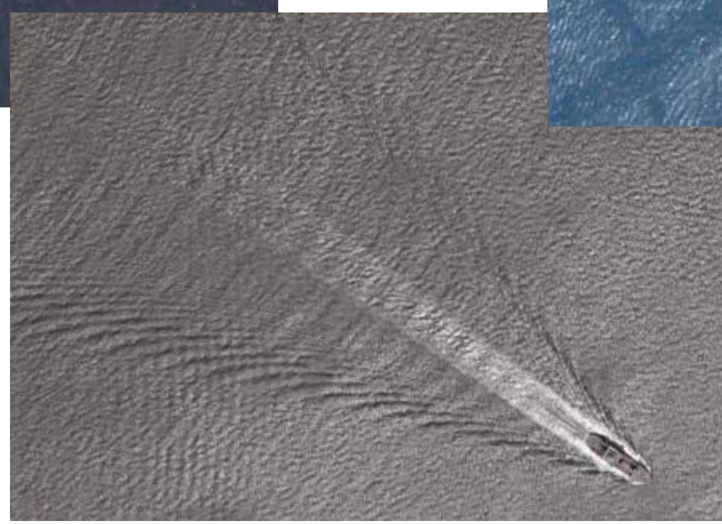
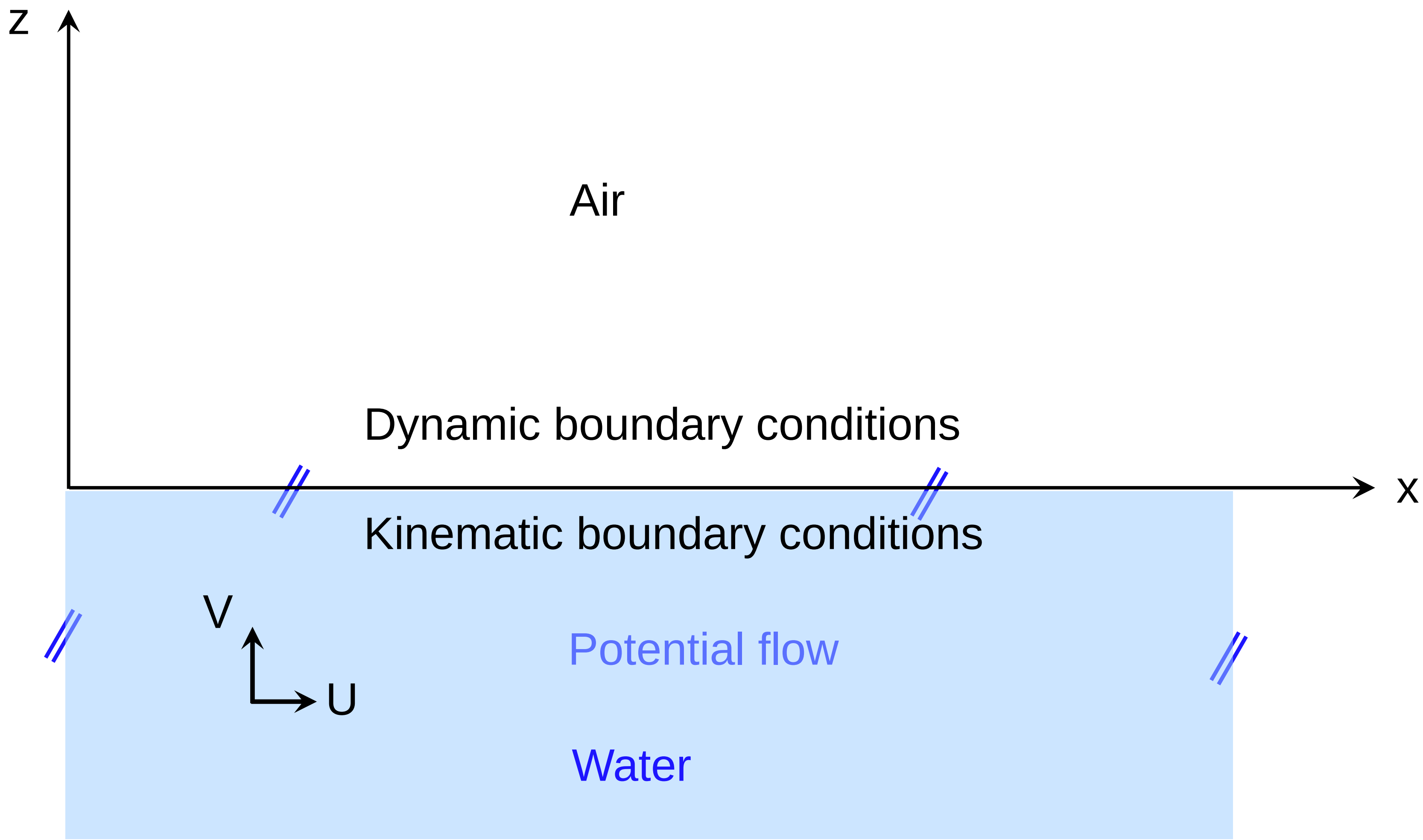
Hydrodynamics 13
Waves

Hydrodynamics 13
Waves

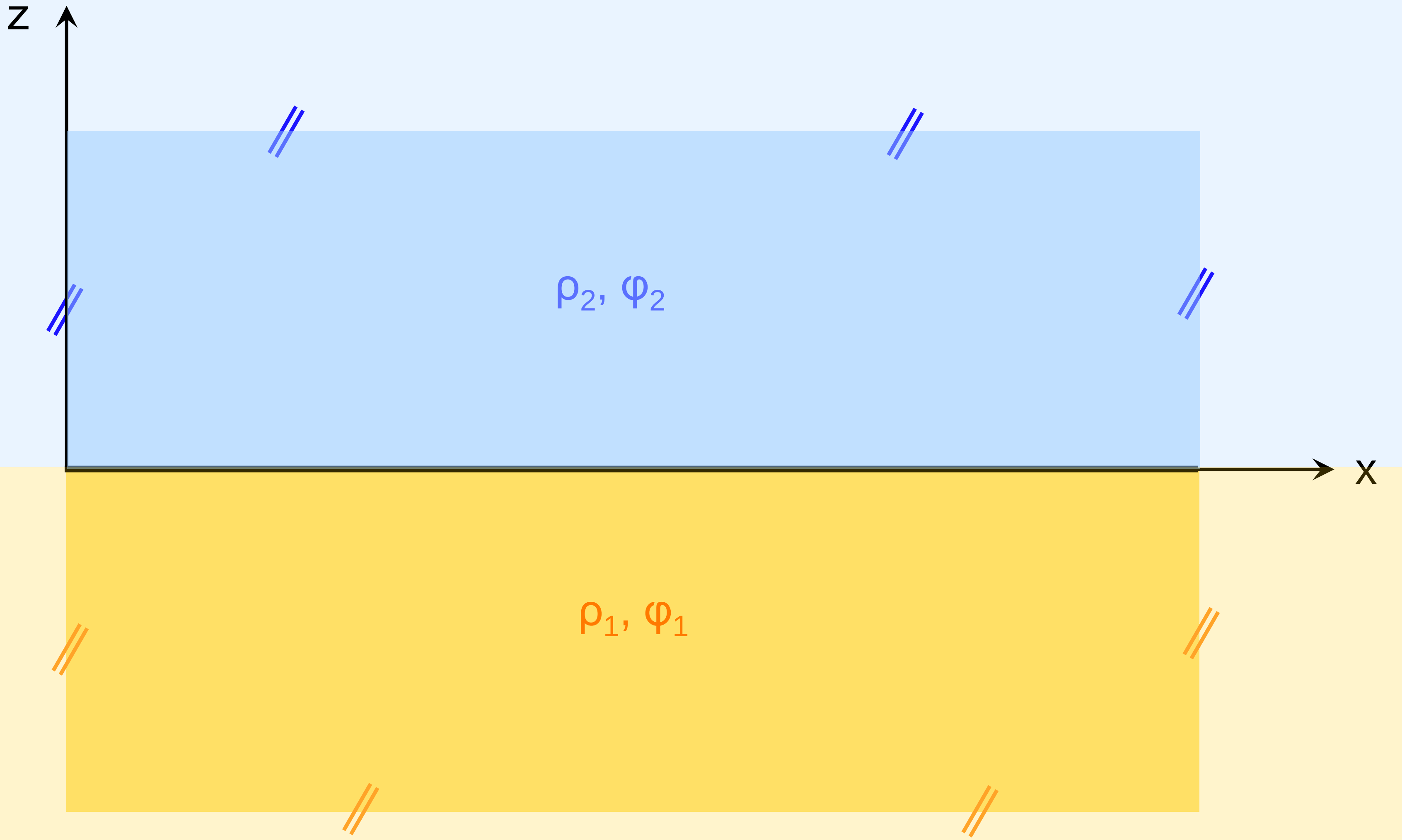


Waves

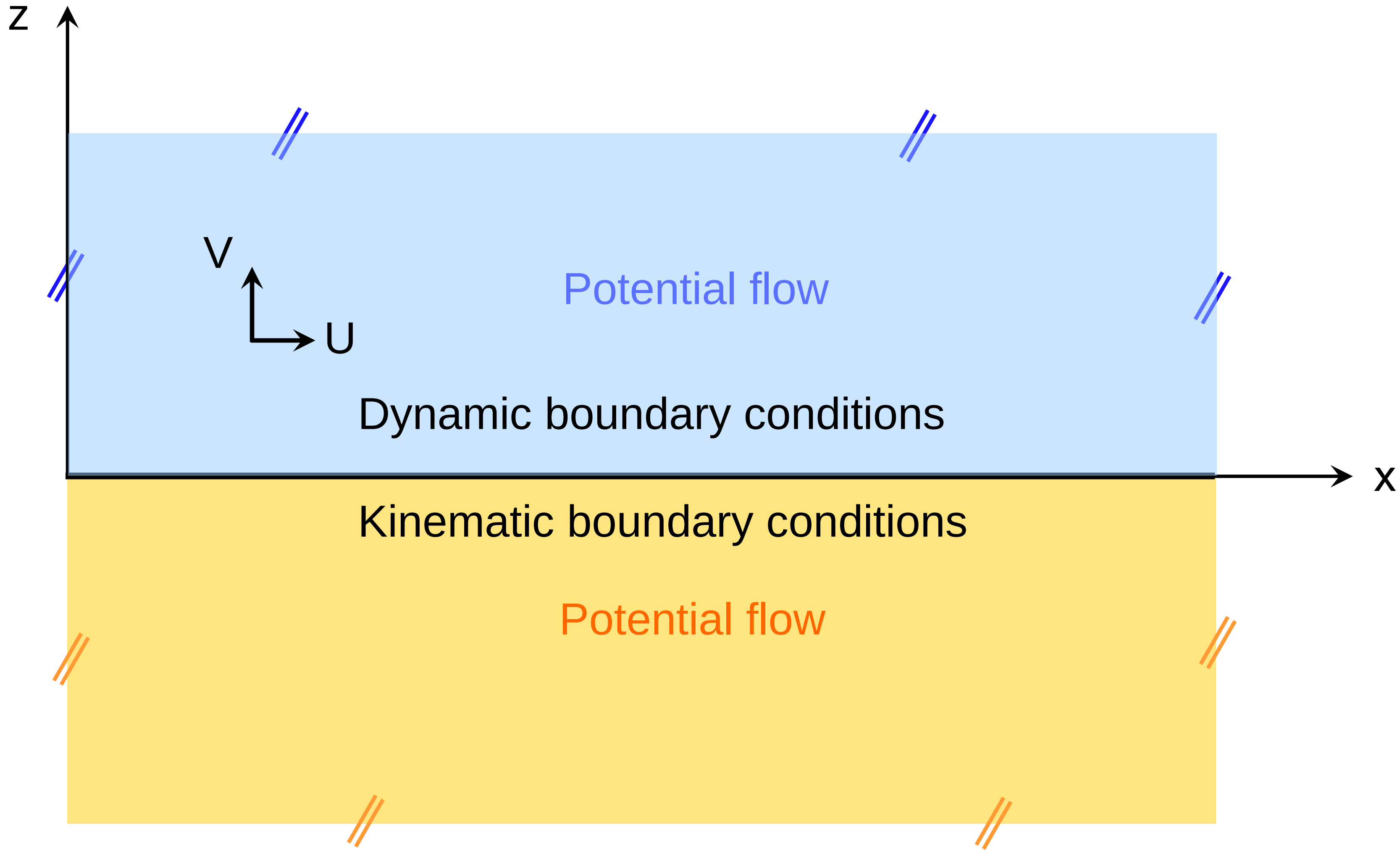


z
Air
Dynamic boundary conditions
x
Kinematic boundary conditions
V
U
Potential flow
Water

General case: two fluids



General case: two fluids



Linear waves dispersion relation

1. Equations and boundary conditions
2. Base state
3. Linearized equations
4. Normal mode expansion
5. Dispersion relation
6. Analysis of the dispersion relation

1. Equations

$$\begin{aligned} \Delta\Phi_1 &= 0 \\ \Delta\Phi_2 &= 0 \end{aligned}$$

Potential flow

$$U_1 = \frac{\partial\Phi_1}{\partial x}, \qquad V_1 = \frac{\partial\Phi_1}{\partial z}$$

$$U_2 = \frac{\partial\Phi_2}{\partial x}, \qquad V_2 = \frac{\partial\Phi_2}{\partial z}$$

Velocity field

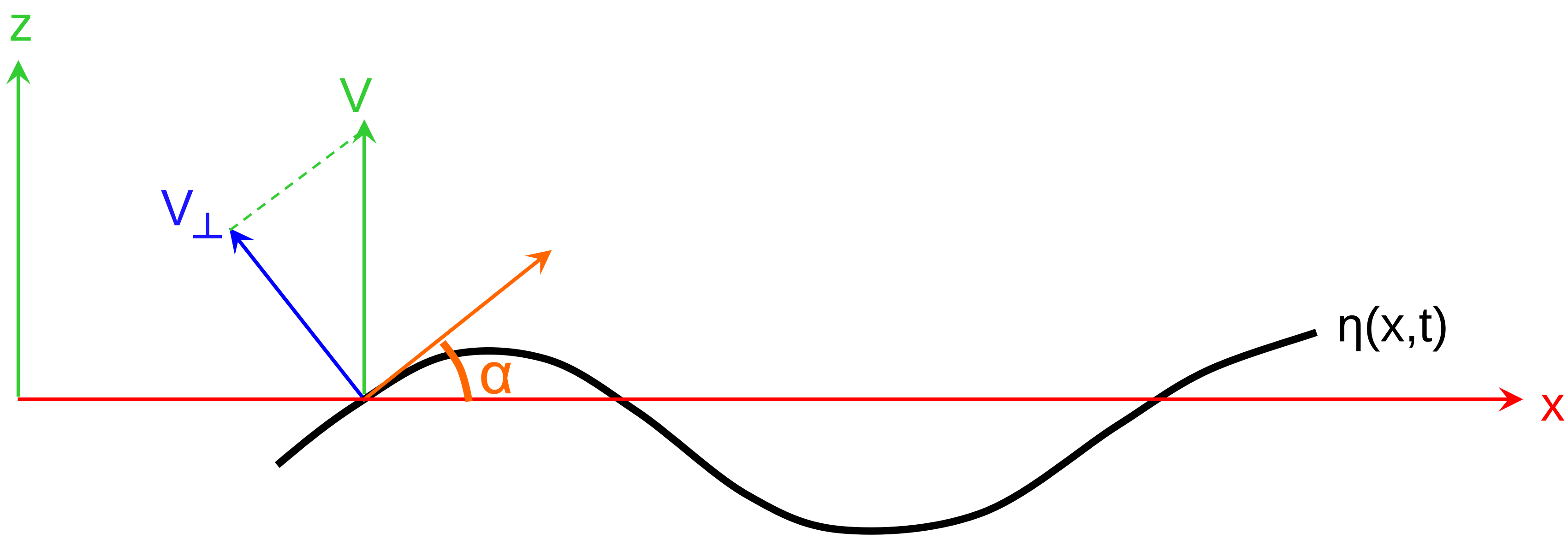
1. Boundary conditions

$$\boxed{\begin{array}{l} \Phi_1 = 0 \text{ at } z = -\infty \\ \Phi_2 = 0 \text{ at } z = +\infty \end{array}}$$

far-field

$\text{at } z = \eta$?

1. Kinematic boundary condition

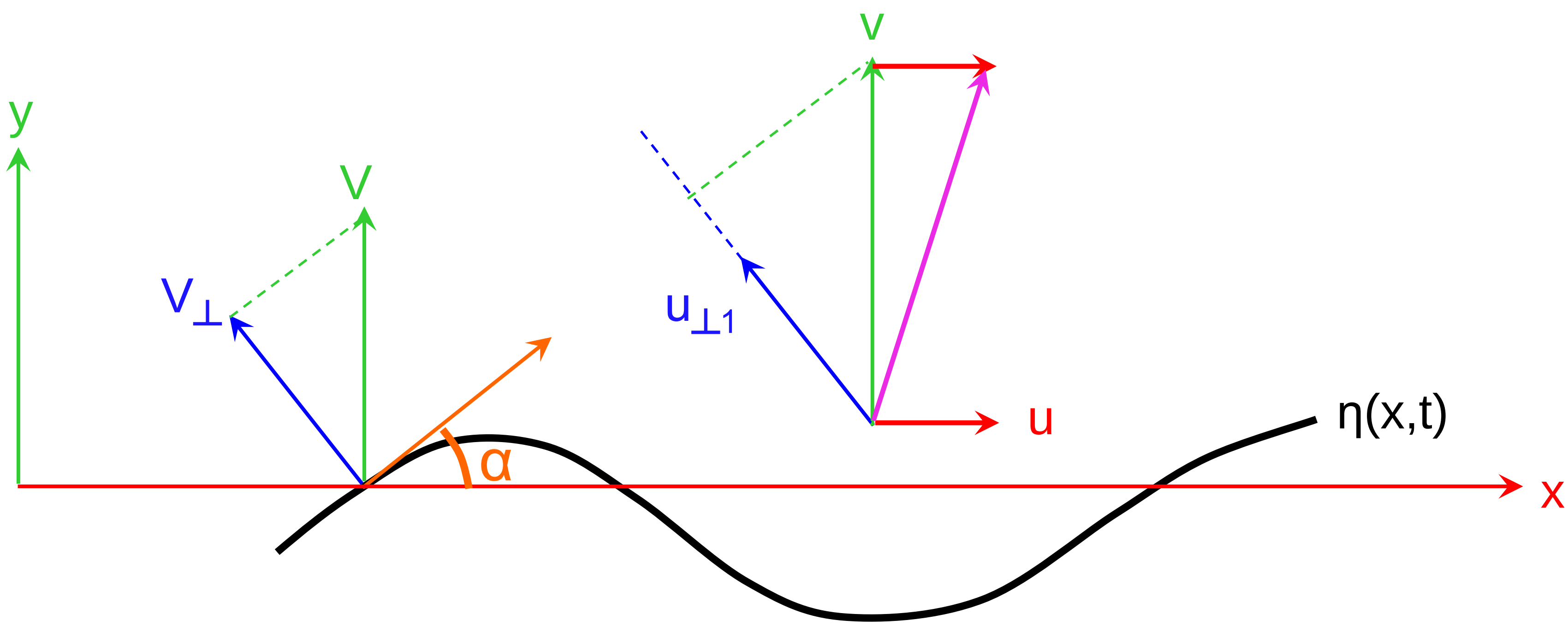


Kinematic condition : impermeability (no penetration)

No fluid particles going across the interface through the normal direction

$$V_{\perp} = \partial\eta/\partial t \cos(\alpha)$$

1. Kinematic boundary condition



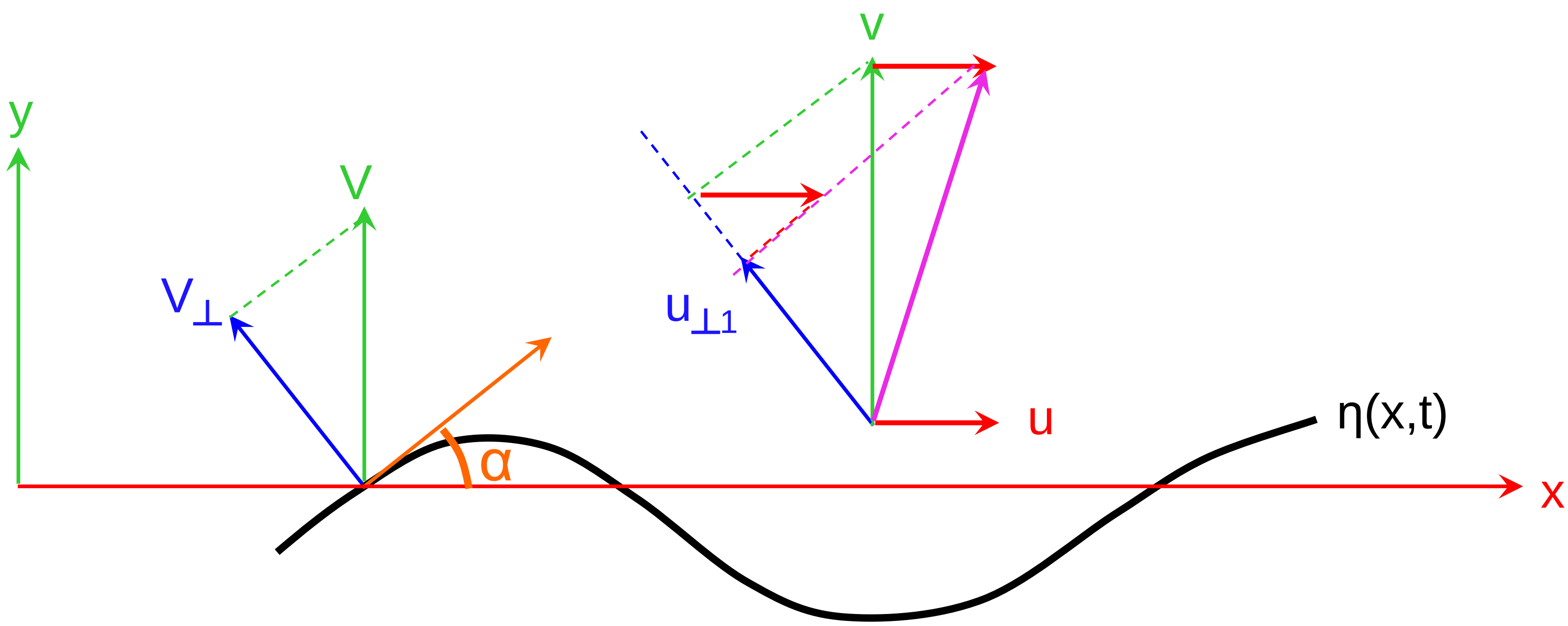
Kinematic condition : impermeability (no penetration)

No fluid particles going across the interface through the normal direction

$V_\perp = \partial\eta/\partial t \cos(\alpha)$

$u_{\perp 1} = v_1 \cos(\alpha) +$

1. Kinematic boundary condition



Kinematic condition : impermeability (no penetration)

No fluid particles going across the interface through the normal direction

$$V_\perp = \partial\eta/\partial t \cos(\alpha)$$

$$u_{\perp 1} = v_1 \cos(\alpha) - u_1 \sin(\alpha)$$

$$\Bigg\} \quad \partial\eta/\partial t = v_1 - u_1 \tan(\alpha) \quad \Rightarrow \quad \boxed{\partial\eta/\partial t = v_1 - u_1 \partial\eta/\partial x}$$

1. Kinematic boundary conditions

$$\Phi_1 = 0 \text{ at } z = -\infty$$
$$\Phi_2 = 0 \text{ at } z = +\infty$$

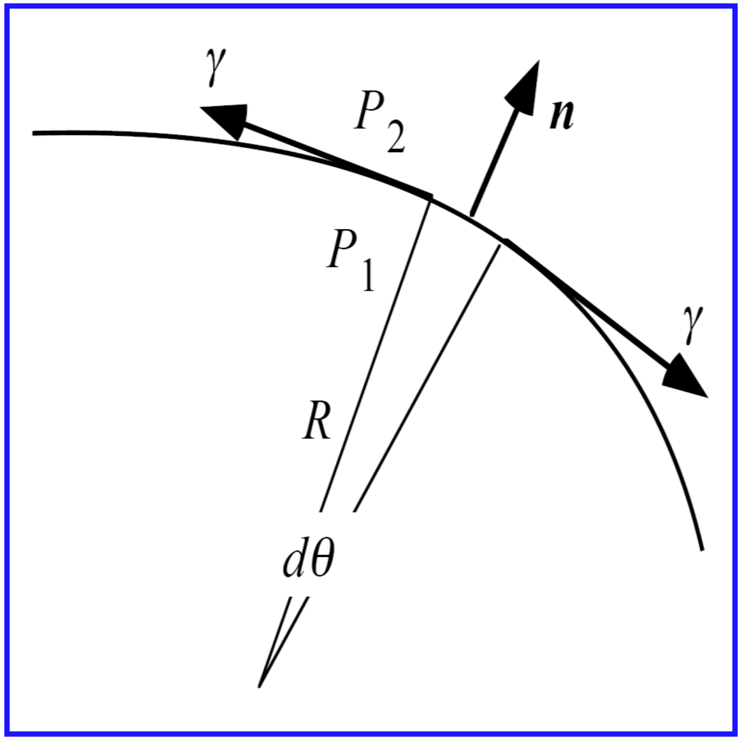
far-field

$$U_1 \frac{\partial \eta}{\partial x} - V_1 = -\frac{\partial \eta}{\partial t}$$
$$U_2 \frac{\partial \eta}{\partial x} - V_2 = -\frac{\partial \eta}{\partial t}$$

$\text{at } z = \eta$

1. Dynamic boundary conditions

$$P_1 - P_2 = -\gamma \frac{\frac{\partial^2 \eta}{\partial x^2}}{\left(1 + \frac{\partial \eta}{\partial x}^2\right)^{3/2}} \text{ at } z = \eta$$



$$\mathbf{n} = \frac{(-\partial_x \eta, 1)}{\sqrt{1 + \partial_x^2 \eta}}$$

$$\mathcal{C} = \nabla.\mathbf{n}$$

1. More equations

$$\frac{\partial \Phi_1}{\partial t} + \frac{U_1^2 + V_1^2}{2} + \frac{P_1}{\rho_1} + gz = C_1(t) \quad = 0$$

$$\frac{\partial \Phi_2}{\partial t} + \frac{U_2^2 + V_2^2}{2} + \frac{P_2}{\rho_2} + gz = C_2(t) \quad = 0$$

2nd Bernouilli relations

2. Base state

$$\Phi_1 = 0,$$

$$\Phi_2 = 0$$

$$\eta = 0$$

$$P_1 = -\rho_1 g z$$

3. Perturb and linearize perturbation expansion

$$
\begin{aligned}
\Phi_1 &= 0 &+\epsilon\phi_1 \\
\Phi_2 &= 0 &+\epsilon\phi_2 \\
U_1 &= 0 &+\epsilon u_1 \\
V_1 &= 0 &+\epsilon v_1 \\
U_2 &= 0 &+\epsilon u_2 \\
V_2 &= 0 &+\epsilon v_2 \\
P_1 &= -\rho_1 g z &+\epsilon p_1 \\
P_2 &= -\rho_2 g z &+\epsilon p_2 \\
\eta &= 0 &+\epsilon\sigma
\end{aligned}
$$

$$\epsilon \ll 1$$

Variables Base state Small perturbation

3. Linearized equations

$$\begin{aligned} \Delta\phi_1 &= 0 \\ \Delta\phi_2 &= 0 \end{aligned}$$

perturbed potential flow

$$u_1 = \frac{\partial\phi_1}{\partial x}, \qquad v_1 = \frac{\partial\phi_1}{\partial z}$$

$$u_2 = \frac{\partial\phi_2}{\partial x}, \qquad v_2 = \frac{\partial\phi_2}{\partial z}$$

3. Perturbed kinematic boundary conditions

$$\phi_1 = 0 \text{ at } z = -\infty$$

$$\phi_2 = 0 \text{ at } z = +\infty$$

$$-\epsilon^2 u_1 \frac{\partial \sigma}{\partial x} + \epsilon v_1 = \epsilon \frac{\partial \sigma}{\partial t} \text{ at } z = \epsilon\sigma$$

$$-\epsilon^2 u_2 \frac{\partial \sigma}{\partial x} + \epsilon v_2 = \epsilon \frac{\partial \sigma}{\partial t} \text{ at } z = \epsilon\sigma$$

3. Perturbed kinematic boundary conditions

$$\phi_1 = 0 \text{ at } z = -\infty$$

$$\phi_2 = 0 \text{ at } z = +\infty$$

$$-\epsilon^2 u_1 \frac{\partial \sigma}{\partial x} + \epsilon v_1 = \epsilon \frac{\partial \sigma}{\partial t} \text{ at } z = \epsilon\sigma$$

$$-\epsilon^2 u_2 \frac{\partial \sigma}{\partial x} + \epsilon v_2 = \epsilon \frac{\partial \sigma}{\partial t} \text{ at } z = \epsilon\sigma$$

$$v_1 = \frac{\partial \sigma}{\partial t} \text{ at } z = \epsilon\sigma$$

$$v_2 = \frac{\partial \sigma}{\partial t} \text{ at } z = \epsilon\sigma$$

3. Flattened kinematic boundary conditions

$$\frac{\partial \phi_1}{\partial z} = \frac{\partial \sigma}{\partial t} \text{ at } z = \epsilon\sigma$$
$$\frac{\partial \phi_2}{\partial z} = \frac{\partial \sigma}{\partial t} \text{ at } z = \epsilon\sigma$$

Taylor expansion around 0: $\phi(\epsilon\sigma) = \phi(0) + (\epsilon\sigma)\frac{\partial \phi}{\partial z}\Big|_0$

$$\frac{\partial \phi_1}{\partial z} = \frac{\partial \sigma}{\partial t} \text{ at } z = 0$$
$$\frac{\partial \phi_2}{\partial z} = \frac{\partial \sigma}{\partial t} \text{ at } z = 0$$

⇨transforms a b.c. at an unkwown interface into a fixed place!

3. Perturbed dynamic boundary conditions

$$\left(P_1 + \epsilon p_1 - P_2 - \epsilon p_2\right)\Big|_{\epsilon\sigma} = -\gamma \epsilon \frac{\partial^2 \sigma}{\partial x^2}\left(1 - 3/2\epsilon^2 \left(\frac{\partial \sigma}{\partial x}\right)^2\right)$$

Replace $P_1 = -g\rho_1 z$, … and linearize

$$g\left(\rho_2 - \rho_1\right)\sigma + \left(p_1 - p_2\right)\Big|_{\epsilon\sigma} = -\gamma \frac{\partial^2 \sigma}{\partial x^2}$$

flatten

$$\left(\rho_2 - \rho_1\right) g \sigma + \left(p_1 - p_2\right)\Big|_0 = -\gamma \frac{\partial^2 \sigma}{\partial x^2}$$

3. Perturbed and linearized Bernouilli

Perturbed 2nd Bernouilli relations

$$\epsilon\frac{\partial\phi_1}{\partial t} + \epsilon^2\frac{u_1^2+v_1^2}{2} + \epsilon\frac{p_1}{\rho_1} = 0$$

$$\epsilon\frac{\partial\phi_2}{\partial t} + \epsilon^2\frac{u_2^2+v_2^2}{2} + \epsilon\frac{p_2}{\rho_2} = 0$$

Linearized 2nd Bernouilli relations

$$\frac{\partial\phi_1}{\partial t} + \frac{p_1}{\rho_1} = 0$$

$$\frac{\partial\phi_2}{\partial t} + \frac{p_2}{\rho_2} = 0$$

4. Normal mode expansion

Fourier transform in x and t

$$\begin{aligned}\phi_1 &= f_1(z)\exp(i(kx-\omega t)),\\ \phi_2 &= f_2(z)\exp(i(kx-\omega t)),\\ \sigma &= C\exp(i(kx-\omega t)),\end{aligned}$$

k is the wavenumber and ω the frequency (in rad/s)

$$\lambda = 2\pi/k \qquad T = 2\pi/\omega$$

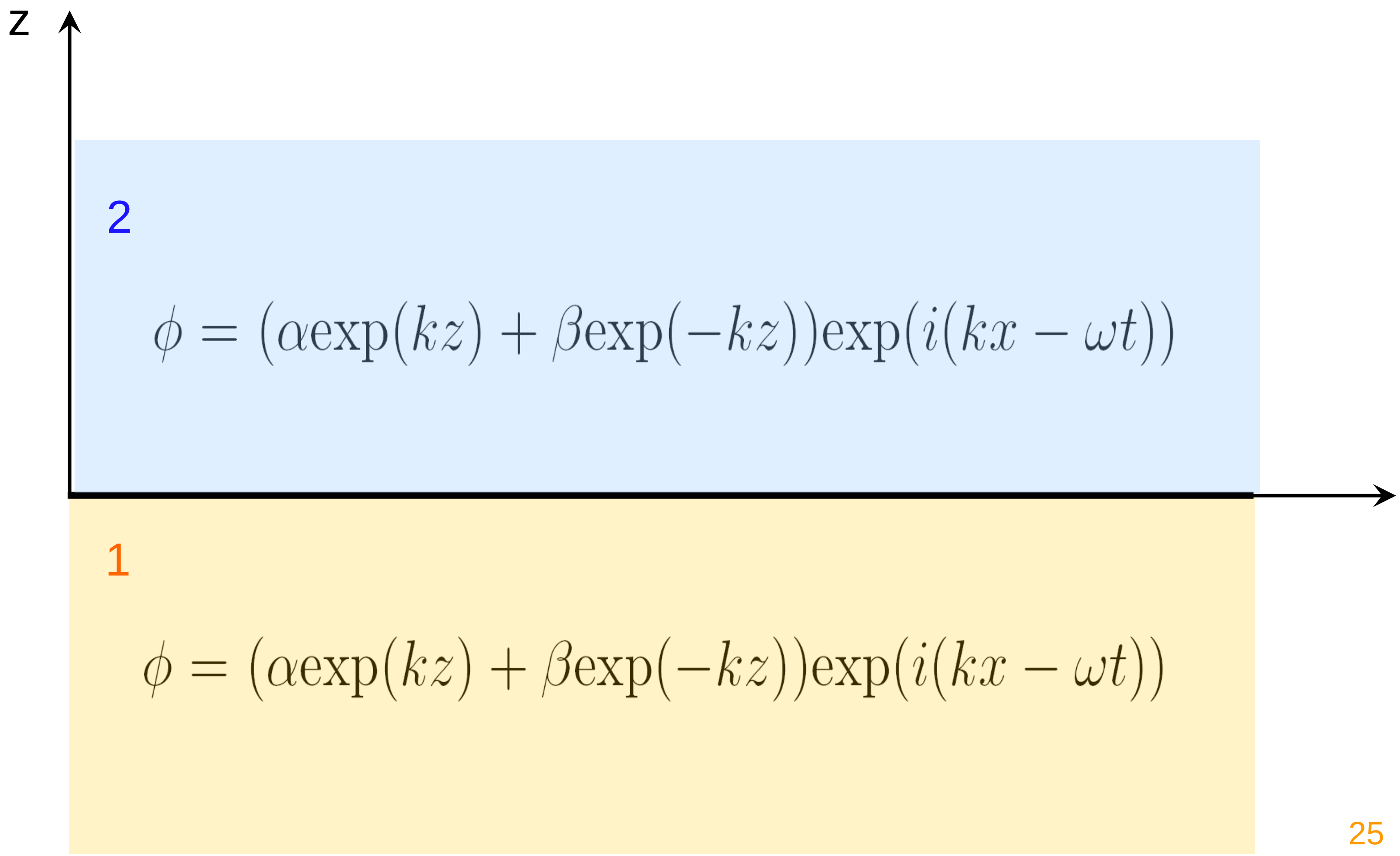
$$f = \omega/(2\pi)$$

4. Normal mode expansion

Solution to Laplace equation:

4. Normal mode expansion

Solution to Laplace equation:



$$\phi = (\alpha \exp(kz) + \beta \exp(-kz))\exp(i(kx - \omega t))$$

$$\phi = (\alpha \exp(kz) + \beta \exp(-kz))\exp(i(kx - \omega t))$$

4. Normal mode expansion

Solution to Laplace equation:

$$
\begin{aligned}
\phi_1 &= A\exp(kz)\exp(i(kx - \omega t)), \\
\phi_2 &= B\exp(-kz)\exp(i(kx - \omega t)), \\
\sigma &= C\exp(i(kx - \omega t)).
\end{aligned}
$$

4. Normal mode expansion

Replace in boundary conditions

$$
\begin{aligned}
g(\rho_2 - \rho_1)C + i\omega\rho_1 A - i\omega\rho_2 B &= \gamma k^2 C \\
kA &= -i\omega C \\
-kB &= -i\omega C
\end{aligned}
$$

This is an eigenvalue problem iωX=MX!

$$
kg(\rho_2 - \rho_1)C + \omega^2\rho_1 C + \omega^2\rho_2 C = \gamma k^3 C.
$$

5. Dispersion relation

$$\omega^2 = \frac{-kg(\rho_2 - \rho_1) + \gamma k^3}{\rho_1 + \rho_2}$$

•Unstable if there exists one ω, Im(ω)>0 $\rho_2 > \rho_1$

•Neutral if for all ω, Im(ω)=0: $\rho_1 > \rho_2$

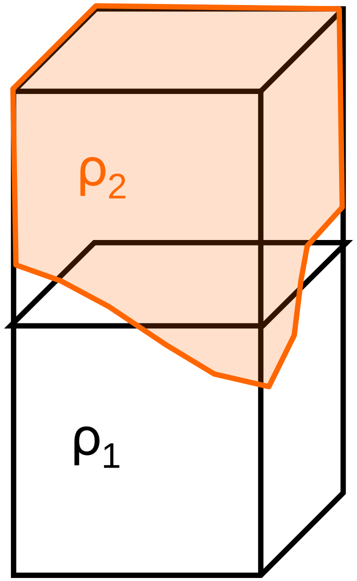
•Stable (or damped) if for all ω, Im(ω)<0:

The flow considered is not damped, we have neglected dissipation by neglecting viscosity

Instability analysis:

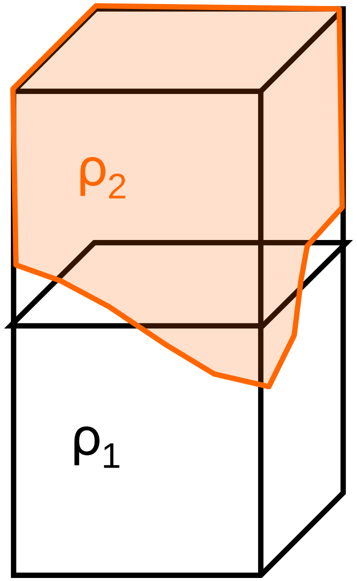
1. Equations and boundary conditions
2. Base state
3. Linearized equations
4. Normal mode expansion
5. Dispersion relation
6. Analysis of the dispersion relation

Dispersion relation



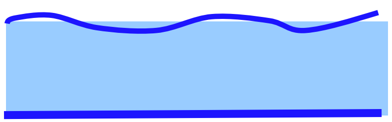
$$\omega^2 = \frac{-kg(\rho_2 - \rho_1) + \gamma k^3}{\rho_1 + \rho_2}$$

Dispersion relation



z

$$\omega^2 = \frac{-kg(\cancel{\rho_2} - \rho_1) + \gamma k^3}{\rho_1 + \cancel{\rho_2}}$$



$$\omega^2 = \tanh(kH)\left(\frac{\gamma k^3}{\rho} + gk\right)$$

Dispersion relation

$$\omega^2 = \tanh(kH)\left(\frac{\gamma k^3}{\rho} + gk\right)$$

Capillary wavenumber: $k_c = \sqrt{\rho g/\gamma}$

Length scale: $\tilde{k} = k/k_c$

Time scale $\tilde{\omega} = \omega/\sqrt{gk_c}$

One single non-dimensional parameter $\tilde{H} = Hk_c$

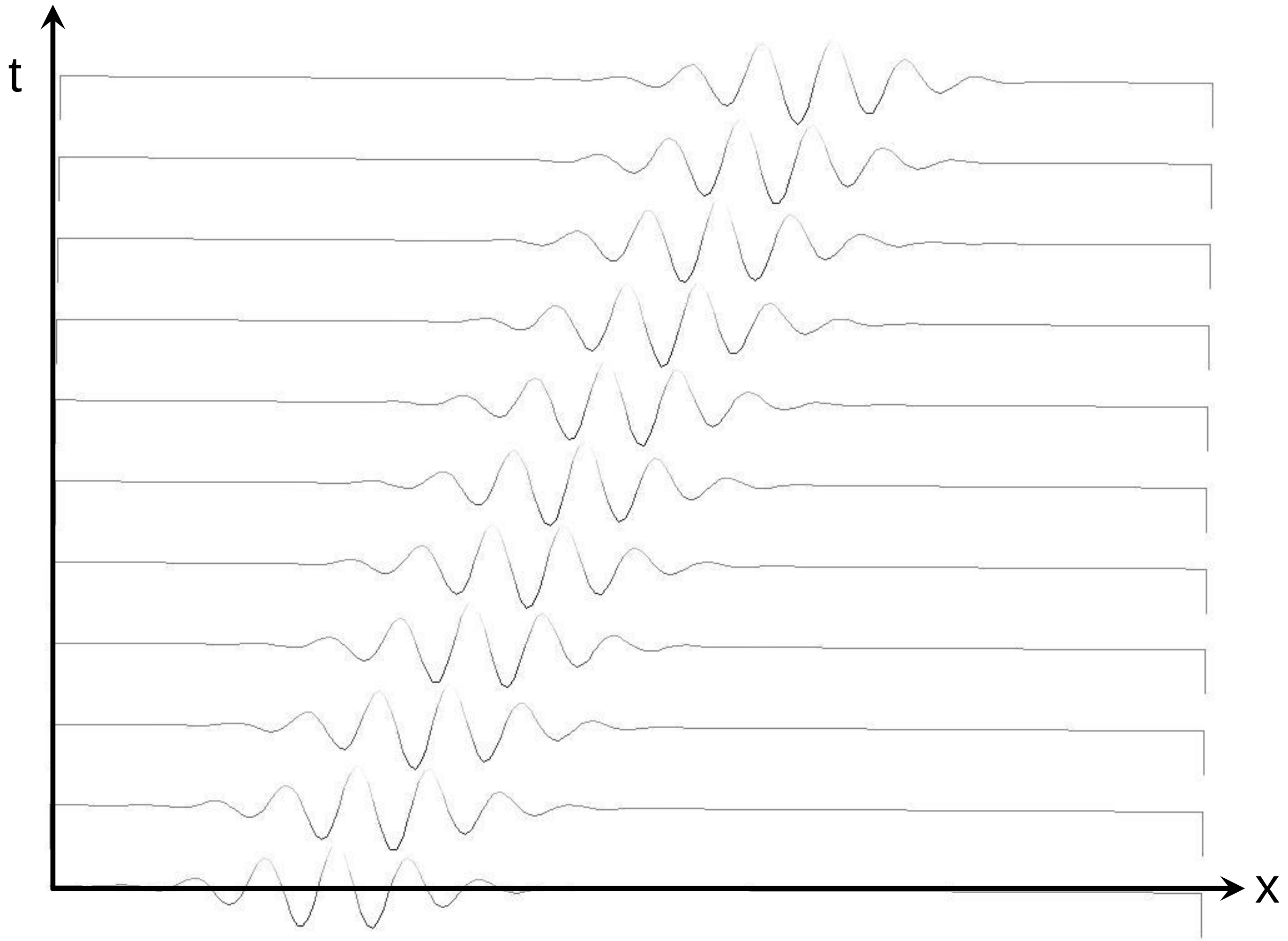
$$\tilde{\omega}^2 = \tanh(\tilde{k}\tilde{H})\left(\tilde{k}^3 + \tilde{k}\right)$$

Dispersion relation

$$\tilde{\omega}^2 = \tanh(\tilde{k}\tilde{H})\left(\tilde{k}^3 + \tilde{k}\right)$$

	gravity $\tilde{k} \ll 1$	capillary $\tilde{k} \gg 1$
shallow water $\tilde{k} \ll 1/\tilde{H}$	$\pm\tilde{k}$	$\pm\tilde{k}^2\sqrt{\tilde{H}}$
Deep water $\tilde{k} \gg 1/\tilde{H}$	$\pm\sqrt{\tilde{k}}$	$\pm\tilde{k}\sqrt{\tilde{k}}$

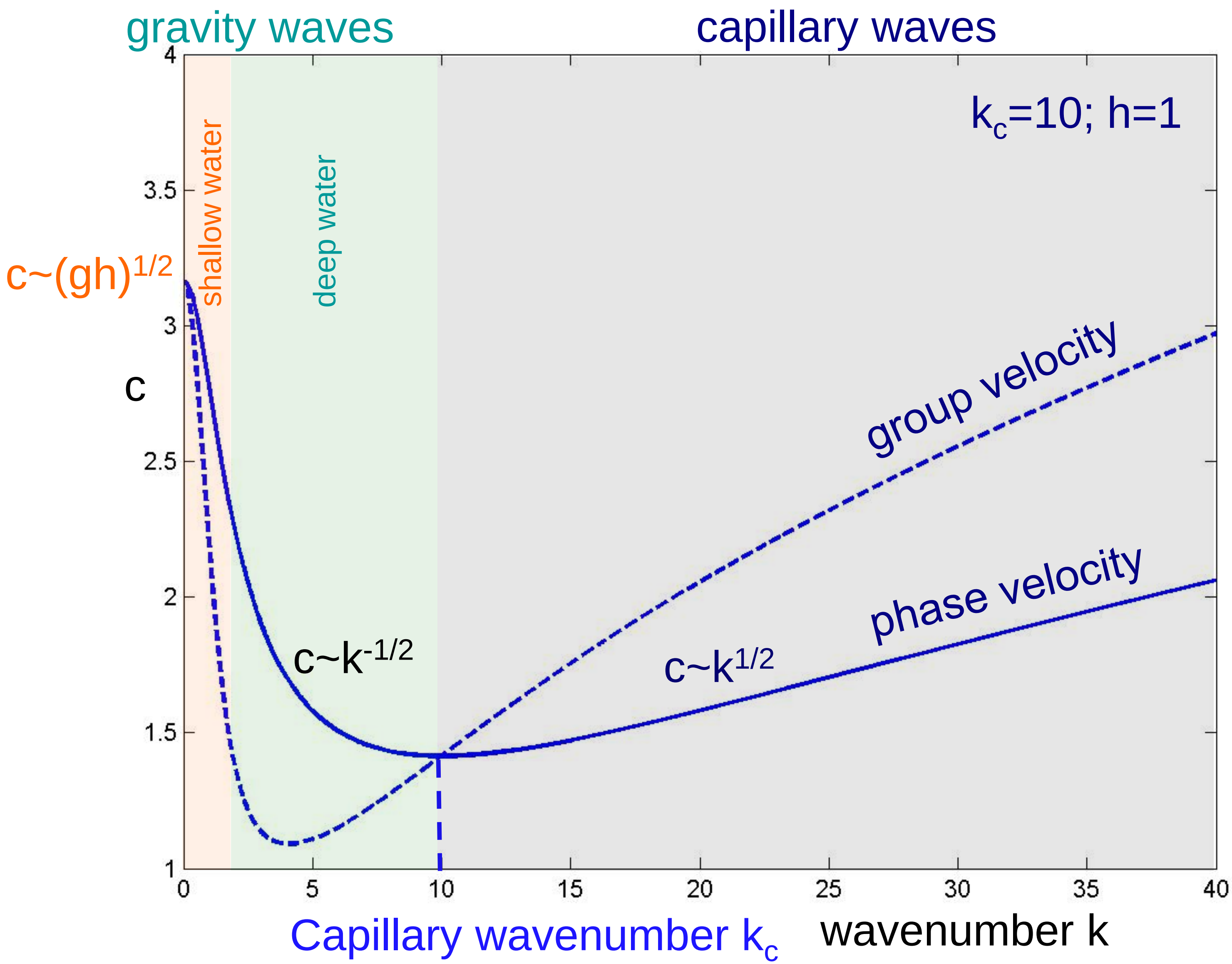
Difference between group velocity v and phase velocity c



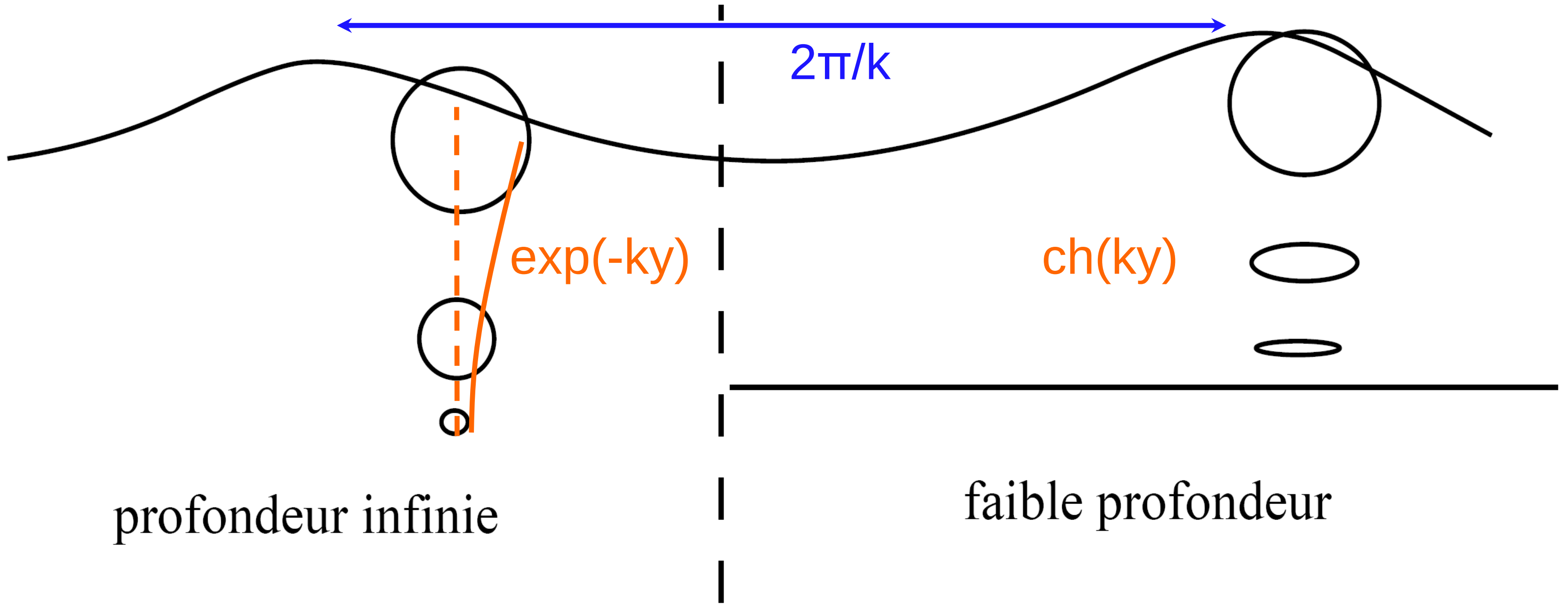
Dispersion relation

	gravity $\tilde{k} \ll 1$	capillary $\tilde{k} \gg 1$
shallow water $\tilde{k} \ll 1/\tilde{H}$	$\omega_{shallow/gravity} \sim \pm k\sqrt{gH}$ $c_{shallow/gravity} \sim \pm\sqrt{gH}$ $v_{shallow/gravity} \sim \pm\sqrt{gH}$	$\omega_{shallow/capillary} \sim \pm k^2\sqrt{\gamma H/\rho}$ $c_{shallow/capillary} \sim \pm k\sqrt{\gamma H/\rho}$ $v_{shallow/capillary} \sim \pm 2k\sqrt{\gamma H/\rho}$
Deep water $\tilde{k} \gg 1/\tilde{H}$	$\omega_{deep/gravity} \sim \pm\sqrt{gk}$ $c_{deep/gravity} \sim \pm\sqrt{\frac{g}{k}}$ $v_{deep/gravity} \sim \pm\frac{1}{2}\sqrt{\frac{g}{k}}$	$\omega_{deep/capillary} \sim \pm k^{3/2}\sqrt{\gamma/\rho}$ $c_{deep/capillary} \sim \pm k^{1/2}\sqrt{\gamma/\rho}$ $v_{deep/capillary} \sim \pm 3/2k^{1/2}\sqrt{\gamma/\rho}$

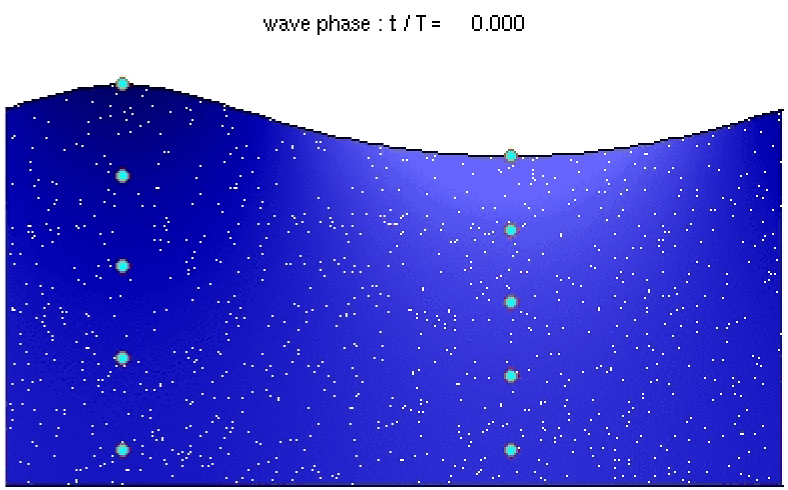
Dispersion relation



Trajectories below the waves



Stokes drift!



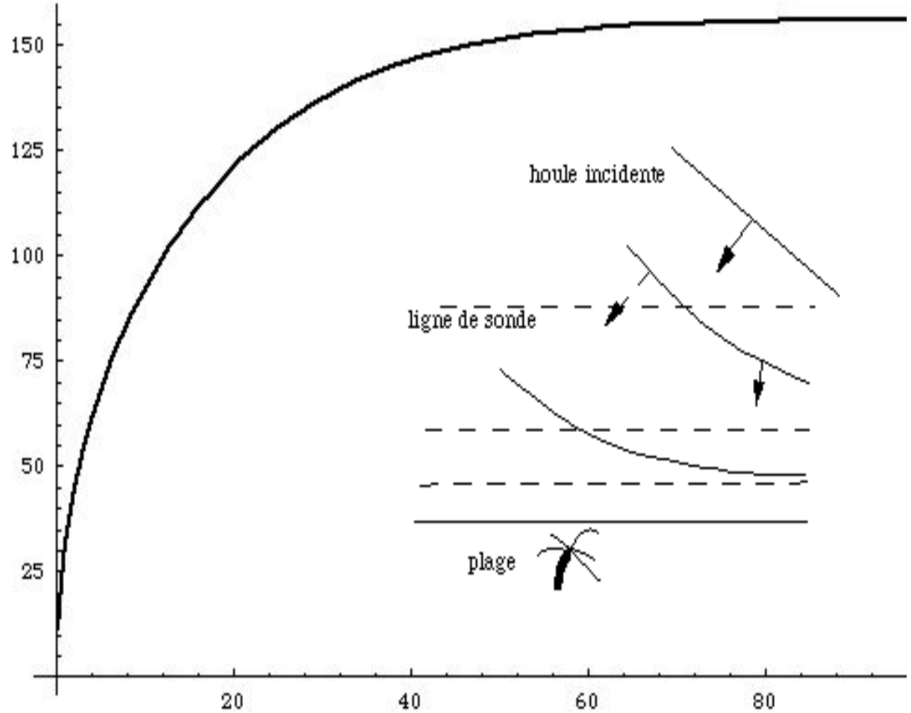
Why are the waves parallel to the shore?

$c \sim (gh)^{1/2}$

$\lambda \sim T(gh)^{1/2}$

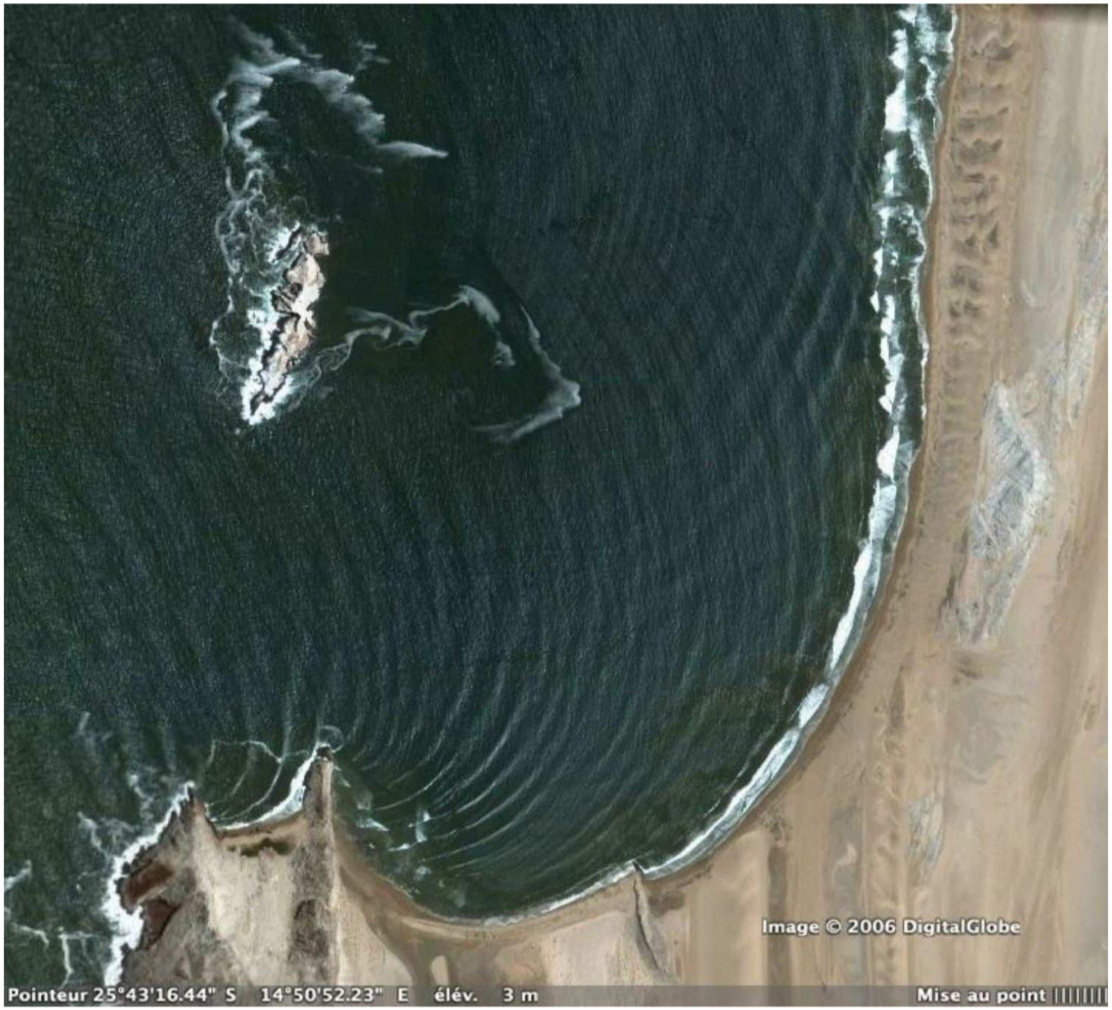
T=10s; ω=0.62

λ



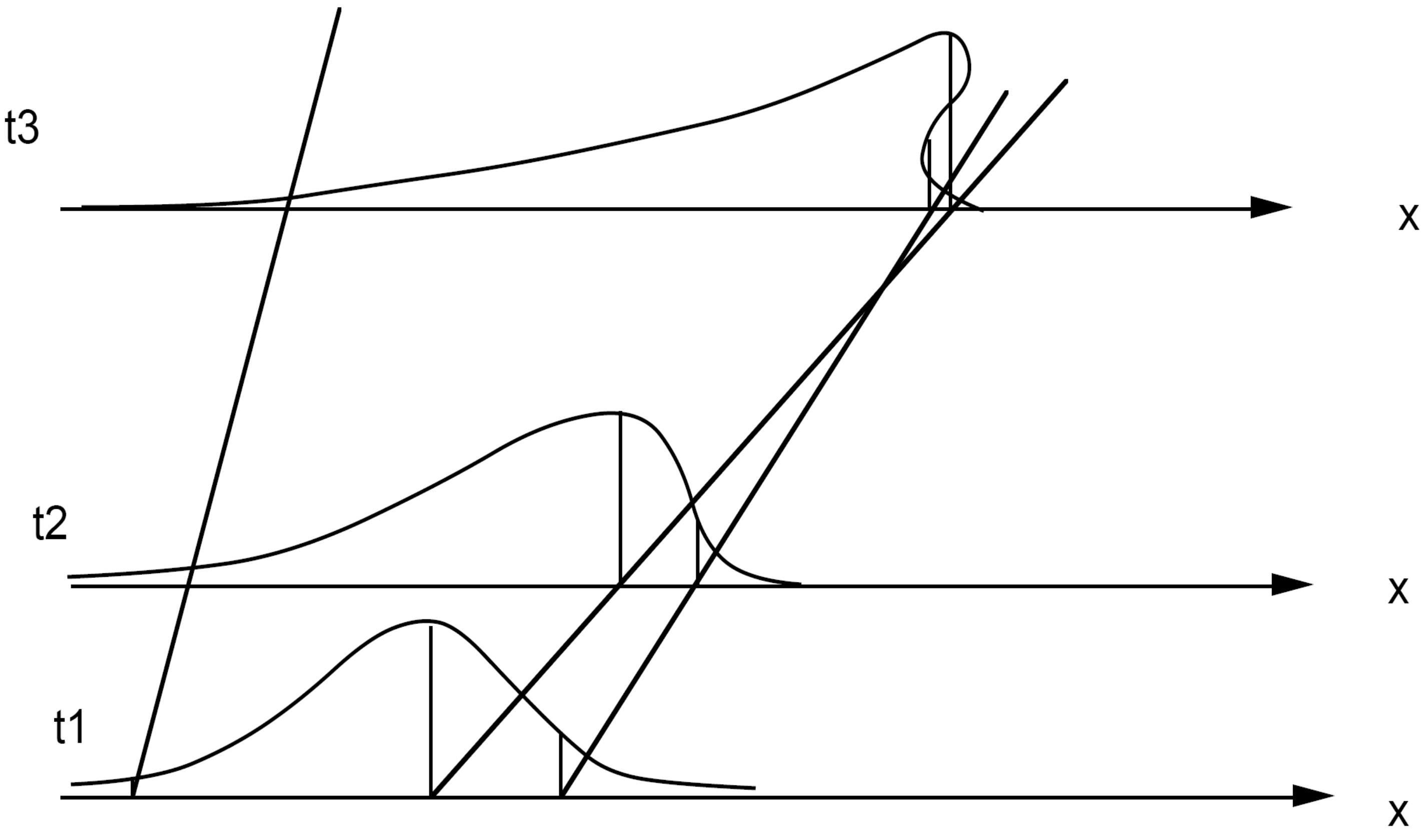
h

Refraction and diffraction of waves



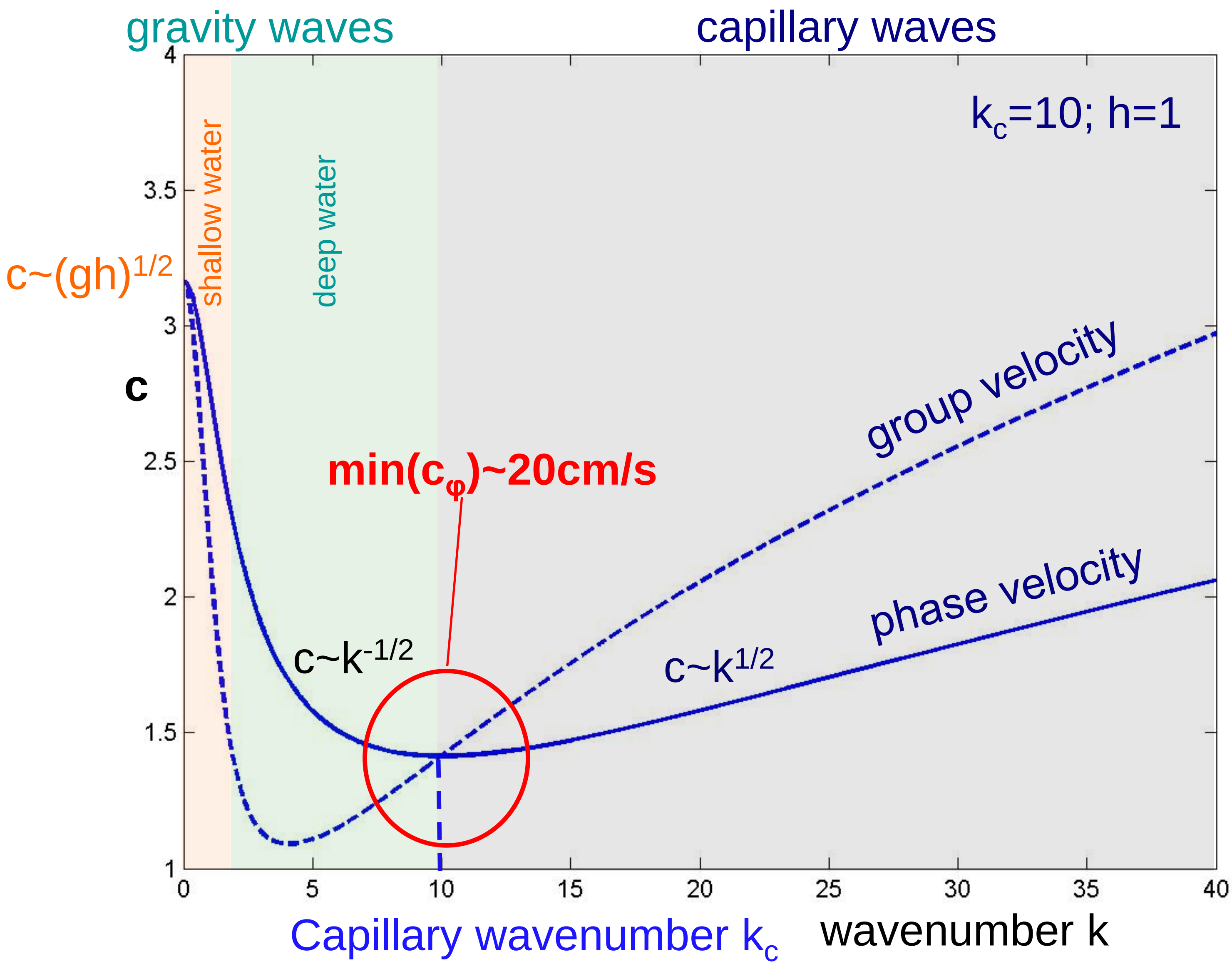
Satellite view Namibian coast

Nonlinear waves, wavebreaking



The celerity increases with the depth

Dispersion relation



Conditions for wave pattern formation?

$$V_{duck} \lesseqgtr c_{min} \quad ?$$

Dispersion relation

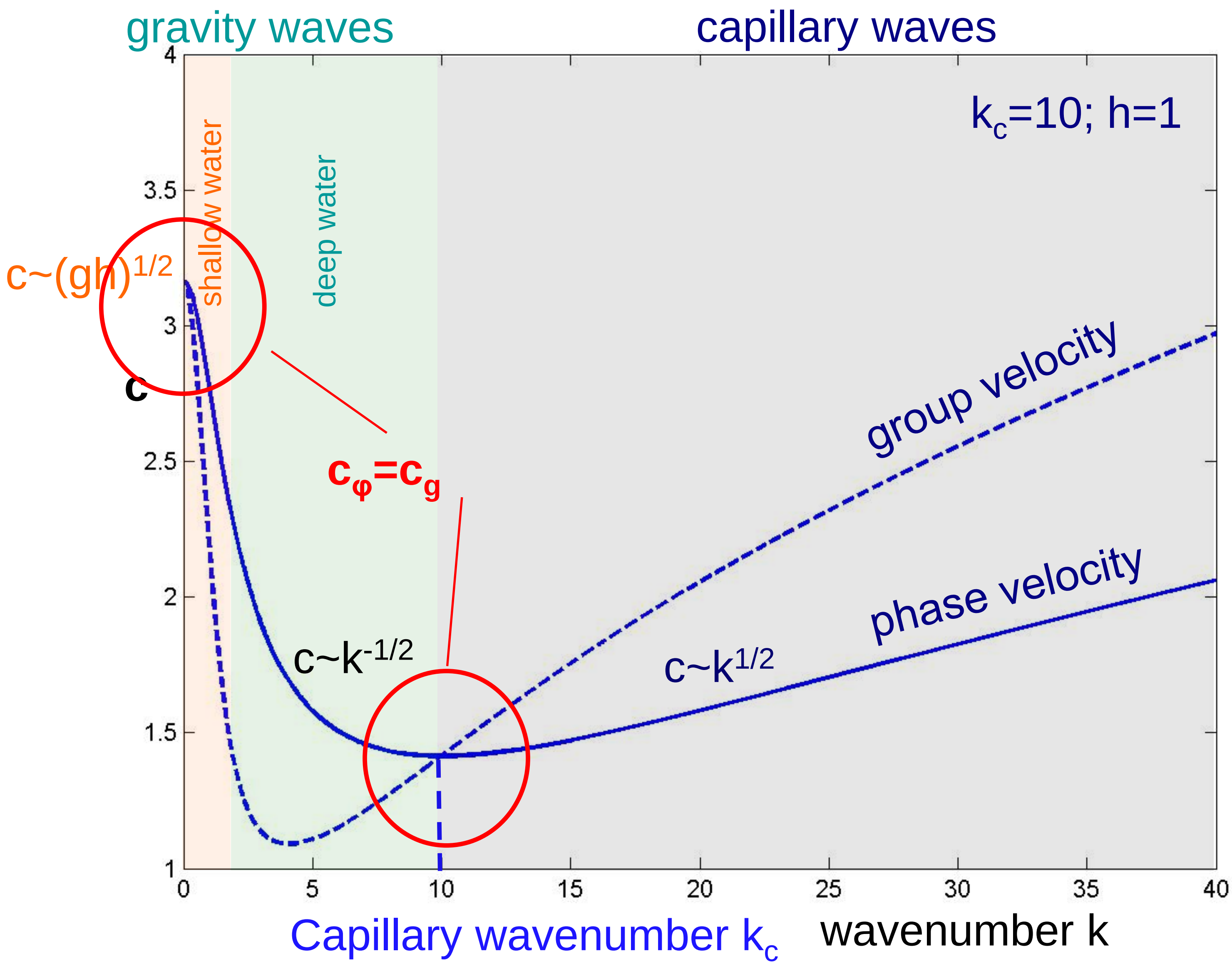
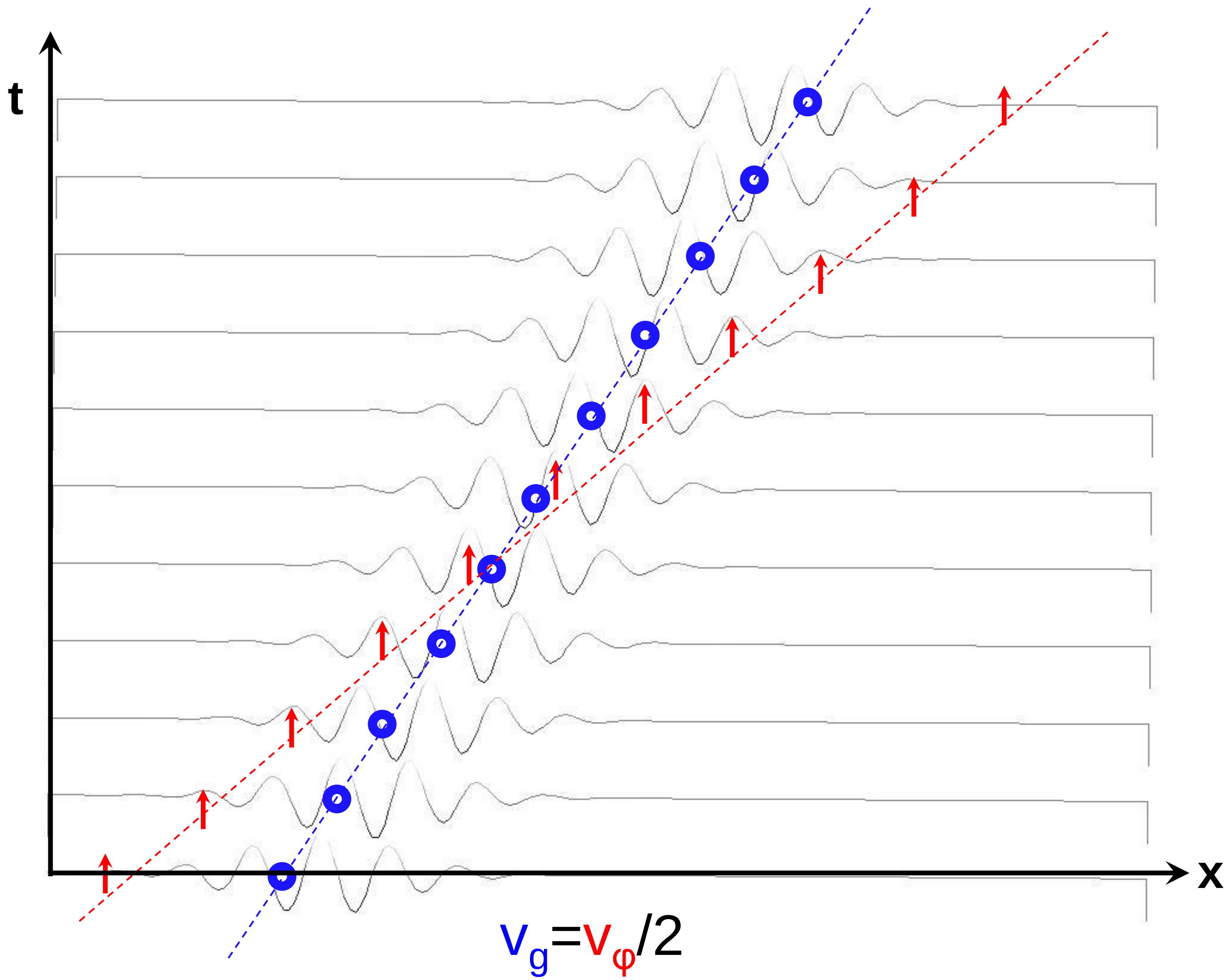


Diagramme spatio-temporel



$v_g = v_\varphi/2$

Diagramme spatio-temporel

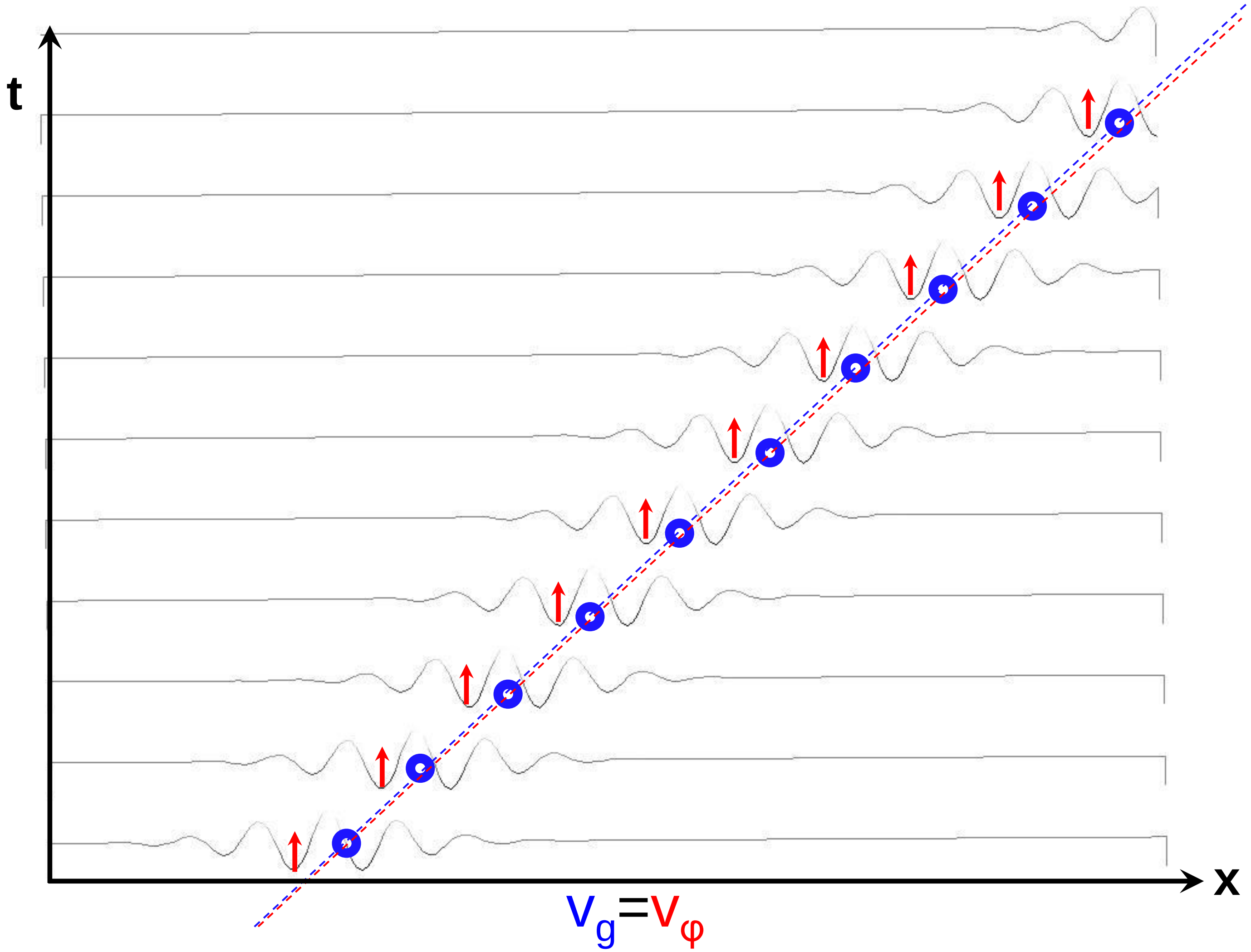
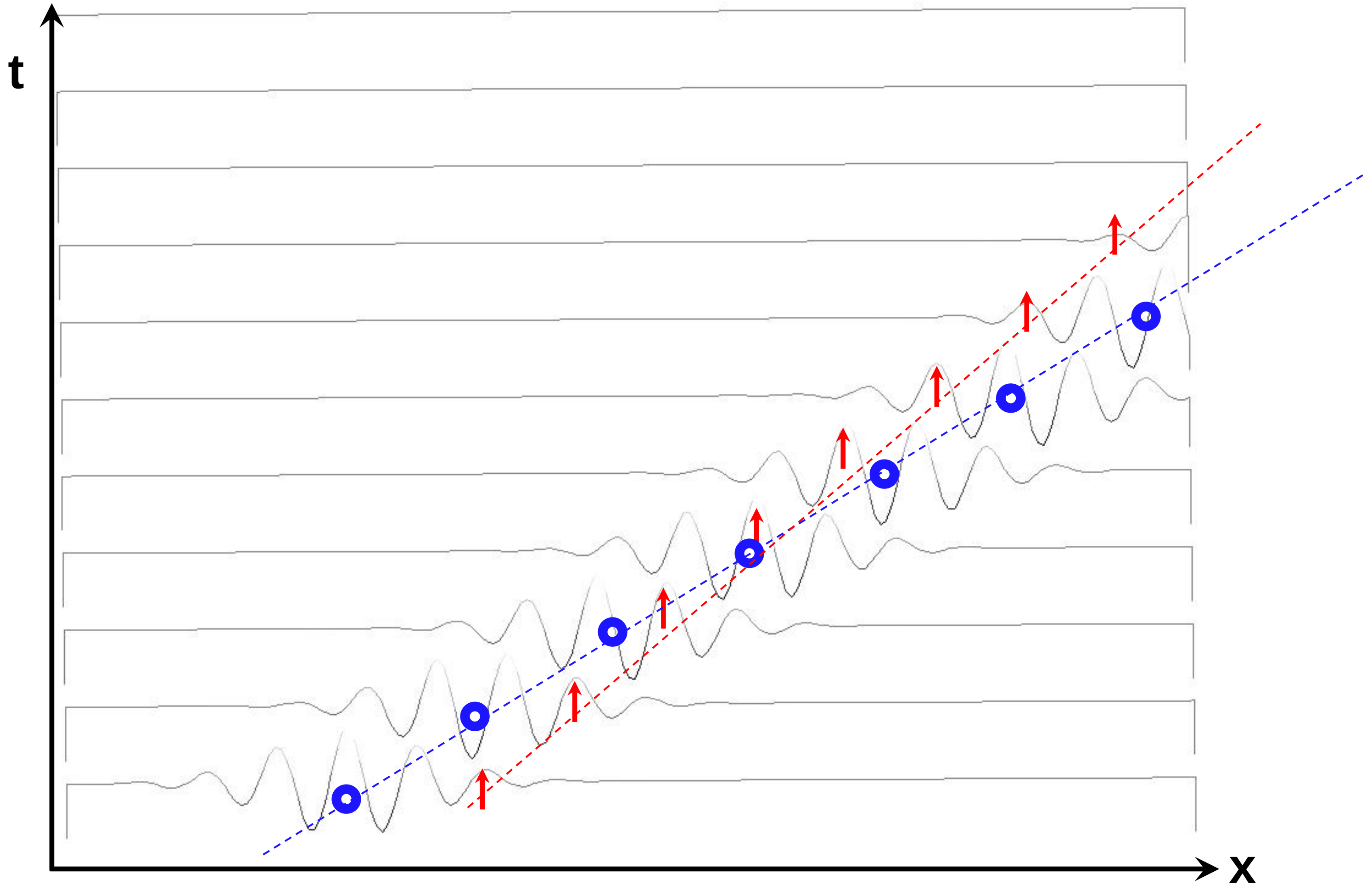


Diagramme spatio-temporel



$v_g = 3/2 v_\varphi$

Spectral analysis

Fourier transform:

$$u(x,t) = \frac{1}{2}\int_0^{+\infty} \hat{u}(k)\,\mathrm{e}^{\mathrm{i}(kx-\omega(k)t)}\mathrm{d}k + c.c.$$

$\hat{u}(k)$ is given by Fourier transform at time t=0

Carrier/enveloppe :

$$u(x,t) = \frac{1}{2}A(x,t)\,\mathrm{e}^{\mathrm{i}(k_0x-\omega_0t)} + c.c.$$

Enveloppe :

$$A(x,t) = \int_0^{\infty} \hat{u}(k)\,\mathrm{e}^{\mathrm{i}(k-k_0)x-\mathrm{i}(\omega-\omega_0)t}\mathrm{d}k.$$

Spectral analysis

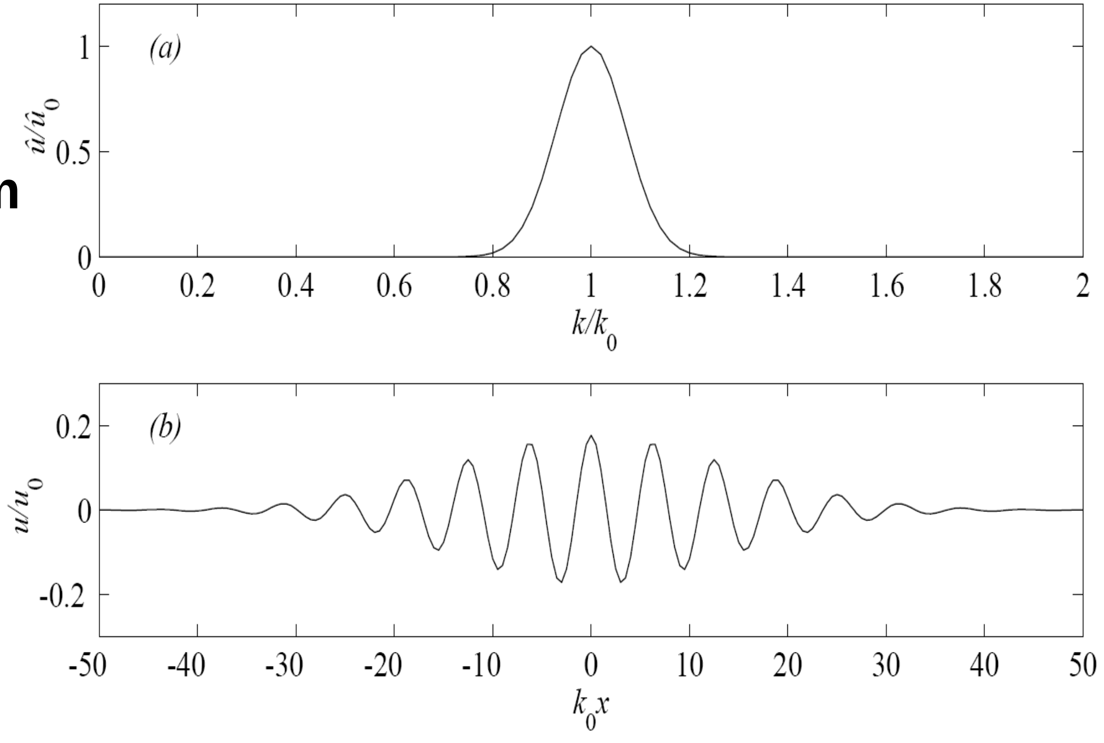
Gaussian spectrum: $$\hat{u}(k) = u_0\, \mathrm{e}^{-\sigma^2 (k-k_0)^2}$$

Initial enveloppe : $$A(x,0) = \frac{u_0\sqrt{\pi}}{2\sigma}\, \mathrm{e}^{-\frac{x^2}{4\sigma^2}}$$

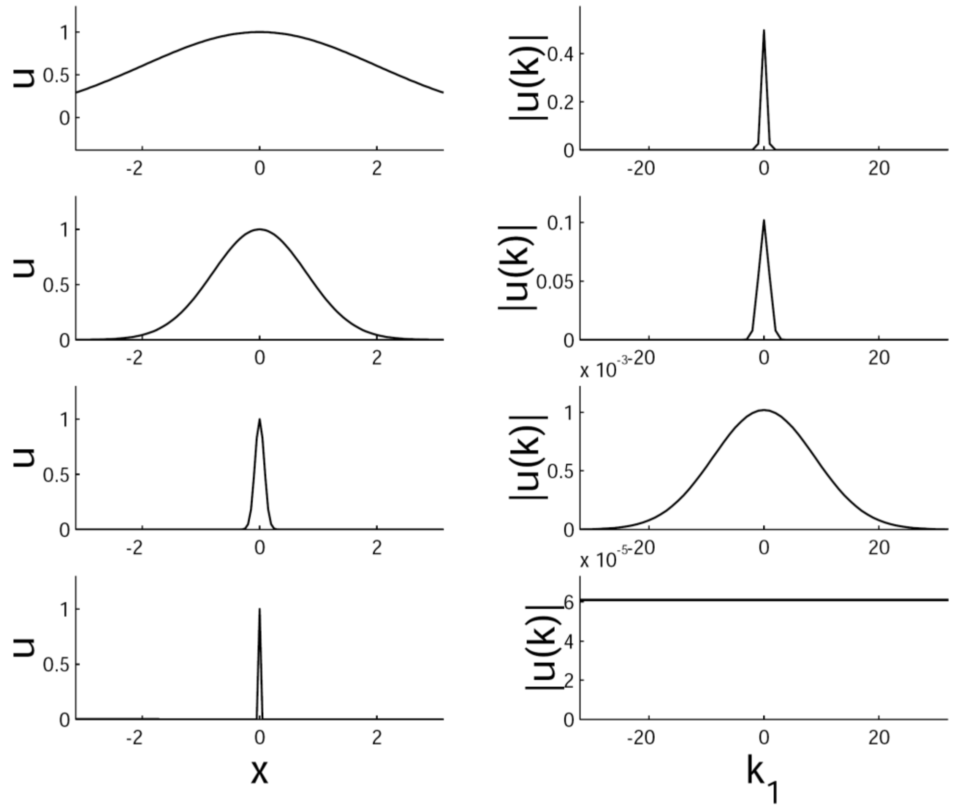
Gaussian spectrum

spectrum

wave

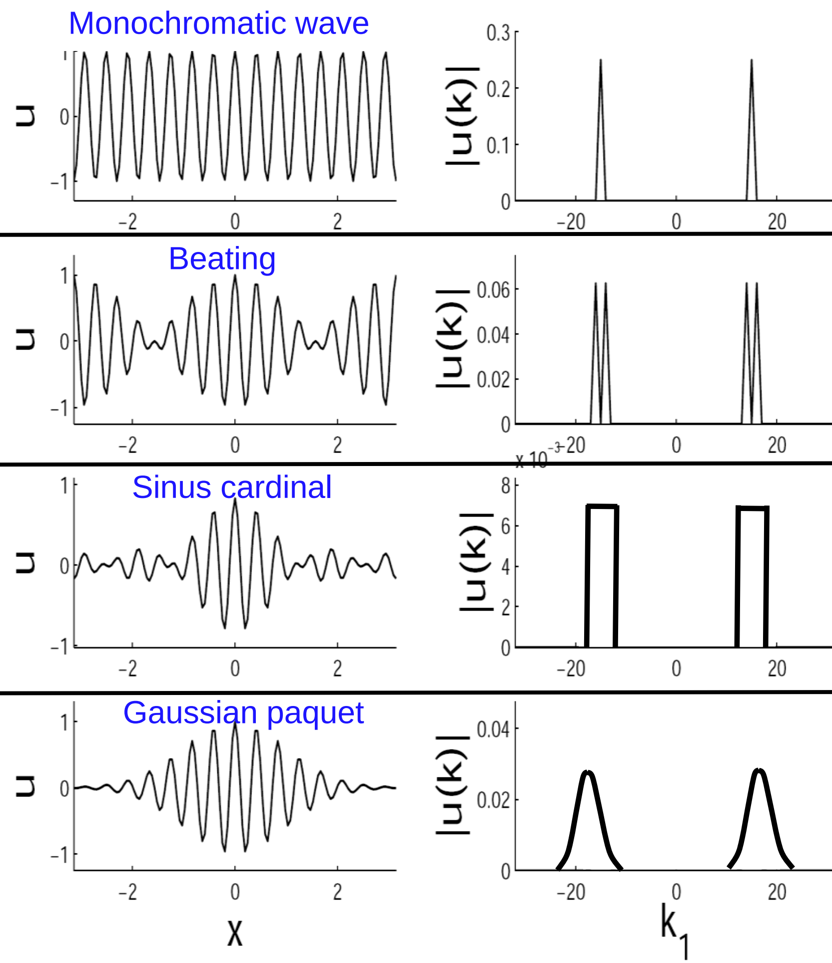


Paquets d’ondes gaussiens



$u_0(x)$	$\widehat{u}_0(k_1)$
$\exp\left(-\frac{x^2}{2\,\sigma^2}\right)$	$\frac{\sigma}{\sqrt{2\pi}}\ \exp\left(-\frac{\sigma^2\,k^2}{2}\right)$
$\delta(x)$	$\frac{1}{2\pi}$

Waves and spectrum



$2\ \cos(k_0\, x)$	$\delta(k_1 + k_0) + \delta(k_1 - k_0)$
$\cos[(k_0 - \kappa)x] + \cos[(k_0 + \kappa)x]$ $= 2\ \cos(k_0\, x)\ \cos(\kappa\, x)$	$\delta(k_1 + k_0 + \kappa) + \delta(k_1 - k_0 - \kappa)$ $+\delta(k_1 + k_0 - \kappa) + \delta(k_1 - k_0 + \kappa)$
$\int_{k_0-\kappa}^{k_0+\kappa} \cos(k_1\, x)\ dk_1$ $= 4\,\kappa\ \cos(k_0\, x)\ \frac{\sin(\kappa\, x)}{\kappa\, x}$	$= 1$ pour $\vert k_1 \pm k_0 \vert \leq \kappa$ $= 0$ sinon
$2\,\kappa\sqrt{2\pi}\ \cos(k_0\, x)\exp\left(-\frac{\kappa^2\, x^2}{2}\right)$	$\exp\left[-\frac{(k_1+k_0)^2}{2\,\kappa^2}\right] + \exp\left[-\frac{(k_1-k_0)^2}{2\,\kappa^2}\right]$

Spectral analysis

$$A(x,t) = \int_0^\infty \hat{u}(k)\,\mathrm{e}^{\mathrm{i}(k-k_0)x-\mathrm{i}(\omega-\omega_0)t}\mathrm{d}k.$$

Spectral analysis

$$A(x,t) = \int_0^\infty \hat{u}(k)\, \mathrm{e}^{\mathrm{i}(k-k_0)x-\mathrm{i}(\omega-\omega_0)t} \mathrm{d}k.$$

Definition of group velocity $\omega - \omega_0 = c_g(k - k_0), \qquad c_g = \frac{\partial \omega}{\partial k}(k_0)$

Spectral analysis

$$A(x,t) = \int_0^\infty \hat{u}(k)\,\mathrm{e}^{\mathrm{i}(k-k_0)x-\mathrm{i}(\omega-\omega_0)t}\mathrm{d}k.$$

$$\hat{u}(k) = u_0\,\mathrm{e}^{-\sigma^2(k-k_0)^2}$$

Spectral analysis

Gaussian spectrum: $$\hat{u}(k) = u_0\, \mathrm{e}^{-\sigma^2 (k-k_0)^2}$$

Initial enveloppe : $$A(x,0) = \frac{u_0\sqrt{\pi}}{2\sigma}\, \mathrm{e}^{-\frac{x^2}{4\sigma^2}}$$

Spectral analysis

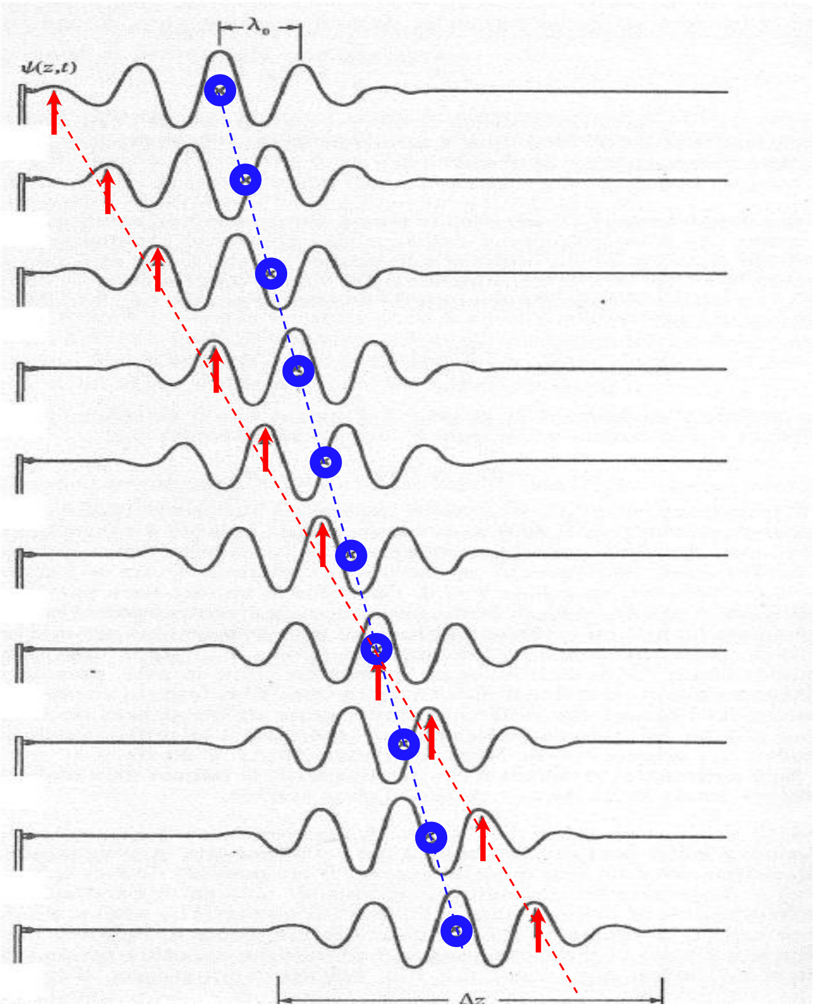
$$A(x,t) = \int_0^\infty \hat{u}(k)\, \mathrm{e}^{\mathrm{i}(k-k_0)x-\mathrm{i}(\omega-\omega_0)t}\mathrm{d}k.$$

Definition of group velocity $\omega - \omega_0 = c_g(k - k_0), \qquad c_g = \frac{\partial \omega}{\partial k}(k_0)$

$$\hat{u}(k) = u_0\, \mathrm{e}^{-\sigma^2(k-k_0)^2}$$

$$A(x,t) = \frac{u_0\sqrt{\pi}}{2\sigma}\, \mathrm{e}^{-\frac{(x-c_g t)^2}{4\sigma^2}}$$

Group velocity



Wave packet

Spectral analysis

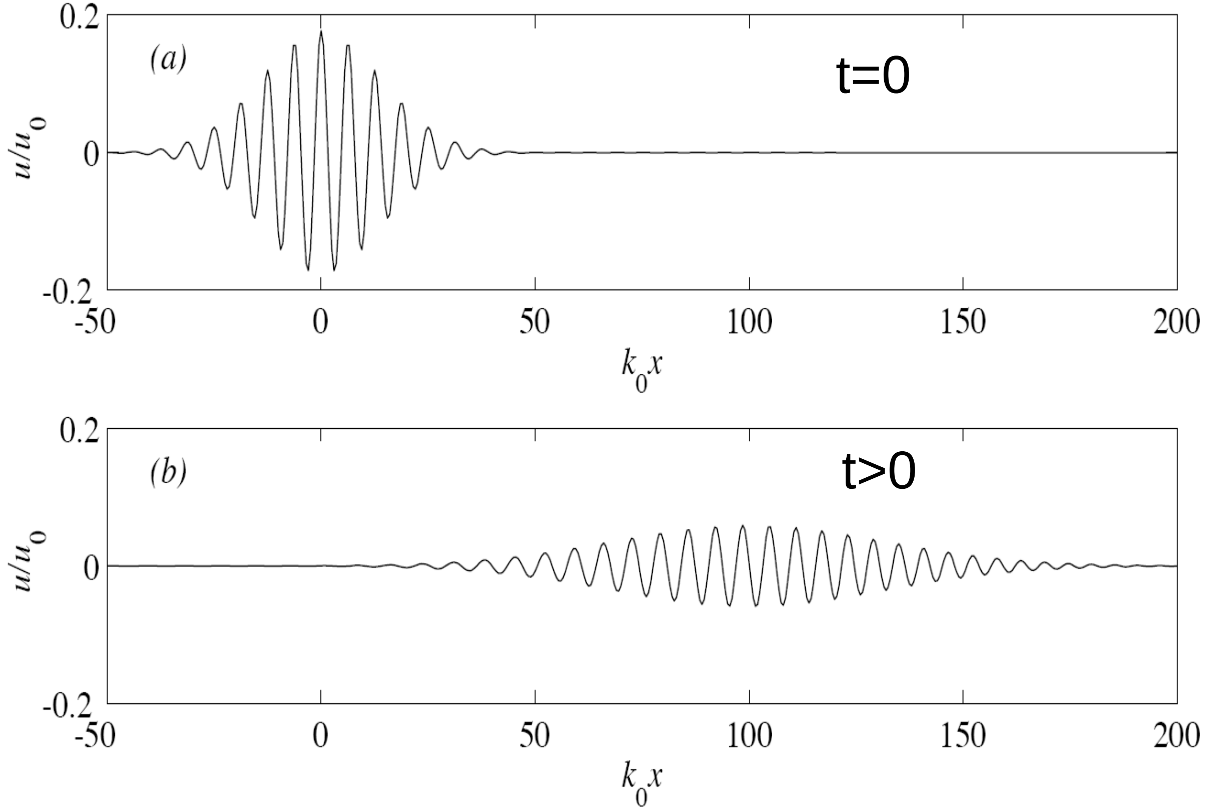
Higher order development

$$\omega - \omega_0 = c_g(k - k_0) + \frac{\omega_0''}{2}(k - k_0)^2$$

$$c_g = \frac{\partial \omega}{\partial k}(k_0), \qquad \omega_0'' = \frac{\partial^2 \omega}{\partial k^2}(k_0)$$

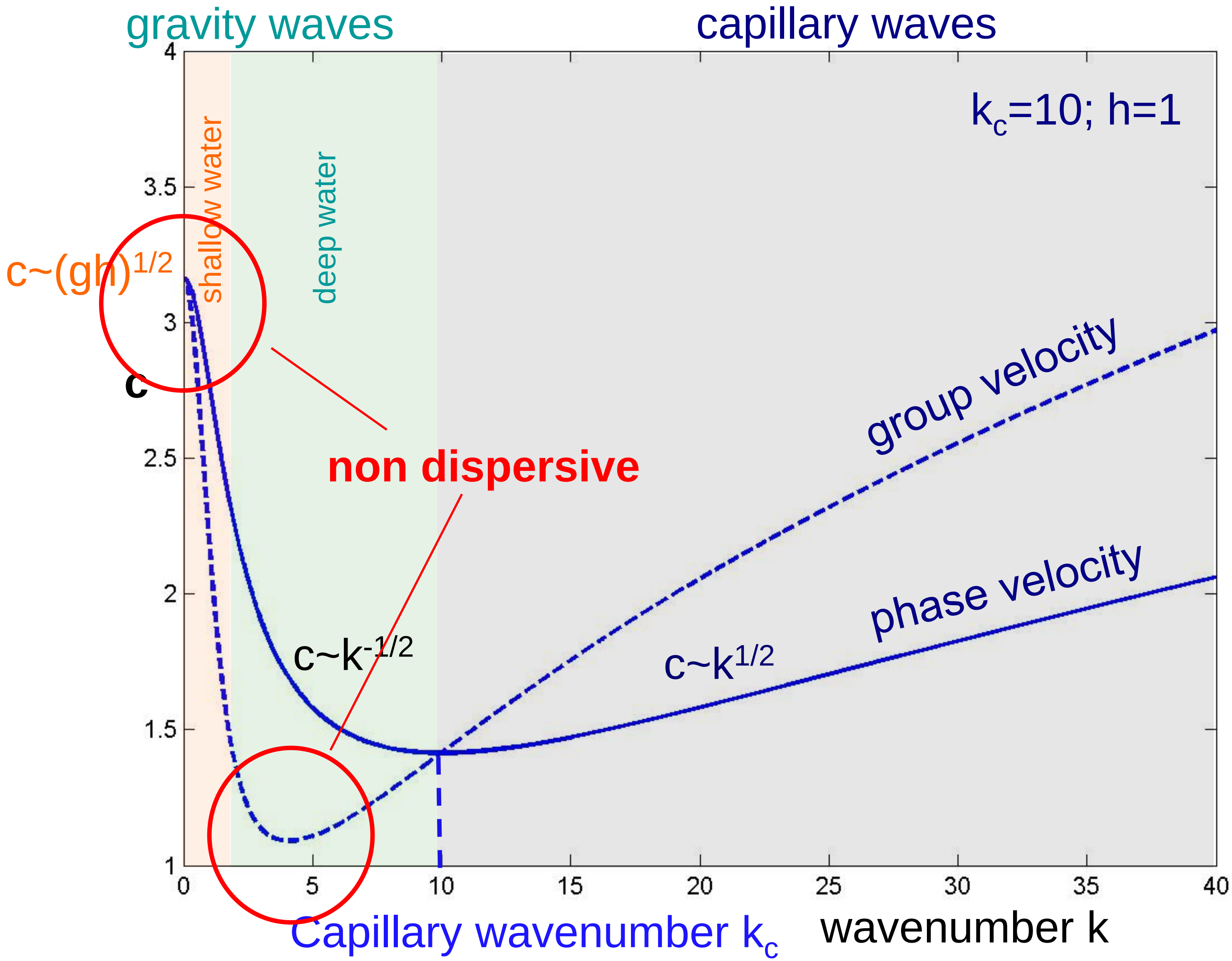
$$A(x,t) = \frac{u_0}{2}\sqrt{\frac{\pi}{\sigma^2 + \frac{1}{2}\mathrm{i}\omega_0'' t}}\exp\left(-\frac{(x - c_g t)^2}{4(\sigma^2 + \frac{1}{2}\mathrm{i}\omega_0'' t)}\right)$$

Wave packet dispersion

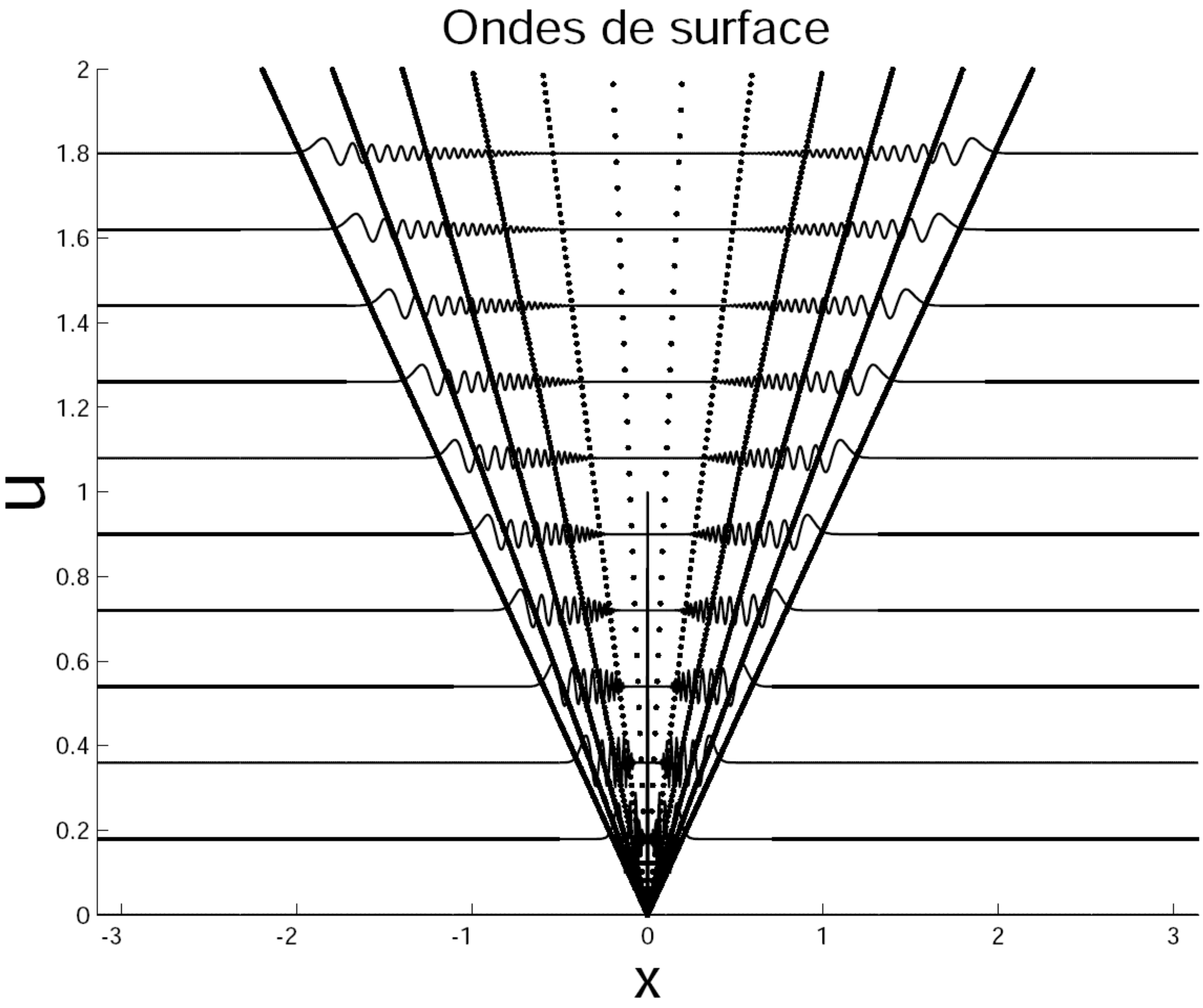


Onde correspondant à l'enveloppe ( ) *pour* $\sigma^{-1}k_0 = 0{,}1$ *et* $\omega_0'' = 4c_g/k_0$: *(a), instant initial* $t = 0$; *(b),* $c_g t = 100/k_0$.

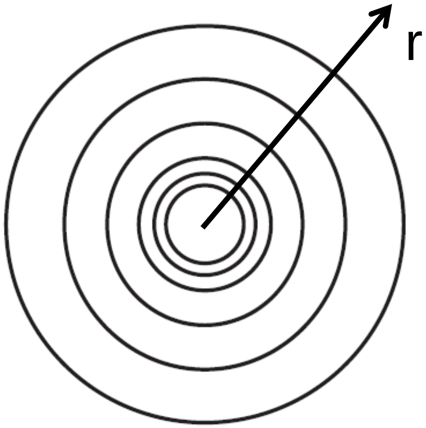
Dispersion relation



Dispersion



Dispersion



Waves with k reach r at time t=r/v(k)

For deep gravity waves: $v_{deep/gravity} \sim \pm\frac{1}{2}\sqrt{\frac{g}{k}}$

$k = gt^2/4r^2$

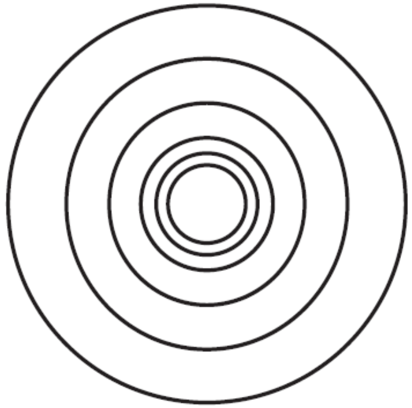
Since $\omega_{deep/gravity} \sim \pm\sqrt{gk}$

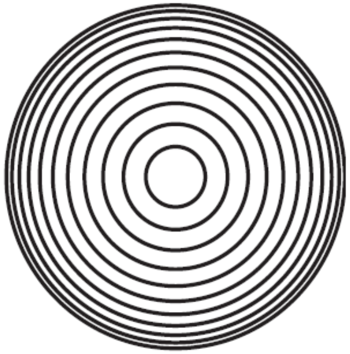
$\omega = gt/2r$

⇨The frequency increases with time

“Ronds dans l’eau”

Gravity waves

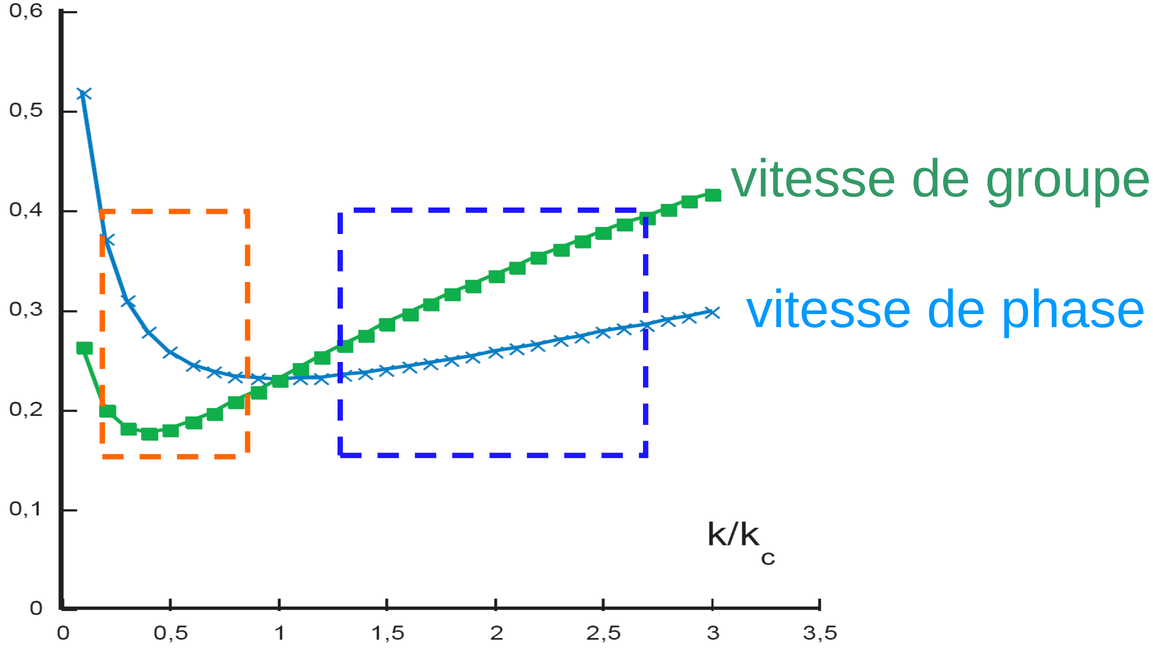




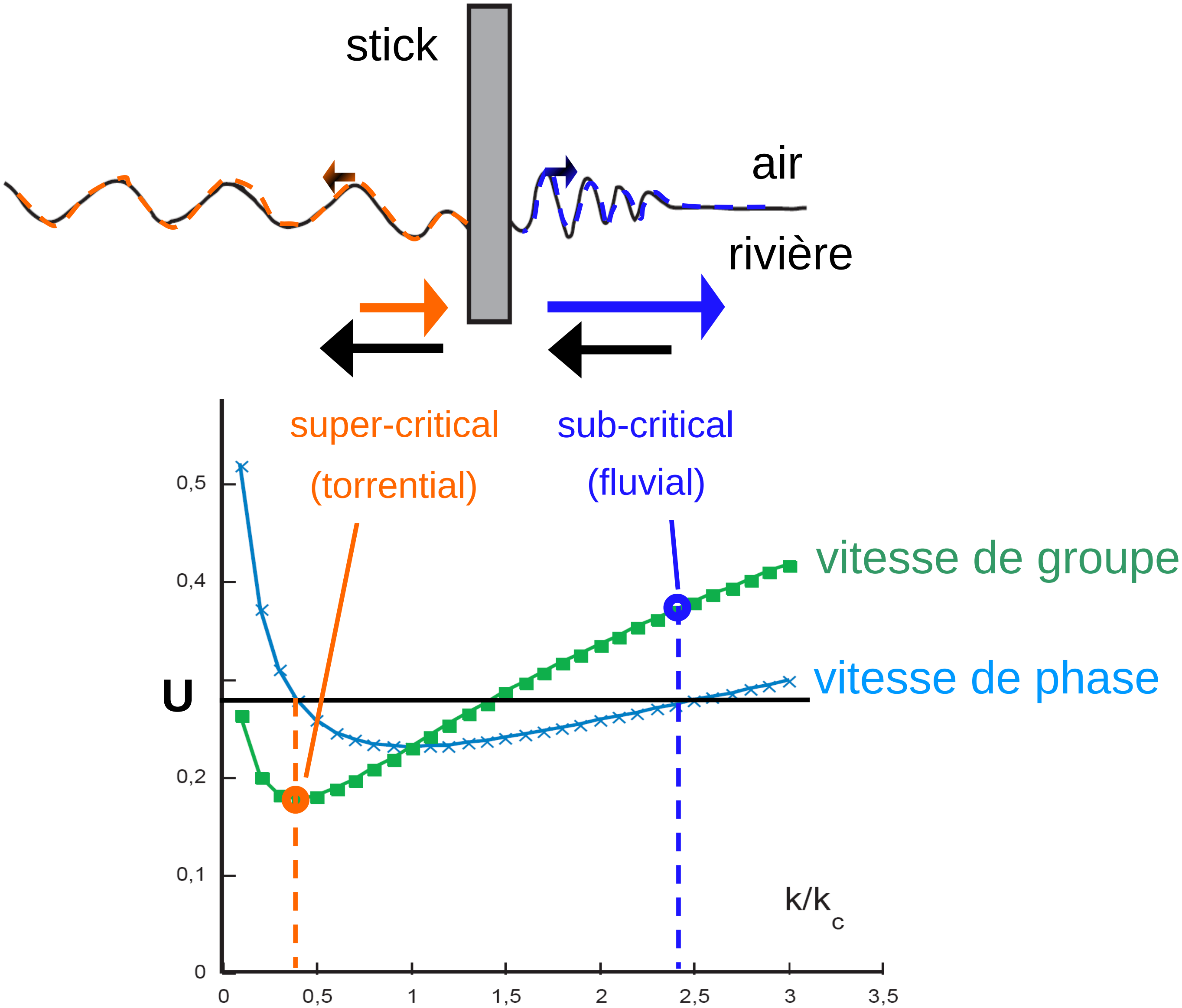
Capillary waves

stone>l_c

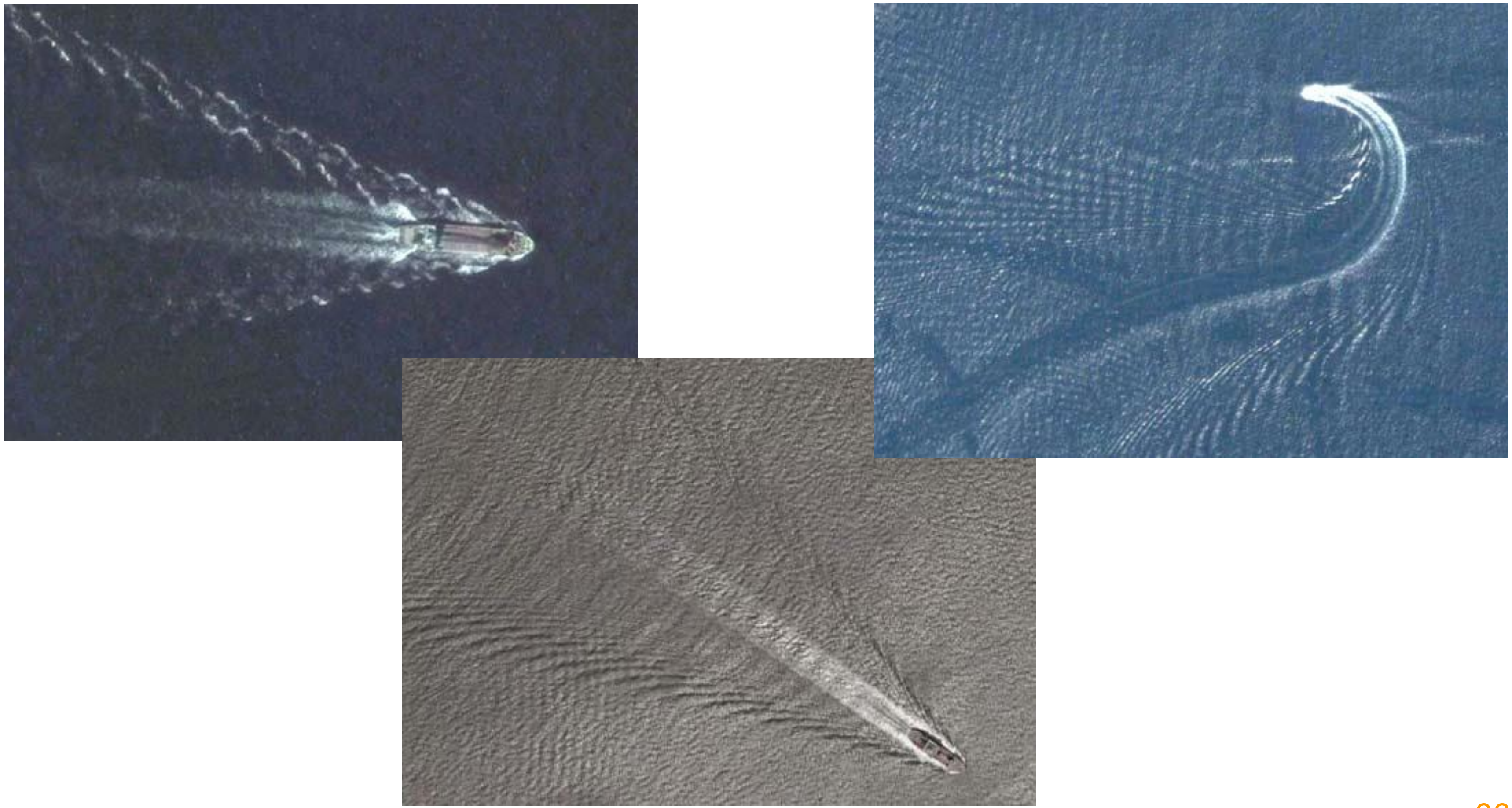
water drop<l_c



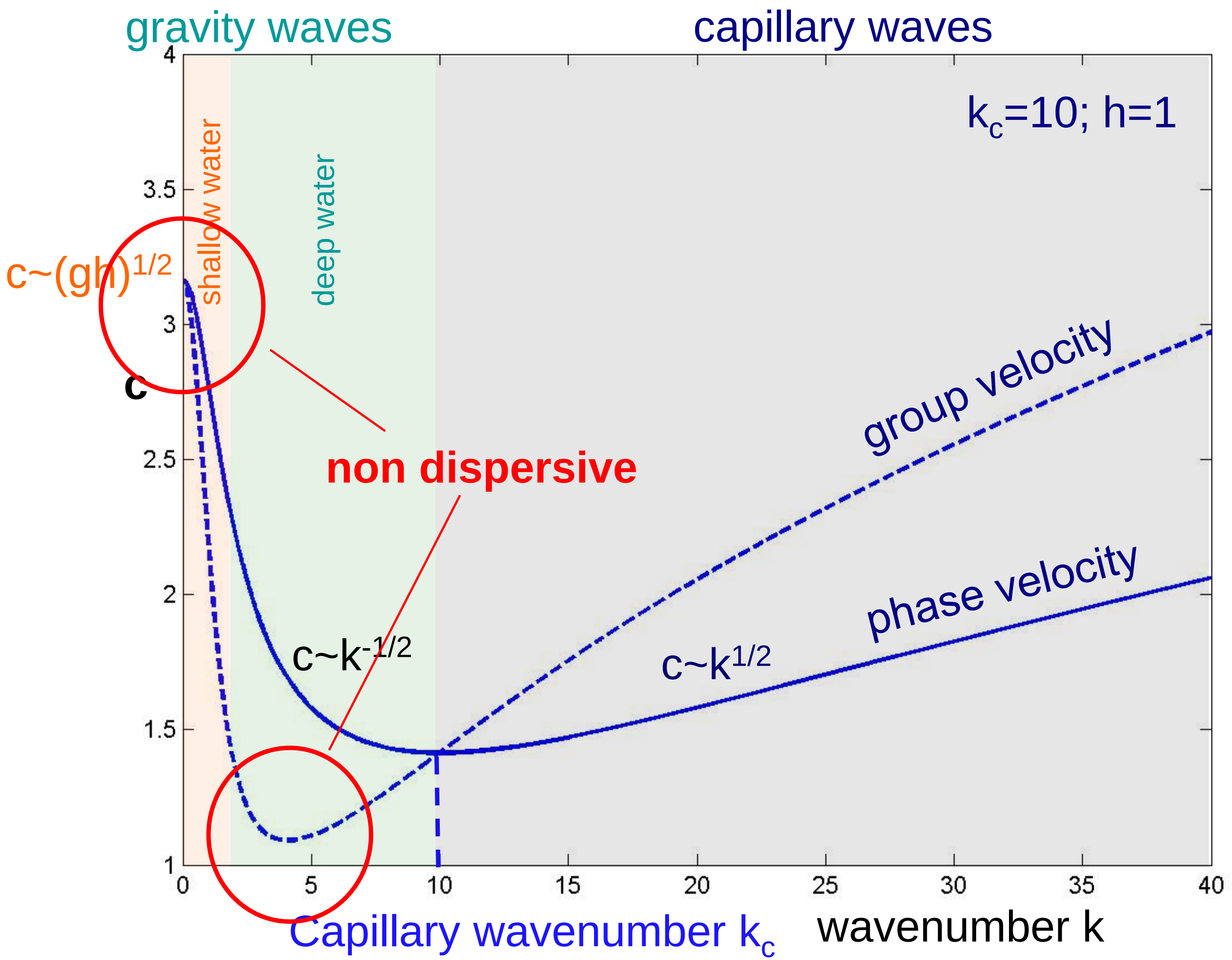
Waves created by an obstacle in a river



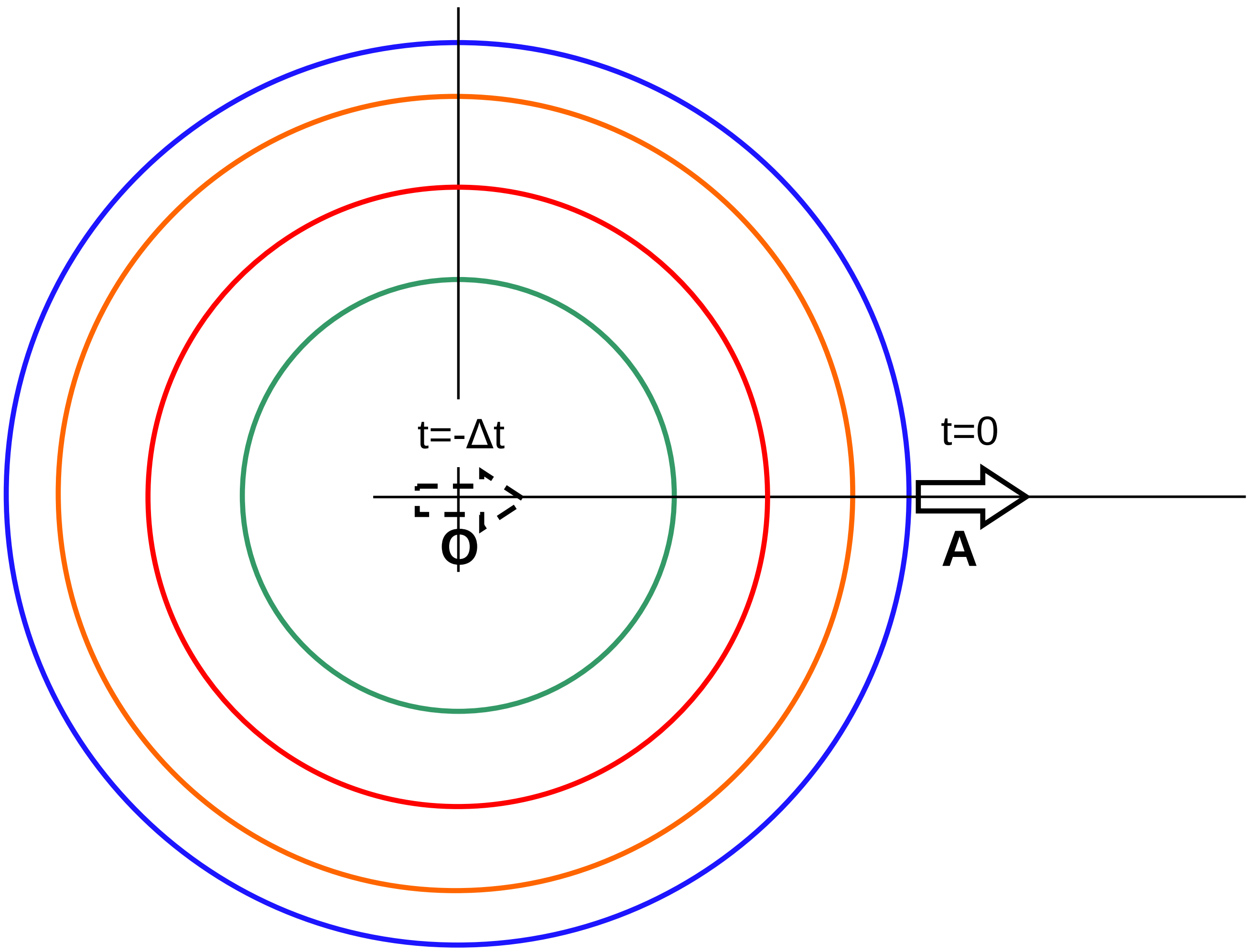
Kelvin wakes



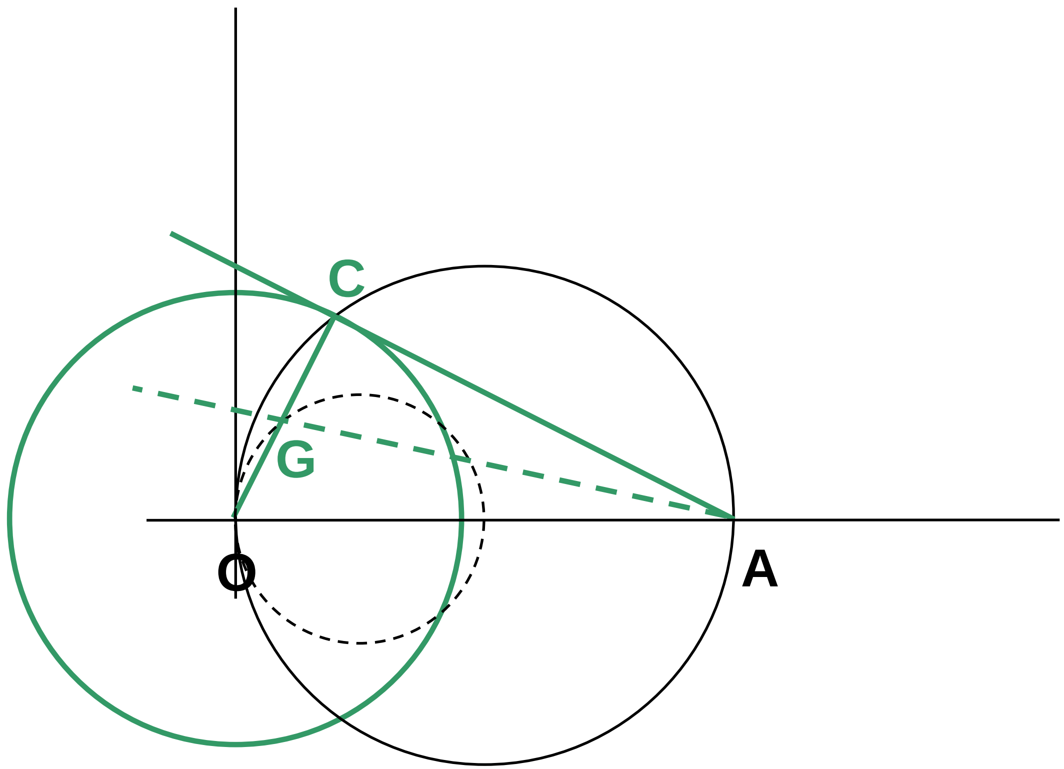
Dispersion relation



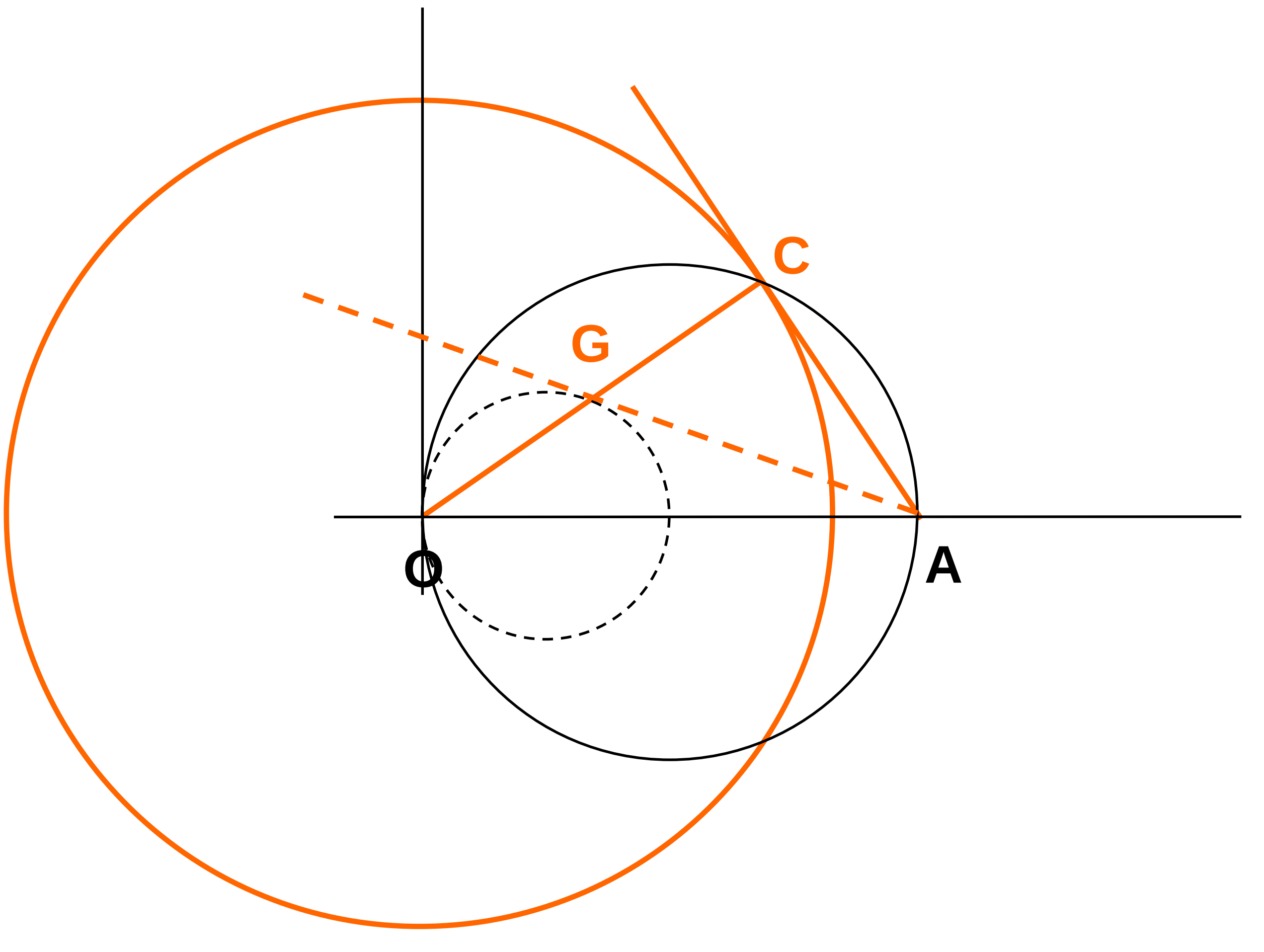
Kelvin wake



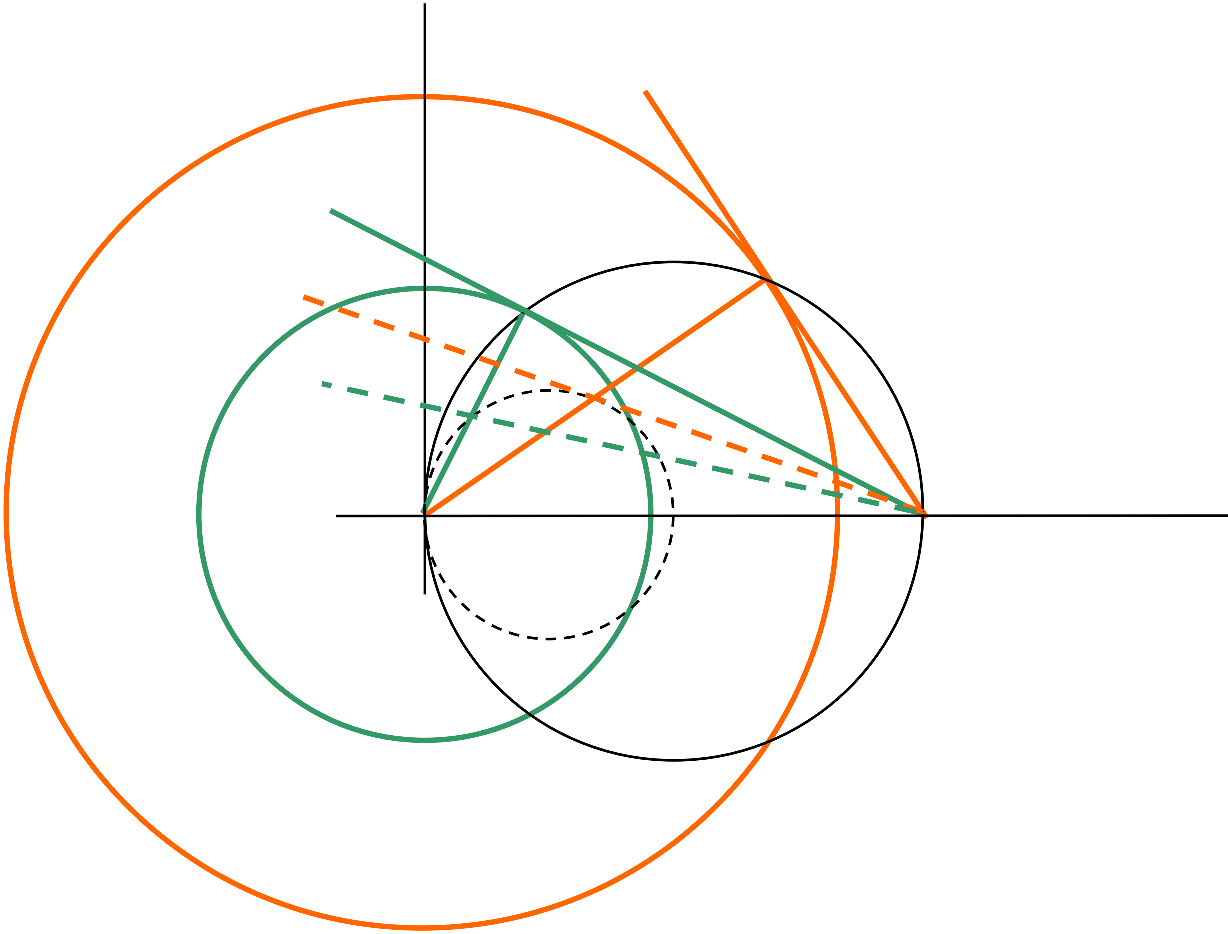
Waves created by a ship



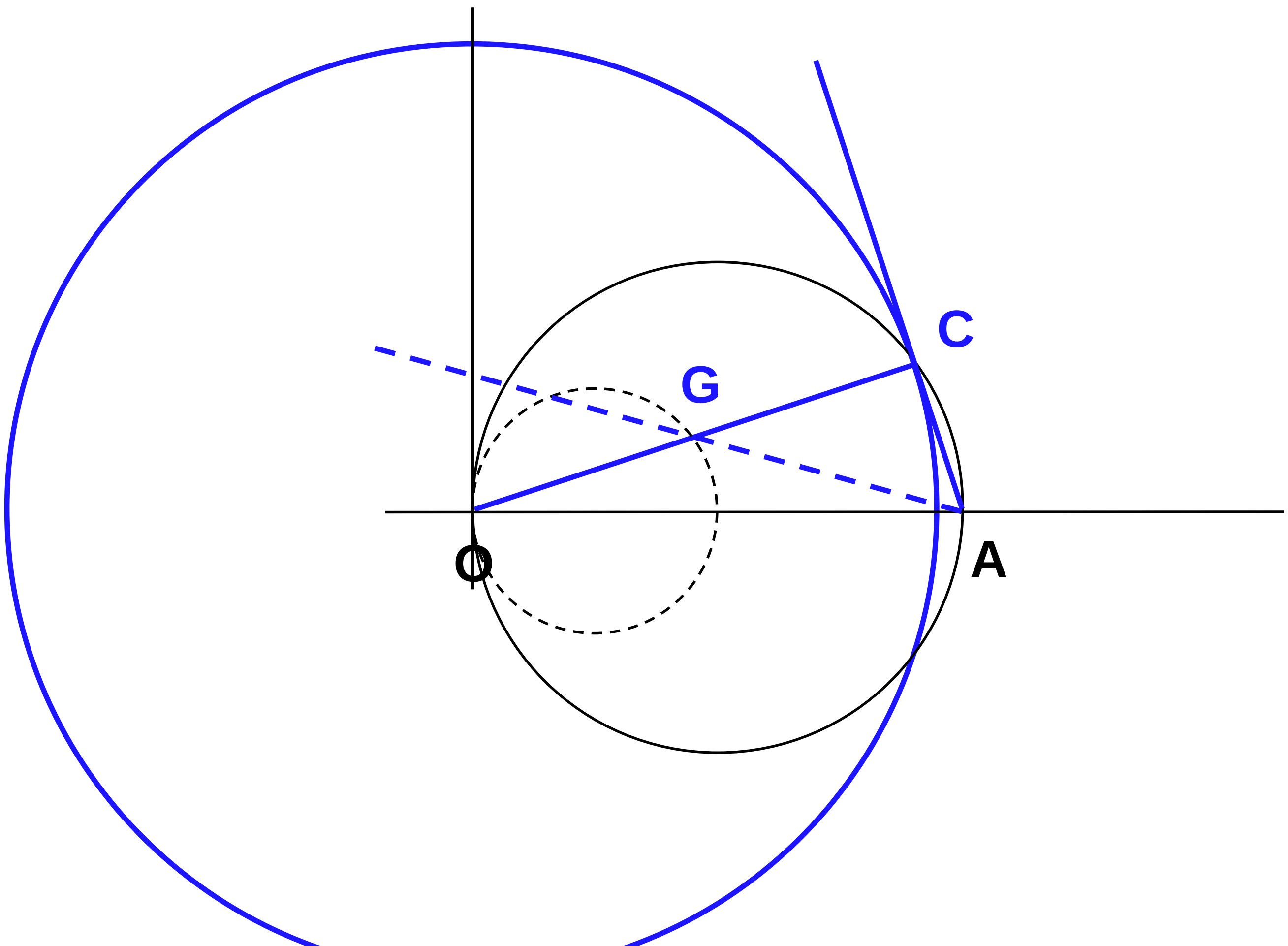
Waves created by a ship



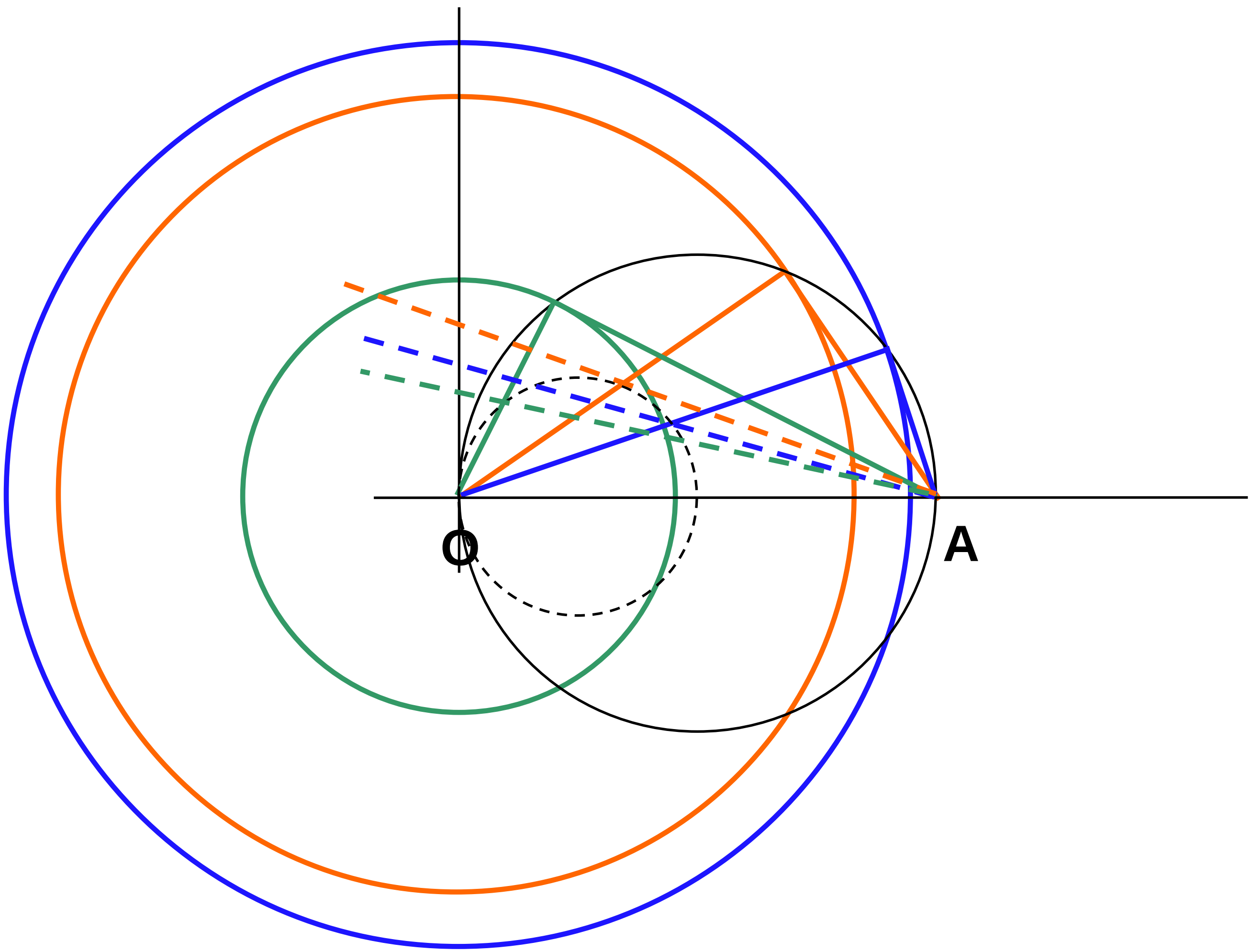
Waves created by a ship



Waves created by a ship



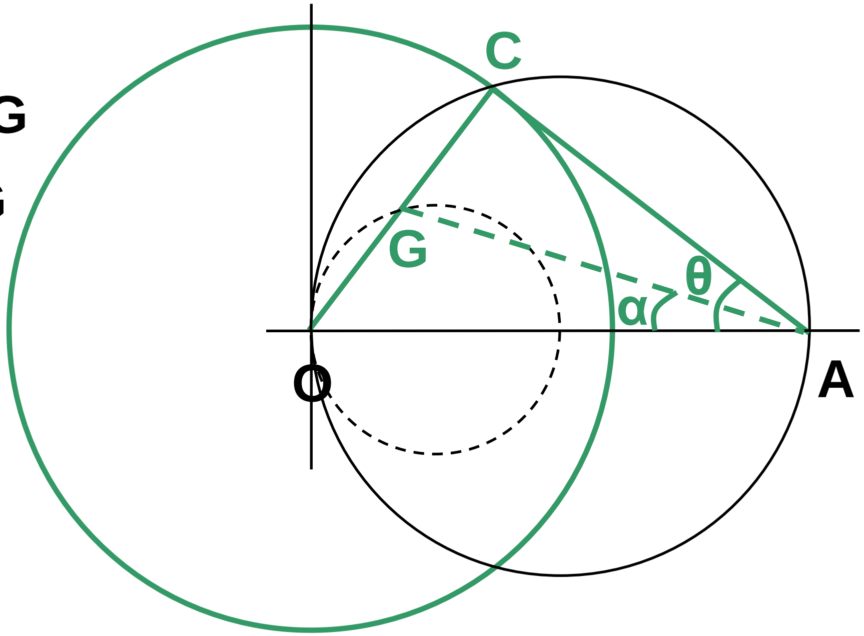
Waves created by a ship



Waves created by a ship

$\sin(\alpha)/OG=\cos(\theta)/AG$

$\sin(\theta-\alpha)AG=GC=OG$

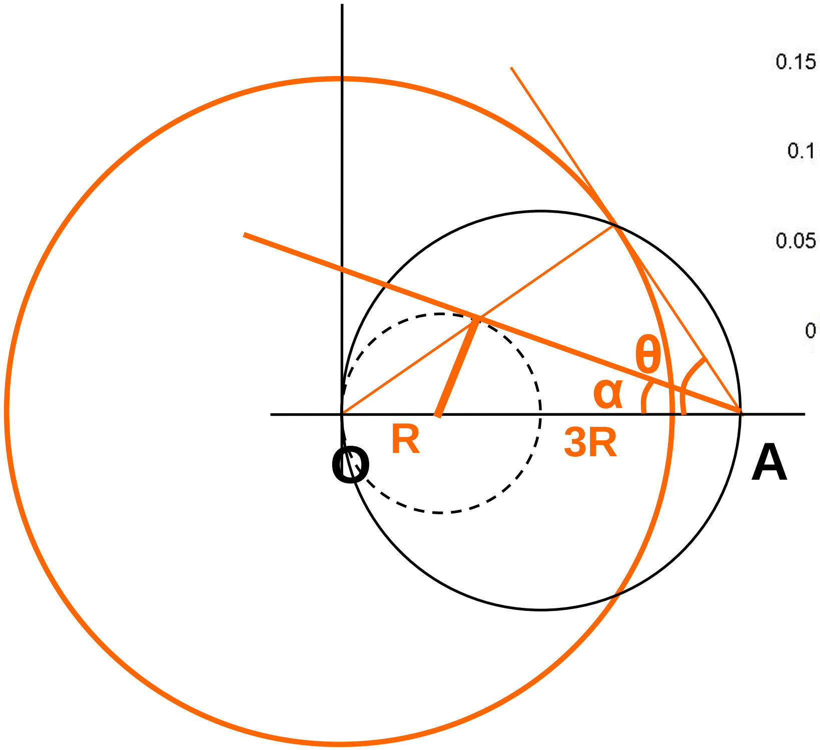


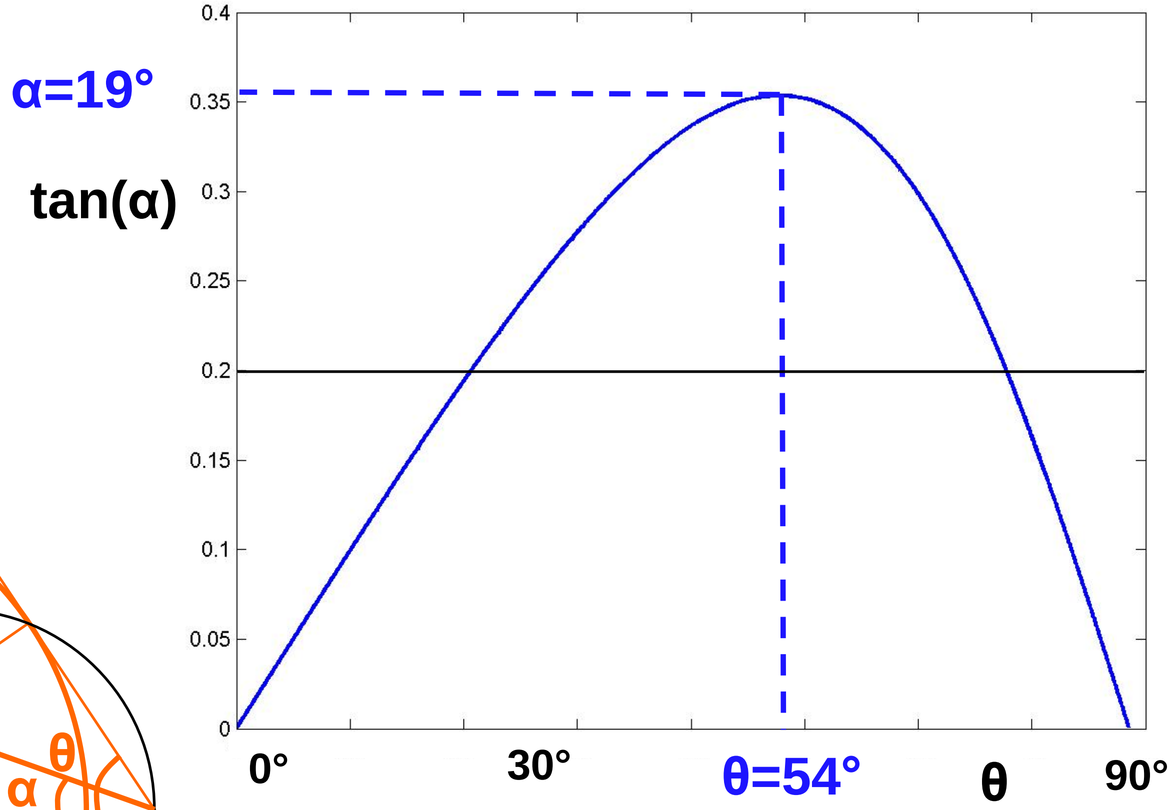
⇨ $\sin(\alpha)=\cos(\theta)\sin(\theta-\alpha)$

⇨ $\sin(\alpha)=\cos(\theta)(\sin(\theta)\cos(\alpha)+\cos(\theta)\sin(\alpha))$

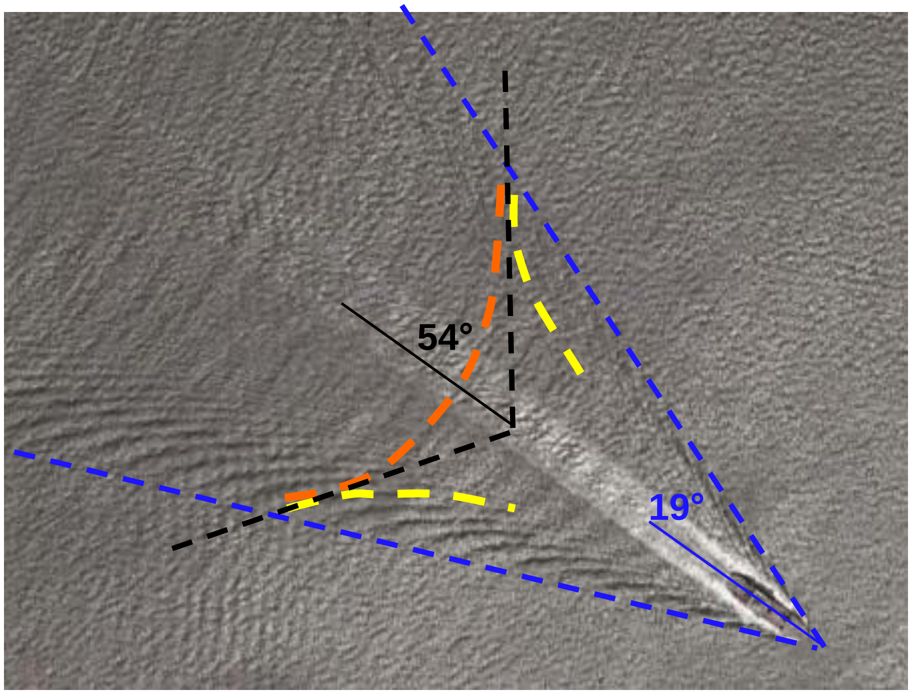
⇨ $\tan(\alpha)=\cos(\theta)\sin(\theta)/(1+\cos^2(\theta))$

Waves created by a ship

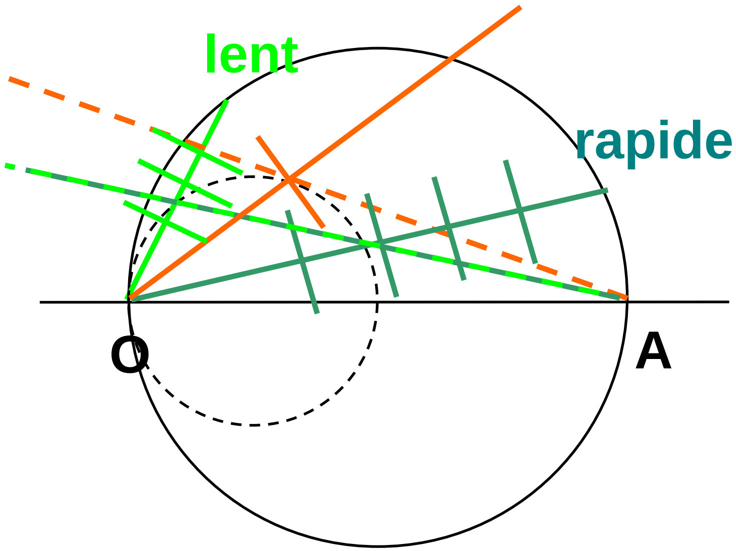


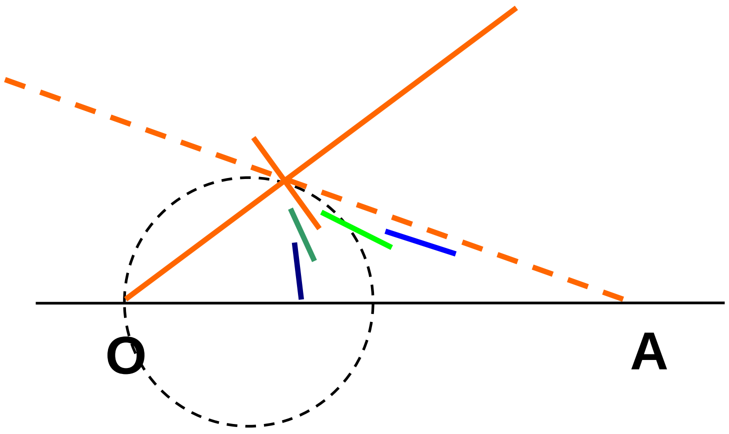


Waves created by a ship

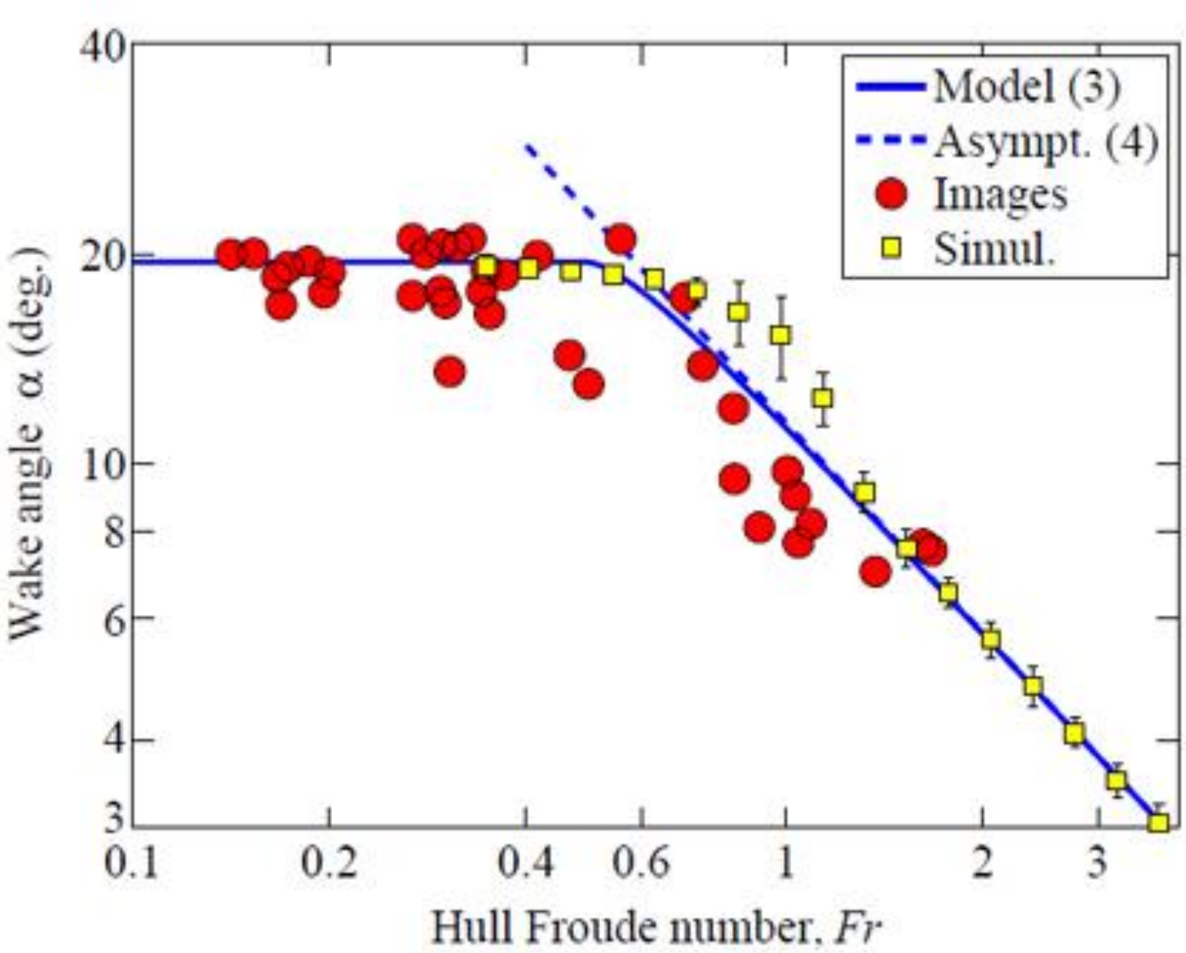


Waves created by a ship

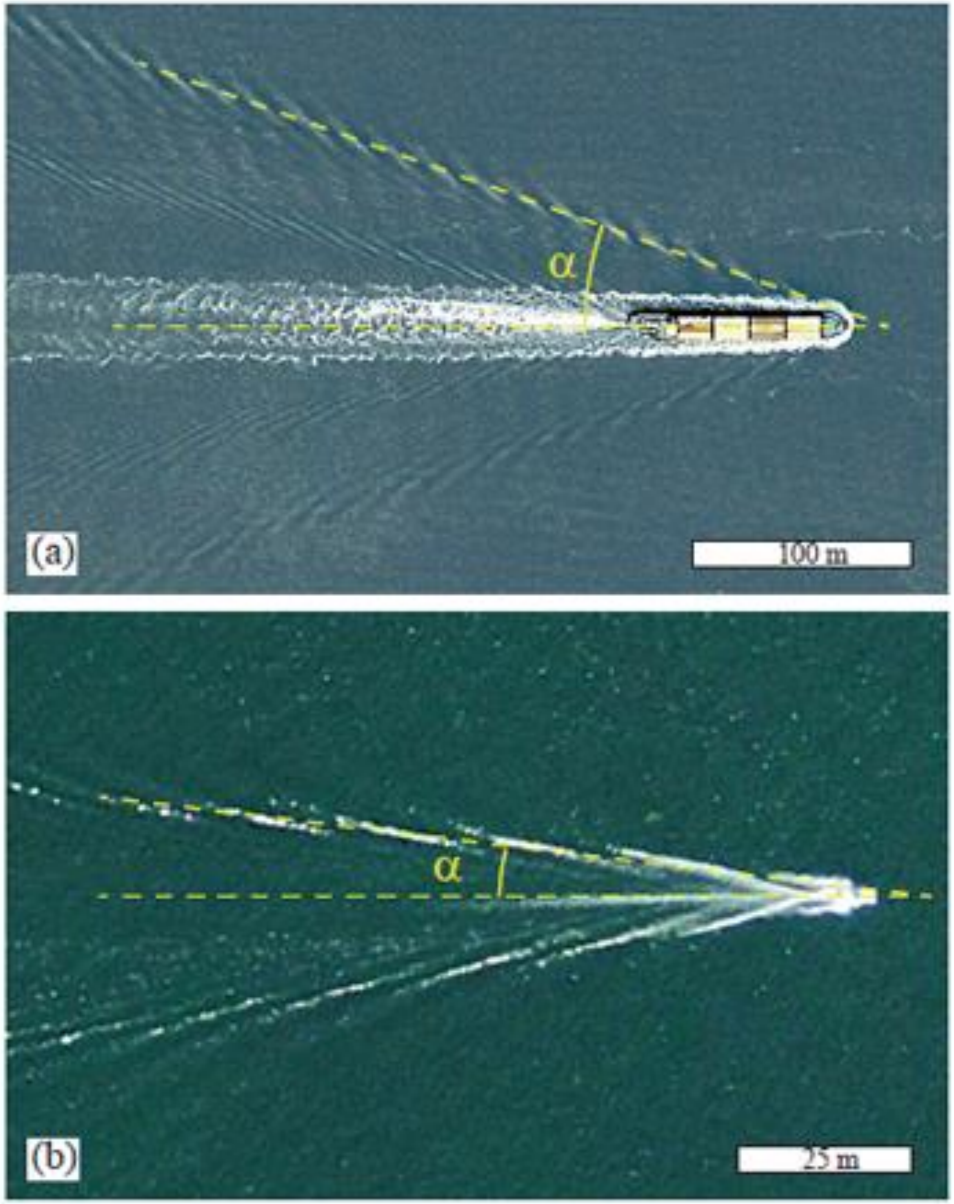




Careful google-earth data analysis by Moisy and Rabaud (2013)



Careful google-earth data analysis by Moisy and Rabaud (2013)



Froude number

$$Fr = \frac{U}{c}$$

c: celerity of wave

$$Fr = \frac{U}{\sqrt{gd}}$$

d: size of ship

$$Fr = \frac{U}{\sqrt{gl}}$$

shallow water

Froude decomposition

$$C_T(F, Re) = C_D(Re) + C_W(F, Re)$$

Total drag

Drag in absence of free surface

Wave drag

Froude hypothesis

$$C_T(F, Re) = C_D(Re) + C_W(F, \cancel{Re})$$

Total drag

Drag in absence of free surface

Wave drag

Alternative expression

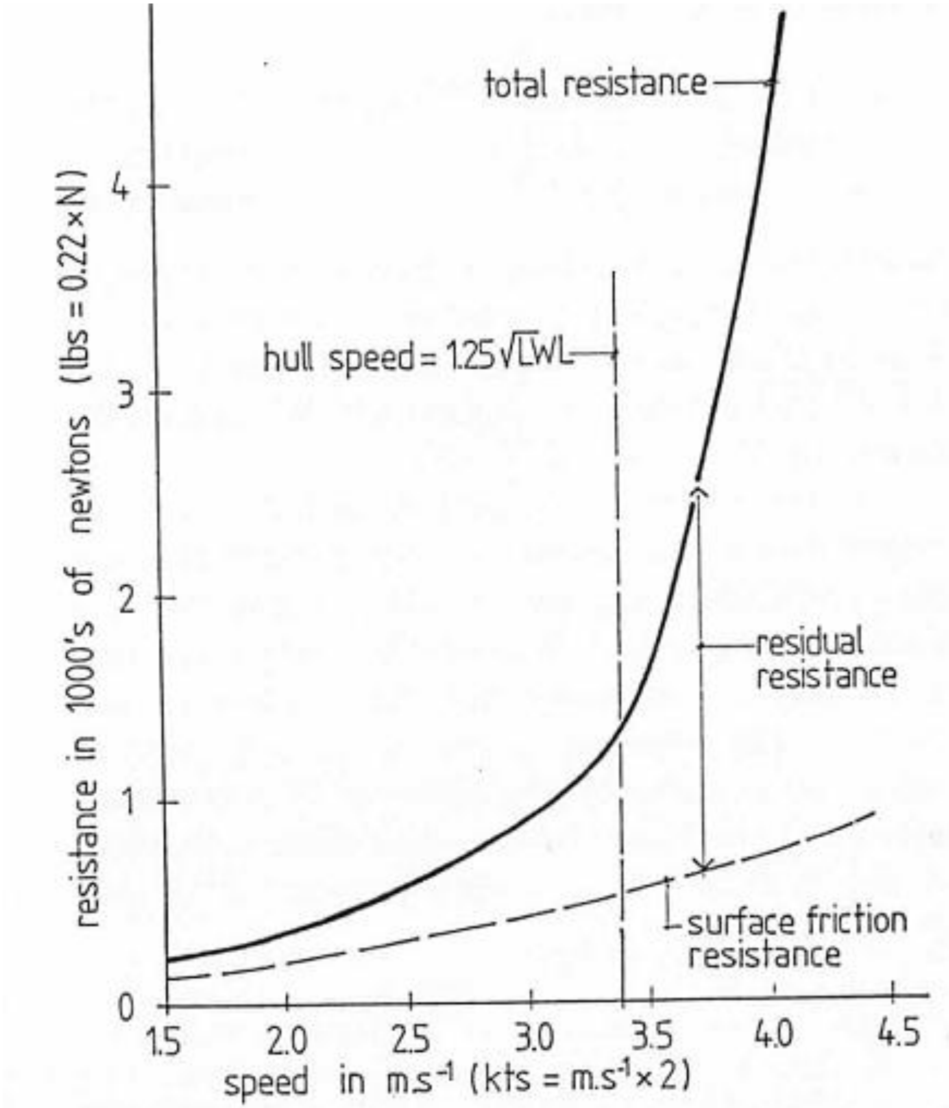
$$C_T(F, Re) = C_F(Re) + C_P(F, Re)$$

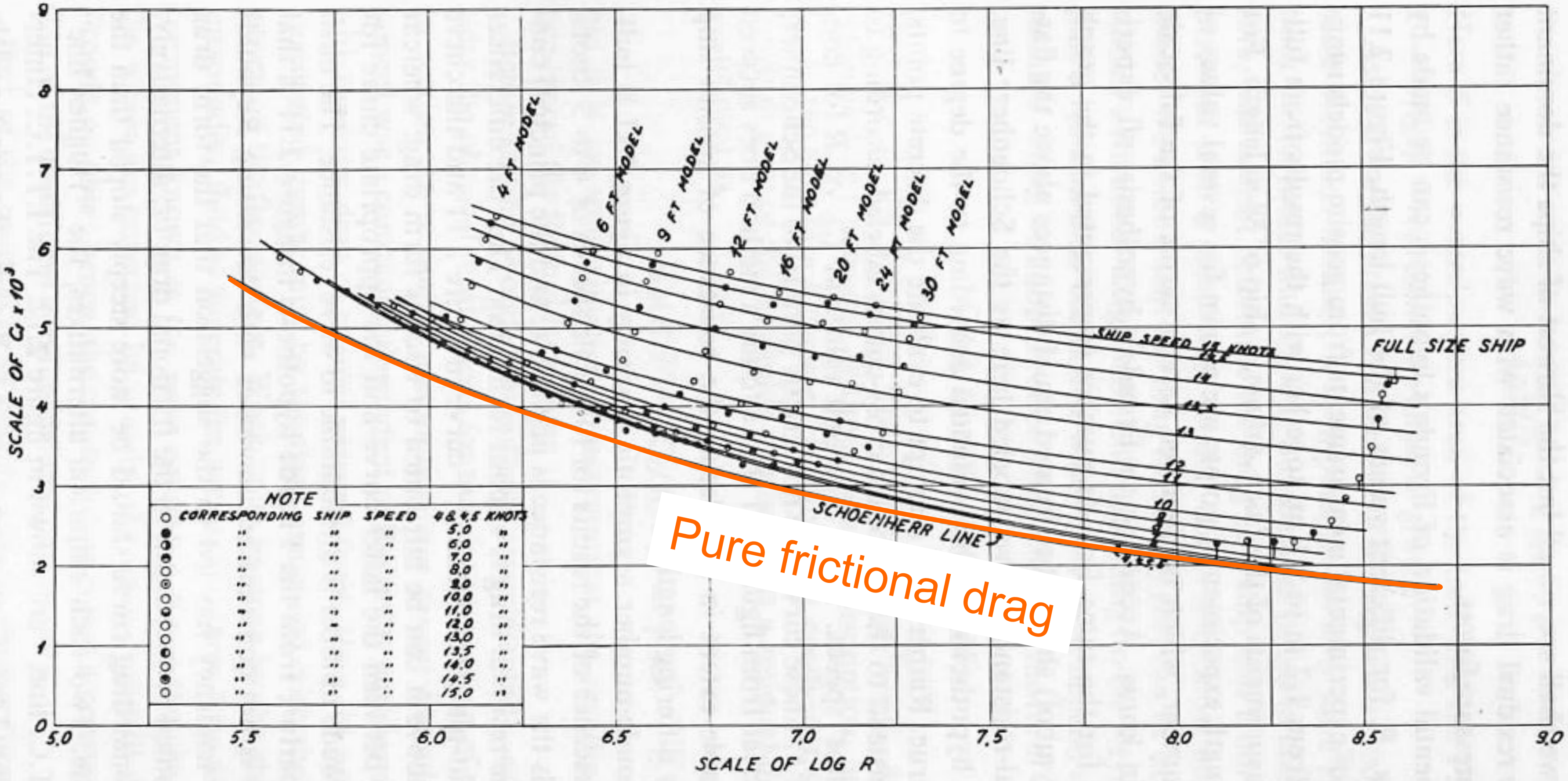
Total drag | Friction drag | Pressure drag

$$C_P(F, Re) \approx C_W(F)$$

Pressure drag | Wave drag (Residual drag)

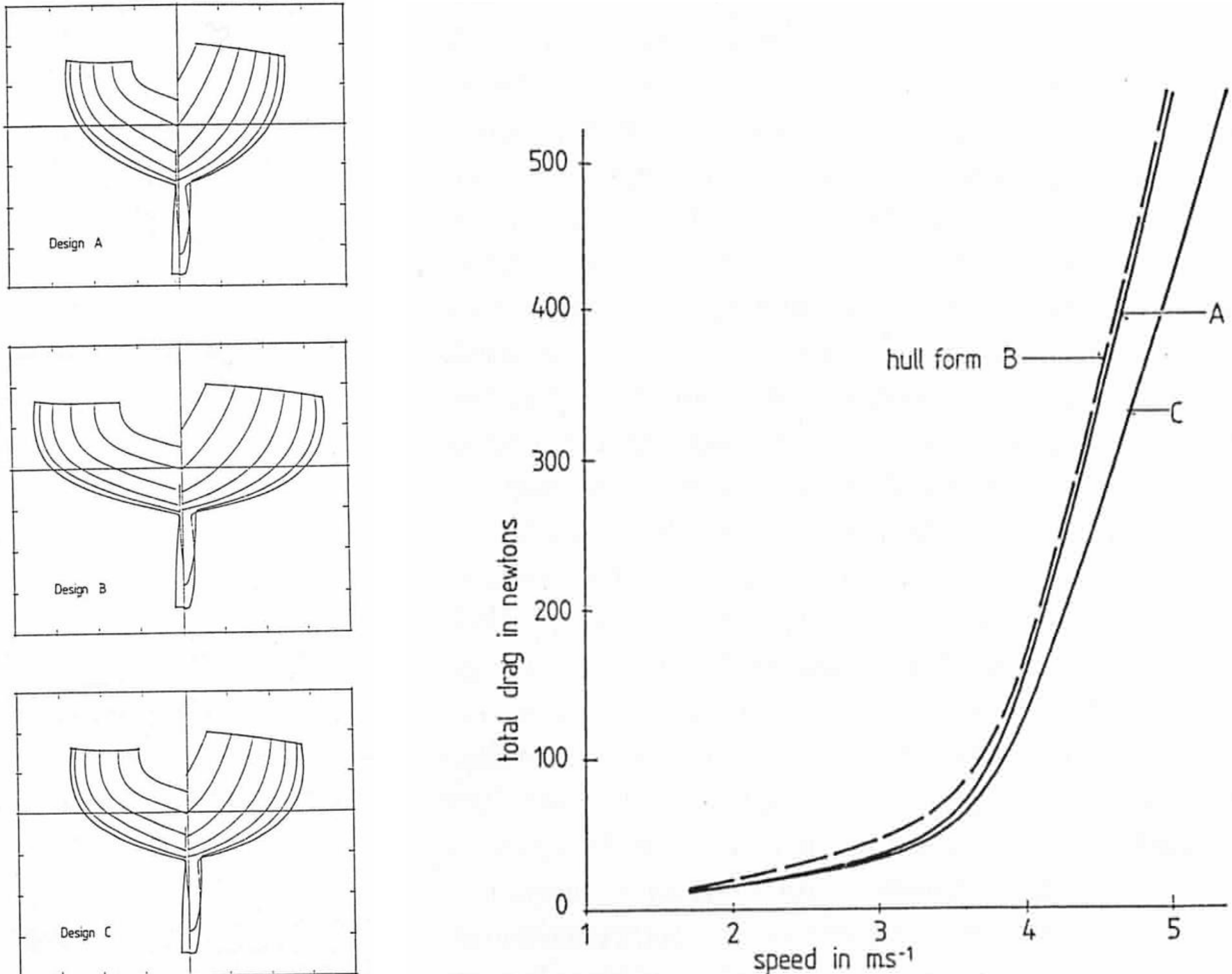
Friction resistance and residual resistance





2.11

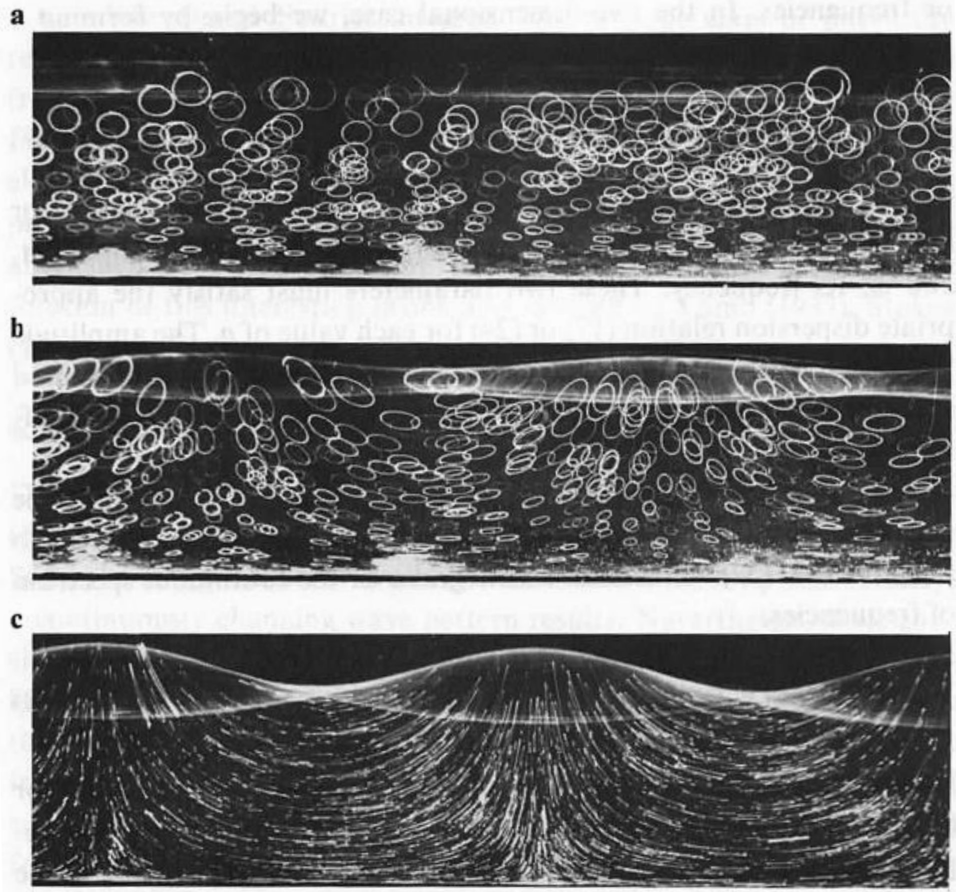
Total drag coefficients of the *Lucy Ashton* and several geosim models of the same vessel (from Troost and Zakay 1951). The faired curves represent constant values of the Froude number and, if Froude's hypothesis were strictly valid, these would be parallel with spacing independent of the Reynolds number. Note that, even for this small full-scale vessel (58 m long), there is a large gap between the largest model results and the full-scale results.


Design A
Design B
Design C
500
400
300
200
100
0
total drag in newtons
1
2
3
4
5
speed in ms-1
hull form B
A
C

Standing waves

$$\eta = A\cos(kx - \omega t) + A\cos(kx + \omega t)$$
$$= 2A\cos kx\cos\omega t,$$

Reflected waves



6.7

Particle trajectories in a plane progressive wave (a), a partial reflected wave (b), and a standing wave (c). These correspond respectively to a reflection coefficient of 0, 0.38, and 1.0 in equation (56). Note that the reflection coefficient can be measured from the maximum and minimum of the envelope, using (56). These photographs are based on time exposures, and are reproduced from a more extensive series of observations made by Ruellan and Wallet (1950).

Reflected waves

$$\eta = A\,\mathrm{Re}[e^{-ikx+i\omega t} + Re^{ikx+i\omega t}]$$

R: complex reflection coefficient

$$\eta = A\,\mathrm{Re}[e^{-ikx+i\omega t}(1 + \cdot Re^{2ikx})]$$

Wave energy

$$KE + PE = \rho \iiint_{\mathcal{V}} \left(\frac{1}{2} V^2 + gy \right) d\mathcal{V}$$

$$E = \rho \int_{-h}^{\eta} \left(\frac{1}{2} V^2 + gy \right) dy = \frac{1}{2} \rho \int_{-h}^{\eta} V^2 \, dy + \frac{1}{2} \rho g \left(\eta^2 - h^2 \right).$$

Neglect potential energy of fluid at rest

Wave energy

$$KE + PE = \rho \iiint_{\mathcal{V}} \left(\frac{1}{2} V^2 + gy \right) d\mathcal{V}$$

$$E = \rho \int_{-h}^{\eta} \left(\frac{1}{2} V^2 + gy \right) dy = \frac{1}{2} \rho \int_{-h}^{\eta} V^2 \, dy + \frac{1}{2} \rho g \left(\eta^2 - \cancel{h^2} \right).$$

Neglect potential energy of fluid at rest

$$E = \frac{\rho \omega^2 A^2}{4k} e^{2k\eta} + \frac{1}{2} \rho g \eta^2.$$

Wave energy

$$KE + PE = \rho \iiint_{\mathcal{V}} \left(\frac{1}{2} V^2 + gy \right) d\mathcal{V}$$

$$E = \rho \int_{-h}^{\eta} \left(\frac{1}{2} V^2 + gy \right) dy = \frac{1}{2} \rho \int_{-h}^{\eta} V^2 \, dy + \frac{1}{2} \rho g \left(\eta^2 - h^2 \right).$$

Neglect potential energy of fluid at rest

$$E = \frac{\rho \omega^2 A^2}{4k} e^{2k\eta} + \frac{1}{2} \rho g \eta^2.$$

$$E = \tfrac{1}{4} \rho g A^2 + \tfrac{1}{2} \rho g A^2 \cos^2(kx - \omega t)$$

Wave energy

$$KE + PE = \rho \iiint_{\mathcal{V}} \left(\frac{1}{2} V^2 + gy \right) d\mathcal{V}$$

$$E = \rho \int_{-h}^{\eta} \left(\frac{1}{2} V^2 + gy \right) dy = \frac{1}{2} \rho \int_{-h}^{\eta} V^2 \, dy + \frac{1}{2} \rho g \left(\eta^2 - \cancel{h^2} \right).$$

Neglect potential energy of fluid at rest

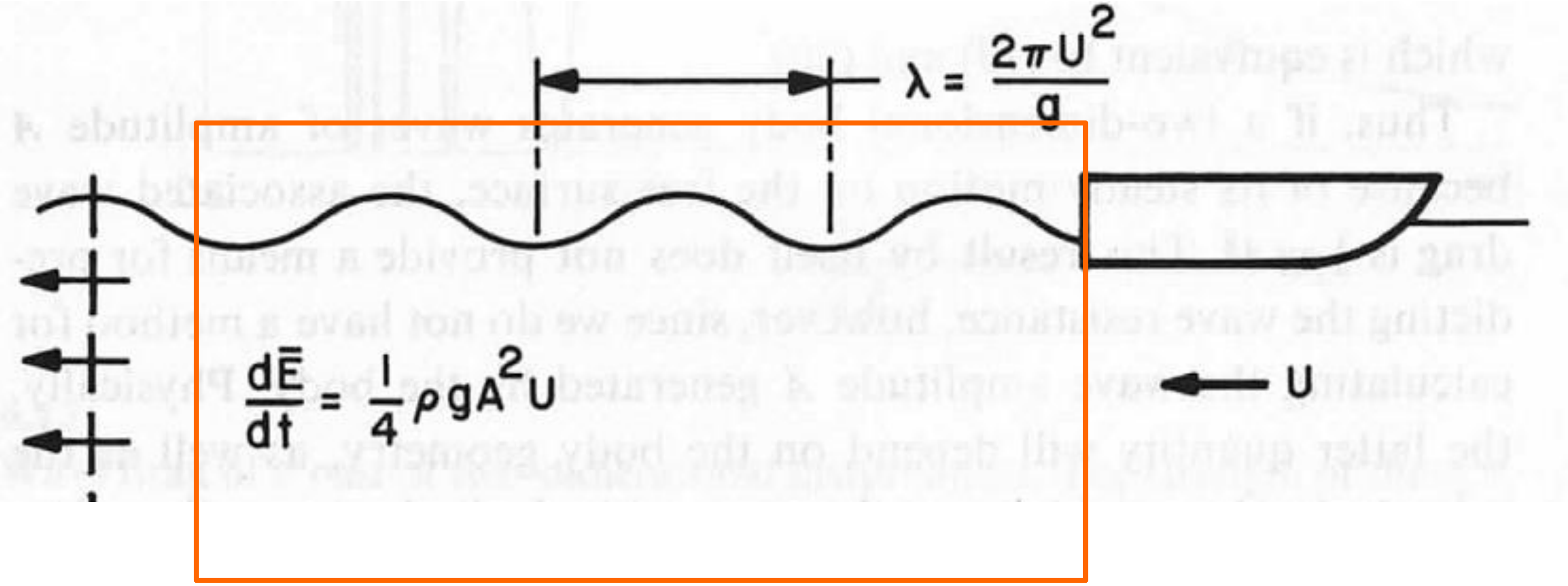
$$E = \tfrac{1}{4} \rho g A^2 + \tfrac{1}{2} \rho g A^2 \cos^2(kx - \omega t)$$

On average: $\bar{E} = \tfrac{1}{2} \rho g A^2$

Wave energy flux

One can show that the energy travels at the group velocity…

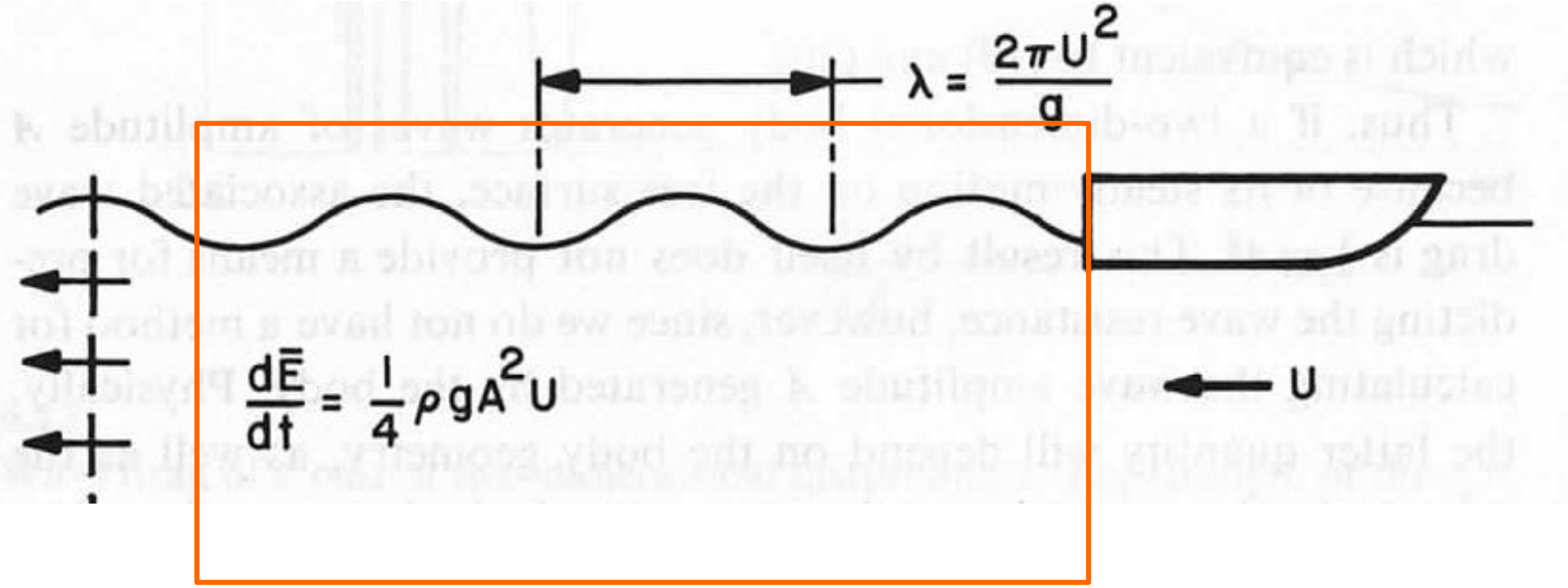
Energy balance (in ship's frame)



Energy input : -work of drag= -D.U

Energy output: (Vg-U)E

Energy balance (in ship's frame)



Energy input : -work of drag= -D.U

Energy output: (Vg-U)E

Deep water: Vg=U/2

D=E/2

$$D = \frac{1}{4}\rho g A^2$$

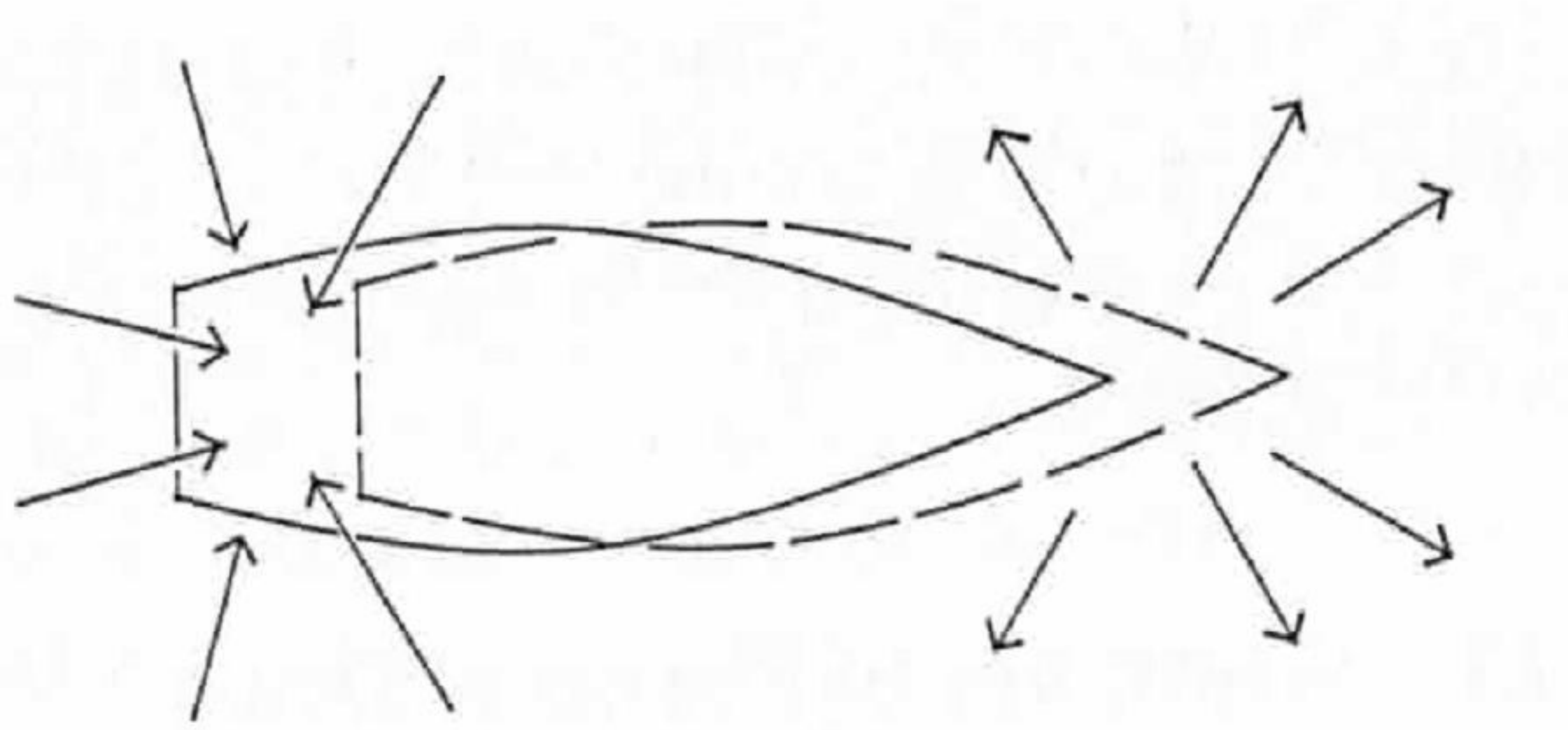
Wave production from bow

displacement $\eta = a \cos(kx + \varepsilon)$

dispersion relation $k = g/U^2$

wave amplitude a

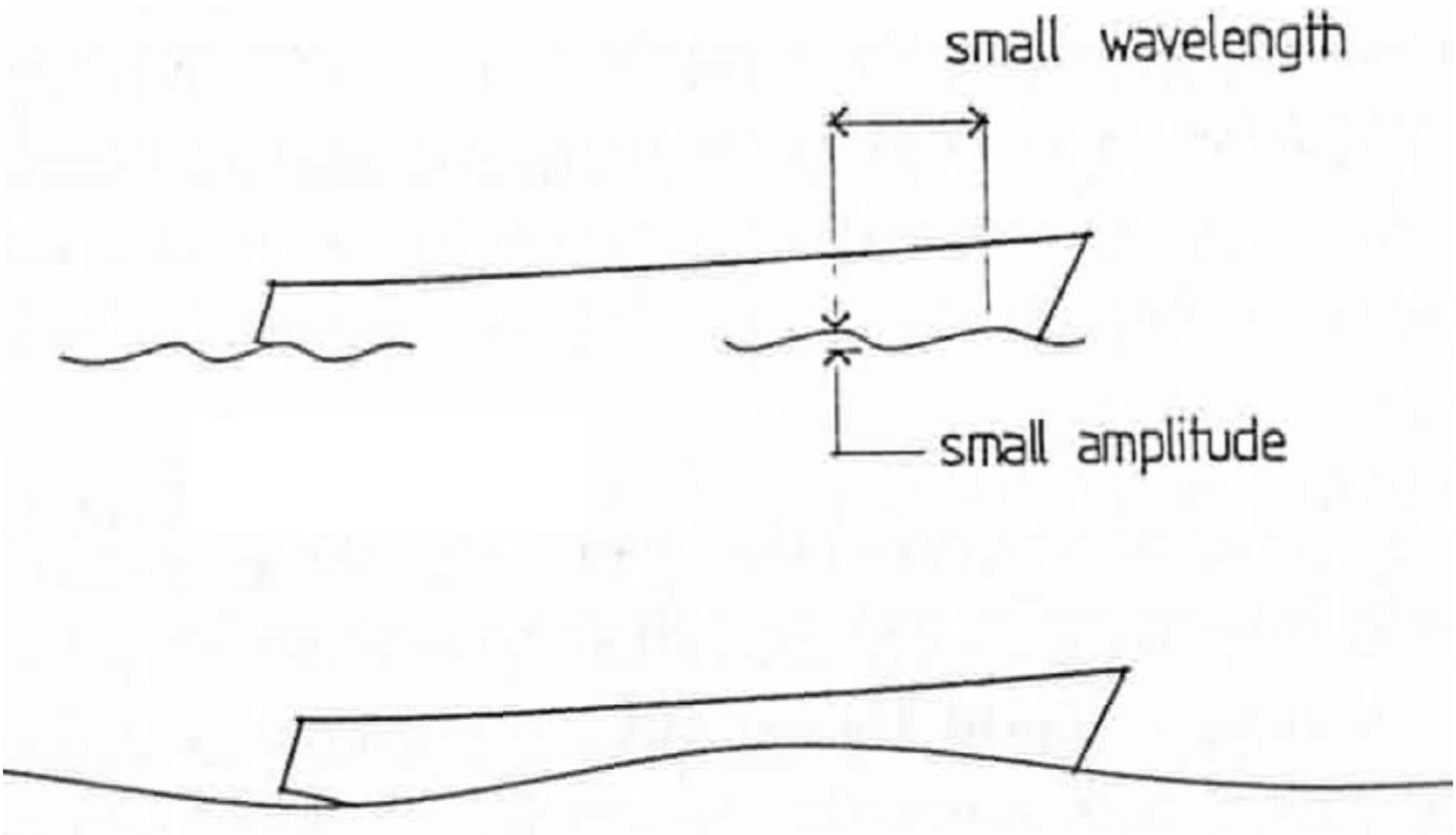
wave energy $\frac{1}{4}\rho g a^2$



Stern
“Hole”

Bow
“Bump”

Wavelength selection as a function of velocity
Which of these two ships has the highest velocity?



Superimpose wave of opposite amplitude from stern located at a distance l

displacement

$$\eta = a\cos(kx+\varepsilon) - a\cos(kx+\varepsilon+kl) = \mathrm{Re}\{ae^{i(kx+\varepsilon)}(1-e^{ikl})\}.$$

dispersion relation

$$k = g/U^2$$

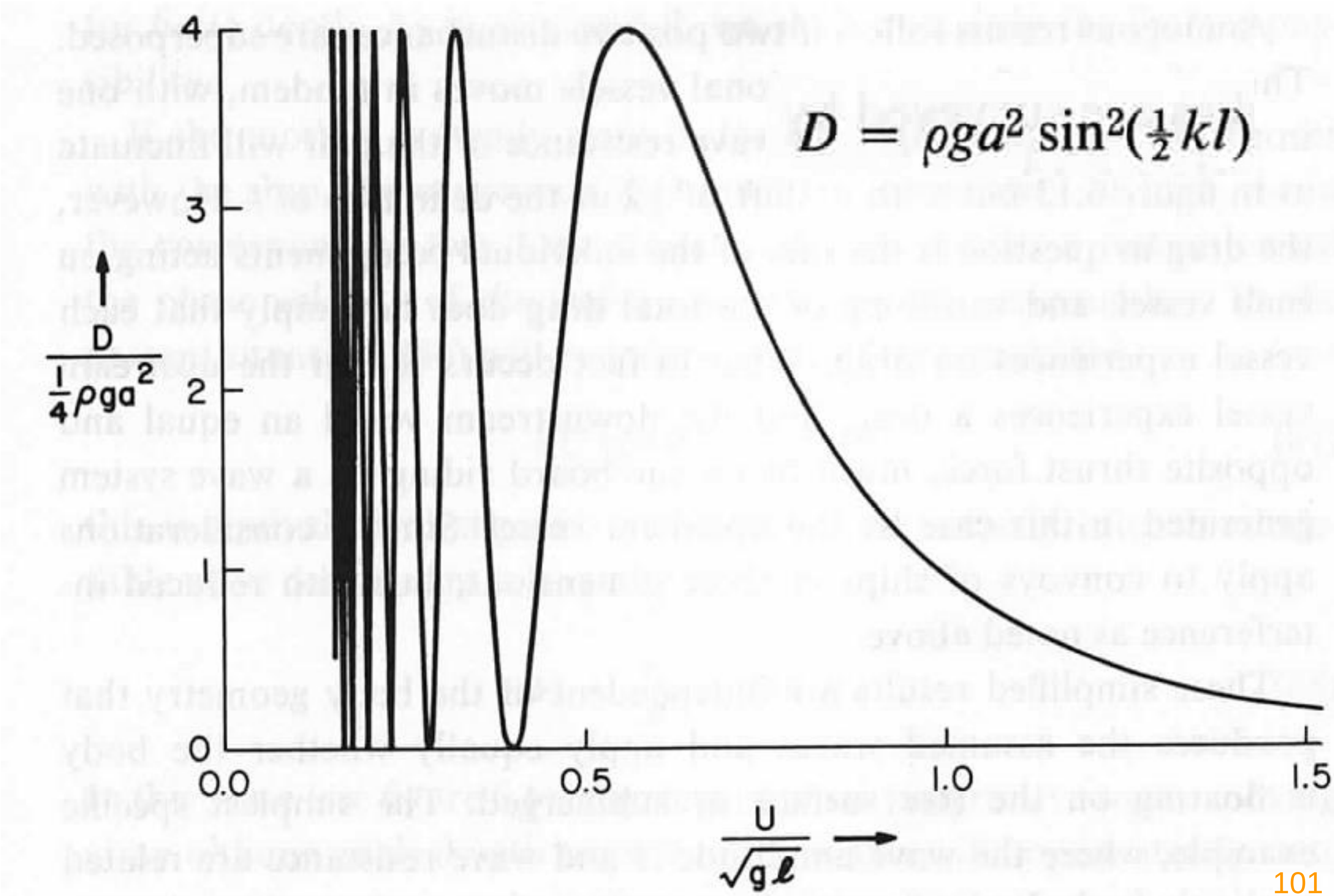
wave amplitude

$$A = a\left|1-e^{ikl}\right| = 2a\left|\sin(\tfrac{1}{2}kl)\right|$$

wave energy

$$D = \rho g a^2 \sin^2(\tfrac{1}{2}kl)$$

Superimpose wave of opposite amplitude from stern located at a distance l



The missing ingredient is now to determine the wave amplitude!

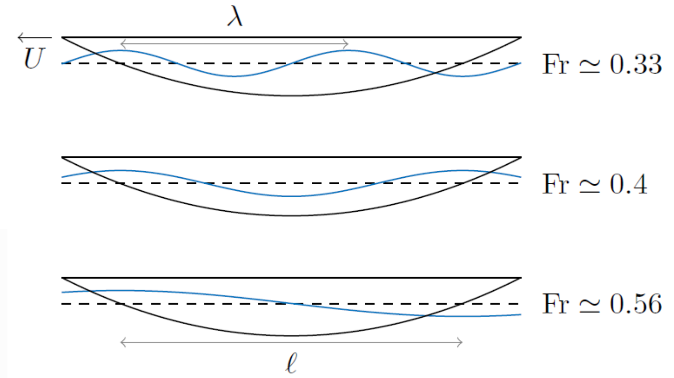
Combination of theoretical and empirical results.

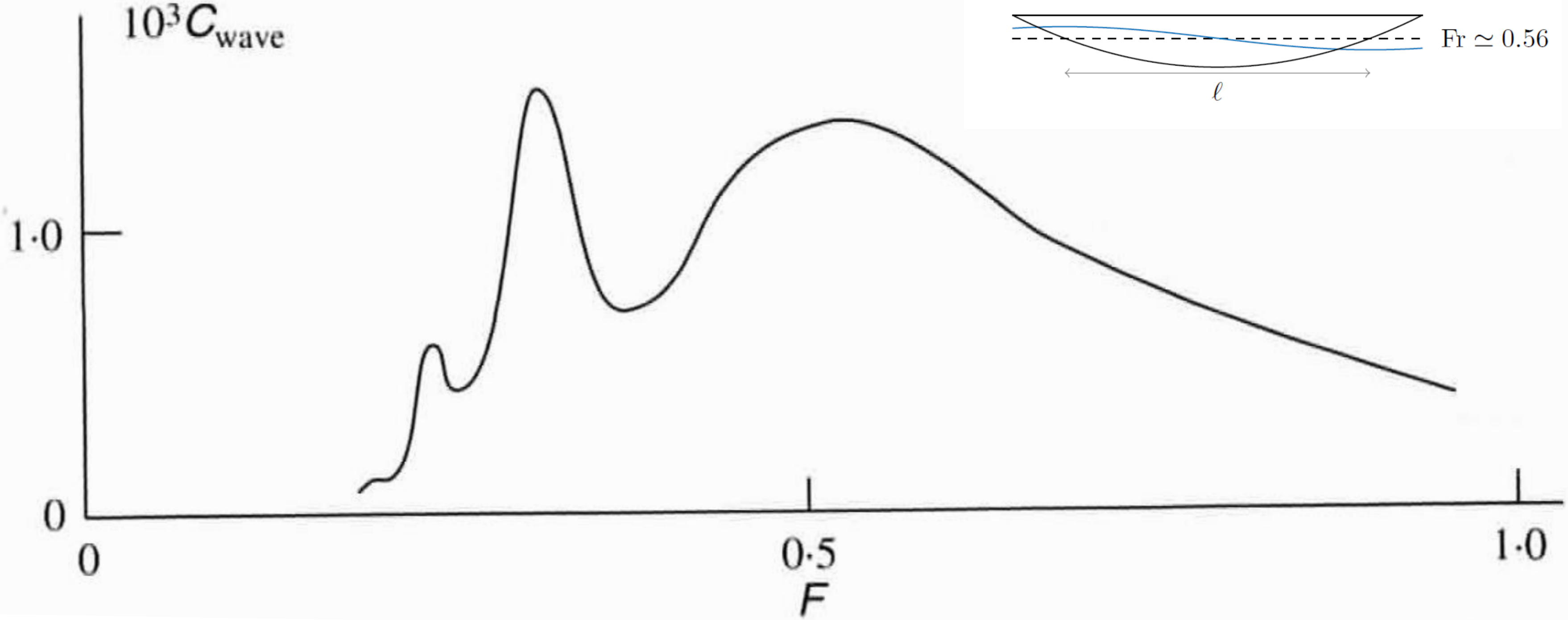
Generalisation to 3D

$$\frac{dE}{dt} = \frac{1}{2}\rho g \int_{-\infty}^{\infty} A^2(V_g \cos\theta - U)\, dz.$$

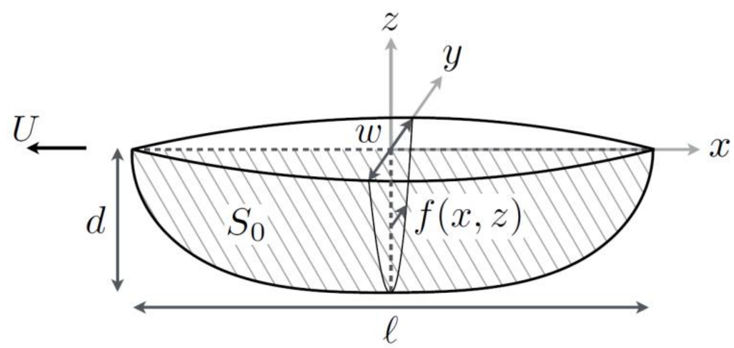
$$D = \frac{1}{2}\pi\rho U^2 \int_{-\pi/2}^{\pi/2} |A(\theta)|^2 \cos^3\theta \, d\theta.$$

Less interference


λ
U
Fr ≃ 0.33
Fr ≃ 0.4
Fr ≃ 0.56
ℓ


$10^3 C_{\text{wave}}$
1·0
0
0
0·5
1·0
F

Michell's theory



$$\mathcal{R}_{\mathrm{w}}(f,h) = \frac{2\rho U^2}{\pi \ell^2 \mathrm{Fr}^4} \int_{k_h}^{+\infty} |\mathcal{J}_f(k,\mathrm{Fr},h)|^2 \frac{k^{3/2}\tanh(kh)}{\sqrt{k - k_0\tanh(kh)}}\,\mathrm{d}k$$

$$\mathcal{J}_f(k,\mathrm{Fr},h) = \int_{-d}^{0} \mathrm{d}z \int_{-\frac{\ell}{2}}^{\frac{\ell}{2}} f(x,z)\frac{\cosh[k(z+h)]}{\cosh(kh)} e^{ix\sqrt{k_0 k\tanh(kh)}}\,\mathrm{d}x\,.$$

$$k_h = k_0 \tanh(k_h h)$$

$$k_0 = g/U^2$$

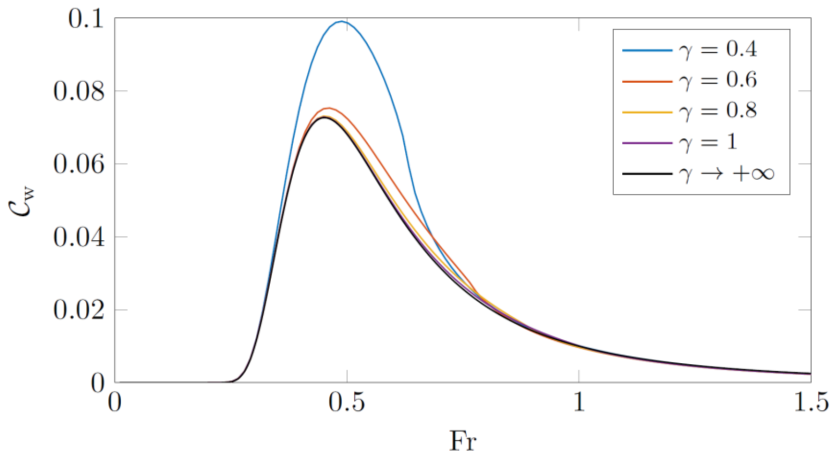


Figure 1.14: Wave drag coefficient $\mathcal{C}_{\mathrm{w}}$ as a function of the Froude number Fr predicted by Michell's model for increasing water depth $\gamma = h/\ell$ (for $\alpha = 6.7$, $\beta = 2.3$ and $\tilde{f} = 1/2\exp(-16\tilde{x}^2)$). The black curve corresponds to the infinite depth case.

J.-P. Boucher PhD thesis

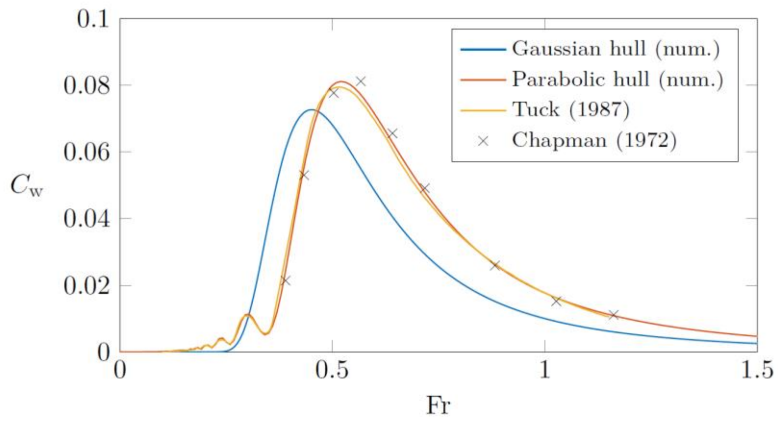


Figure 3.4: Wave-drag coefficient C_w as function of the Froude number Fr for a gaussian hull and a parabolic hull for $\alpha = 6.7$ and $\beta = 2.3$. These results are compared to the theoretical curve from [32] and experimental data points from [31] (black crosses).

Figure 4.2: Pictures of (a) a sprint canoe, (b) a sprint kayak, and (c) a single scull rowing boat. The three boats are moving from right to left. Pictures have been rescaled (see Table 2.1 for the characteristics of rowing boats and Table 5.1 for the characteristics of sprint canoes and sprint kayaks).

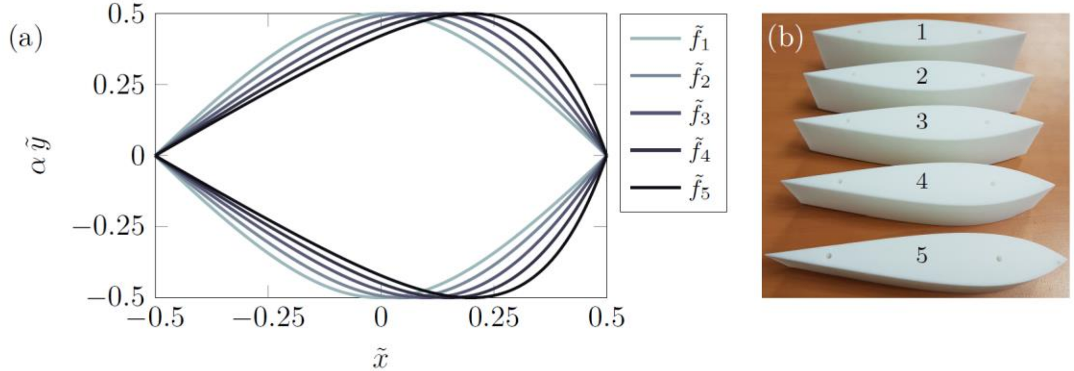


Figure 4.5: (a) Profile $\alpha\tilde{y} = \pm\tilde{f}(\tilde{x})$ for five different sets of the parameters $(a,\ b,\ \alpha,\ \beta)$ in Eq. (4.3), with increasing asymmetry. (b) Picture of the five 3d-printed model hulls defined by the functions $\tilde{f}_1$ to $\tilde{f}_5$. They are 18 cm-long, 3 cm-wide and 5 cm-high.

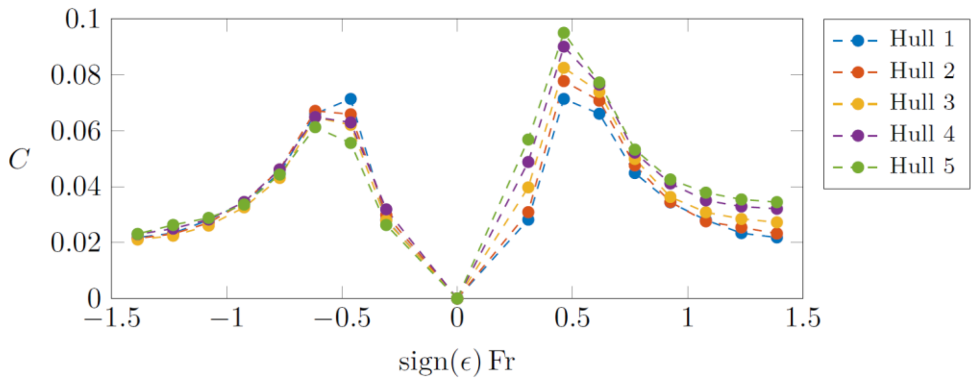


Figure 4.8: Total drag coefficient as a function of the signed Froude number $\mathrm{sign}(\epsilon)\,\mathrm{Fr}$ for the five hulls for $d/D = 0.75$. The use of the parameter $\mathrm{sign}(\epsilon)\,\mathrm{Fr}$ allows us to compare a given hull moving with the rounded part first ($\epsilon > 0$, right part of the plot) or with the pointed part first ($\epsilon < 0$, left part of the plot) (see Fig. 4.4).

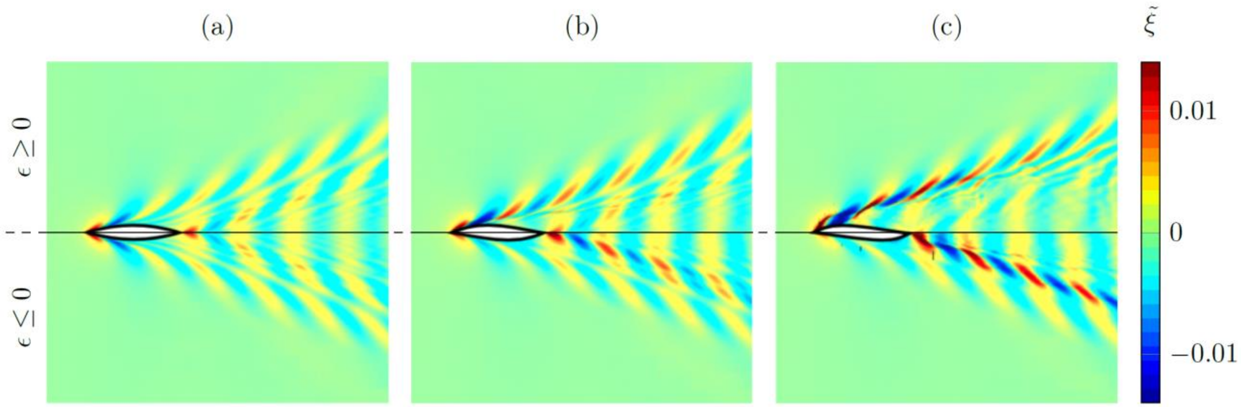
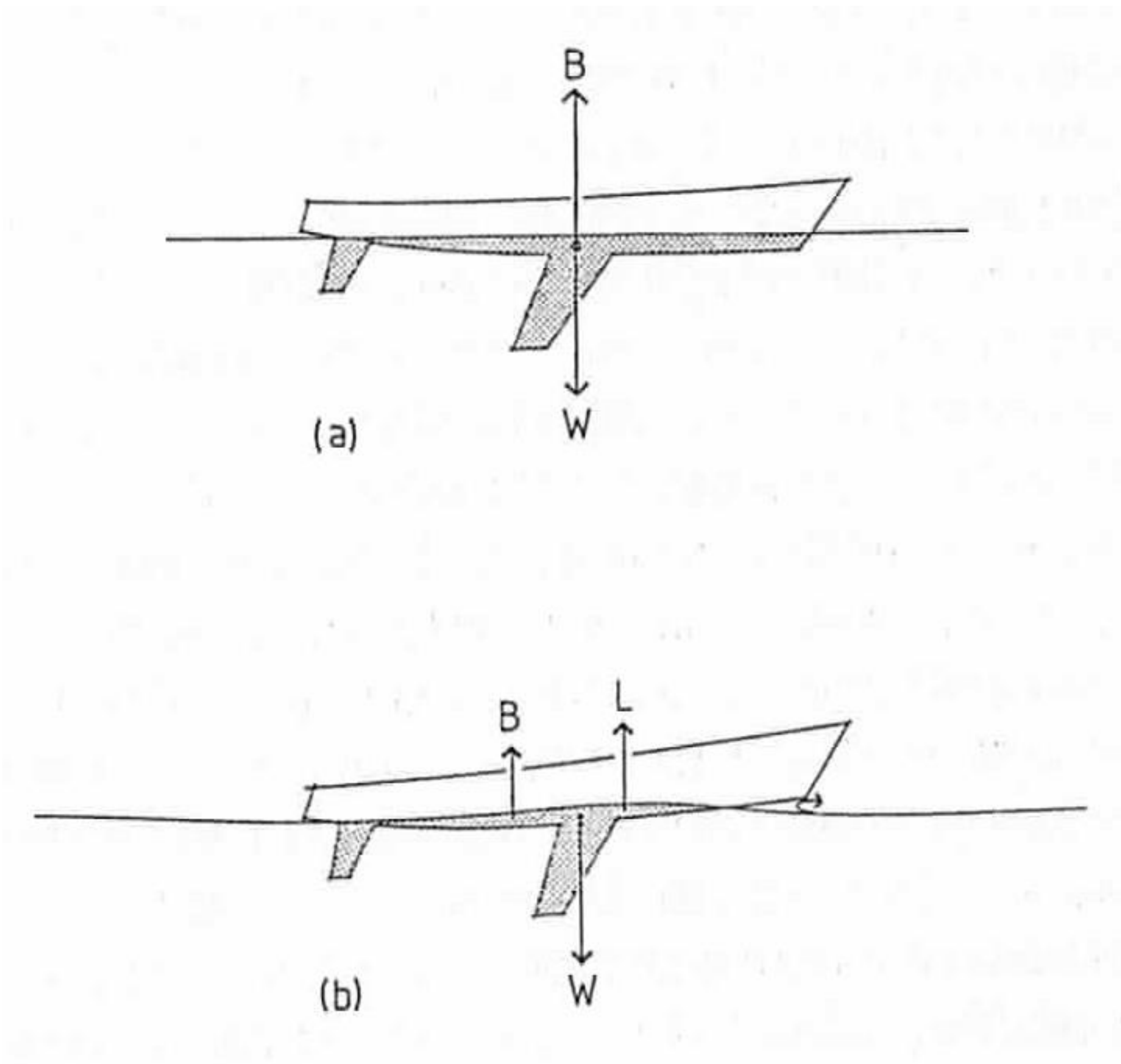
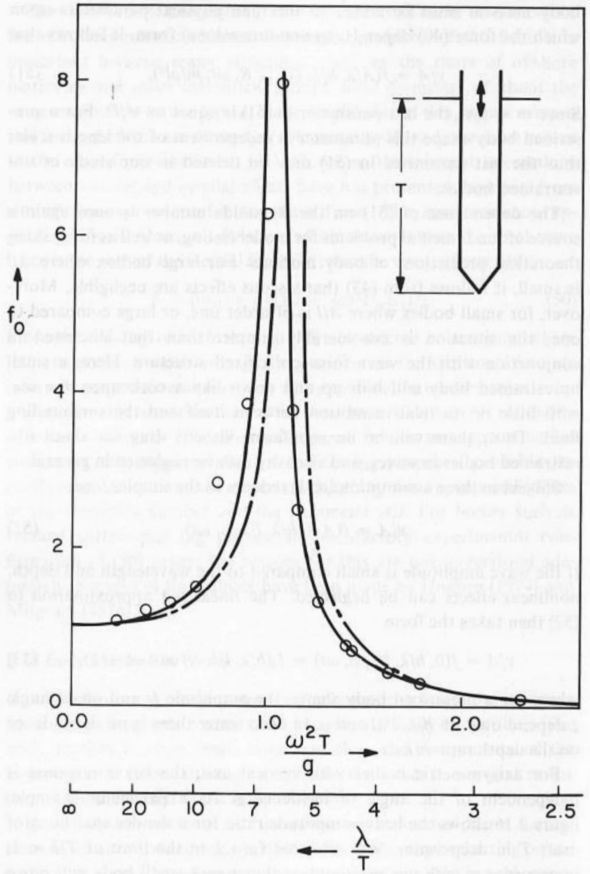


Figure 4.15: Dimensionless wave elevation $\tilde{\xi}$ obtained numerically for hulls 1, 3 and 5 (for $\mathrm{Fr} = 0.3$ and $d/D = 0.5$). For each subplot, the upper half corresponds to the hull moving with the rounded part first ($\epsilon \geq 0$), while the lower half corresponds to the hull moving with the pointed part first ($\epsilon \leq 0$).

This picture is in reality complicated by the floating position of the ship



Which opens the field of dynamical reaction of ships to waves….



2.16
Heave response of a slender spar buoy in regular waves, from Adee and Bai (1970). The ordinate is the ratio of heave amplitude to wave amplitude, and the buoy is a circular cylinder, with a conical bottom, as shown to scale in the sketch. The dashed line is a theoretical prediction that neglects the hydrodynamic forces due to the motions of the body. The solid line includes a correction for the added mass. The circles denote experimental measurements.