t}$$

$$\text{electricity} = E = \sum_b E_{b,t} + \sum_u E_{u,t}$$

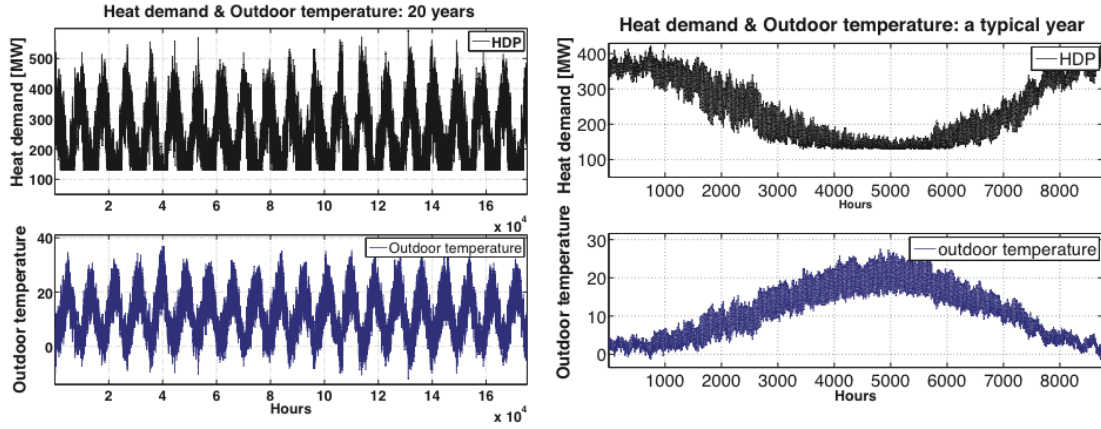


$$\dot{m}_{r,t} = \sum_u^U f_{u,t} \dot{m}_{r,u}$$

- Annual operating cost:

$$OPEX = \int_0^{t^{op}} \sum_r^R c_r(t) \dot{m}_r(t) dt + \sum_u^U c_u S_u + \int_0^{t^{op}} c_e^+(t) \dot{E}^+(t) dt - \int_0^{t^{op}} c_e^-(t) \dot{E}^-(t) dt \quad (2)$$

$$\approx \sum_t^T \sum_r^R c_{r,t} \dot{m}_{r,t} \Delta t_t + \sum_u^U c_u^{mt} S_u + \sum_t^T c_{e,t}^+ \dot{E}_t^+ \Delta t_t - \sum_t^T c_{e,t}^- \dot{E}_t^- \Delta t_t \quad [CHF/year] \quad (3)$$

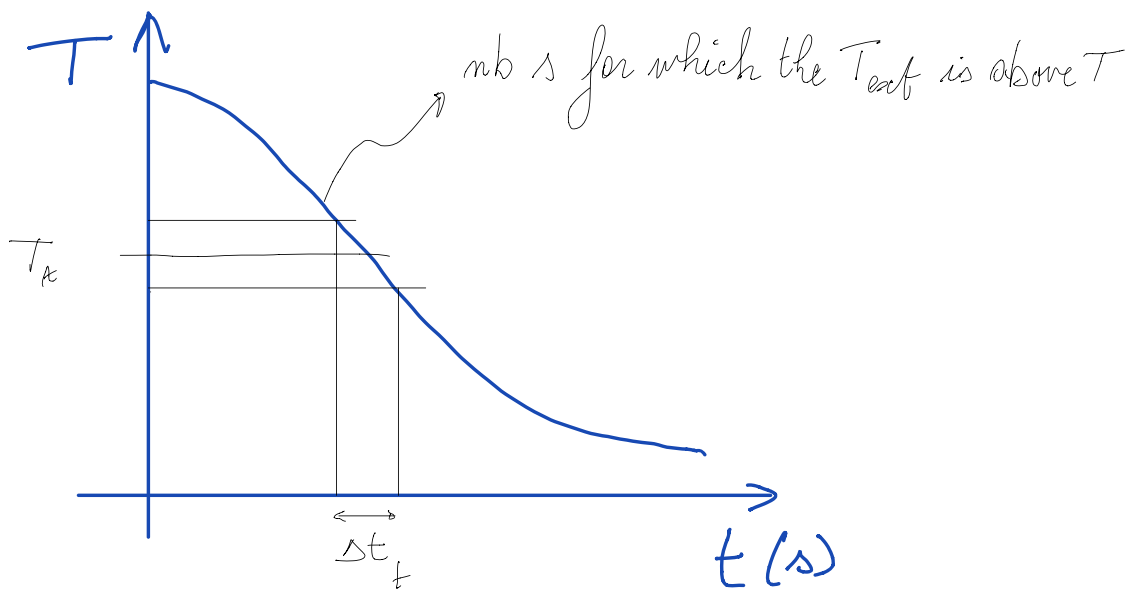


$$\dot{Q}_{b,i} = \sum_{t=1}^{nt} \dot{Q}_{b,i,t} \cdot \Delta t_t$$

$$\Delta t_t = [\text{s/year}]$$

Probability of appearance of conditions t during a year

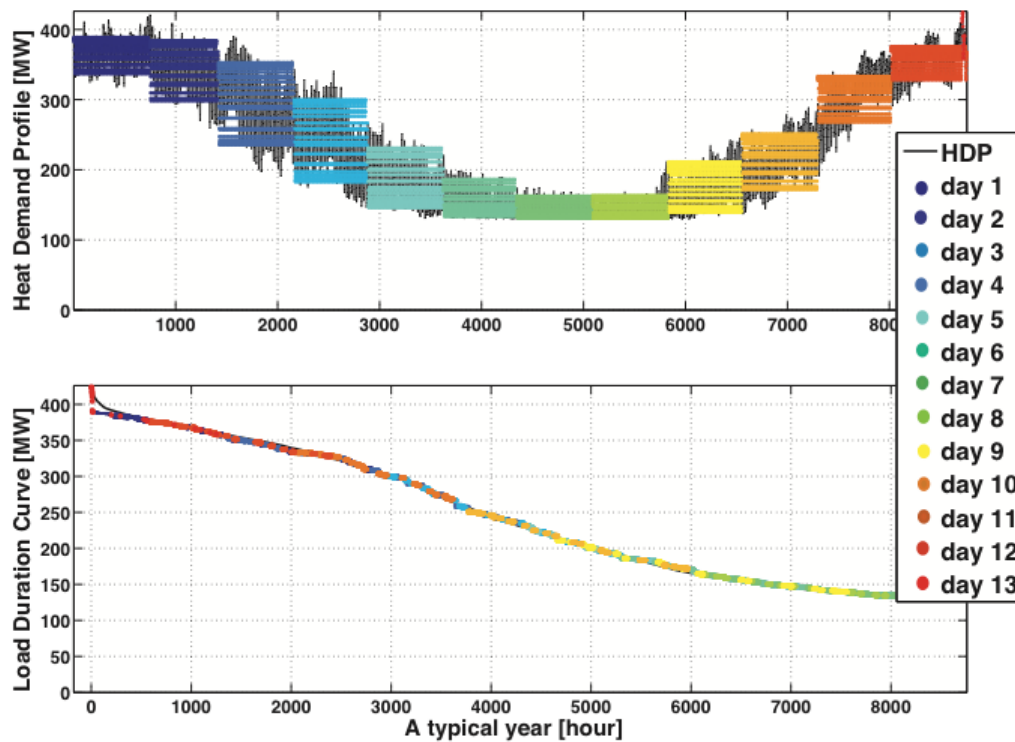
TEMPERATURE DISTRIBUTION CURVE



RANKING DAYS

1 DAY / MONTH

S. Fazlollahi et al. / Computers and Chemical Engineering 65 (2014) 54–66



Typical days definition

CLUSTERING

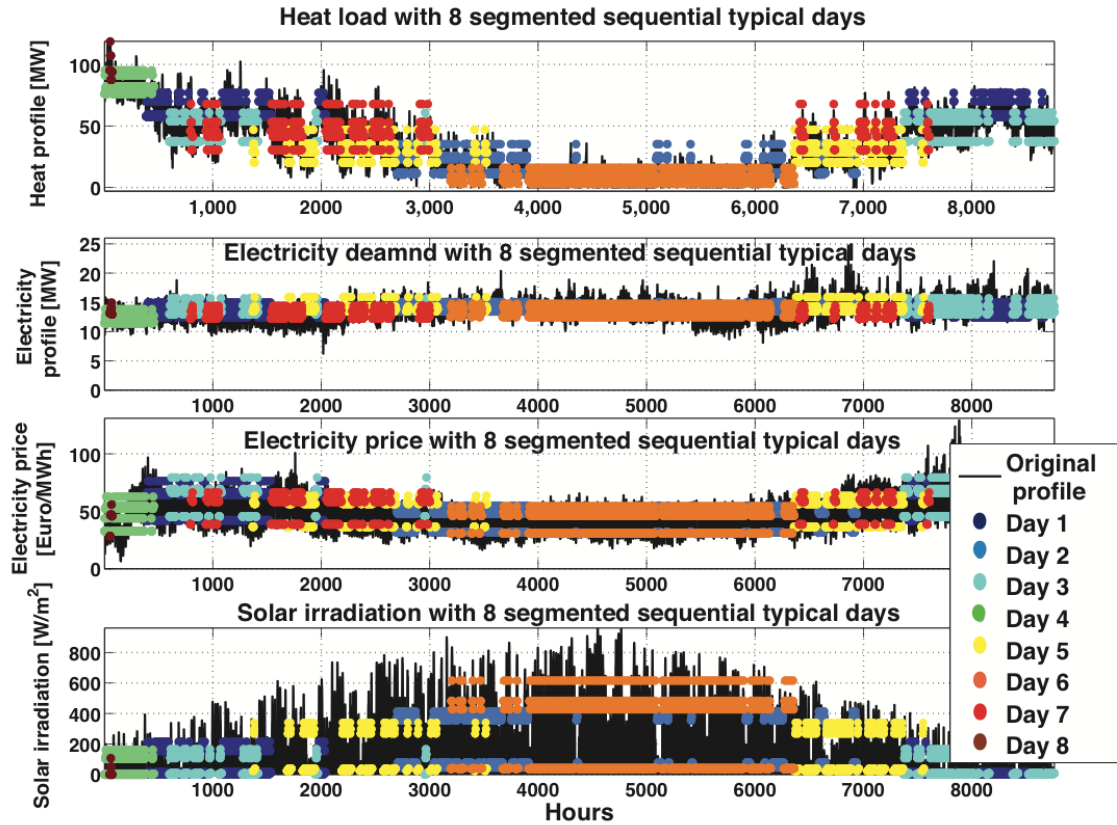
k -means is a **greedy** optimization algorithm, which minimizes the squared error over all N_k clusters (Eq. (1)):

$$\min \left[\sum_{k=1}^{N_k} \sum_{i=1}^{N_i} d(\hat{\mu}_k, \hat{x}_i) \times z_{i,k} \right] \quad (1)$$

$$\sum_{k=1}^{N_k} z_{i,k} = 1, \quad \forall i \quad (2)$$

$$d(\hat{\mu}_k, \hat{x}_i) = \sum_{a=1}^{N_a} \sum_{g=1}^{N_g} (\hat{\mu}_{k,a,g} - \hat{x}_{i,a,g})^2 \quad \forall k, i \quad (3)$$

$$\hat{x}_{i,a,g} = \frac{x_{i,a,g} - \min\{x_{i,a,g} \mid \forall i, \forall g\}}{\max\{x_{i,a,g} \mid \forall i, \forall g\} - \min\{x_{i,a,g} \mid \forall i, \forall g\}} \quad \forall i, a, g \quad (4)$$

**Table 5**

The comparison between the reference case and the optimization results from 2010 to 2012 with regards to the size of the peak boiler, the fuel consumption and the operating costs.

	Ref. case	13 Empirical periods	7 typical periods using	
			The typical year	The 20 years
Municipal waste [GWh]	7415	7570 (−2.1%) ^a	7593 (−2.4%)	7489 (−1.0%)
Biomass [GWh]	663	638 (+3.8%)	654 (+1.4%)	659 (+0.6%)
Coal [GWh]	1992	2032 (−2.0%)	2006 (−0.7%)	1989 (+0.15%)
Natural gas [GWh]	112	8.9 (+92%)	6.72 (+94%)	85 (+24.0%)
Peak gas boiler [MW _{th}]	175	34 (+80%)	34 (+80%)	200 (−14%)
Under estimated periods ^b	0	4	4	0
Operating costs [M€]	119.7	117.5 (1.8%)	117 (2.3%)	119.4 (0.2%)
Resolution time [s]	2700	85	23	23
No. constraint	183 × 10 ⁴	65320	7427	7427
No. variables	152 × 10 ⁴	54423	6225	6225
No. integer variables	11 × 10 ⁴	3756	432	432

^a The relative differences between the reference case and the typical periods optimization.

^b Number of time steps from 2010 to 2012 when the maximum original heat demands are higher than the maximum typical values.

MILP formulation

Objective function

The objective is to minimize the **total annualized cost** function:

$$TOTEX = OPEX + CAPEX + ENVEX \quad [CHF/year] \quad (1)$$

where:

- **Annual operating cost:**

$$OPEX = \int_0^{t^{op}} \sum_r c_r(t) \dot{m}_r(t) dt + \sum_u c_u S_u + \int_0^{t^{op}} c_e^+(t) \dot{E}^+(t) dt - \int_0^{t^{op}} c_e^-(t) \dot{E}^-(t) dt \quad (2)$$

$$\approx \sum_t \sum_r c_{r,t} \dot{m}_{r,t} \Delta t_t + \sum_u c_u^{mt} S_u + \sum_t c_{e,t}^+ \dot{E}_t^+ \Delta t_t - \sum_t c_{e,t}^- \dot{E}_t^- \Delta t_t \quad [CHF/year] \quad (3)$$

- **Annualized investment cost:**

$$CAPEX = \frac{1}{\tau} \sum_u I(s_u) \approx \frac{1}{\tau} \sum_u \left(c_u^{inv1} y_u + c_u^{inv2} s_u \right) \quad [CHF/year] \quad (4)$$

- **Annual cost related to emissions:**

$$ENVEX = c_{co2} \sum_t \left[\dot{m}_{co2,t} \Delta t_t + k_{co2,t} (\dot{E}_t^+ - \dot{E}_t^-) \Delta t_t \right] \quad [CHF/year] \quad (5)$$

Technology constraints and modeling equations

- **Inequality constraints** $\forall t \in \mathbf{T}, \forall u \in \mathbf{U}$

$$f_u^{\min} y_u \leq f_u \leq f_u^{\max} y_u \quad (6)$$

$$f_u^{\min} y_{u,t} \leq f_{u,t} \leq f_u^{\max} y_{u,t} \quad (7)$$

$$f_{u,t} \leq f_u \quad (8)$$

- **Modeling equations**

example: heat pump

$u = \text{heat pump}$

$$\dot{e}_u^+ = \frac{\dot{q}_u^-}{\text{COP}} \quad (9)$$

$$\text{COP} = \left(\frac{\tilde{T}^{\text{sink}}}{\tilde{T}^{\text{sink}} - \tilde{T}^{\text{source}}} \right) \eta_{\text{carnot}} \quad \text{with} \quad \eta_{\text{carnot}} \approx 0.55 \quad (10)$$

The heat pump COP can be calculated as a function of the temperature at each time step t and distribution system d considered:

$u = \text{heat pump}, \forall t \in \mathbf{T}, \forall d \in \mathbf{D}$

$$\dot{e}_{u,d,t}^+ = \frac{\dot{q}_{u,d,t}^-}{\text{COP}} \quad (11)$$

$$\text{COP}_{d,t} = \left(\frac{\tilde{T}_{d,t}^{\text{sink}}}{\tilde{T}_{d,t}^{\text{sink}} - \tilde{T}_{d,t}^{\text{source}}} \right) \eta_{\text{carnot}} \quad (12)$$

Energy and mass balances

- **Heat distribution** $\forall t \in \mathbf{T}, \forall d \in \mathbf{D}$ with $T_{d+1,t} \geq T_{d,t}$

$$\sum_u^{\mathbf{U}} \dot{q}_{u,d} f_{u,t} - \dot{Q}_{d,t} = \emptyset \quad (13)$$

$$\dot{Q}_{d+1,t}^{\text{res}} + \dot{Q}_{d,t} - \sum_b^{\mathbf{B}} \dot{Q}_{b,d,t} = \dot{Q}_{d-1,t}^{\text{res}} \quad (14)$$

$$\dot{Q}_{d,t} = \dot{m}_{d,t} \text{cp}_d (T_{d,t}^{\text{s}} - T_{d,t}^{\text{r}}) \quad (15)$$

$$\dot{Q}_{0,t}^{\text{res}} = \emptyset, \quad \dot{Q}_{n_{d+1},t}^{\text{res}} = \emptyset \quad (16)$$

$$\dot{Q}_{d,t}^{\text{res}} \geq \emptyset \quad (17)$$

- **Electricity balance** $\forall t \in \mathbf{T}$

$$\dot{E}_t^+ - \dot{E}_t^- + \sum_u^{\mathbf{U}} f_{u,t} \dot{e}_u^- - \sum_u^{\mathbf{U}} f_{u,t} \dot{e}_u^+ - \sum_b^{\mathbf{B}} \dot{E}_{b,t} = 0 \quad (18)$$

- **Resource and material balance** $\forall t \in \mathbf{T}, \forall r \in \mathbf{R}$

$$\dot{m}_{r,t} = \sum_u^{\mathbf{U}} f_{u,t} \dot{m}_{r,u} \quad (19)$$

example: emissions of CO₂ $\forall t \in \mathbf{T}, r = \text{co}_2$

$$\dot{m}_{\text{co}_2,t}^- = \sum_u^{\mathbf{U}} f_{u,t} \dot{m}_{\text{co}_2,u} \quad (20)$$

The goal is to calculate the set of variables $f_{u,t}, y_{u,t}, f_u, y_u, \dot{E}_t^-, \dot{E}_t^+, \dot{m}_{r,t}, \dot{Q}_{d,t}, \dot{Q}_{d,t}^{\text{res}}$ that minimizes the objective TOTEX.

Index and set	Description
$t \in \mathbf{T}$	Time $\mathbf{T} = \{t_1 \dots t_{n_t}\}$
$u \in \mathbf{U}$	Utility $\mathbf{U} = \{\text{boiler, heat pump, refrigeration, ...}\}$
$r \in \mathbf{R}$	Resource $\mathbf{R} = \{\text{hydrogen, natural gas, chocolate, ...}\}$
$d \in \mathbf{D}$	Distribution system $\mathbf{D} = \{d_1 \dots d_{n_d}\}$
$b \in \mathbf{B}$	Building $\mathbf{B} = \{b_1 \dots b_{n_b}\}$
Parameter	Description
t^{op}	Total operating time per year [h/year]
Δt_t	Duration of time interval t [h]
$c_{r,t}$	Specific cost of resource r at time t [CHF/kg]
$c_{e,t}^+$	Price for purchased electricity at time t [CHF/kWh]
$c_{e,t}^-$	Price for sold electricity at time t [CHF/kWh]
c_u^{inv1}	Fixed investment cost of utility u [CHF]
c_u^{inv2}	Variable investment cost of utility u [CHF/size]
c_u^{mt}	Maintenance specific cost of utility per year u [CHF/size/year]
$\frac{1}{\tau}$	Annualization factor of investment [1/year]
f_u^{min}	Minimum sizing factor of utility u [-]
f_u^{max}	Maximum sizing factor of utility u [-]
$k_{\text{CO}_2,t}$	CO ₂ equivalent emissions of the grid at time t [ton _{CO2} /kWh]
$\dot{m}_{r,u}$	Reference mass flow of resource r in utility u [kg/s]
\dot{q}_u^-	Reference heat load produced by utility u [kW]
\dot{q}_u^+	Reference heat load consumed by utility u [kW]
\dot{e}_u^-	Reference electrical power produced by utility u [kW]
\dot{e}_u^+	Reference electrical power consumed by utility u [kW]
$T_{d,t}$	Logarithmic mean temperature of the distribution system d at time t [K]
$T_{d,t}^s$	Supply temperature of the distribution system d at time t [K]
$T_{d,t}^r$	Return temperature of the distribution system d at time t [K]
$\dot{m}_{d,t}$	Mass flow in the distribution system d [kg/s]
cp_d	Heat capacity of the fluid in the distribution system d [kJ/K/kg]
$\dot{q}_{u,d}$	Reference heat load of utility u fed in the distribution system d [kW]
$\dot{Q}_{b,d,t}$	Heat load demand of building b at time t in the distribution system d [kW]
$\dot{E}_{b,t}$	Electricity consumption of building b at time t [kW]
COP	Coefficient Of Performance of the heat pump [-]
\tilde{T}_{sink}	Logarithmic mean temperature of the heat sink of the heat pump [K]
$\tilde{T}_{\text{source}}$	Logarithmic mean temperature of the heat source of the heat pump [K]
η_{carnot}	Carnot efficiency [-]
Variable	Description
f_u	Sizing factor of utility u [-]
$f_{u,t}$	Sizing factor of utility u at time t [-]
y_u	Binary variable to use or not utility u [-]
$y_{u,t}$	Binary variable to use or not utility u at time t [-]
\dot{E}_t^+	Purchased electrical power at time t [kW]
\dot{E}_t^-	Sold electrical power at time t [kW]
$\dot{m}_{r,t}$	Mass flow of resource r at time t [kg/s]
$\dot{Q}_{d,t}$	Total utility heat load at time t in the distribution system d [kW]
$\dot{Q}_{d,t}^{\text{res}}$	Heat load cascaded from distribution system d to $d + 1$ at time t [kW]