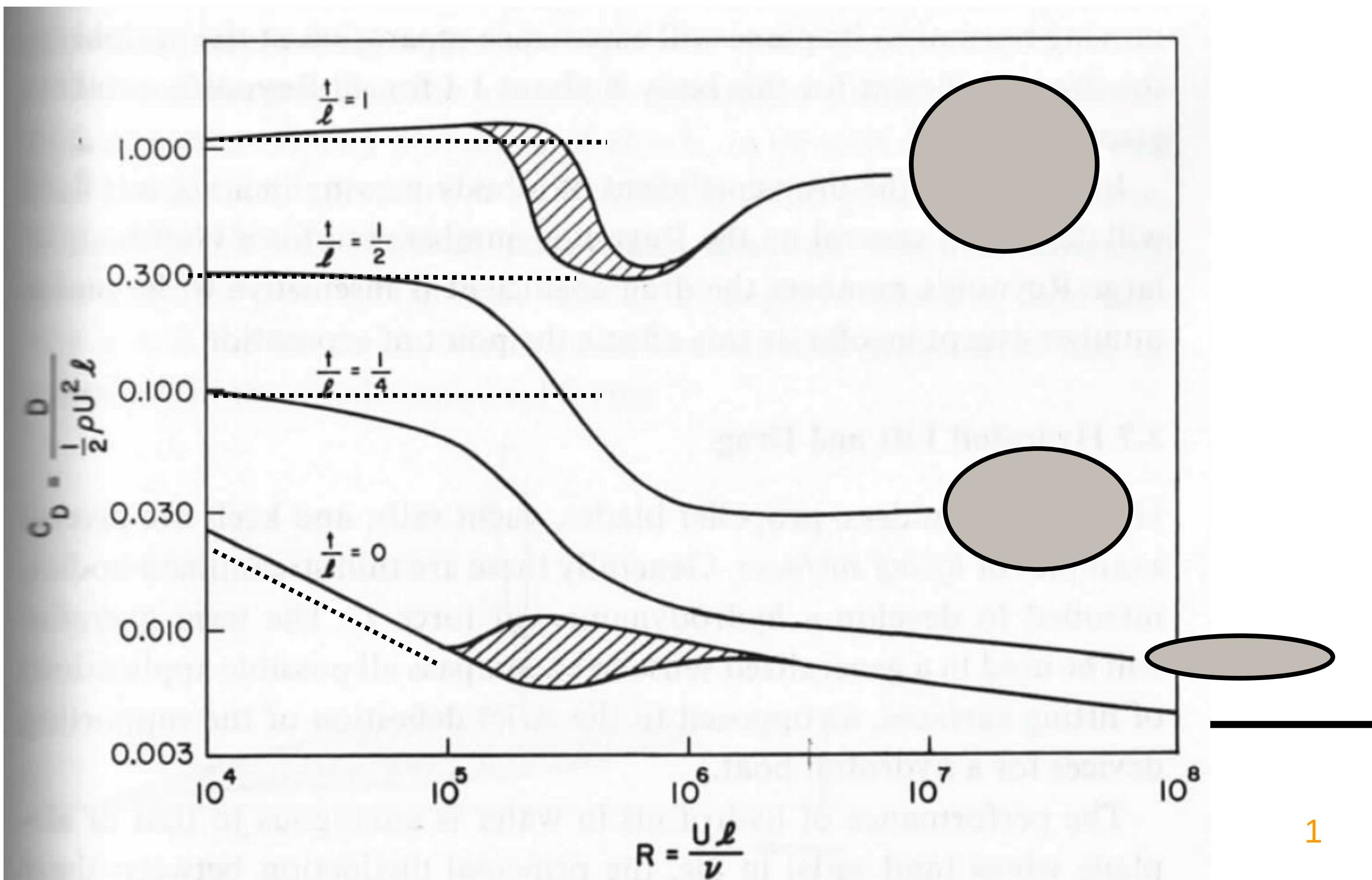
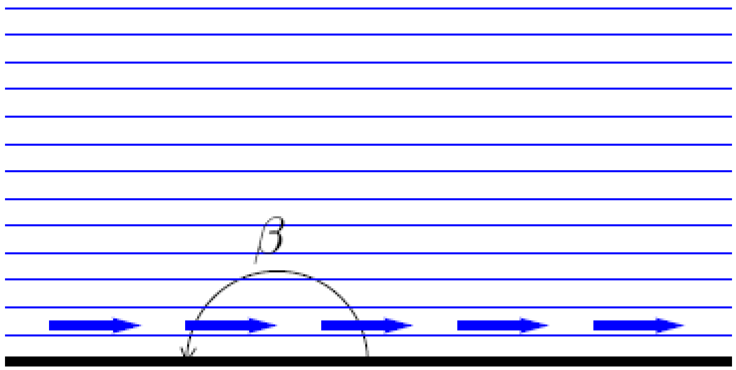
But this does still not explain the aerodynamic drag scaling: $\frac{1}{2}\rho_\infty U_\infty{}^2 S$



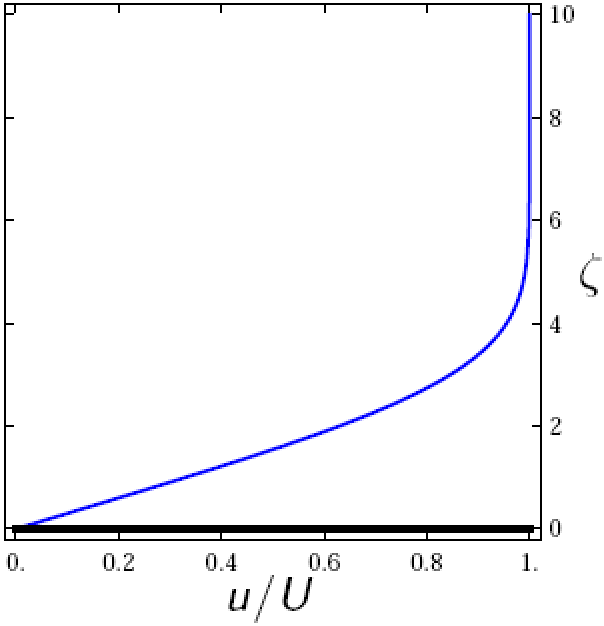
Pressure gradient effect

Le cas de référence est le cas $m = 0$ i.e. $\beta = \pi/(m+1) = \pi$ correspondant à l'**écoulement au dessus d'une plaque plane** :



écoulement uniforme

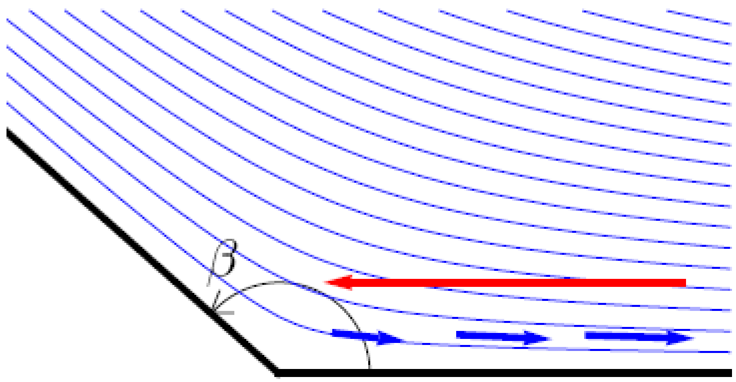
∇p **nul**

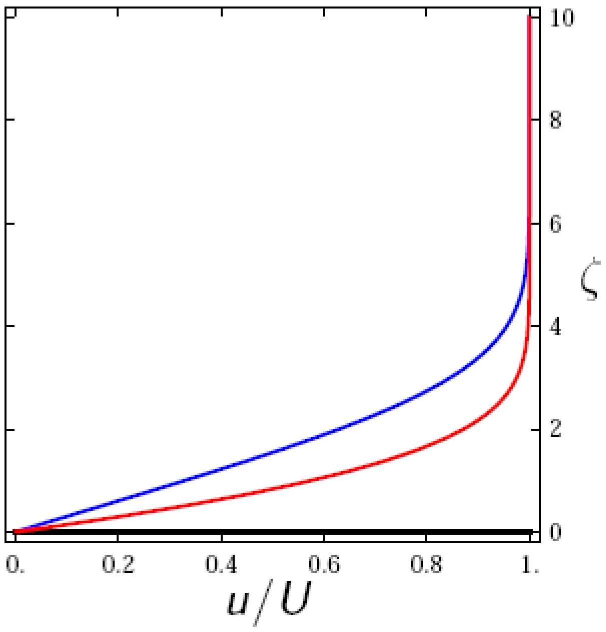


$\longrightarrow$ **couche limite de Blasius**

Pressure gradient effect

Si $m > 0$ on a $\beta = \pi/(m+1) < \pi$ correspondant à l'**écoulement au dessus d'un dièdre rentrant** :



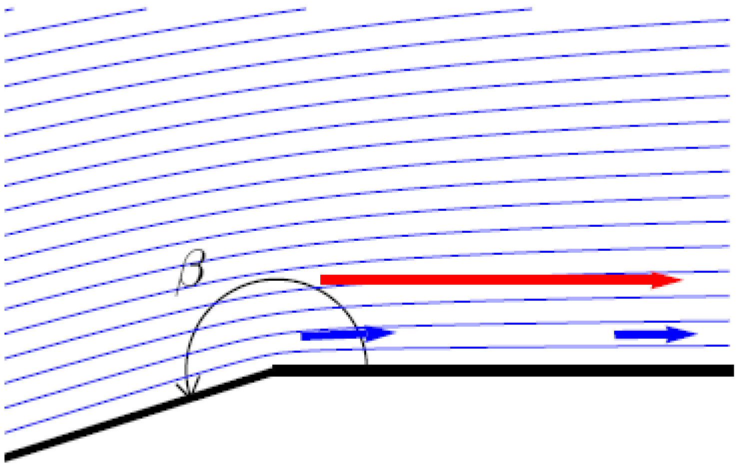


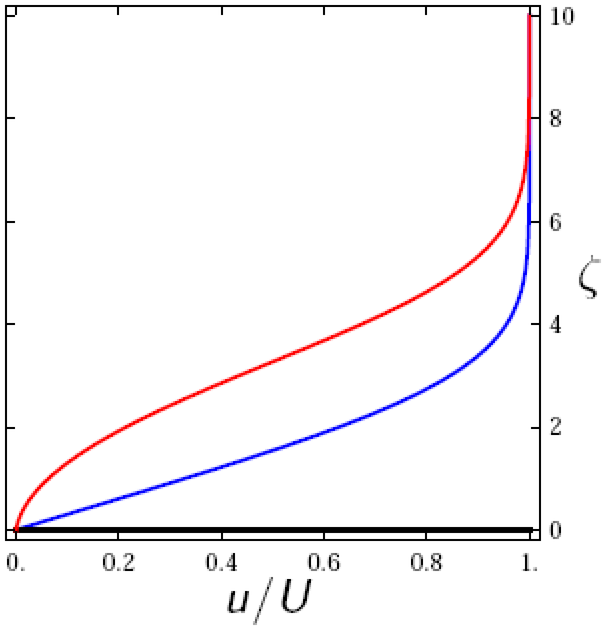
écoulement accéléré

$\overline{\nabla} p$ **accélérateur** → **couche limite + collée**

Pressure gradient effect

Si $m < 0$ on a $\beta = \pi/(m+1) > \pi$ correspondant à l'**écoulement au dessus d'un dièdre saillant** :





écoulement ralenti

$\overline{\nabla} p$ **décélérateur**

$\longrightarrow$ **couche limite + épaisse**

Pressure gradient effect

Adverse pressure gradient : $$\frac{\partial p}{\partial x} > 0$$

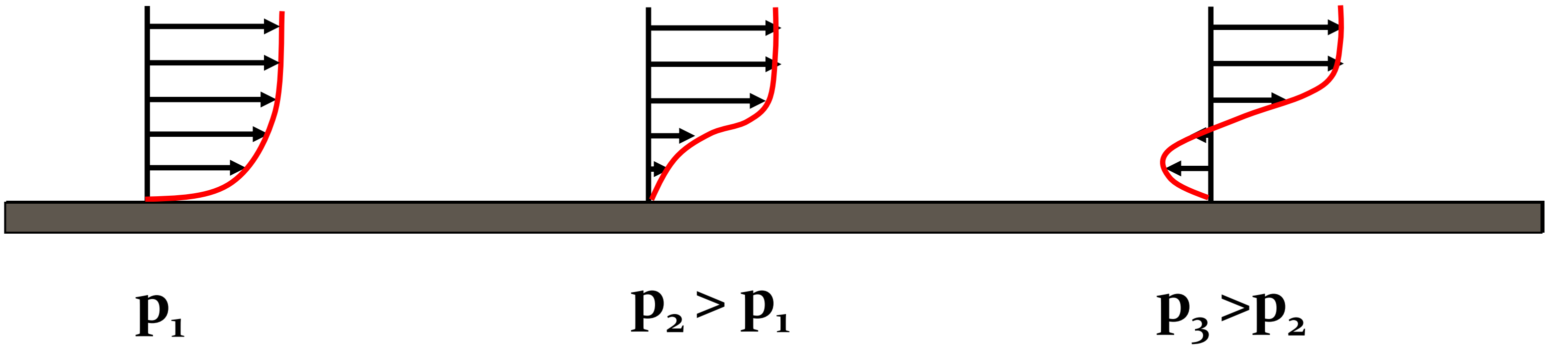
p_1 $p_2 > p_1$ $p_3 > p_2$

$p(x)$ $p(x+dx)$

Resulting pressure force

Pressure gradient effect

Adverse pressure gradient : $\frac{\partial p}{\partial x} > 0$



Close to the wall, the viscous effects dominate

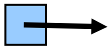
The pressure gradient further decreases the velocity

⇨Detachement

Pressure gradient effect

Favorable pressure gradient: $\frac{\partial p}{\partial x} < 0$

p_1 $p_2 < p_1$ $p_3 < p_2$

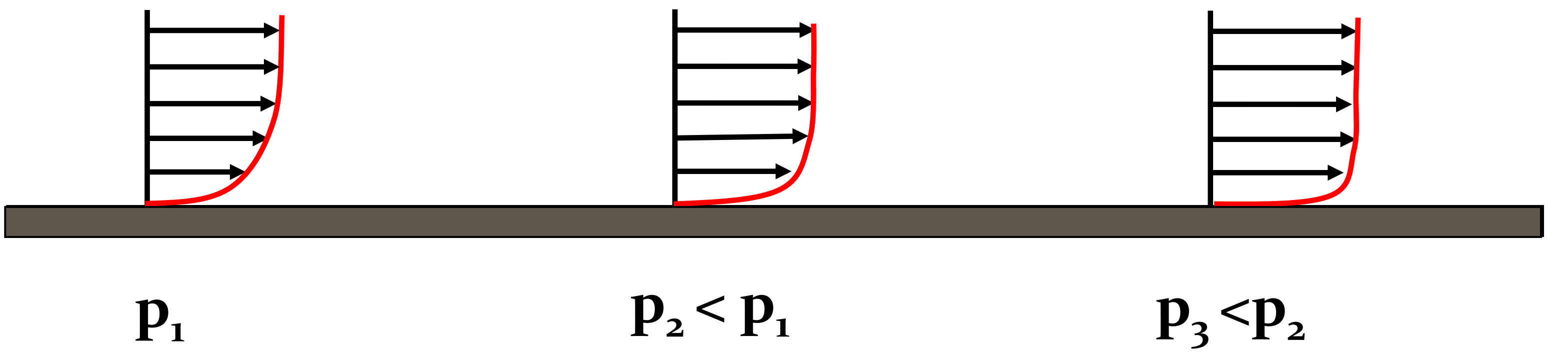


$p(x)$ $p(x+dx)$

Resulting pressure force

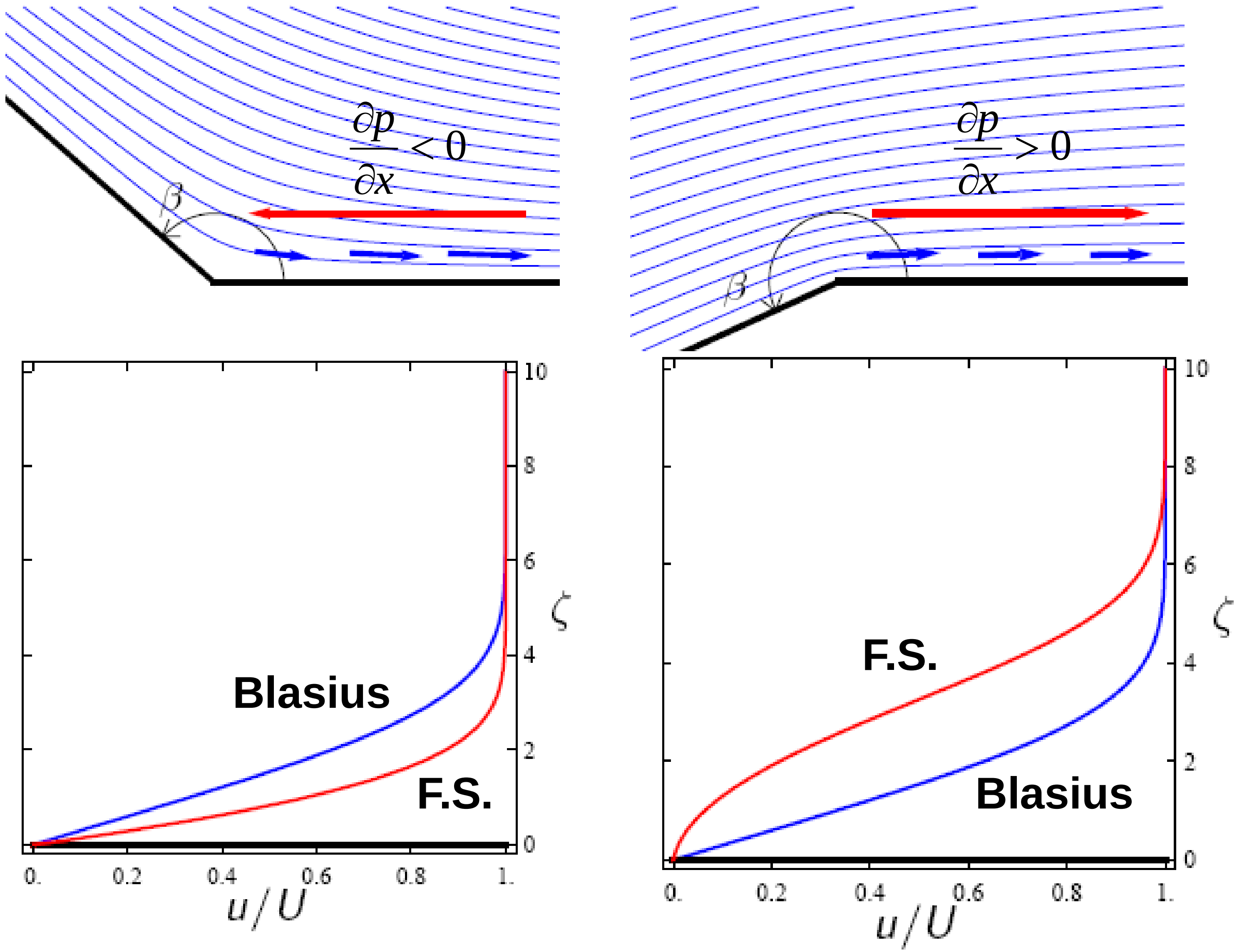
Pressure gradient effect

Favorable pressure gradient: $\frac{\partial p}{\partial x} < 0$



Close to the wall, the pressure gradient further increases the velocity of the flow ⇨ no detachement

Pressure gradient effect



Falkner-Skan solutions

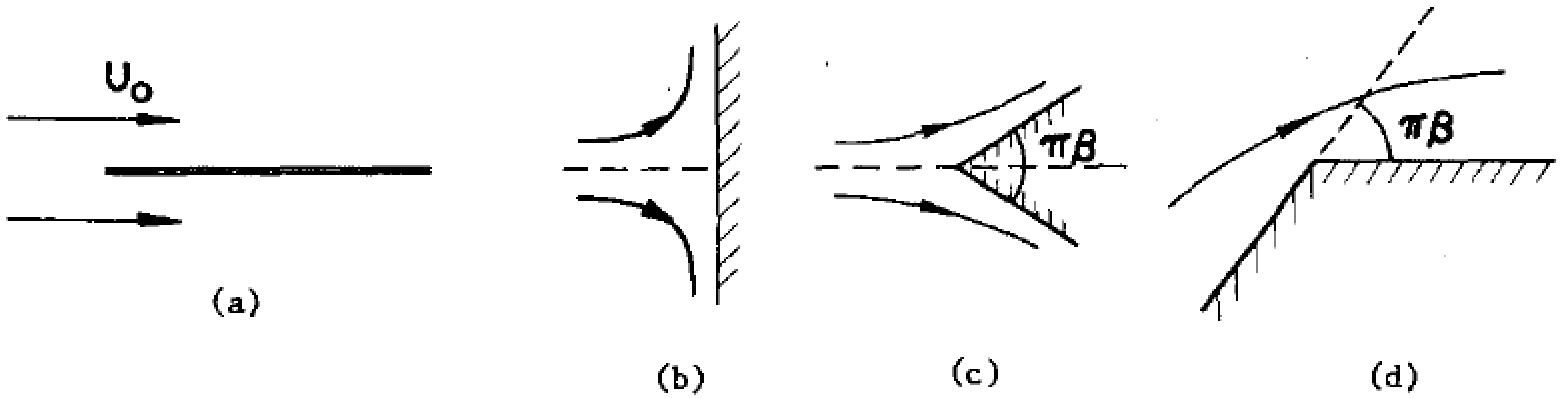
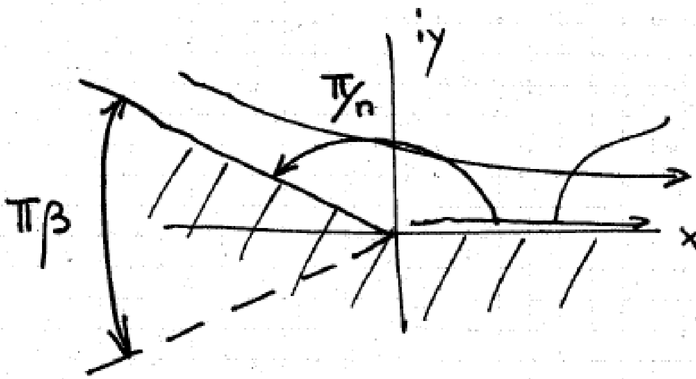


Figure 5.2 Boundary layer flows represented by solutions of the Falkner-Skan equation for different values of the parameter m: (a) $m = 0$; (b) $m = 1$; (c) $0 < m < 1$; (d) $-1/2 < m < 0$

Falkner-Skan far field solutions

pot. complexe

$$F(z) = C z^{n}$$



$U \propto x^{m}$

$$\frac{dF}{dz} = C n z^{n-1} = v_x - i v_y \longrightarrow \boxed{n = 1+m}$$

$$\frac{\pi}{n} + \frac{\pi\beta}{2} = \pi \longrightarrow \frac{1}{1+m} + \frac{\beta}{2} = 1$$

$$\boxed{\beta = \frac{2m}{1+m}}$$

Falkner-Skan boundary layer equations

1. Prandtl equations

$$\hat{\psi}_{\hat{y}}\,\hat{\psi}_{x\hat{y}} - \hat{\psi}_x\,\hat{\psi}_{\hat{y}\hat{y}} = U\frac{\mathrm{d}U}{\mathrm{d}x} + \hat{\psi}_{\hat{y}\hat{y}\hat{y}},$$

$$\hat{\psi} = \hat{\psi}_{\hat{y}} = 0 \text{ on } \hat{y} = 0, \quad \hat{\psi}_{\hat{y}} \to U(x) \text{ as } \hat{y} \to \infty.$$

Falkner-Skan boundary layer equations

1. Prandtl equations

$$\hat{\psi}_{\hat{y}}\,\hat{\psi}_{x\hat{y}} - \hat{\psi}_x\,\hat{\psi}_{\hat{y}\hat{y}} = U\frac{\mathrm{d}U}{\mathrm{d}x} + \hat{\psi}_{\hat{y}\hat{y}\hat{y}},$$

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2. Self-similar solution

$$\hat{\psi}(x, \hat{y}) = (Ax^{m+1})^{1/2} f(\eta) \text{ where } \eta = \hat{y}(Ax^{m-1})^{1/2}.$$

Falkner-Skan boundary layer equations

1. Prandtl equations

$$\hat{\psi}_{\hat{y}}\,\hat{\psi}_{x\hat{y}} - \hat{\psi}_x\,\hat{\psi}_{\hat{y}\hat{y}} = U\frac{\mathrm{d}U}{\mathrm{d}x} + \hat{\psi}_{\hat{y}\hat{y}\hat{y}},$$

$$\hat{\psi} = \hat{\psi}_{\hat{y}} = 0 \text{ on } \hat{y} = 0, \quad \hat{\psi}_{\hat{y}} \to U(x) \text{ as } \hat{y} \to \infty.$$

2. Self-similar solution

$$\hat{\psi}(x, \hat{y}) = (Ax^{m+1})^{1/2} f(\eta) \text{ where } \eta = \hat{y}(Ax^{m-1})^{1/2}.$$

3. Falkner-Skan equation

$$f''' + \tfrac{1}{2}(m+1)\,ff'' + m(1 - f'^2) = 0$$

$$f(0) = f'(0) = 0, \quad f'(\infty) = 1$$

Falkner-Skan boundary layer solutions

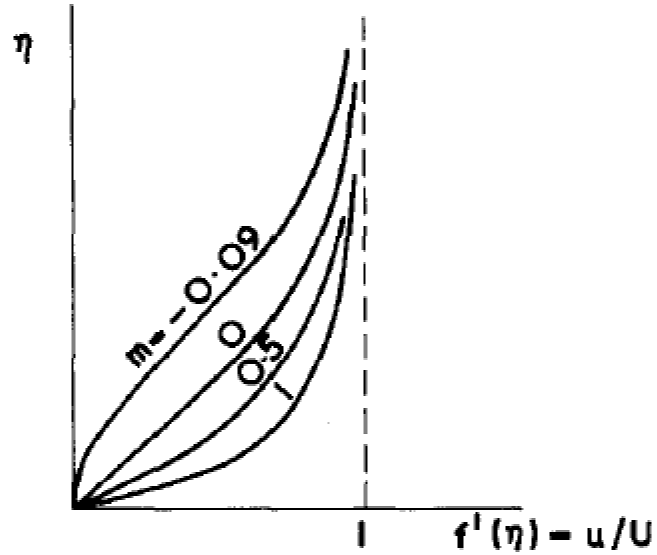
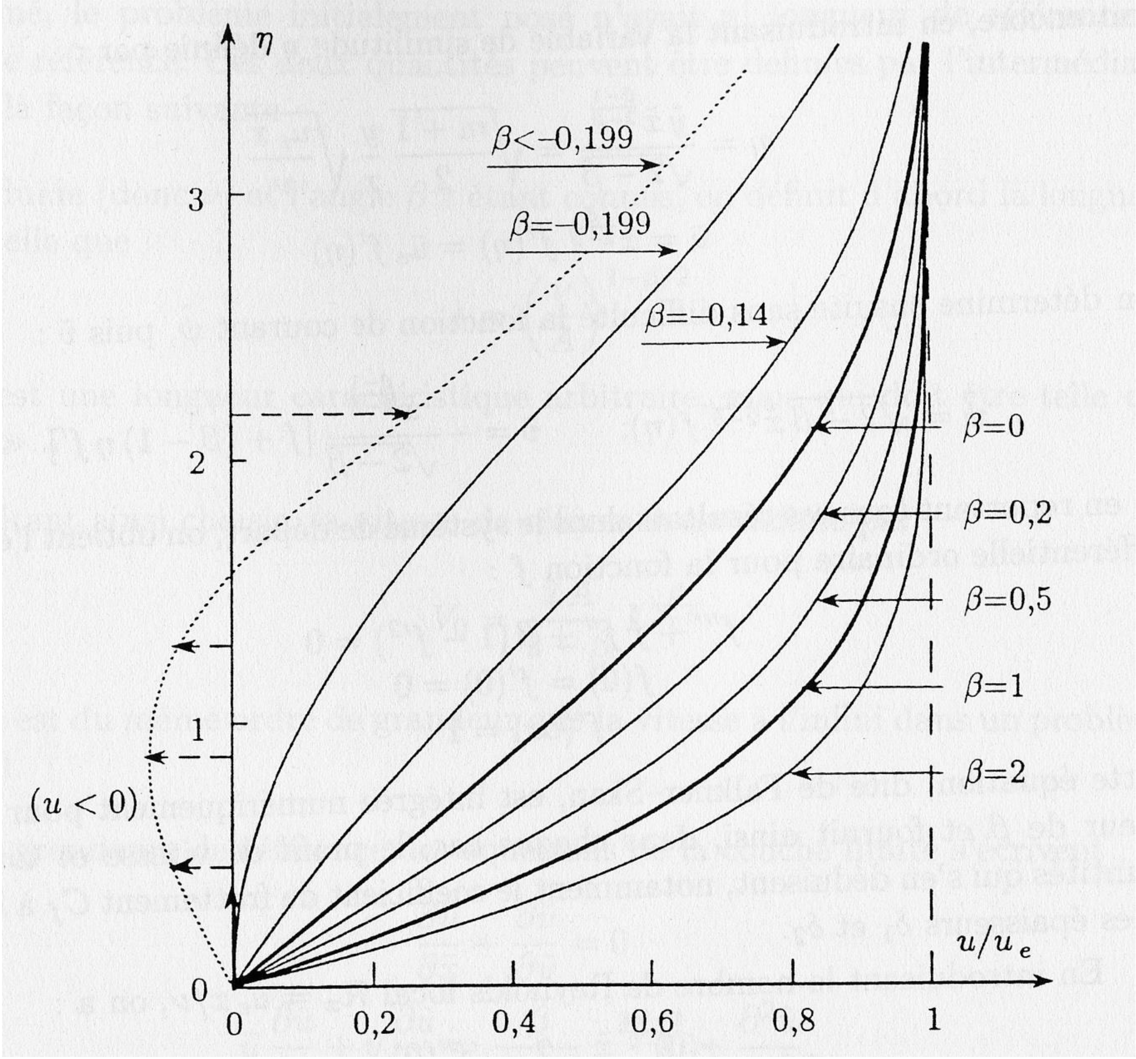
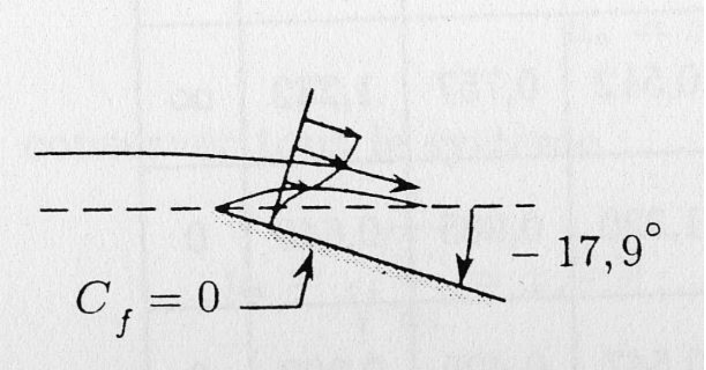
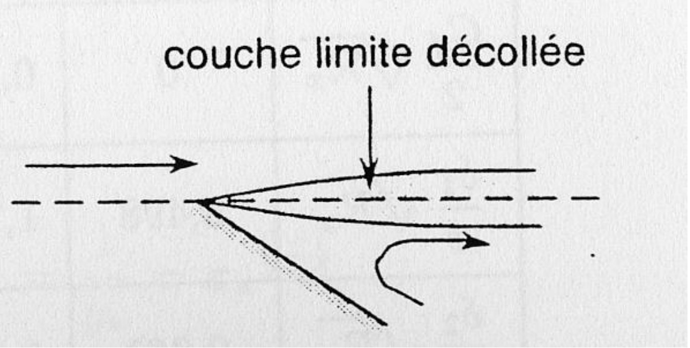


Figure 5.3 Sketch of velocity profiles given by solutions of the Falkner–Skan equation

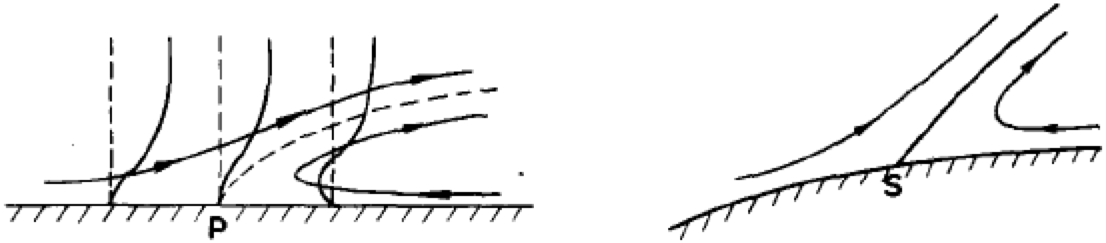
Falkner-Skan boundary layer solutions







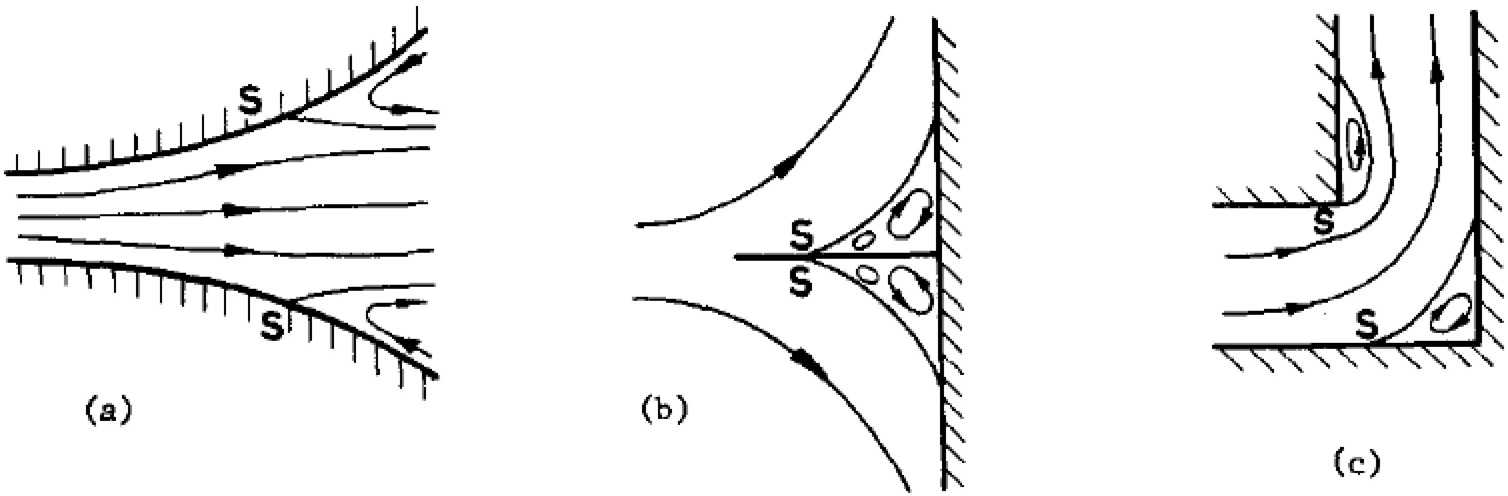
Boundary layer separation



$$\hat{\psi} \sim (x - x_S)^{1/2}, \quad \text{so that } \frac{\partial \hat{\psi}}{\partial x} \sim (x - x_S)^{-1/2} \text{ as } x \to x_S.$$

Boundary layer separation

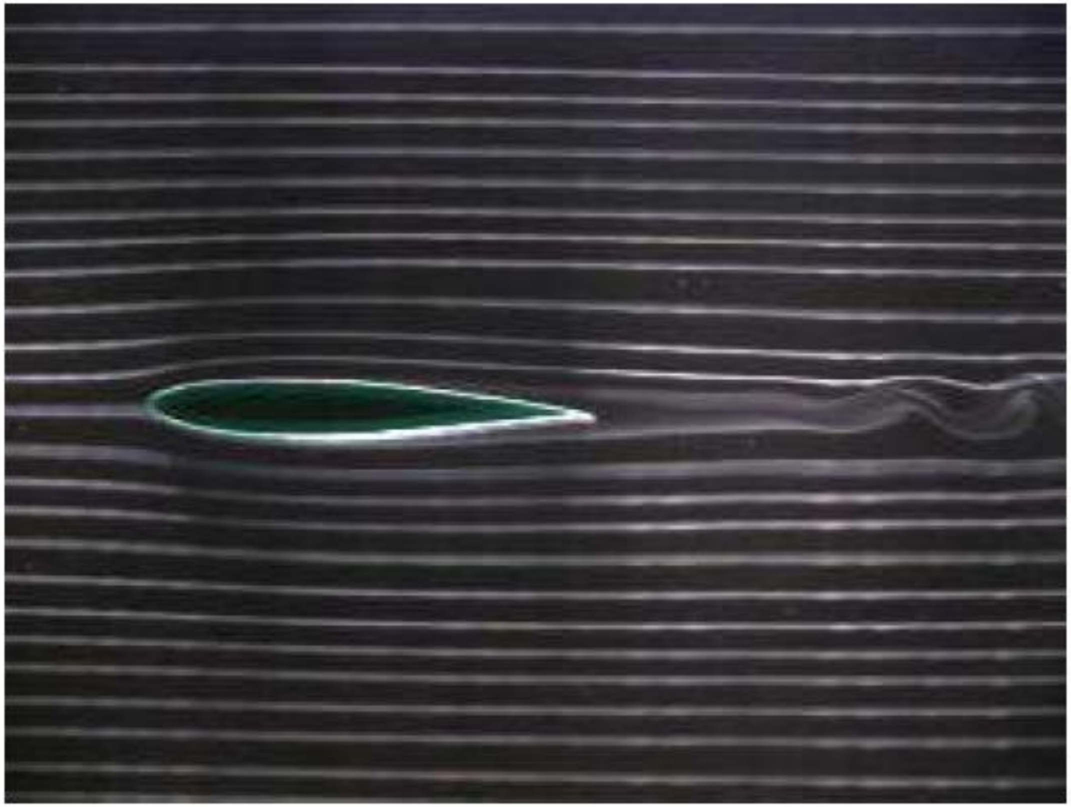
Boundary layer separation



Decollement sur un profil d'aile

Expériences en soufflerie menées à l'université de Stanford, l'écoulement est visualisé grâce à des fumées :

angle d'incidence $\boxed{\gamma = 0°}$:

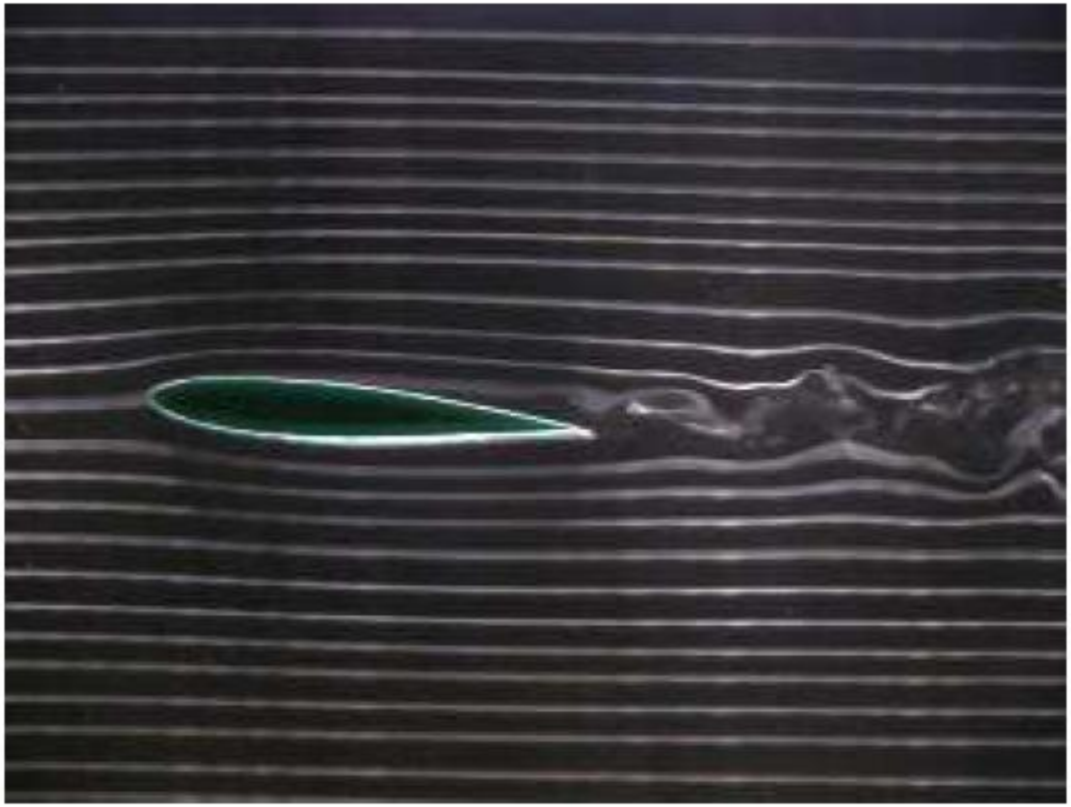


[DVD 'Multimedia Fluid Mechanics', Homsy et al. 2004, Cambridge University Press]

Décollement sur un profil d'aile

Expériences en soufflerie menées à l'université de Stanford, l'écoulement est visualisé grâce à des fumées :

angle d'incidence $\boxed{\gamma = 5^\circ}$:

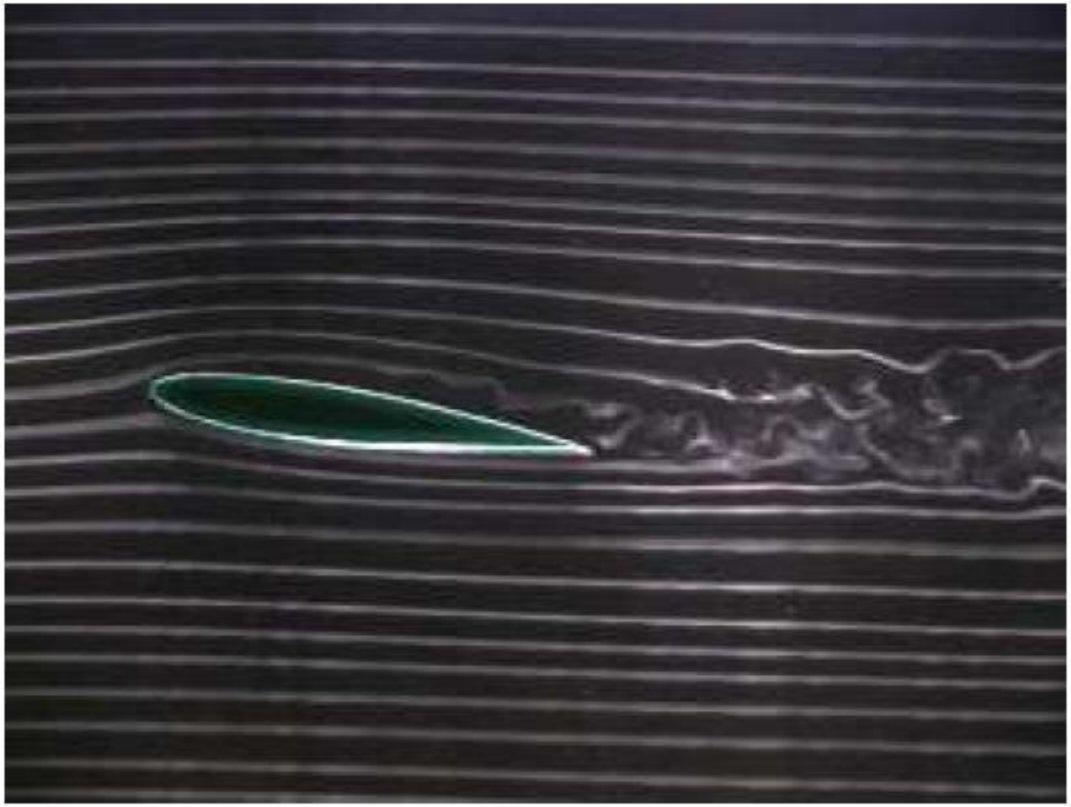


[DVD 'Multimedia Fluid Mechanics', Homsy et al. 2004, Cambridge University Press]

Décollement sur un profil d'aile

Expériences en soufflerie menées à l'université de Stanford, l'écoulement est visualisé grâce à des fumées :

angle d'incidence $\boxed{\gamma = 10°}$:

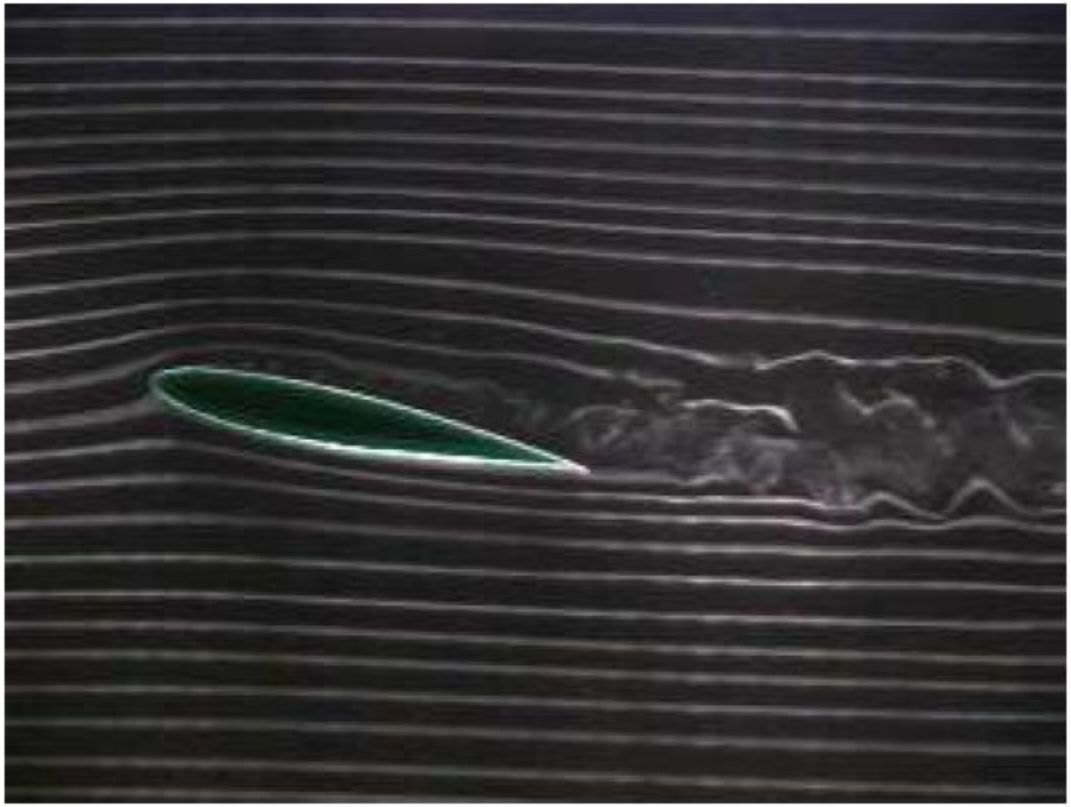


[DVD 'Multimedia Fluid Mechanics', Homsy et al. 2004, Cambridge University Press]

Décollement sur un profil d'aile

Expériences en soufflerie menées à l'université de Stanford, l'écoulement est visualisé grâce à des fumées :

angle d'incidence $\boxed{\gamma = 15^\circ}$:

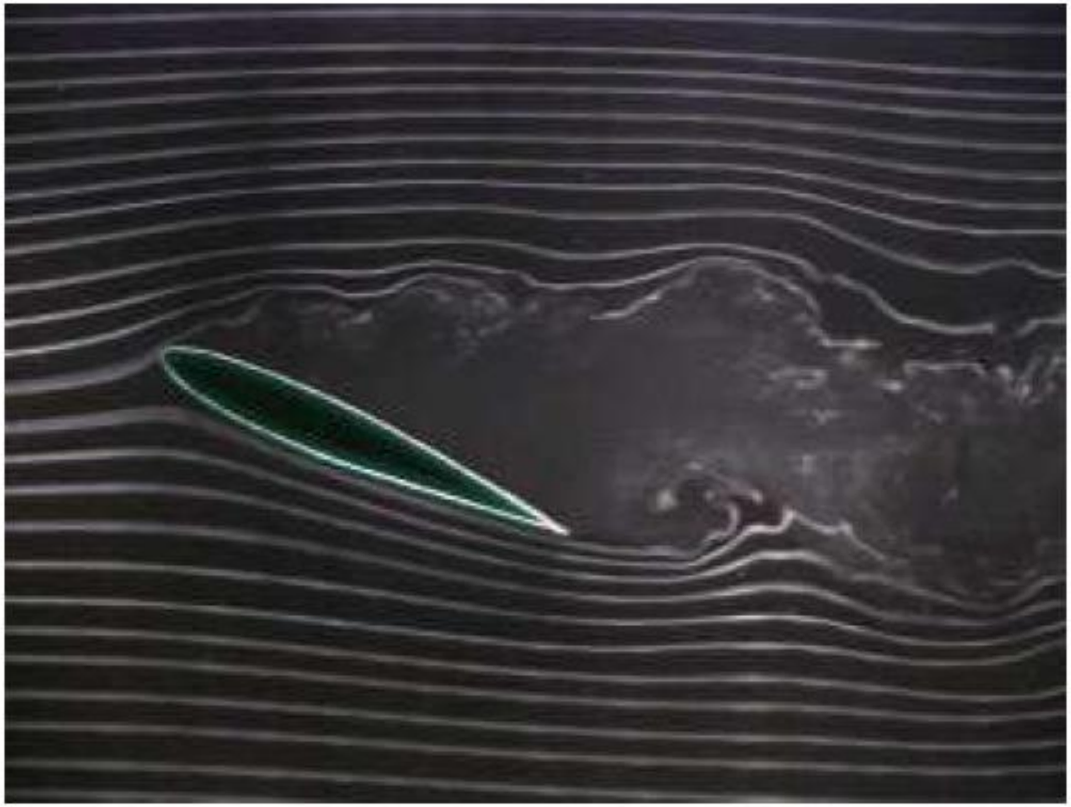


[DVD 'Multimedia Fluid Mechanics', Homsy et al. 2004, Cambridge University Press]

Décollement sur un profil d'aile

Expériences en soufflerie menées à l'université de Stanford, l'écoulement est visualisé grâce à des fumées :

angle d'incidence $\boxed{\gamma = 25^o}$:



[DVD 'Multimedia Fluid Mechanics', Homsy et al. 2004, Cambridge University Press]

Décollement sur un profil d'aile

Expériences en soufflerie menées à l'université de Stanford, l'écoulement est visualisé grâce à des fumées :

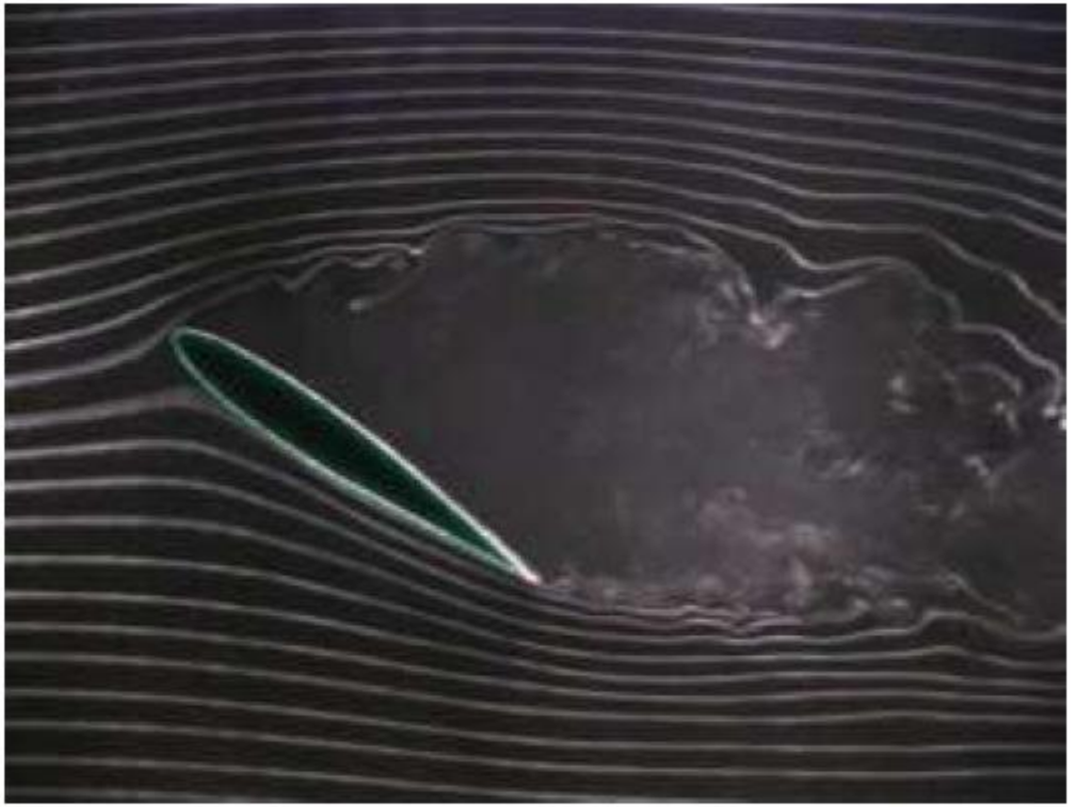
angle d'incidence $\boxed{\gamma = 30^\circ}$:

[DVD 'Multimedia Fluid Mechanics', Homsy et al. 2004, Cambridge University Press]

Décollement sur un profil d'aile

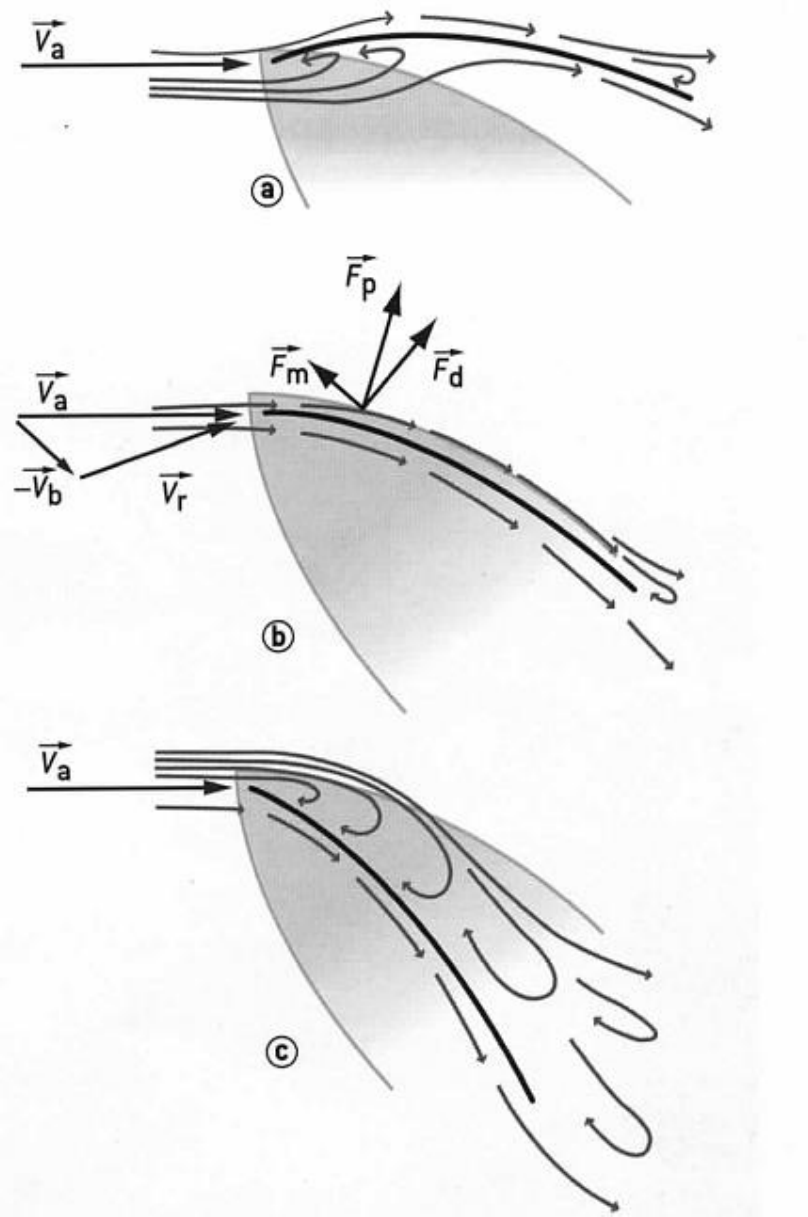
Expériences en soufflerie menées à l'université de Stanford, l'écoulement est visualisé grâce à des fumées :

angle d'incidence $\boxed{\gamma = 35^\circ}$:



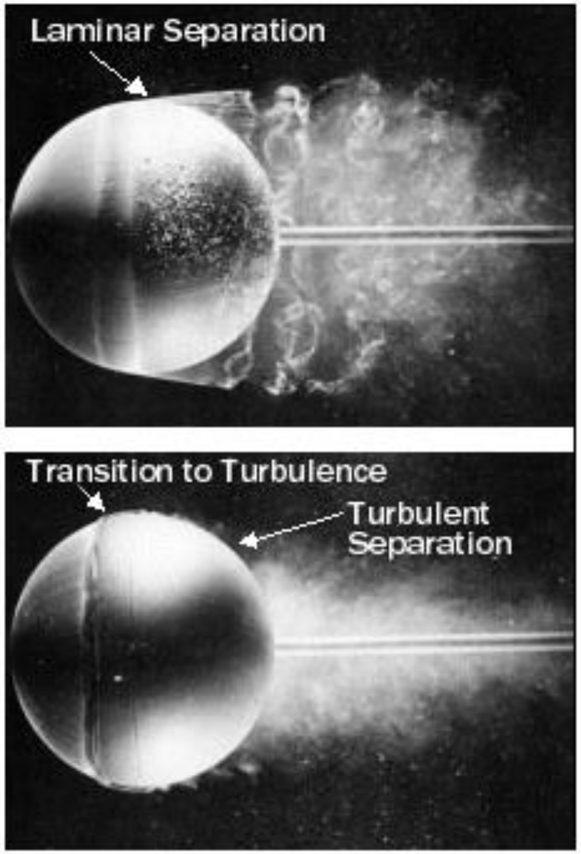
[DVD 'Multimedia Fluid Mechanics', Homsy et al. 2004, Cambridge University Press]

Application to sailing



Application to sailing

Example: Flow around a sphere



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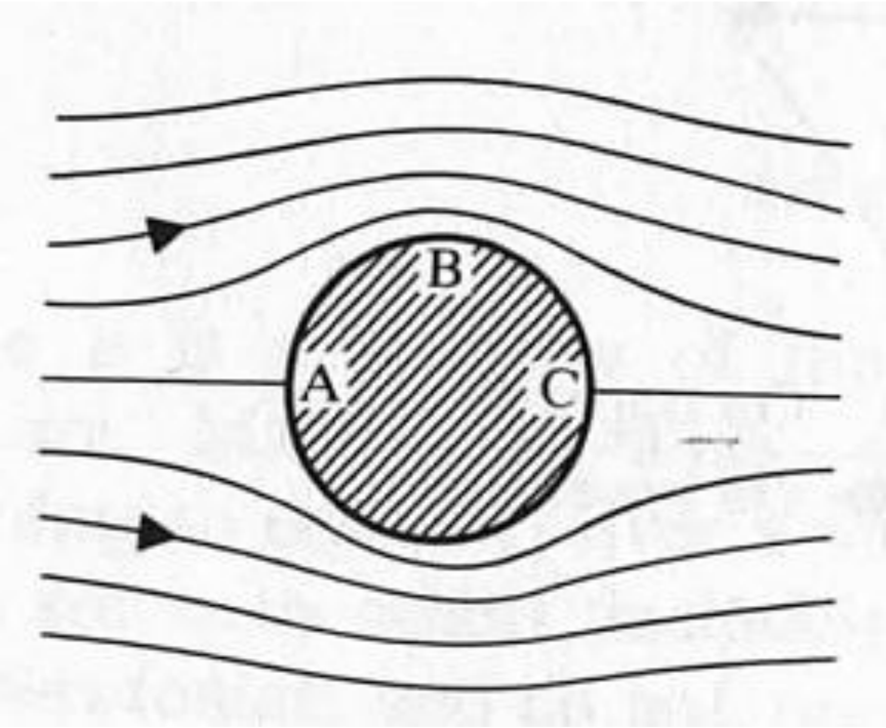
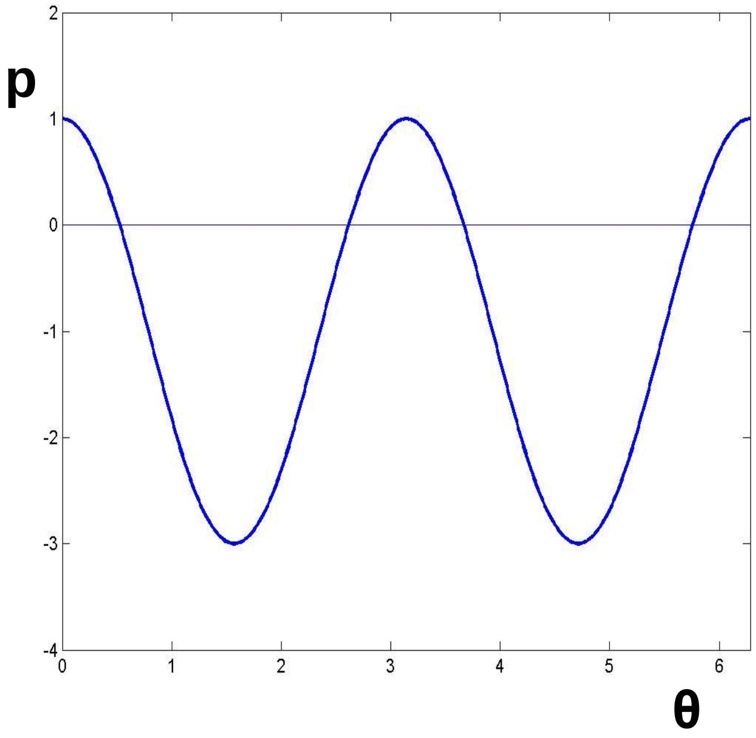
Flow around a cylinder

$$p(a,\theta) = \frac{1}{2}\rho U_\infty^2 (1 - 4\sin\theta^2)$$





Flow around a cylinder

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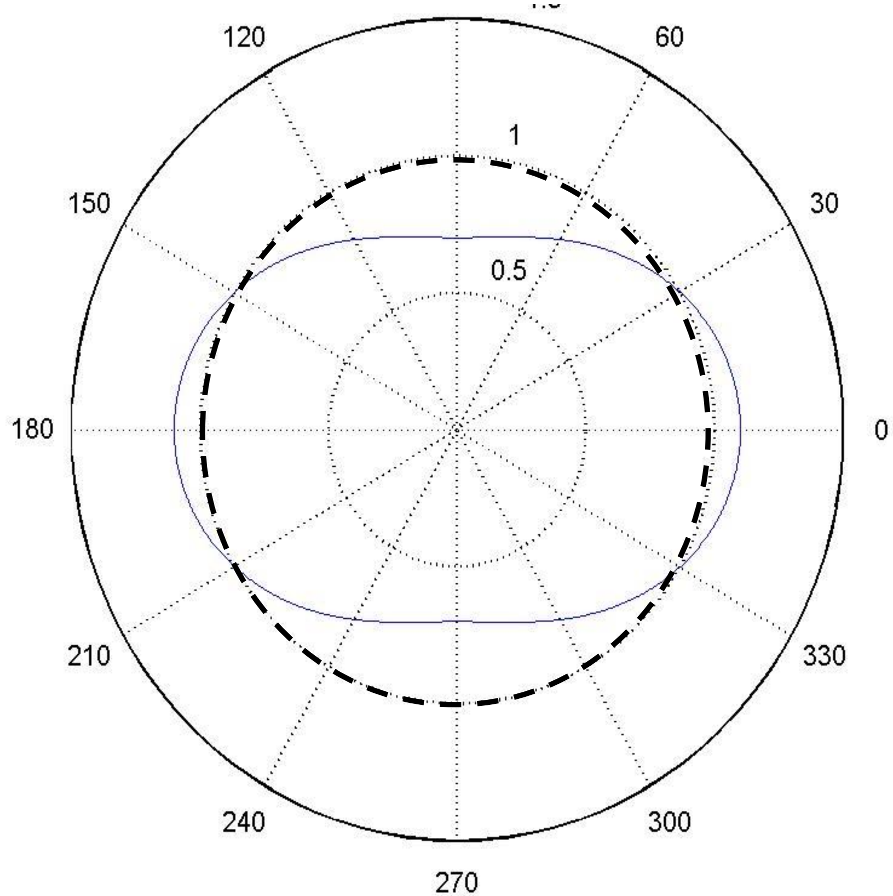
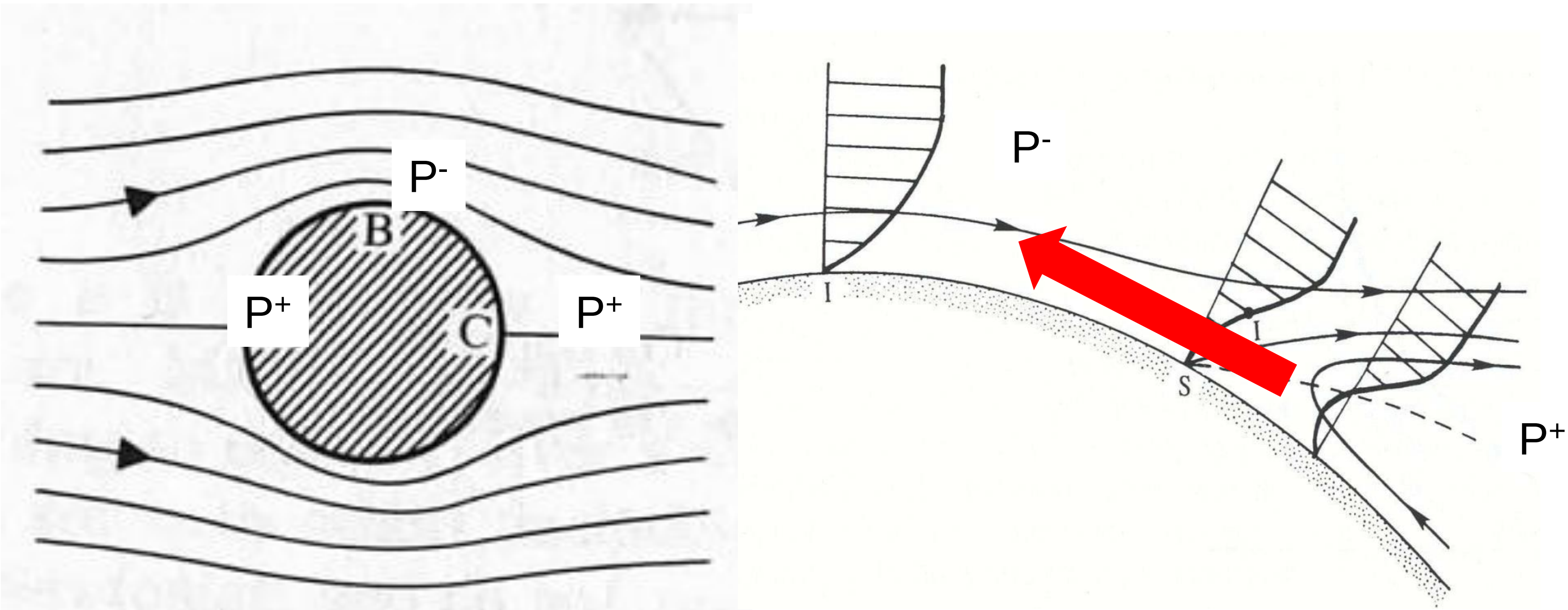
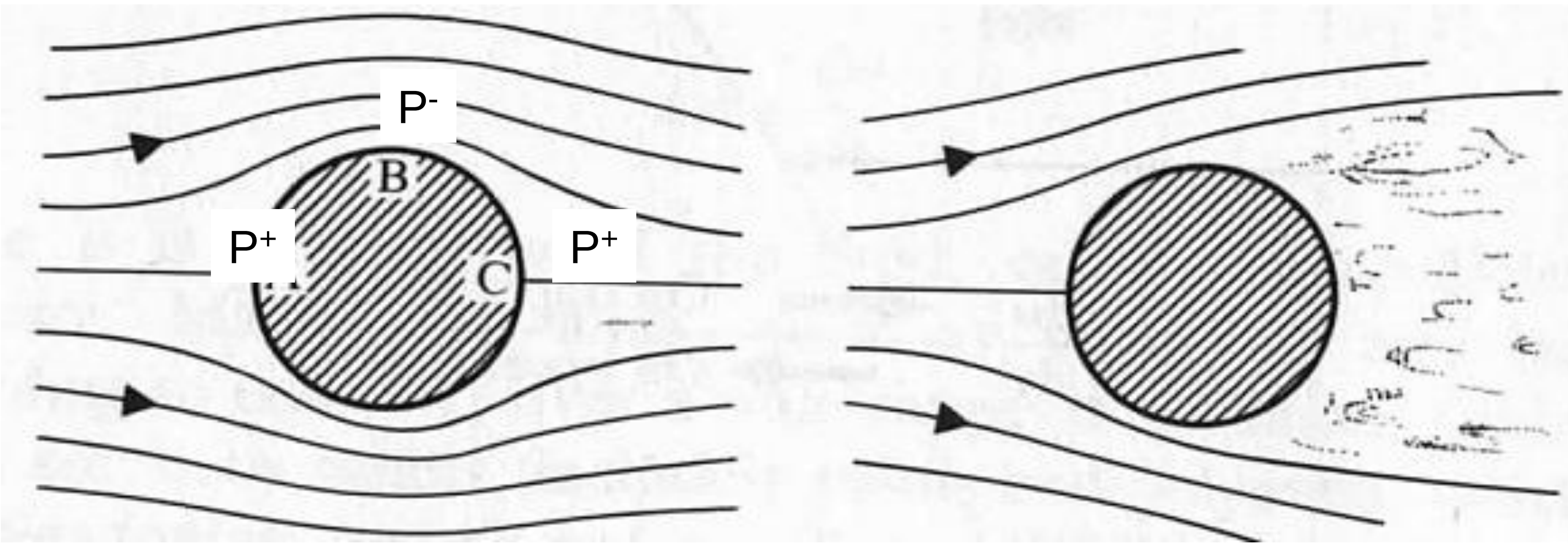


Origin of detachment: pressure gradient

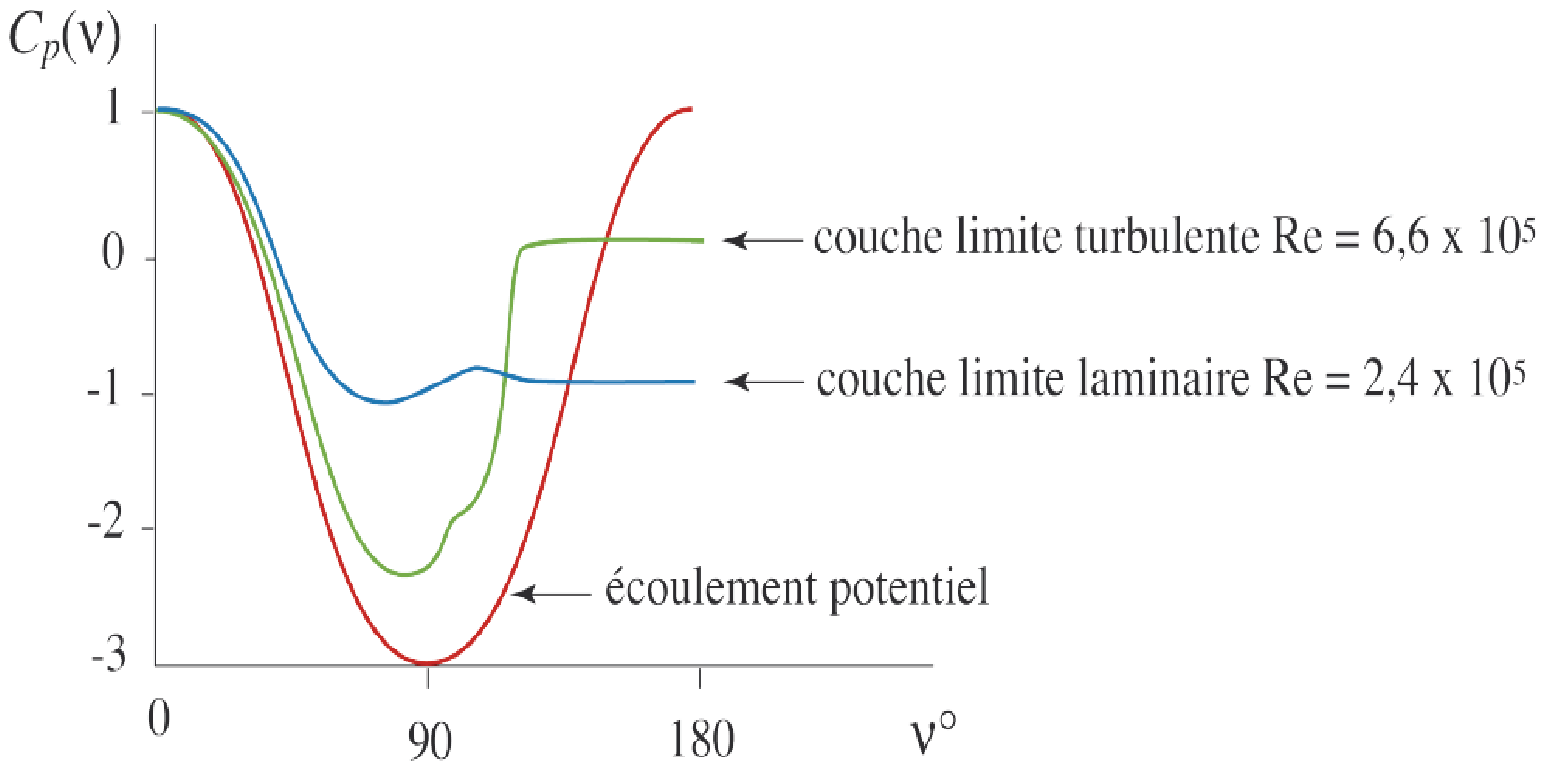


A viscous flow close to the wall opposes the free-stream

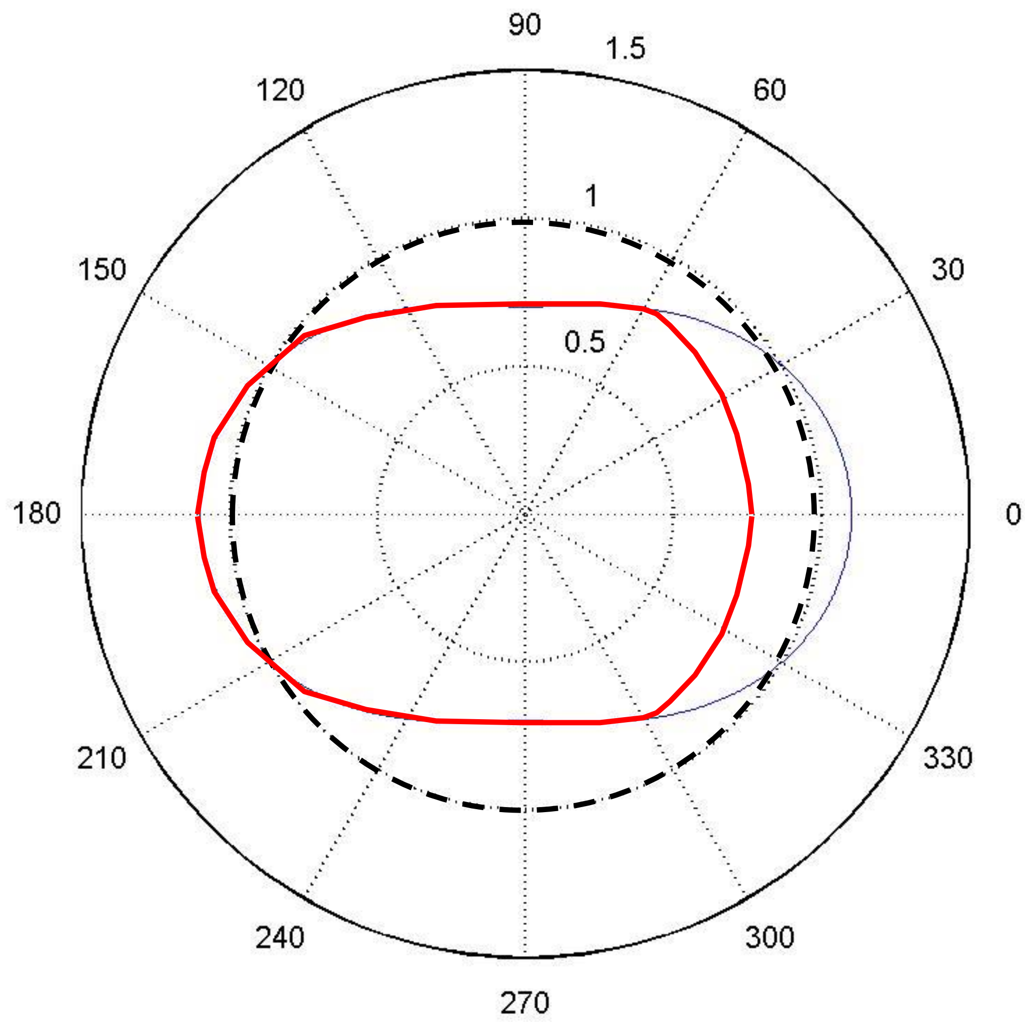


P-
B
P+
C
P+

Pressure coefficient

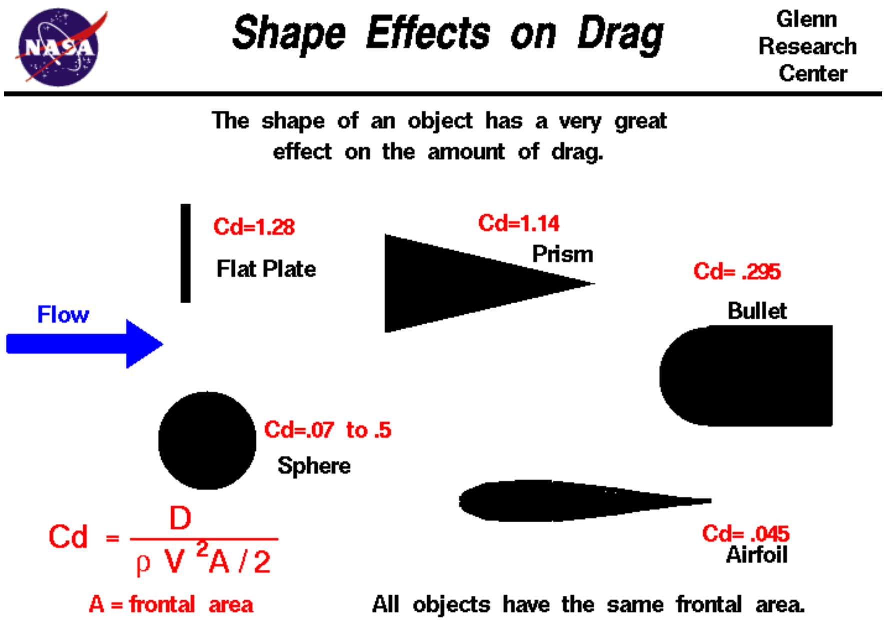


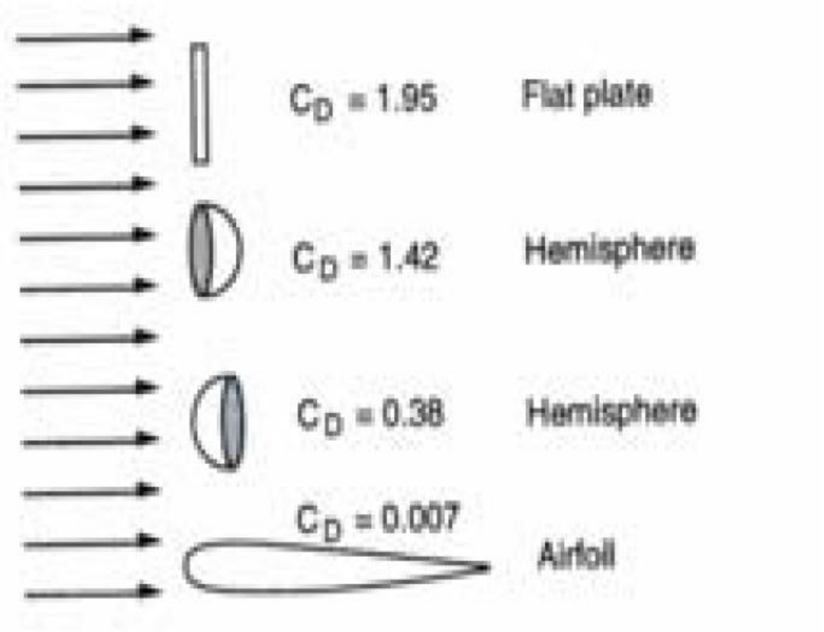
Form drag



Drag coefficient

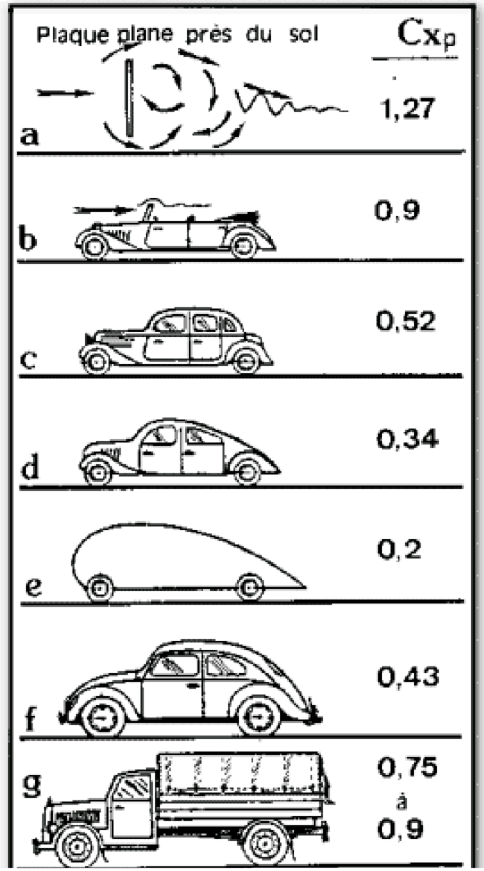
$$C_X = \frac{trainée}{\frac{1}{2}\rho U^2 A}$$



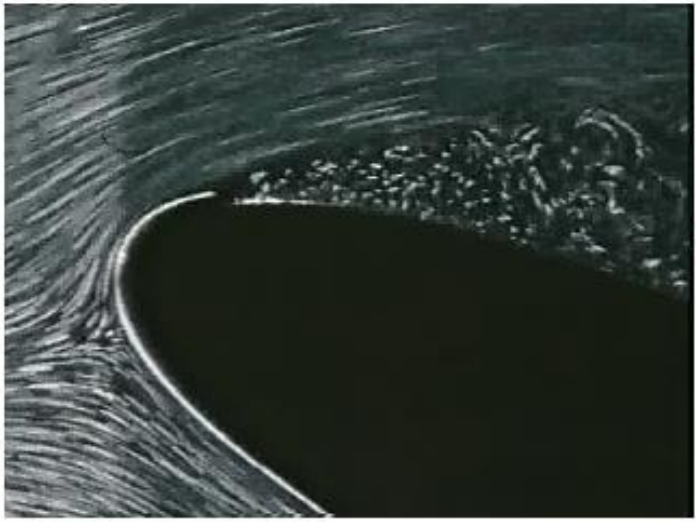


$$C_{\mathbf{x}} = \frac{\textbf{drag}}{\frac{1}{2}\rho U^2 A}$$

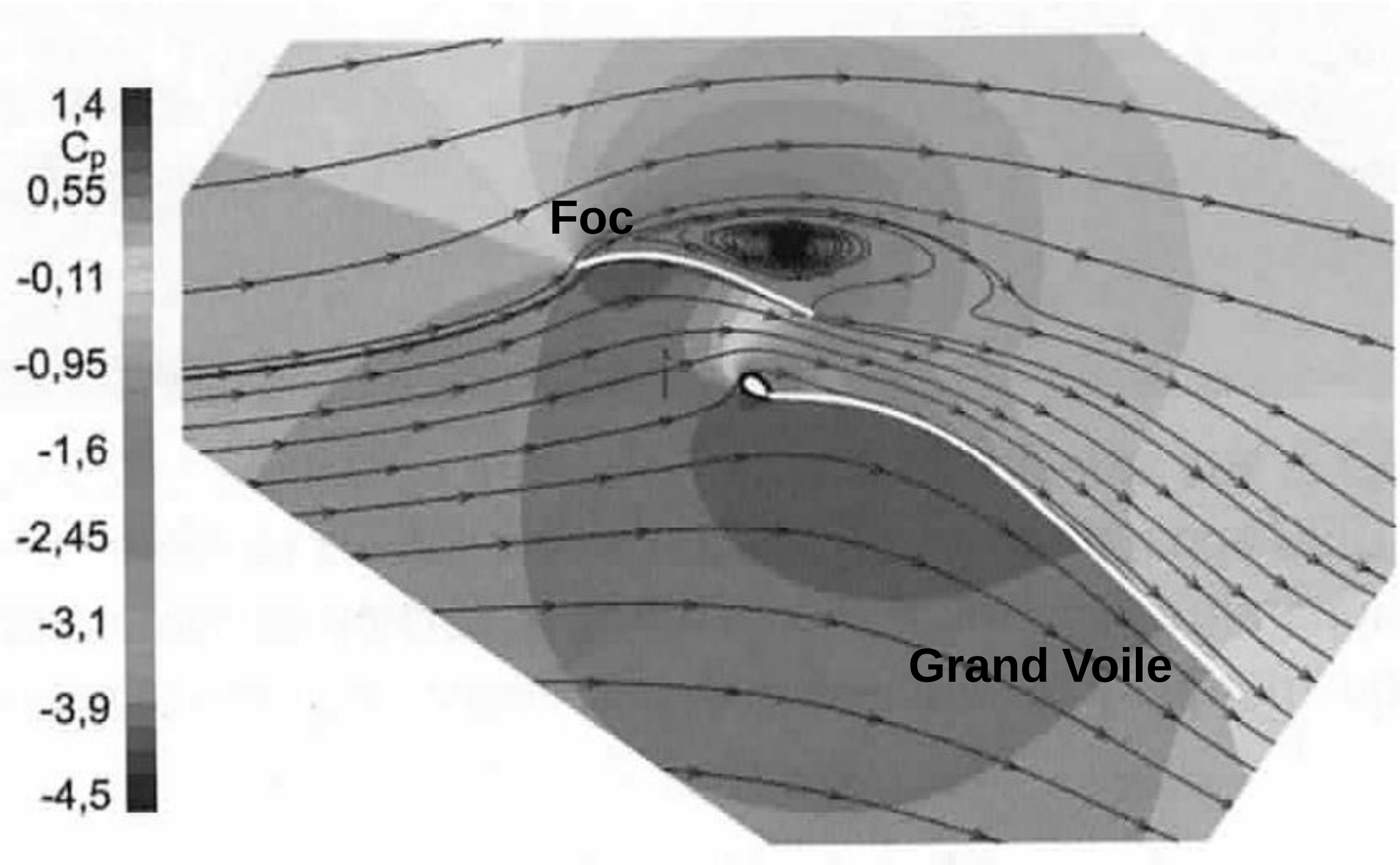
section, somewhat arbitrary…



Separation control



Application to sailing

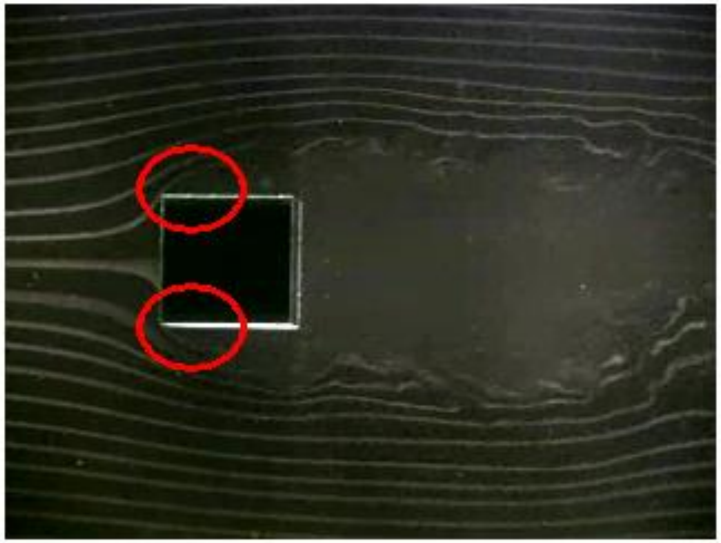


Thickness effect

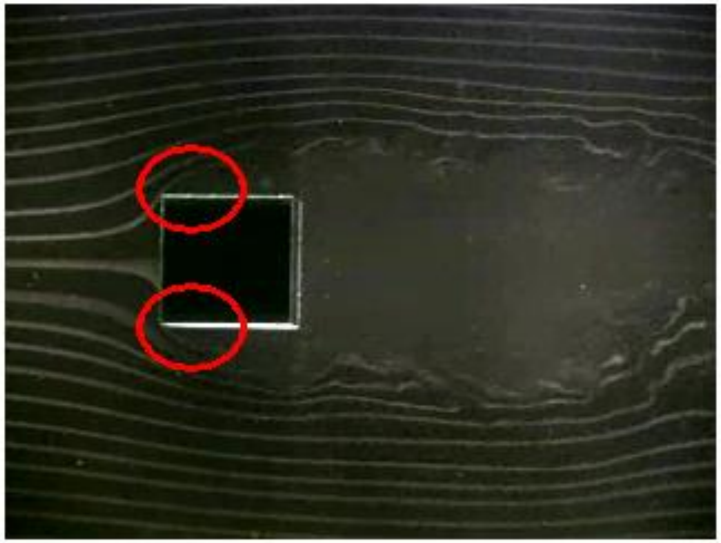
Attached

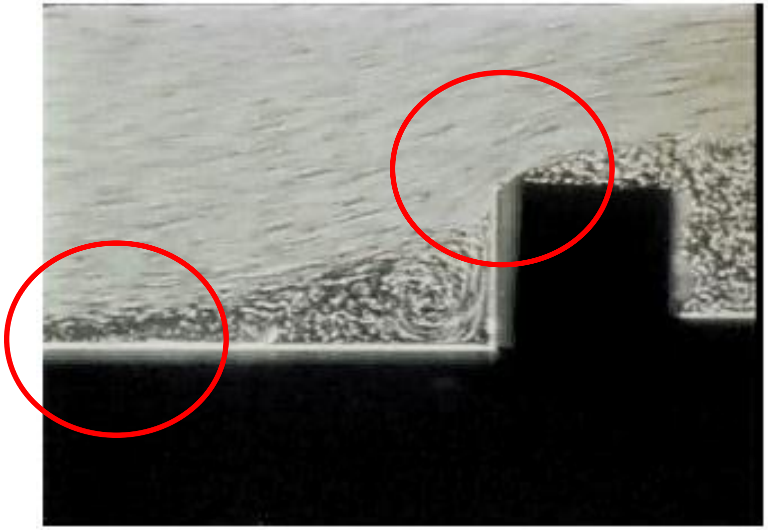
Detached

A gallery of detached flows

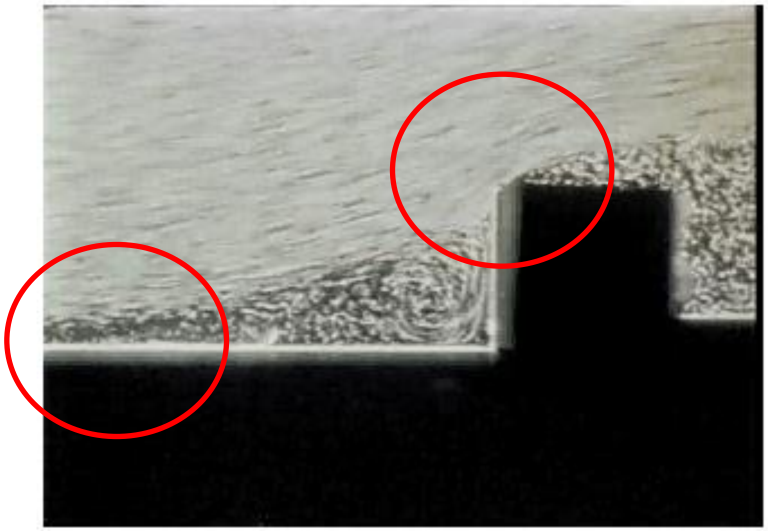


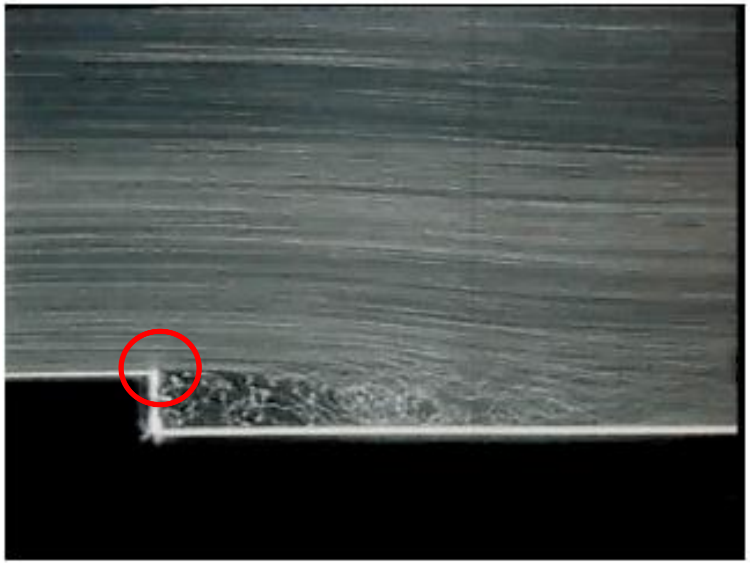
A gallery of detached flows



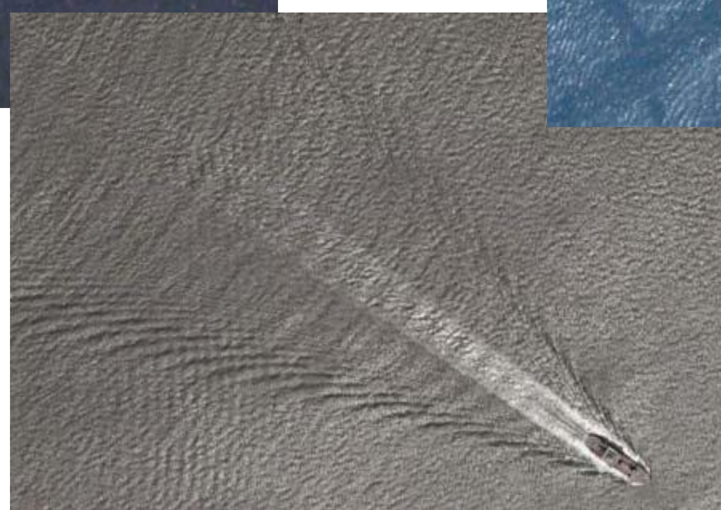


A gallery of detached flows

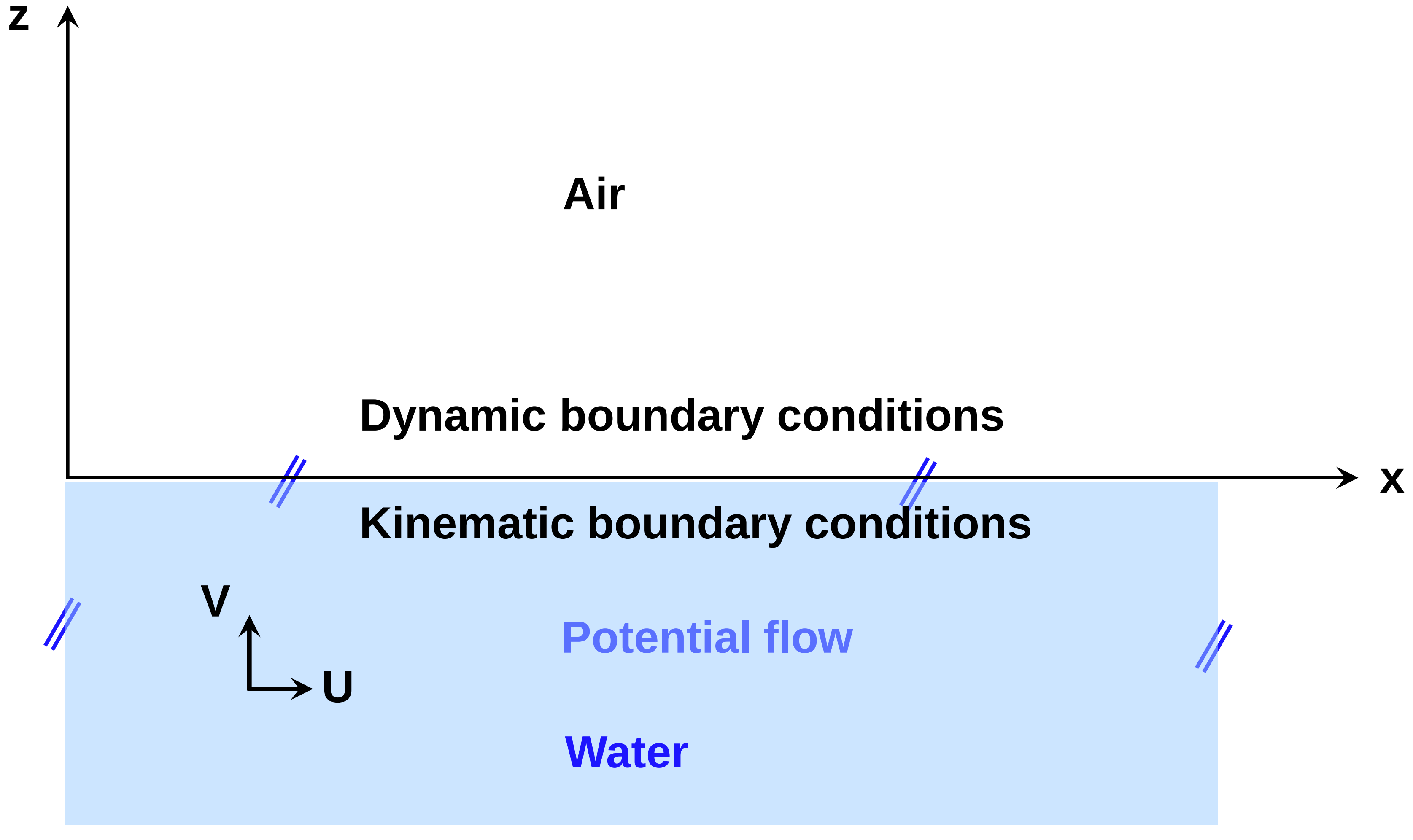




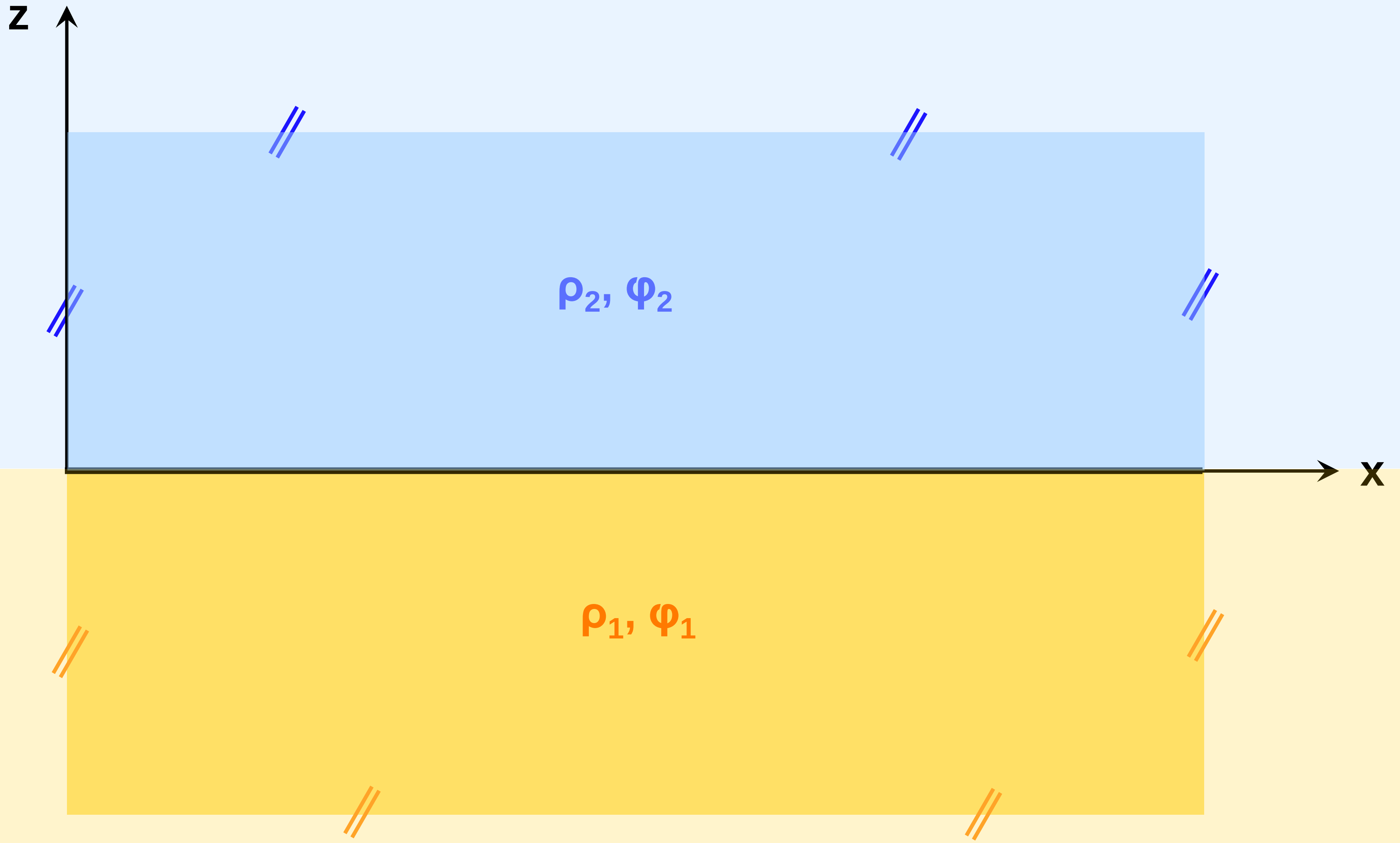
Hydrodynamics 13
Waves



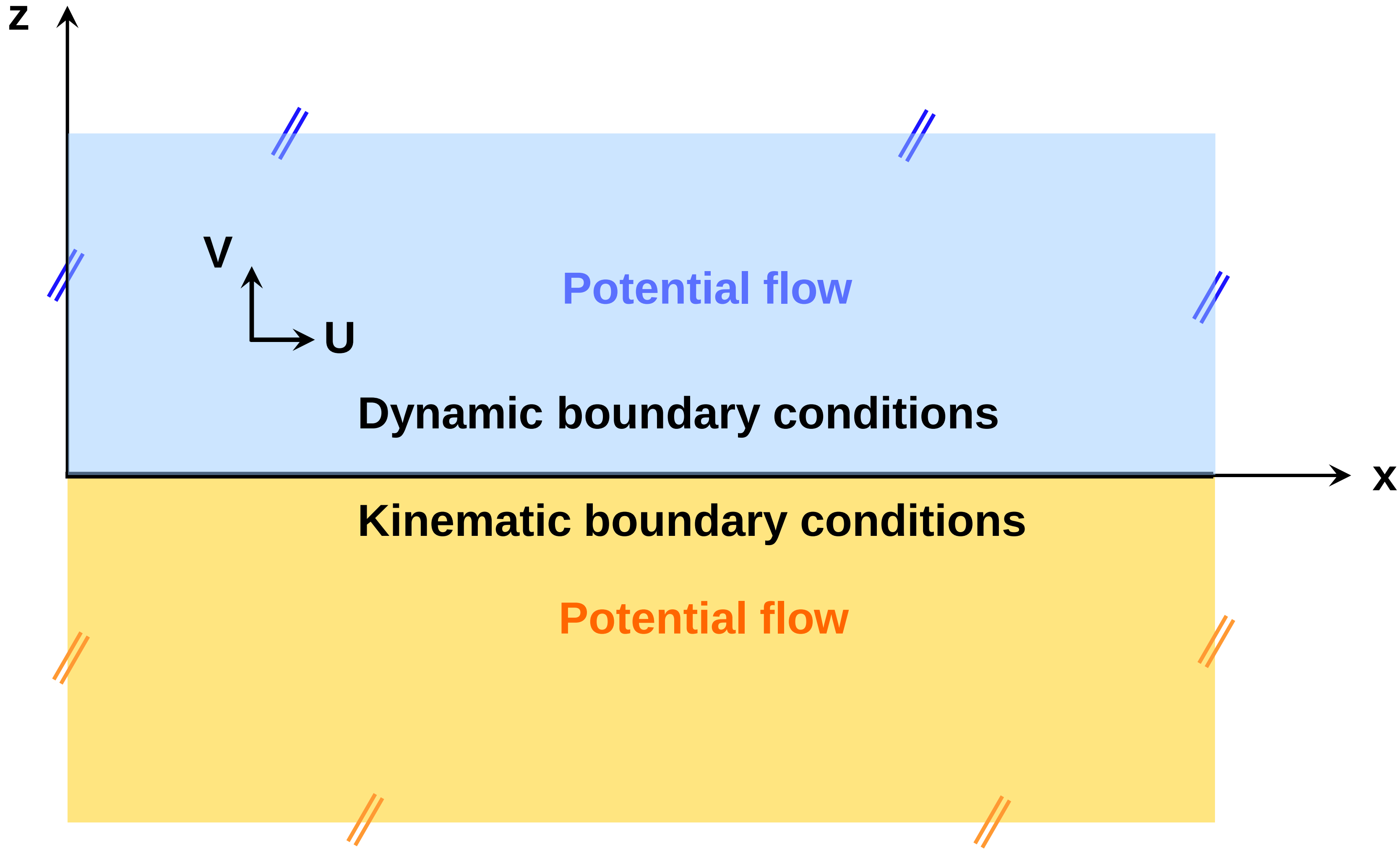
Waves


z
Air
Dynamic boundary conditions
x
Kinematic boundary conditions
V
U
Potential flow
Water

General case: two fluids



General case: two fluids



Linear waves dispersion relation

1. **Equations and boundary conditions**
2. **Base state**
3. **Linearized equations**
4. **Normal mode expansion**
5. **Dispersion relation**
6. **Analysis of the dispersion relation**

1. Equations

$$\begin{aligned}\Delta\Phi_1 &= 0\\ \Delta\Phi_2 &= 0\end{aligned}$$

Potential flow

$$U_1 = \frac{\partial\Phi_1}{\partial x}, \qquad V_1 = \frac{\partial\Phi_1}{\partial z}$$

$$U_2 = \frac{\partial\Phi_2}{\partial x}, \qquad V_2 = \frac{\partial\Phi_2}{\partial z}$$

Velocity field

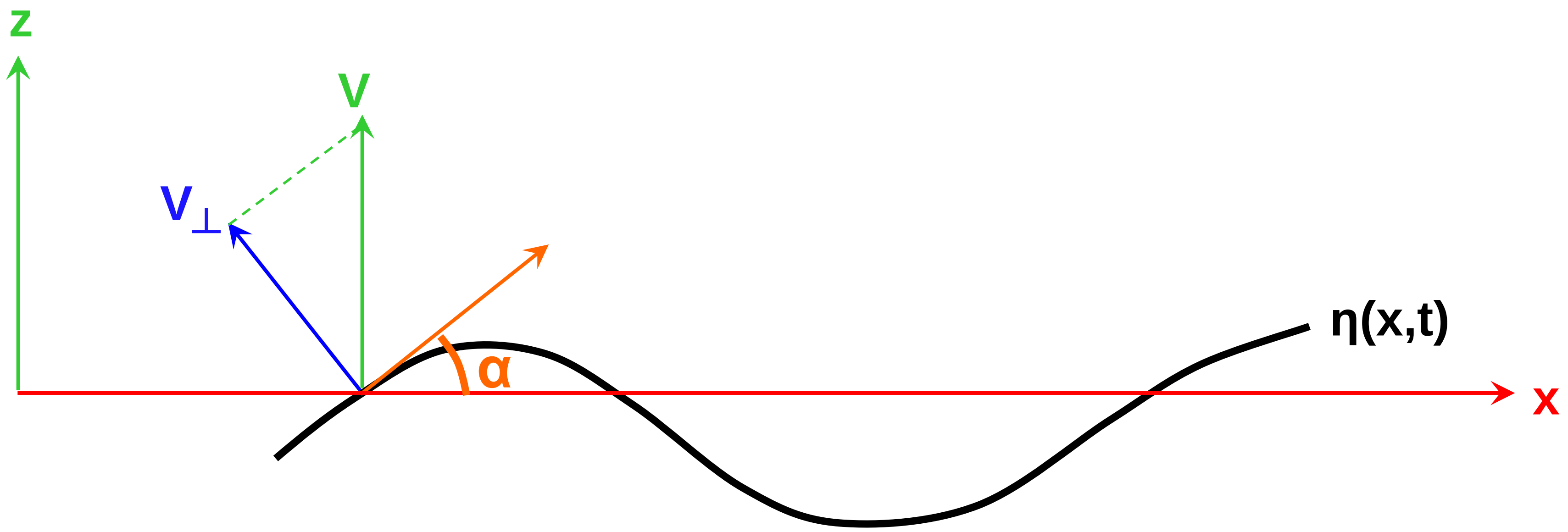
1. Boundary conditions

$$\boxed{\begin{aligned} \Phi_1 &= 0 \text{ at } z = -\infty \\ \Phi_2 &= 0 \text{ at } z = +\infty \end{aligned}}$$

far-field

$\text{at } z = \eta$ **?**

1. Kinematic boundary condition

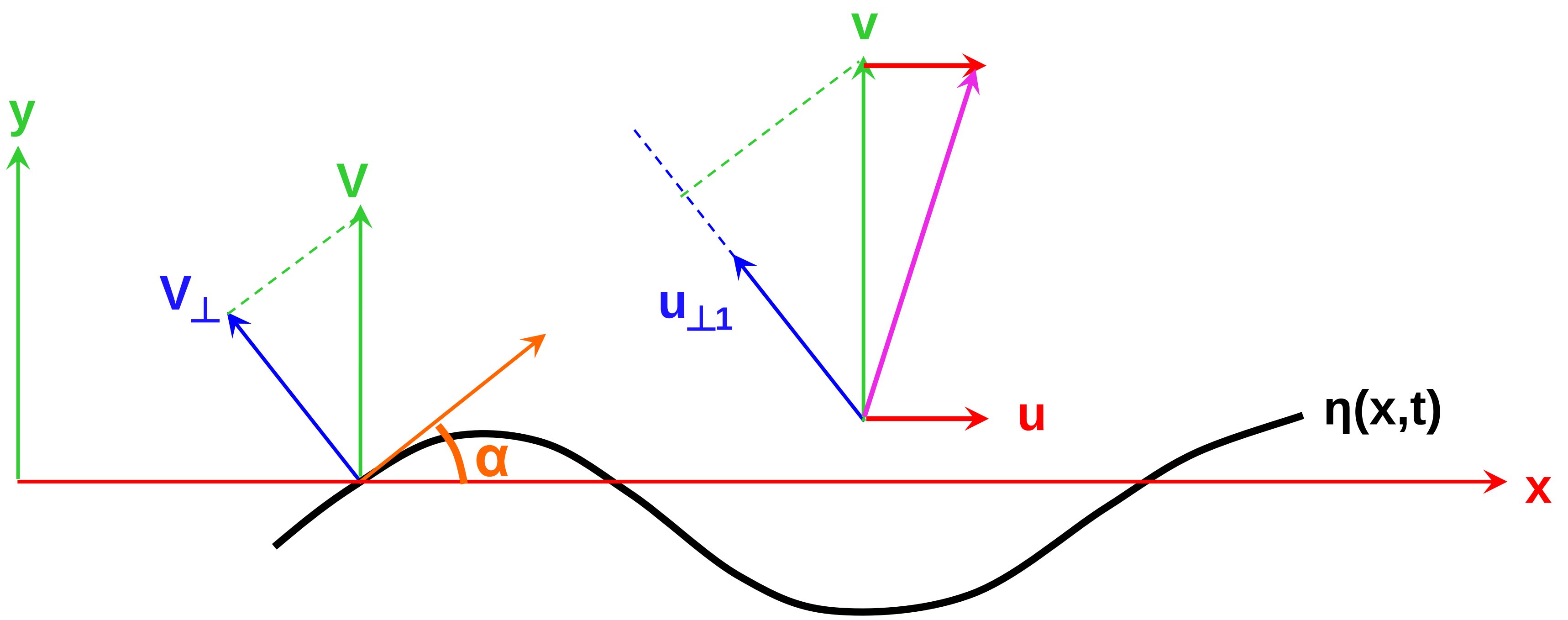


Kinematic condition : impermeability (no penetration)

No fluid particles going across the interface through the normal direction

$V_{\perp} = \partial\eta/\partial t \cos(\alpha)$

1. Kinematic boundary condition



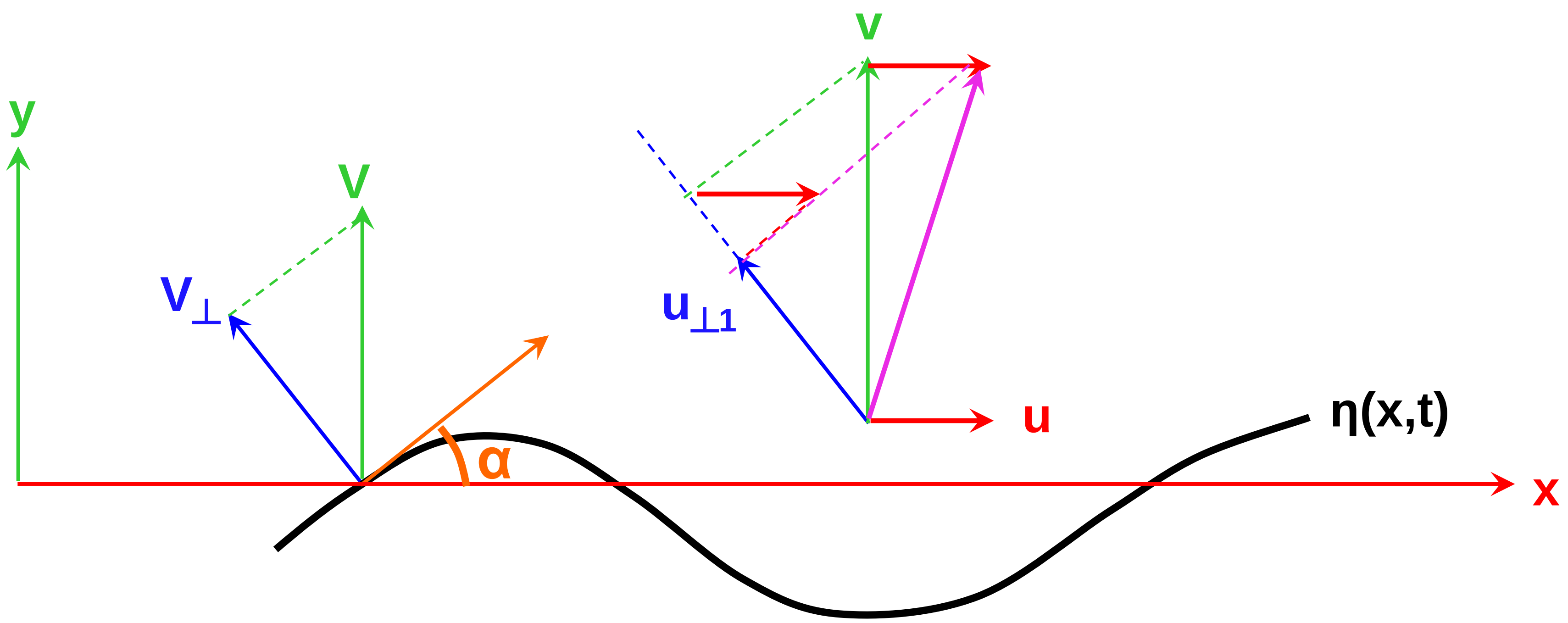
Kinematic condition : impermeability (no penetration)

No fluid particles going across the interface through the normal direction

$V_{\perp} = \partial\eta/\partial t \cos(\alpha)$

$u_{\perp 1} = v_1 \cos(\alpha) +$

1. Kinematic boundary condition



Kinematic condition : impermeability (no penetration)

No fluid particles going across the interface through the normal direction

$$V_\perp = \partial\eta/\partial t \cos(\alpha)$$

$$u_{\perp 1} = v_1 \cos(\alpha) - u_1 \sin(\alpha)$$

$$\partial\eta/\partial t = v_1 - u_1 \tan(\alpha) \Rightarrow \boxed{\partial\eta/\partial t = v_1 - u_1 \partial\eta/\partial x}$$

1. Kinematic boundary conditions

$$\boxed{\begin{aligned} \Phi_1 &= 0 \text{ at } z = -\infty \\ \Phi_2 &= 0 \text{ at } z = +\infty \end{aligned}}$$

far-field

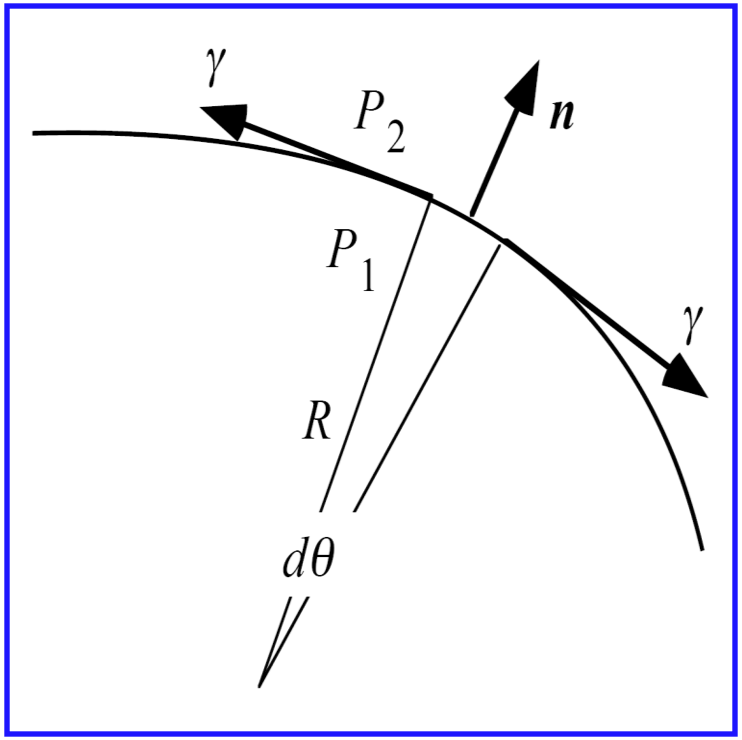
$$U_1 \frac{\partial \eta}{\partial x} - V_1 = - \frac{\partial \eta}{\partial t}$$

$$U_2 \frac{\partial \eta}{\partial x} - V_2 = - \frac{\partial \eta}{\partial t}$$

$$\text{at } z = \eta$$

1. Dynamic boundary conditions

$$P_1 - P_2 = -\gamma \frac{\frac{\partial^2 \eta}{\partial x^2}}{\left(1 + \frac{\partial \eta}{\partial x}^2\right)^{3/2}} \text{ at } z = \eta$$



$$\mathbf{n} = \frac{(-\partial_x \eta, 1)}{\sqrt{1 + \partial_x^2 \eta}}$$

$$\mathcal{C} = \nabla.\mathbf{n}$$

1. More equations

$$\frac{\partial \Phi_1}{\partial t} + \frac{U_1^2 + V_1^2}{2} + \frac{P_1}{\rho_1} + gz = C_1(t) \quad = 0$$

$$\frac{\partial \Phi_2}{\partial t} + \frac{U_2^2 + V_2^2}{2} + \frac{P_2}{\rho_2} + gz = C_2(t) \quad = 0$$

2nd Bernouilli relations

2. Base state

$$\Phi_1 = 0,$$

$$\Phi_2 = 0$$

$$\eta = 0$$

$$P_1 = -\rho_1 g z$$

3. Perturb and linearize
perturbation expansion

$$
\begin{aligned}
\Phi_1 &= 0 && +\epsilon\phi_1 \\
\Phi_2 &= 0 && +\epsilon\phi_2 \\
U_1 &= 0 && +\epsilon u_1 \\
V_1 &= 0 && +\epsilon v_1 \\
U_2 &= 0 && +\epsilon u_2 \\
V_2 &= 0 && +\epsilon v_2 \\
P_1 &= -\rho_1 g z && +\epsilon p_1 \\
P_2 &= -\rho_2 g z && +\epsilon p_2 \\
\eta &= 0 && +\epsilon\sigma
\end{aligned}
$$

$$\epsilon \ll 1$$

Variables **Base state** **Small perturbation**

3. Linearized equations

$$\begin{aligned} \Delta\phi_1 &= 0 \\ \Delta\phi_2 &= 0 \end{aligned}$$

perturbed potential flow

$$u_1 = \frac{\partial \phi_1}{\partial x}, \qquad v_1 = \frac{\partial \phi_1}{\partial z}$$

$$u_2 = \frac{\partial \phi_2}{\partial x}, \qquad v_2 = \frac{\partial \phi_2}{\partial z}$$

3. Perturbed kinematic boundary conditions

$$\phi_1 = 0 \text{ at } z = -\infty$$

$$\phi_2 = 0 \text{ at } z = +\infty$$

$$-\epsilon^2 u_1 \frac{\partial \sigma}{\partial x} + \epsilon v_1 = \epsilon \frac{\partial \sigma}{\partial t} \text{ at } z = \epsilon\sigma$$

$$-\epsilon^2 u_2 \frac{\partial \sigma}{\partial x} + \epsilon v_2 = \epsilon \frac{\partial \sigma}{\partial t} \text{ at } z = \epsilon\sigma$$

3. Perturbed kinematic boundary conditions

$$\phi_1 = 0 \text{ at } z = -\infty$$

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$$-\epsilon^2 u_2 \frac{\partial \sigma}{\partial x} + \epsilon v_2 = \epsilon \frac{\partial \sigma}{\partial t} \text{ at } z = \epsilon\sigma$$

$$v_1 = \frac{\partial \sigma}{\partial t} \text{ at } z = \epsilon\sigma$$

$$v_2 = \frac{\partial \sigma}{\partial t} \text{ at } z = \epsilon\sigma$$

3. Flattened kinematic boundary conditions

$$\frac{\partial \phi_1}{\partial z} = \frac{\partial \sigma}{\partial t} \text{ at } z = \epsilon\sigma$$
$$\frac{\partial \phi_2}{\partial z} = \frac{\partial \sigma}{\partial t} \text{ at } z = \epsilon\sigma$$

Taylor expansion around 0: $\phi(\epsilon\sigma) = \phi(0) + (\epsilon\sigma)\frac{\partial \phi}{\partial z}\Big|_{\mathbf{0}}$

$$\frac{\partial \phi_1}{\partial z} = \frac{\partial \sigma}{\partial t} \text{ at } z = 0$$
$$\frac{\partial \phi_2}{\partial z} = \frac{\partial \sigma}{\partial t} \text{ at } z = 0$$

⇨transforms a b.c. at an unkwown interface into a fixed place!

3. Perturbed dynamic boundary conditions

$$(P_1 + \epsilon p_1 - P_2 - \epsilon p_2)\big|_{\epsilon\sigma} = -\gamma\epsilon\frac{\partial^2\sigma}{\partial x^2}\left(1 - 3/2\epsilon^2\left(\frac{\partial\sigma}{\partial x}\right)^2\right)$$

Replace P_1=-gρ_1z, … **and linearize**

$$\mathbf{g}(\rho_2 - \rho_1)\sigma + (p_1 - p_2)\big|_{\epsilon\sigma} = -\gamma\frac{\partial^2\sigma}{\partial x^2}$$

flatten

$$(\rho_2 - \rho_1)g\sigma + (p_1 - p_2)\big|_0 = -\gamma\frac{\partial^2\sigma}{\partial x^2}$$

3. Perturbed and linearized Bernouilli

Perturbed 2nd Bernouilli relations

$$\epsilon\frac{\partial\phi_1}{\partial t} + \epsilon^2\frac{u_1^2+v_1^2}{2} + \epsilon\frac{p_1}{\rho_1} = 0$$

$$\epsilon\frac{\partial\phi_2}{\partial t} + \epsilon^2\frac{u_2^2+v_2^2}{2} + \epsilon\frac{p_2}{\rho_2} = 0$$

Linearized 2nd Bernouilli relations

$$\frac{\partial\phi_1}{\partial t} + \frac{p_1}{\rho_1} = 0$$

$$\frac{\partial\phi_2}{\partial t} + \frac{p_2}{\rho_2} = 0$$

4. Normal mode expansion

Fourier transform in x and t

$$
\begin{aligned}
\phi_1 &= f_1(z)\exp(i(kx-\omega t)), \\
\phi_2 &= f_2(z)\exp(i(kx-\omega t)), \\
\sigma &= C\exp(i(kx-\omega t)),
\end{aligned}
$$

k is the wavenumber and ω the frequency (in rad/s)

$$\lambda = 2\pi/k \qquad T = 2\pi/\omega$$

$$f = \omega/(2\pi)$$

4. Normal mode expansion

Solution to Laplace equation:

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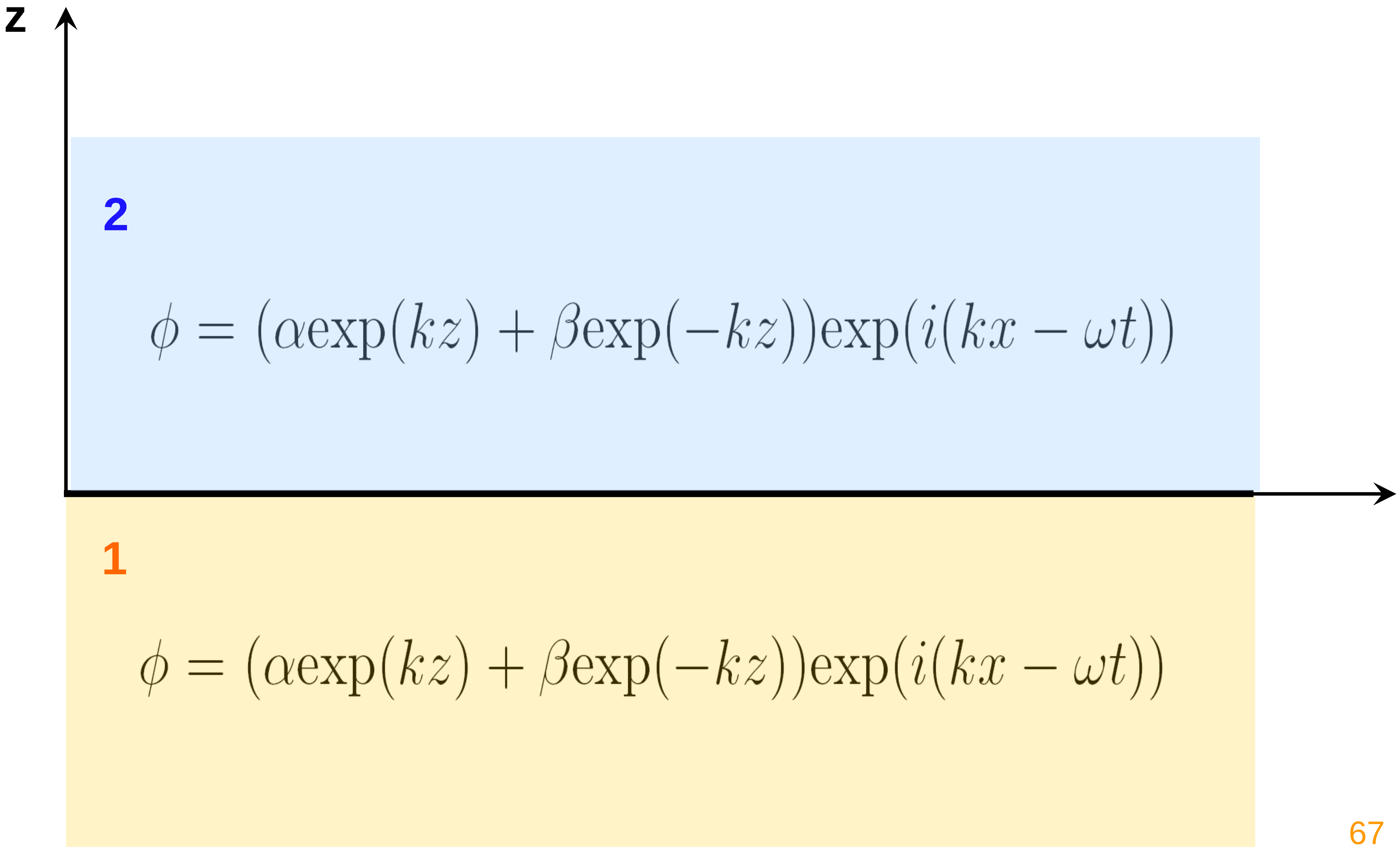
Solution to Laplace equation:



4. Normal mode expansion

Solution to Laplace equation:

$$
\begin{aligned}
\phi_1 &= A\exp(kz)\exp(i(kx - \omega t)), \\
\phi_2 &= B\exp(-kz)\exp(i(kx - \omega t)), \\
\sigma &= C\exp(i(kx - \omega t)).
\end{aligned}
$$

4. Normal mode expansion

Replace in boundary conditions

$$\begin{aligned} g(\rho_2-\rho_1)C + i\omega\rho_1 A - i\omega\rho_2 B &= \gamma k^2 C \\ kA &= -i\omega C \\ -kB &= -i\omega C \end{aligned}$$

This is an eigenvalue problem iωX=MX!

$$kg(\rho_2-\rho_1)C + \omega^2\rho_1 C + \omega^2\rho_2 C = \gamma k^3 C.$$

5. Dispersion relation

$$\omega^2 = \frac{-kg(\rho_2 - \rho_1) + \gamma k^3}{\rho_1 + \rho_2}$$

•Unstable if there exists one ω, Im(ω)>0 $\boxed{\rho_2 > \rho_1}$

•Neutral if for all ω, Im(ω)=0: $\rho_1 > \rho_2$

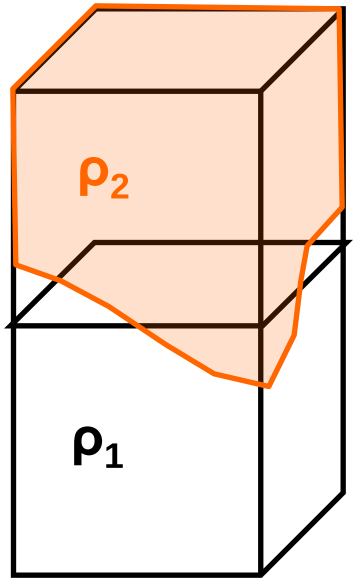
•Stable (or damped) if for all ω, Im(ω)<0:

The flow considered is not damped, we have neglected dissipation by neglecting viscosity

Instability analysis:

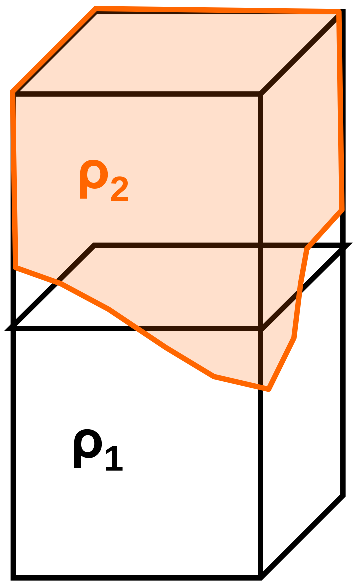
1. **Equations and boundary conditions**
2. **Base state**
3. **Linearized equations**
4. **Normal mode expansion**
5. **Dispersion relation**
6. **Analysis of the dispersion relation**

Dispersion relation



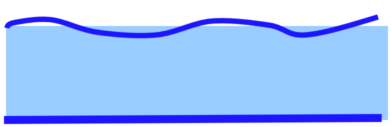
$$\omega^2 = \frac{-kg(\rho_2 - \rho_1) + \gamma k^3}{\rho_1 + \rho_2}$$

Dispersion relation



z

$$\omega^2 = \frac{-kg(\cancel{\rho_2} - \rho_1) + \gamma k^3}{\rho_1 + \cancel{\rho_2}}$$



$$\omega^2 = \tanh(kH)\left(\frac{\gamma k^3}{\rho} + gk\right)$$

Dispersion relation

$$\omega^2 = \tanh(kH)\left(\frac{\gamma k^3}{\rho} + gk\right)$$

Capillary wavenumber: $k_c = \sqrt{\rho g/\gamma}$

Length scale: $\tilde{k} = k/k_c$

Time scale $\tilde{\omega} = \omega/\sqrt{gk_c}$

One single non-dimensional parameter $\tilde{H} = Hk_c$

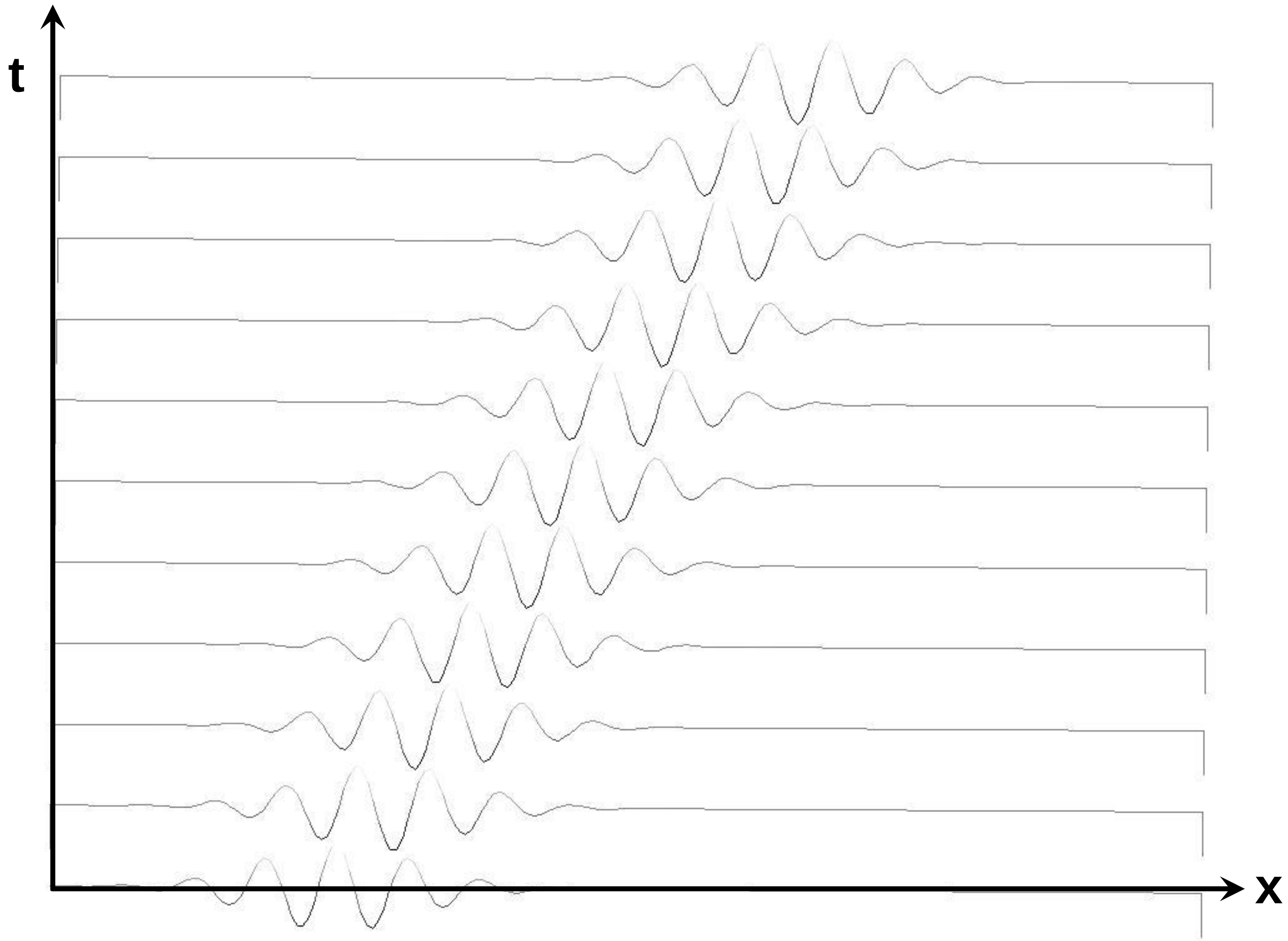
$$\tilde{\omega}^2 = \tanh(\tilde{k}\tilde{H})\left(\tilde{k}^3 + \tilde{k}\right)$$

Dispersion relation

$$\tilde{\omega}^2 = \tanh(\tilde{k}\tilde{H})\left(\tilde{k}^3 + \tilde{k}\right)$$

	gravity $\tilde{k} \ll 1$	capillary $\tilde{k} \gg 1$
shallow water $\tilde{k} \ll 1/\tilde{H}$	$\pm\tilde{k}$	$\pm\tilde{k}^2\sqrt{\tilde{H}}$
Deep water $\tilde{k} \gg 1/\tilde{H}$	$\pm\sqrt{\tilde{k}}$	$\pm\tilde{k}\sqrt{\tilde{k}}$

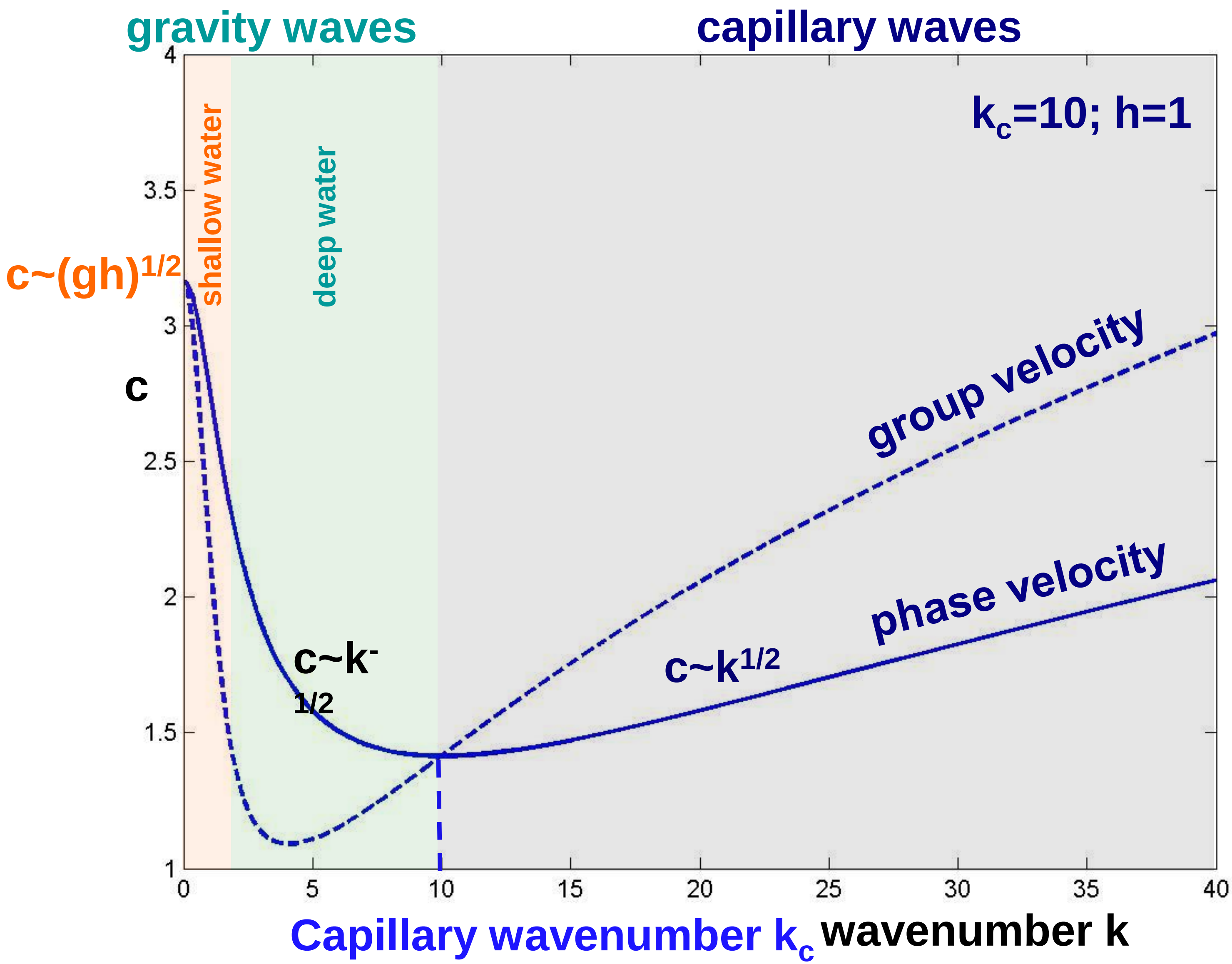
Difference between group velocity v and phase velocity c



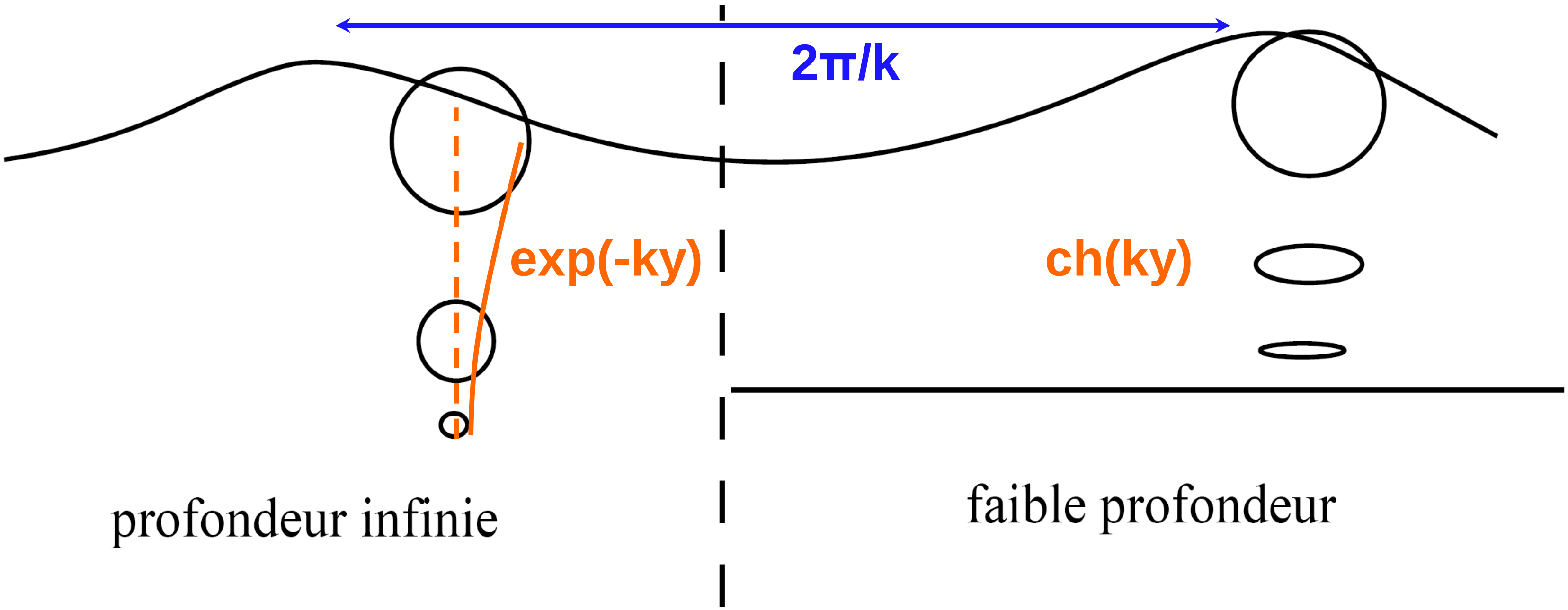
Dispersion relation

	gravity $\tilde{k} \ll 1$	capillary $\tilde{k} \gg 1$
shallow water $\tilde{k} \ll 1/\tilde{H}$	$\omega_{shallow/gravity} \sim \pm k\sqrt{gH}$ $c_{shallow/gravity} \sim \pm\sqrt{gH}$ $v_{shallow/gravity} \sim \pm\sqrt{gH}$	$\omega_{shallow/capillary} \sim \pm k^2\sqrt{\gamma H/\rho}$ $c_{shallow/capillary} \sim \pm k\sqrt{\gamma H/\rho}$ $v_{shallow/capillary} \sim \pm 2k\sqrt{\gamma H/\rho}$
Deep water $\tilde{k} \gg 1/\tilde{H}$	$\omega_{deep/gravity} \sim \pm\sqrt{gk}$ $c_{deep/gravity} \sim \pm\sqrt{\frac{g}{k}}$ $v_{deep/gravity} \sim \pm\frac{1}{2}\sqrt{\frac{g}{k}}$	$\omega_{deep/capillary} \sim \pm k^{3/2}\sqrt{\gamma/\rho}$ $c_{deep/capillary} \sim \pm k^{1/2}\sqrt{\gamma/\rho}$ $v_{deep/capillary} \sim \pm 3/2k^{1/2}\sqrt{\gamma/\rho}$

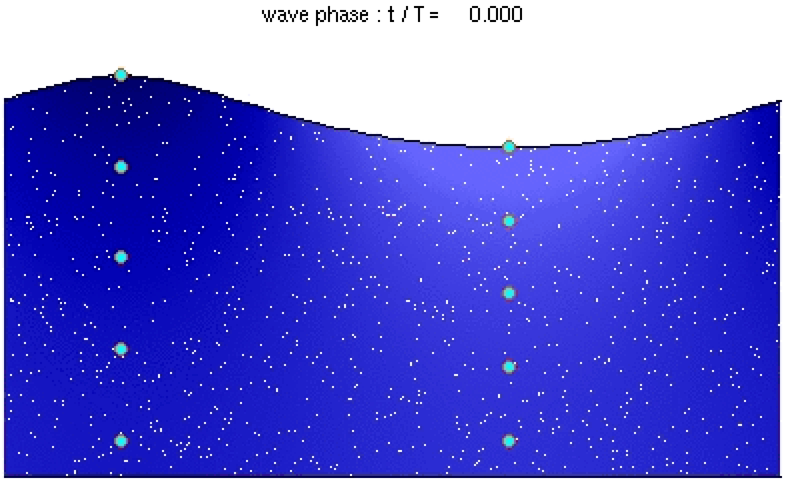
Dispersion relation



Trajectories below the waves



Stokes drift!

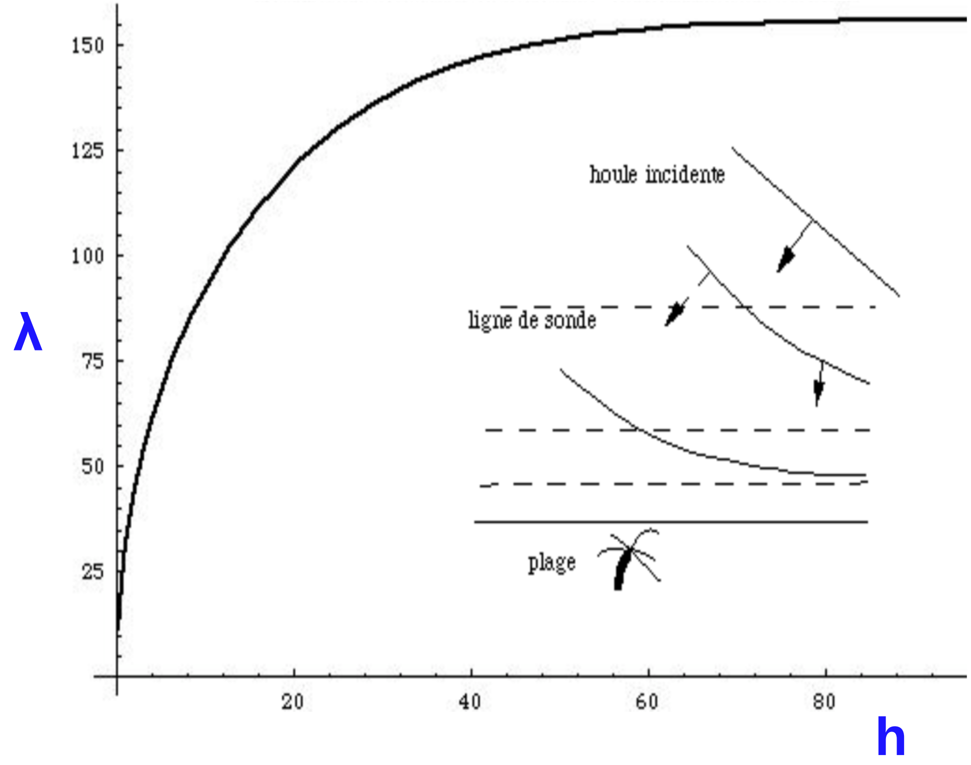


Why are the waves parallel to the shore?

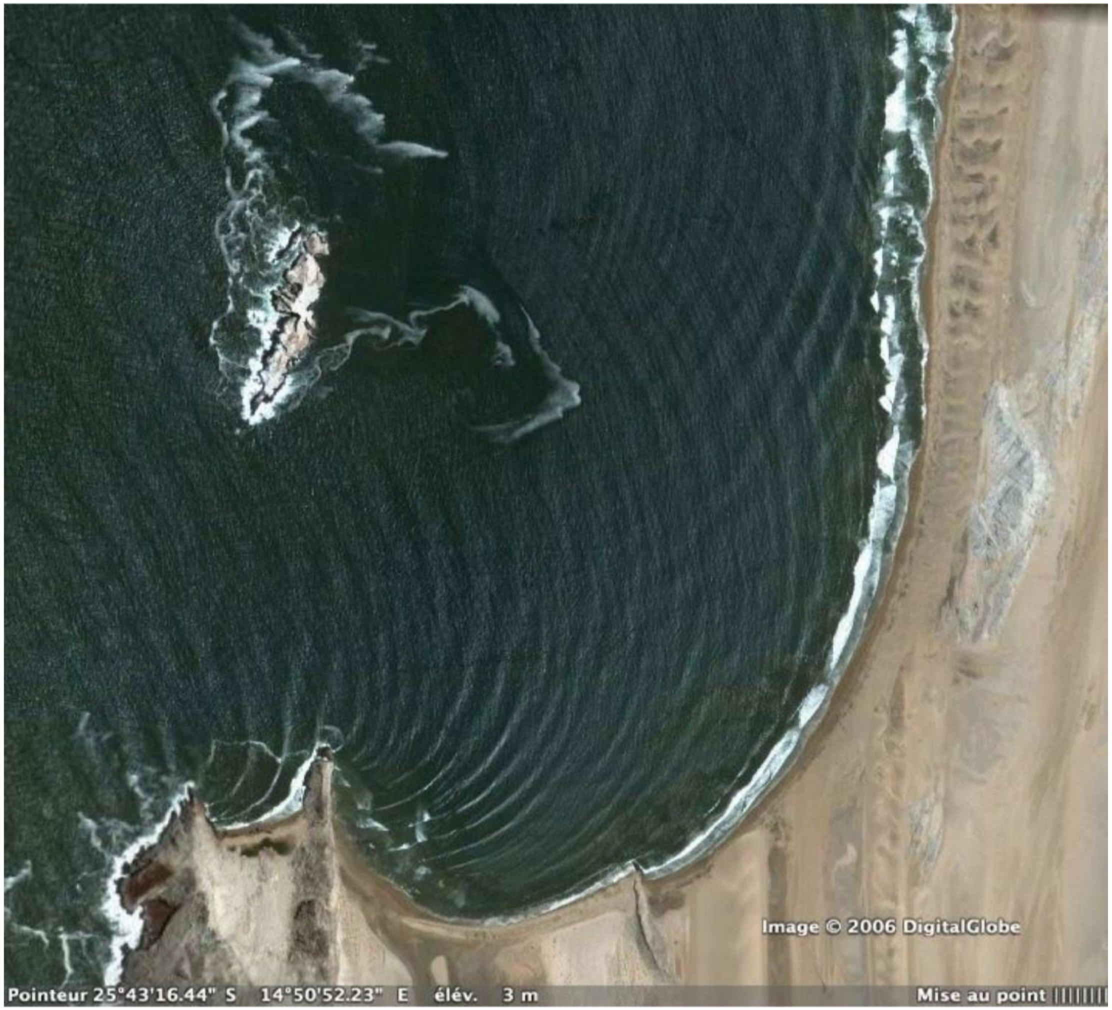
$c \sim (gh)^{1/2}$

$\lambda \sim T(gh)^{1/2}$

T=10s; ω=0.62

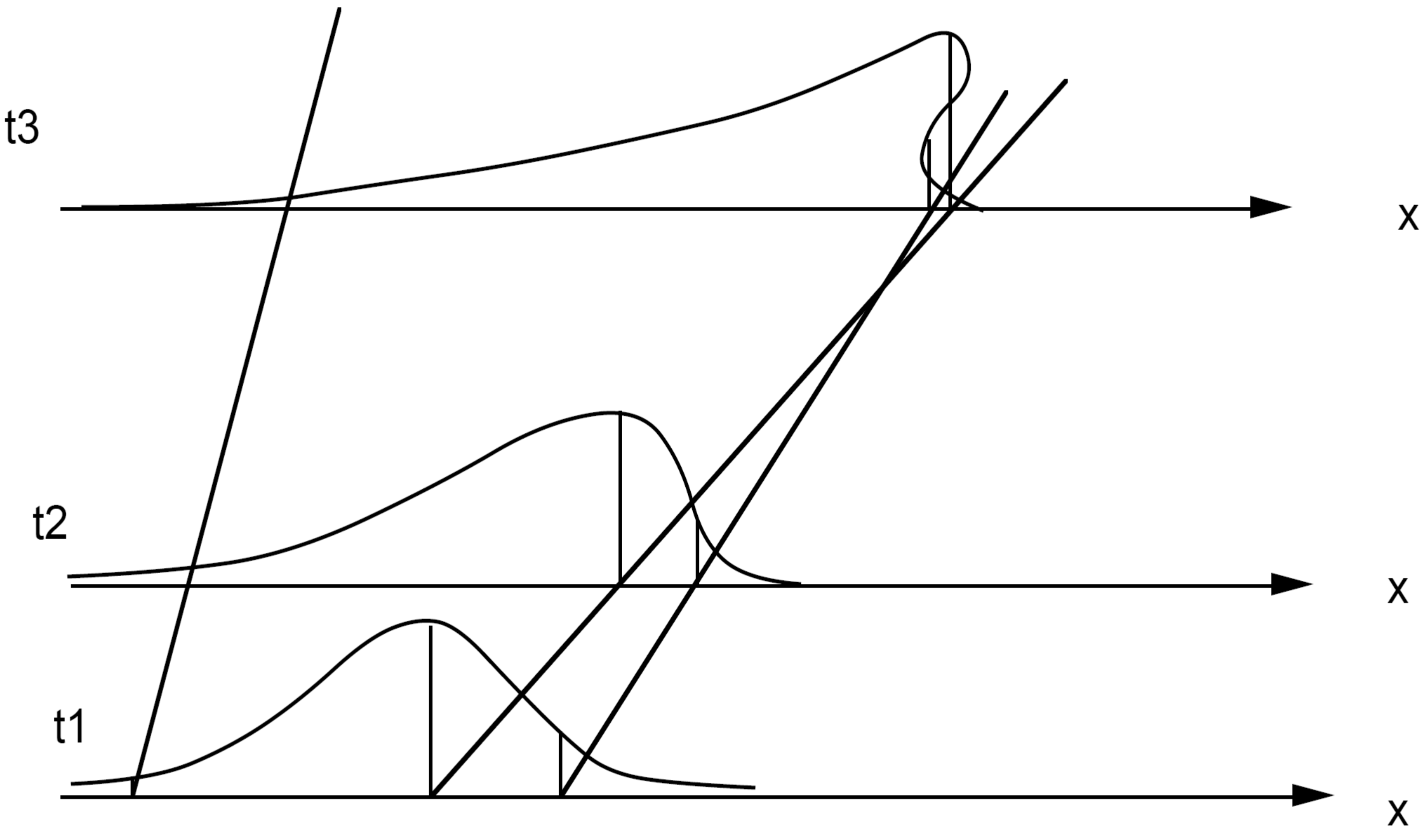


Refraction and diffraction of waves



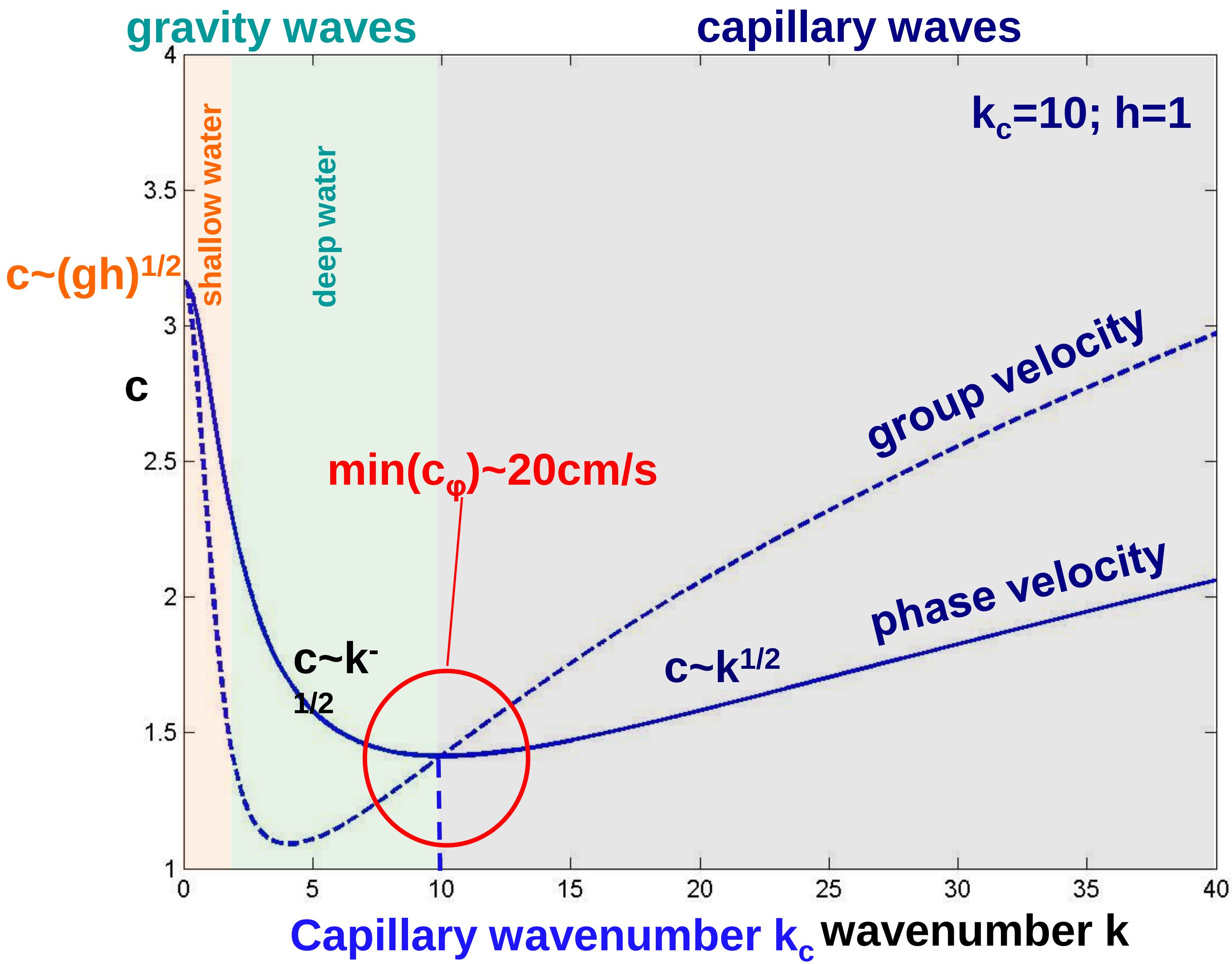
Satellite view Namibian coast

Nonlinear waves, wavebreaking



The celerity increases with the depth

Dispersion relation



Conditions for wave pattern formation?

$$V_{duck} \lessgtr c_{min} \quad ?$$