ME-446 Liquid-Gas Interfacial Heat and Mass Transfer

Homework 8 - Solution

Problem 1: Hydrodynamic instabiltiy

Part A)

When $a > 0$, $\lim_{t\to\infty} |e^{\beta t}| = \infty$ and the amplitude will grow in time.

When $a < 0$, $\lim_{t\to\infty} |e^{\beta t}| = 0$, and the perturbation will vanish.

Part B)

Only the positive β will grow. Set the first derivative with regard to α to zero:

$$\frac{d}{d\alpha}\left[\frac{(\rho_l - \rho_v)g\alpha - \sigma\alpha^3}{\rho_l - \rho_v}\right]^{\frac{1}{2}} = 0$$

$$\frac{d}{d\alpha}[(\rho_l - \rho_v)g\alpha - \sigma\alpha^3] = 0$$

$$(\rho_l - \rho_v)g = 3\sigma\alpha^2$$

$$\alpha = \sqrt{\frac{(\rho_l - \rho_v)g}{3\sigma}}$$

It's easy to verify that this α corresponds to the maximum of the positive β. The perturbation wavelength with the fastest growth can then be written as

$$\lambda = \frac{2\pi}{\alpha} = 2\pi\sqrt{\frac{3\sigma}{(\rho_l - \rho_v)g}}$$

Problem 2: Maxima of number of isolated bubbles

Part A)

Number of isolated bubbles N_{iso}:

$$N_{iso} = \sum_{N=0}^{\infty} N \cdot P_{iso} \cdot P_0(N, N_0) = \sum_{N=1}^{\infty} \frac{N_0^N}{(N-1)!} e^{\left(-N_0 - \frac{\pi N D_b^2}{A}\right)} \tag{1}$$

Merging the power of N and taking out e^{-N_0} of the sum:

$$N_{iso} = e^{-N_0} \sum_{N=1}^{\infty} \frac{[N_0 e^{-\frac{\pi D_b^2}{A}}]^N}{(N-1)!} \tag{2}$$

$$N_{iso} = N_0 e^{-\frac{\pi D_b^2}{A}} e^{-N_0} \sum_{N=1}^{\infty} \frac{[N_0 e^{-\frac{\pi D_b^2}{A}}]^{N-1}}{(N-1)!} \tag{3}$$

$$N_{iso} = N_0 e^{-\frac{\pi D_b^2}{A}} e^{-N_0} \sum_{N=1}^{\infty} \frac{[N_0 e^{-\frac{\pi D_b^2}{A}}]^{N-1}}{(N-1)!} e^{-N_0 e^{-\frac{\pi D_b^2}{A}}} e^{N_0 e^{-\frac{\pi D_b^2}{A}}} \tag{4}$$

We recognize a Poisson distribution:

$$\sum_{N=1}^{\infty} \frac{[N_0 e^{-\frac{\pi D_b^2}{A}}]^{N-1}}{(N-1)!} e^{-N_0 e^{-\frac{\pi D_b^2}{A}}} = 1 \tag{5}$$

$$N_{iso} = N_0 e^{-\frac{\pi D_b^2}{A}} e^{-N_0} e^{N_0 e^{-\frac{\pi D_b^2}{A}}} \tag{6}$$

At the CHF point, we expect $\frac{\partial N_{iso}}{\partial T} = 0$.

$$\frac{\partial}{\partial T} \left[N_0 e^{-\frac{\pi D_b^2}{A}} e^{-N_0} e^{N_0 e^{-\frac{\pi D_b^2}{A}}} \right] = 0 \tag{7}$$

Taking out the constant term and applying the product rule, we have:

$$e^{-N_0} e^{N_0 e^{-\frac{\pi D_b^2}{A}}} \frac{\partial N_0}{\partial T} + N_0 e^{N_0 e^{-\frac{\pi D_b^2}{A}}} (-e^{N_0}) \frac{\partial N_0}{\partial T} + N_0 e^{-N_0} e^{N_0 e^{-\frac{\pi D_b^2}{A}}} e^{-\frac{\pi D_b^2}{A}} \frac{\partial N_0}{\partial T} = 0 \tag{8}$$

Assuming $\frac{\partial N_0}{\partial T} \neq 0$ and simplifying, we end up with:

$$1 - N_0 + N_0 e^{-\frac{\pi D_b^2}{A}} = 0 \tag{9}$$

For $D << A$ we can use the Taylor expansion of $e^{-\frac{\pi D_b^2}{A}}$ at 0:

$$e^{-\frac{\pi D_b^2}{A}} \approx 1 - \frac{\pi D_b^2}{A} \tag{10}$$

Then we obtain:

$$1 - N_0 + N_o \left(1 - \frac{\pi D_b^2}{A} \right) = 0 \tag{11}$$

$$\frac{N_0}{A} \pi D_b^2 = 1 \tag{12}$$

Part B)

Substituing Equation (12) into Equation (6), we have

$$N_{iso} = N_0 e^{-\frac{1}{N_0}} e^{-N_0} e^{N_0 e^{-\frac{1}{N_0}}}$$

Using the same Taylor expansion as before, we have

$$N_{iso} = N_0 e^{-N_0} e^{N_0(1-\frac{1}{N_0})} e^{-\frac{1}{N_0}}$$

Simplifying and assuming $e^{-\frac{1}{N_0}} \approx 1$ if N_0 is large, we obtain

$$N_{iso} = \frac{N_0}{e}$$

Problem 3: Kandlikar Model

CHF prediction using the Kandlikar model for contact angles β between 5° and 50° are shown in Figure 1. The CHF decreases when the contact angle increases.

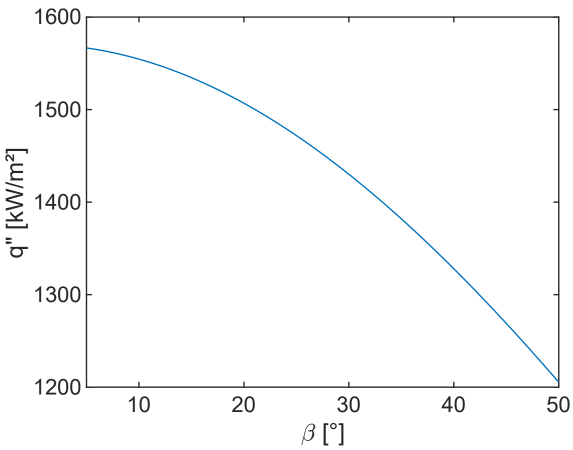

Figure 1: CHF Kandlikar Model for Water at 1 atm