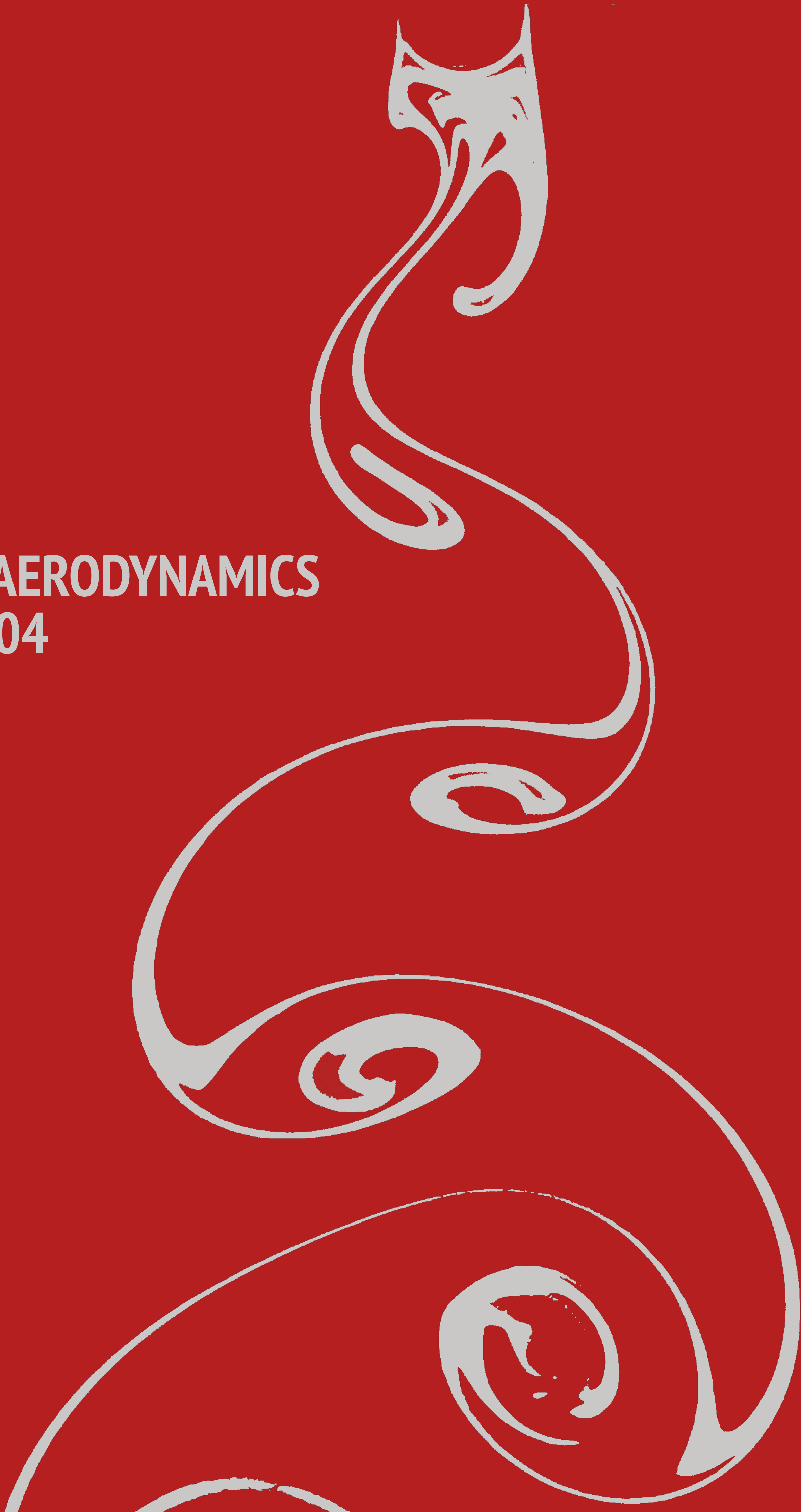
ME-445 AERODYNAMICS
Exercise 04
Week 3



Formula sheet

Cylindrical coordinates

$$\nabla \vec{u} = \left(\frac{\partial v_r}{\partial r}, \frac{1}{r}\frac{\partial v_\theta}{\partial \theta}, 0 \right)$$

$$\nabla \cdot \vec{u} = \frac{1}{r}\frac{\partial (r v_r)}{\partial r} + \frac{1}{r}\frac{\partial v_\theta}{\partial \theta}$$

$$\nabla \times \vec{u} = \left(0, 0, \frac{1}{r}\left[\frac{\partial (r v_\theta)}{\partial r} - \frac{\partial v_r}{\partial \theta} \right] \right)$$

Potential flow

$$v_r = \frac{\partial \phi}{\partial r} = \frac{1}{r}\frac{\partial \psi}{\partial \theta}, \quad v_\theta = \frac{1}{r}\frac{\partial \phi}{\partial \theta} = -\frac{\partial \psi}{\partial r}$$

Uniform parallel flow $\quad w = \phi + i\psi = \mathrm{U}_\infty e^{-i\alpha} z$

Potential vortex in z_0 $\quad w = -\frac{i\gamma}{2\pi}\ln(z - z_0)$

Point source or sink in z_0 $\quad w = \frac{Q}{2\pi}\ln(z - z_0)$

Source-sink doublet in z_0 $\quad w = \frac{\mu}{2\pi(z - z_0)}$

$$\frac{\mathrm{d}w}{\mathrm{d}z} = u - iv$$

Milne-Thomson circle theorem:

$$g(z) = w(z) + \overline{w\left(\frac{a^2}{\bar{z}}\right)}$$

Thin airfoil theory

For a camber line with:

$$\frac{\mathrm{d}y_c}{\mathrm{d}x} = A_0 + \sum_{n=1}^{\infty} A_n \cos n\theta$$

$$\frac{x}{c} = \frac{(1-\cos\theta)}{2}$$

we know:

$$k = 2\mathrm{U}_\infty \left[(\alpha - A_0)\frac{\cos\theta + 1}{\sin\theta} + \sum_{n=1}^{\infty} A_n \sin n\theta \right]$$

$$A_0 = \frac{1}{\pi}\int_0^\pi \frac{\mathrm{d}y_c}{\mathrm{d}x}\mathrm{d}\theta$$

$$A_n = \frac{2}{\pi}\int_0^\pi \frac{\mathrm{d}y_c}{\mathrm{d}x}\cos n\theta \mathrm{d}\theta$$

$$C_l = 2\pi\alpha + \pi(A_1 - 2A_0)$$

$$C_{m,1/4} = -\frac{\pi}{4}(A_1 - A_2)$$

$$x_{cp} = \frac{1}{4} + \frac{\pi}{4C_l}(A_1 - A_2)$$

Finite wings with AR=b^2/S

Sign convention:

if induced velocity points downward: $w(y) > 0$, $\alpha_i(y) > 0$

if induced velocity points upward: $w < 0$, $\alpha_i < 0$

Prandtl's lifting-line theory

$$\mathrm{U}_\infty \alpha_i(y_0) = w(y_0) = -\frac{1}{4\pi}\int_{-b/2}^{b/2} \frac{(\mathrm{d}\Gamma/\mathrm{d}y)}{y - y_0}\mathrm{d}y$$

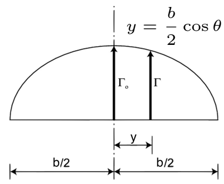
$$\alpha(y_0) = \alpha_{\text{eff}}(y_0) + \alpha_i(y_0)$$

Elliptical loading $\Gamma(y) = \Gamma_0\sqrt{1 - \left(\frac{2y}{b}\right)^2}$

$$w = \frac{\Gamma_0}{2b}$$

$$\alpha_i = \frac{C_L}{\pi \mathrm{AR}}$$

$$C_{D,i} = \frac{C_L^2}{\pi \mathrm{AR}}$$



General loading $\Gamma(\theta) = 2b\mathrm{U}_\infty \sum_{n=1}^{\infty} A_n \sin n\theta$

$$w(\theta) = \mathrm{U}_\infty \sum_{n=1}^{\infty} n A_n \frac{\sin n\theta}{\sin\theta}$$

$$C_L = \pi A_1 \mathrm{AR}$$

$$C_{D,i} = \frac{C_L^2}{\pi \mathrm{AR}}(1 + \delta) \text{ with } \delta = \sum_{n=2}^{\infty} n\left(A_n/A_1\right)^2$$

Boundary Layer

Flat plate **laminar** boundary layer

$\frac{\delta}{x} = \frac{5}{\sqrt{Re_x}}$ boundary layer growth

$C_f = \frac{1.328}{\sqrt{Re_x}}$ skin friction drag coefficient

Flat plate **turbulent** boundary layer

$\frac{\delta}{x} = \frac{0.37}{Re_x^{1/5}}$ boundary layer growth

$C_f = \frac{0.074}{Re_x^{1/5}}$ skin friction drag coefficient

Miscellanous

θ	$0°$	$30°$	$45°$	$60°$	$90°$
$\sin\theta$	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1
$\cos\theta$	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0

water

kinematic viscosity	$\nu = 1 \times 10^{-6}\,\mathrm{m^2\,s^{-1}}$
density	$\rho = 1000\,\mathrm{kg\,m^{-3}}$

air

kinematic viscosity	$\nu = 1.5 \times 10^{-5}\,\mathrm{m^2\,s^{-1}}$
density	$\rho = 1.2\,\mathrm{kg\,m^{-3}}$

$$\sin(x \pm y) = \sin x \cos y \pm \cos x \sin y$$
$$\cos(x \pm y) = \cos x \cos y \mp \sin x \sin y$$

$$\cos 2\theta = 2\cos^2\theta - 1$$
$$\sin 2\theta = 2\sin\theta\cos\theta$$
$$\sin 3\theta = 3\sin\theta - 4\sin^3\theta$$
$$\cos 3\theta = 4\cos^3\theta - 3\cos\theta$$

$$\int_0^\pi \cos\theta \mathrm{d}\theta = 0$$
$$\int_0^\pi \sin\theta \mathrm{d}\theta = 2$$
$$\int_0^\pi \cos^2\theta \mathrm{d}\theta = \int_0^\pi \sin^2\theta \mathrm{d}\theta = \frac{\pi}{2}$$

$$\int_0^\pi \frac{\cos n\theta}{\cos\theta - \cos\theta_1}\mathrm{d}\theta = \pi\frac{\sin n\theta_1}{\sin\theta_1} \qquad n = 0, 1, 2, \ldots$$
$$\int_0^\pi \frac{\sin n\theta \sin\theta}{\cos\theta - \cos\theta_1}\mathrm{d}\theta = -\pi\cos n\theta_1 \qquad n = 1, 2, 3, \ldots$$

1. During the design and development of the McDonnell Douglas F/A-18 Hornet (Figure 1), several wind tunnel tests were conducted. A $16\,\%$ scaled down model was tested at low speed ($\mathrm{Ma}_\infty = 0.08$), providing measurements for the lift coefficient C_L as a function of the angle of attack α, the lift versus drag coefficient, and the coefficient of moment $C_{\mathrm{M,c/4}}$ with respect to the $25\,\%$ mean chord as a function of α (Figure 2). Consider the final version of the airplane in horizontal flight with constant velocity U_∞ and assume that the model was properly scaled and dynamic similarity is assured. The F/A-18 has a maximum take-off mass $m = 23\,500\,\mathrm{kg}$, a wing area $S = 38\,\mathrm{m}^2$, a wingspan $b = 12.3\,\mathrm{m}$, a length $l = 17.1\,\mathrm{m}$, and each of its two engines provide a maximum thrust $T = 79.2\,\mathrm{kN}$. The $25\,\%$ mean chord is located at a distance from the front of $60\,\%$ of the airplane length. Assume that the air density at the flight altitude is $\rho = 1.23\,\mathrm{kg\,m^{-3}}$.

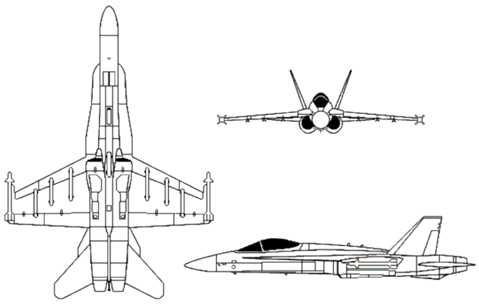


Figure 1: Schematic representation of the McDonnell Douglas F/A-18 Hornet

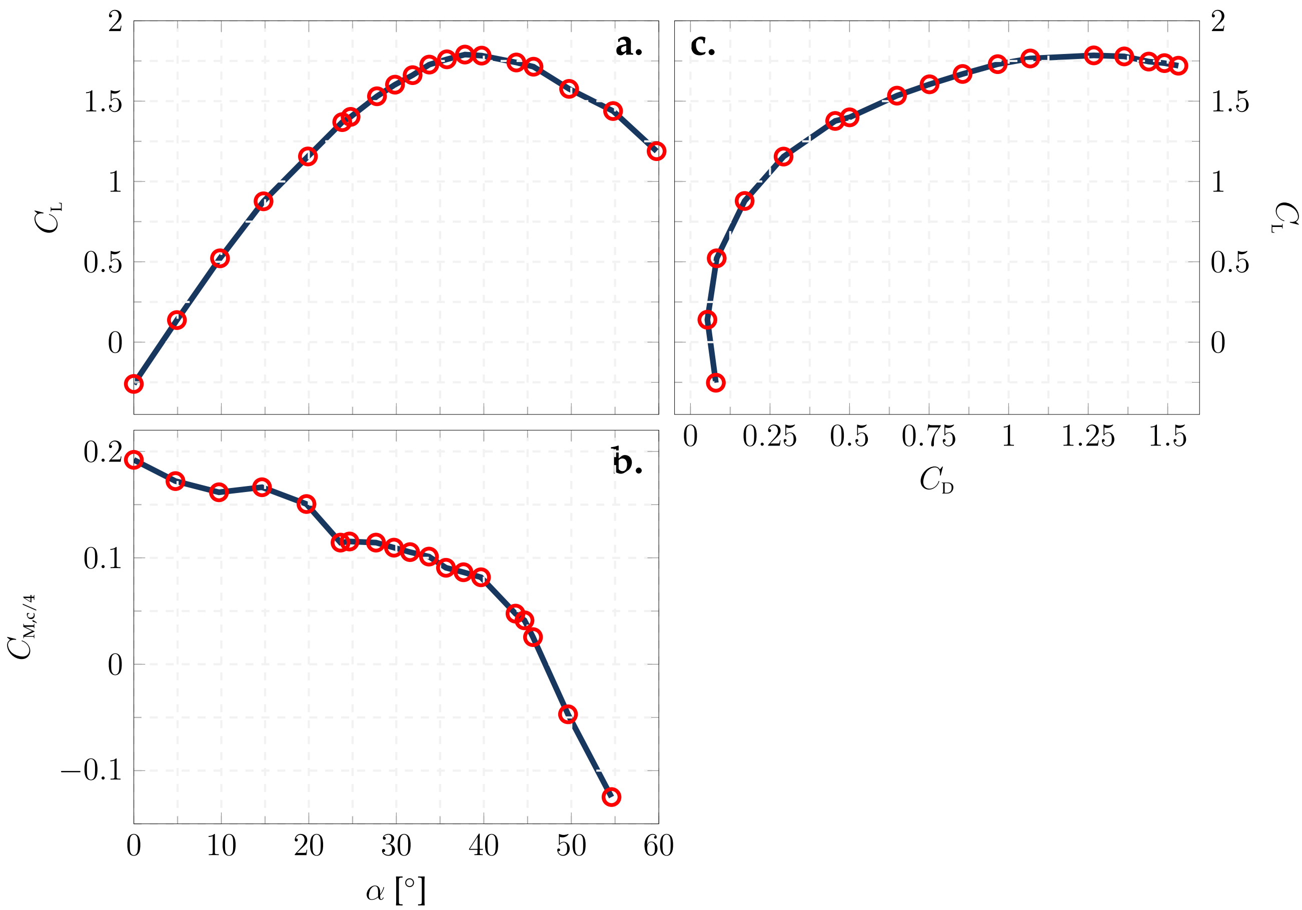


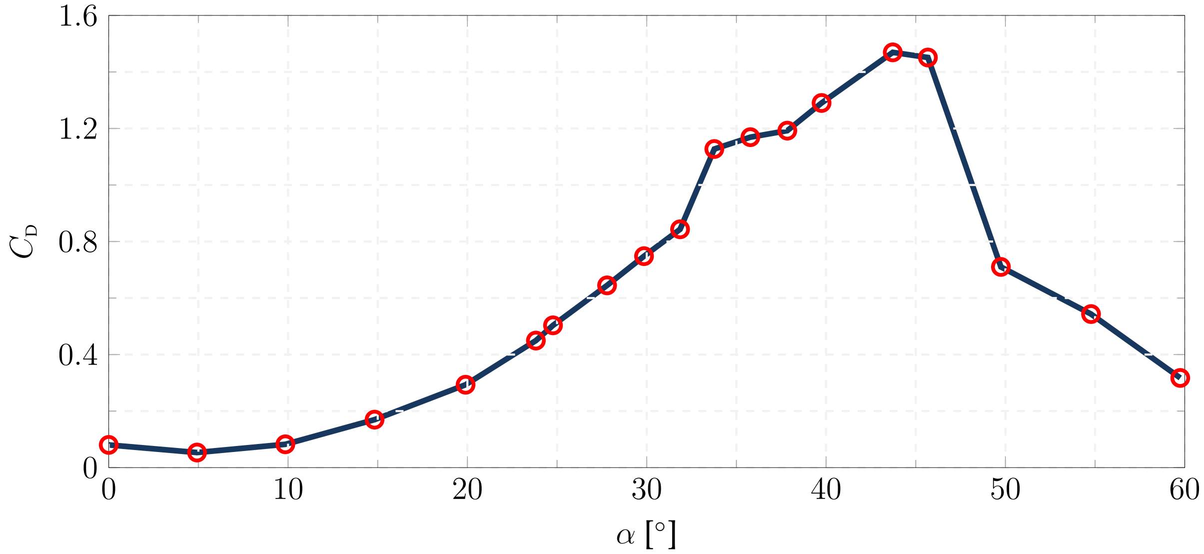
Figure 2: a.) Lift coefficient in function of the angle of attack, b.) pitching moment coefficient with respect to the $25\,\%$ mean chord, c.) lift versus drag coefficient.

(a) Retrieve the data from the graphs in Figure 2 and load them into Matlab.
Note: You can use `image_digitizer.m` *to digitize the data points from the plots or use the file* `F18Hornet.mat` *from Moodle directly.*

(b) Compute and plot drag coefficient C_D as a function of the angle of attack α. What is the highest C_D of the airfoil?
Note: The data points are spaced unequally and need to be interpolated onto the same α spacing before they can be multiplied with one another. Use `interp1(x,y,xq,'linear','extrap')` *to convert between the different graphs.*

Solution: By combining together the plots for the polar curve and the C_L as a function of α, we obtain the C_D data:

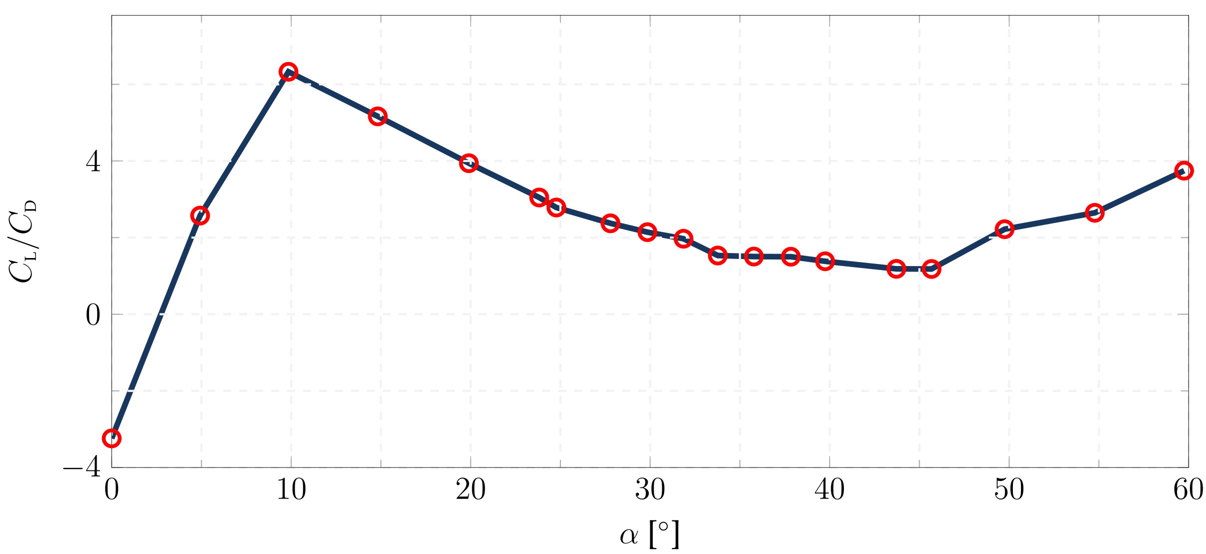
$$C_{D,max} = 1.47 \quad \text{at} \quad \alpha = 43.72°$$



(c) Compute and plot the aerodynamic efficiency η_{aero} in terms of the lift-to-drag ratio C_L/C_D. At which angle does the airplane have the highest C_L/C_D-ratio?

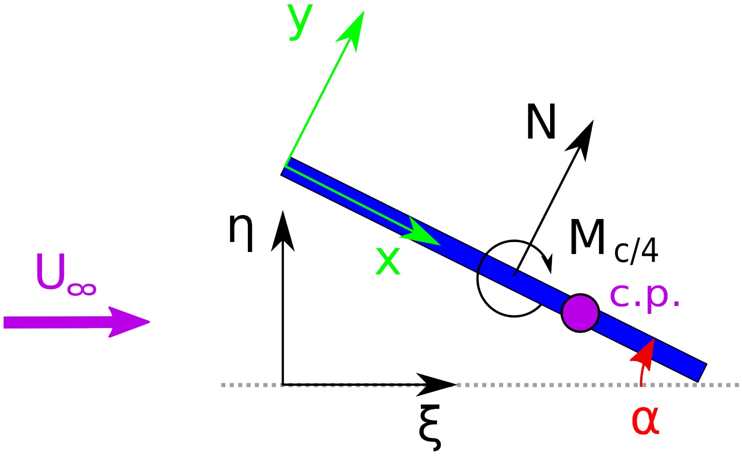
Solution: The aerodynamic efficiency is defined as $\eta_{aero} = C_L/C_D$. By combining the C_L and C_D plots we get η_{aero} as a function of α.

$$\eta_{aero,max} = 6.33 \quad \text{at} \quad \alpha = 9.85°$$



(d) Compute and plot the center of pressure relative to the length of the airplane x_{cp}/l as a function of the angle of attack α. Where is the center of pressure located for $\alpha = 0°$?

Solution: Consider the following sketch for the geometry of the problem:



The origin of the x- and y-axis is located at the front of the airplane. Following the definition of the center of pressure:

$$M_{\text{cp}} = N(x_{\text{cp}} - x_{\text{c/4}}) + M_{\text{c/4}} = 0$$

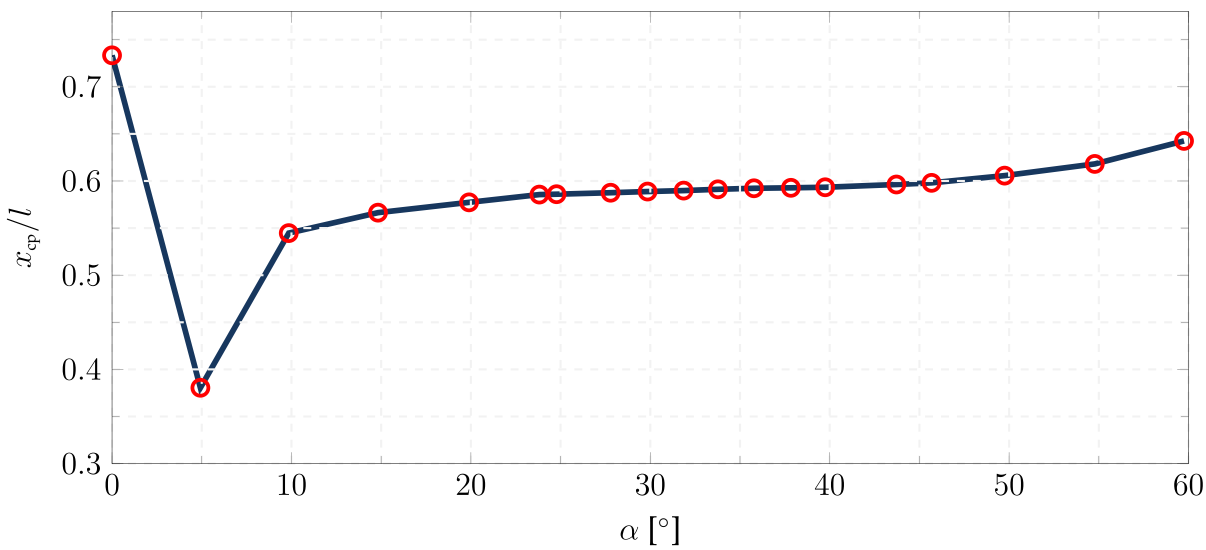
By defining the average chord length as $c = S/b$ and using:

$$C_{\text{N}} = C_{\text{L}} \cos\alpha + C_{\text{D}} \sin\alpha,$$

we find that the location of the center of pressure is

$$x_{\text{cp}} = 0.6l - \frac{S C_{\text{M,c/4}}}{b C_{\text{N}}}.$$

$$x_{\text{cp}}(\alpha = 0°) = 0.73\,l$$



(e) Calculate the minimum steady flight speed, the stalling velocity u_{stall}, of the airplane. What is the value of the stall angle α_{stall}? Calculate the thrust needed to maintain flight at these conditions.

Solution:

$$C_{\text{L,max}} = 1.79$$
$$u_{\text{stall}} = \sqrt{\frac{2W}{\rho S C_{\text{L,max}}}} = 74.2\,\text{m}\,\text{s}^{-1} = 267.1\,\text{km}\,\text{s}^{-1}$$
$$\alpha_{\text{stall}} = 37.84°$$
$$C_{\text{D,stall}} = 1.19$$
$$T_{\text{stall}} = D_{\text{stall}} = \frac{1}{2}\rho u_{\text{stall}}^2 S C_{\text{D,stall}} = 151\,700\,\text{N}$$

(f) Calculate the maximum velocity of the airplane. Determine the Mach number Ma, the angle of attack α, and the lift L at these flight conditions? Can you see any problems arising from these results?

Solution: The airplane has two engines and thus the maximum thrust is $T_{\text{max}} = 2T$.

$$C_{\text{D,min}} = 0.053$$
$$u_{\text{max}} = \sqrt{\frac{2T_{\text{max}}}{\rho S C_{\text{D,min}}}} = 360.6\,\text{m}\,\text{s}^{-1} = 1298.2\,\text{km}\,\text{s}^{-1}$$
$$\text{Ma}_{\text{u,max}} = 1.05$$
$$\alpha_{C_{\text{D,min}}} = 4.93°$$
$$C_{\text{L},C_{\text{D,min}}} = 0.14$$
$$L_{C_{\text{D,min}}} = \frac{1}{2}\rho u_{\text{max}}^2 S C_{\text{L},C_{\text{D,min}}} = 413\,596\,\text{N} \neq W$$

The maximum velocity reached is supersonic $\text{Ma}_{\text{u,max}} > 1$ where compressible flow effects have to be taken into account. However, the C_{L}, C_{D}, and $C_{\text{M,c/4}}$ distribution were determined at low Mach numbers ($\text{Ma}_\infty = 0.08$) where the flow is considered incompressible. The wind tunnel test were not properly scaled and dynamic similarity is not warranted. If the prediction holds true, the lift generated at these flight conditions does not match the weight and thus the airplane cannot maintain altitude.

(g) What is the angle of attack that maximises $C_{\text{L}}/C_{\text{D}}$? Calculate the lift-to-drag ratio, velocity, lift L, and drag D of the airplane.

Solution:

$$\alpha_{\text{max eff}} = 9.85°$$
$$C_{\text{L}}/C_{\text{D}}|_{\text{max}} = 6.33$$
$$u_{\text{max eff}} = \sqrt{\frac{2W}{\rho S C_{\text{L,max eff}}}} = 137.4\,\text{m}\,\text{s}^{-1} = 494.6\,\text{km}\,\text{s}^{-1}$$
$$L_{\text{max eff}} = W = 230\,535\,\text{N}$$
$$D_{\text{max eff}} = \frac{W}{C_{\text{L}}/C_{\text{D}}|_{\text{max}}} = 36\,332\,\text{N}$$

(h) At an altitude of $h = 4000\,\text{m}$ both engines are turned off ($T = 0\,\text{N}$) and the airplane glides to the ground. Calculate the distance that the airplane is able to glide at maximum lift-to-drag ratio.

Solution: If x is the traveled distance:

$$
\begin{aligned}
C_{\mathrm{L}}/C_{\mathrm{D}} &= \frac{x}{h} = \frac{1}{\tan\alpha} \\
x_{\text{max eff}} &= C_{\mathrm{L}}/C_{\mathrm{D}}\big|_{\text{max}}\, h = 25\,381\,\mathrm{m}
\end{aligned}
$$

2. To analyse the aerodynamic performance of an airfoil wind tunnel experiments are conducted and pressure measurements for a range of angles α at different chordwise positions taken. A total of 36 pressure sensors is distributed over the surface of the airfoil (see Figure 3). The measurements were carried out for 52 angular positions of $\alpha = [-21°, 21°]$. The airfoil is a NACA0015 with a chord length of $c = 0.3\,\mathrm{m}$. The incoming flow has a velocity of $U_\infty = 30\,\mathrm{m\,s^{-1}}$ with a density of $\rho = 1.2\,\mathrm{kg\,m^{-3}}$.

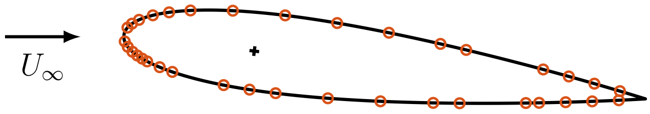
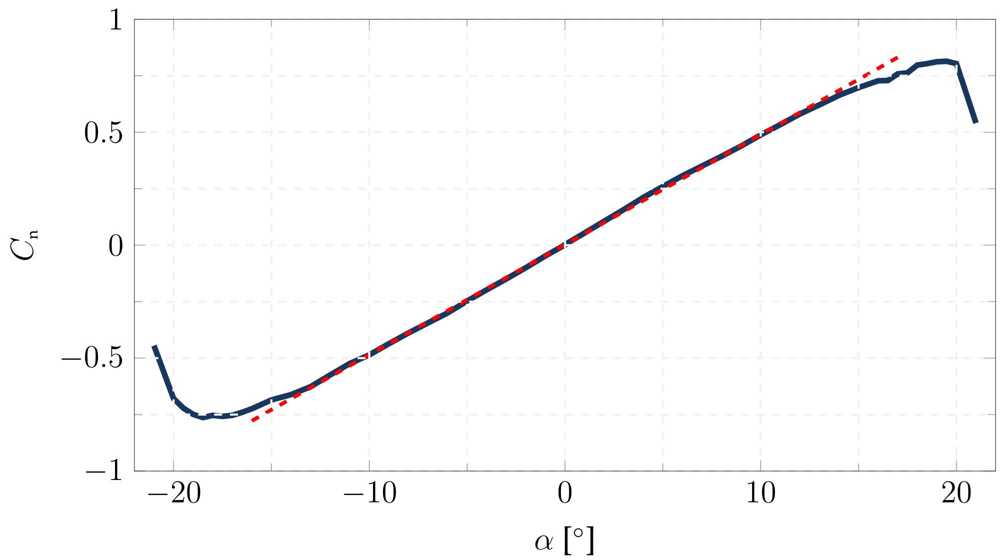


Figure 3: NACA0015 airfoil with integrated pressure sensors in red

(a) Load the file `NACA0015.mat` into Matlab containing the 36 chordwise positions of the pressure sensors of the lower and upper side of the airfoil, the angle of attack variations, and the recorded pressure data for each sensors and angle. Compute and plot the resulting normal force coefficient C_n as a function of α based on the pressure data. What is the slope $\partial C_n/\partial\alpha$ of the linear part of the curve?

Solution:

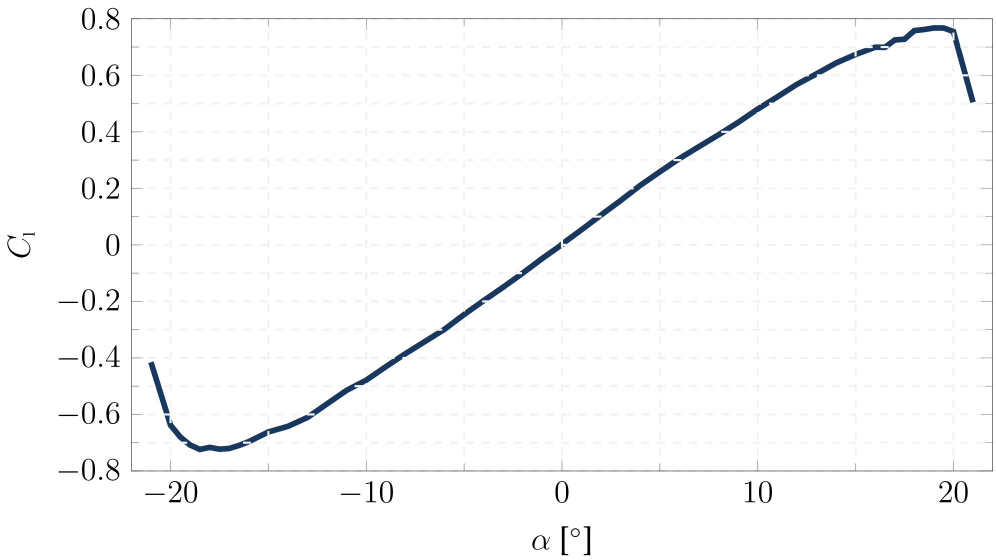
$$\frac{\partial C_n}{\partial \alpha} = 8.51 \times 10^{-4}\,\mathrm{rad}^{-1}$$



(b) Compute and plot the lift coefficient polar. What is the maximal lift coefficient of this airfoil and at what angle of attack does it occur?

Solution:

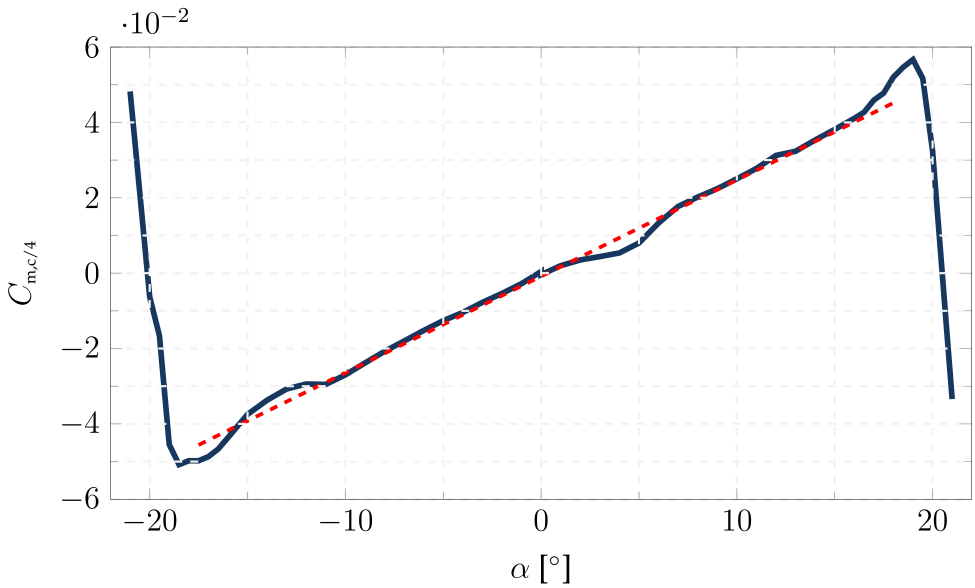
$$C_{l,\max} = 0.77 \quad \text{at} \quad \alpha = 19°$$



(c) Compute and plot the pitching moment coefficient at quarter chord $C_{m,c/4}$. What is the slope $\partial C_m/\partial \alpha$ of the linear part of the curve?

Solution:

$$\frac{\partial C_{m,c/4}}{\partial \alpha} = 4.46 \times 10^{-5}\,\text{rad}^{-1}$$



(d) Determine the aerodynamic centre of the airfoil.

Solution: Moment balance between quarter chord and the aerodynamic centre:

$$C_{m,AC} = C_{m,c/4} - \left(\frac{1}{4} - AC\right) C_n$$

deriving in function of α:

$$\frac{\partial C_{m,AC}}{\partial \alpha} = \frac{\partial C_{m,c/4}}{\partial \alpha} - \left(\frac{1}{4} - AC\right) \frac{\partial C_n}{\partial \alpha}$$

with $\quad m_0 := \frac{\partial C_{\mathrm{m,c/4}}}{\partial \alpha}, a_0 := \frac{\partial C_{\mathrm{n}}}{\partial \alpha}$ and $\frac{\partial C_{\mathrm{m,AC}}}{\partial \alpha} = 0$ follows:

$$0 = m_0 - \left(\frac{1}{4} - AC\right) a_0$$

$$AC = \frac{1}{4} - \frac{m_0}{a_0} = 0.198$$