Hydrodynamics

Marmottant and Villermaux (2004)

Pr. François Gallaire
LFMI
MED2 2926
Tel: 3 3365
Email: francois.gallaire@epfl.ch

Chapter 1: Introduction

Outline

1. Introduction
2. Fluid: Definition and models
3. Fluid Kinematics

Introduction: Detachment on modern cars

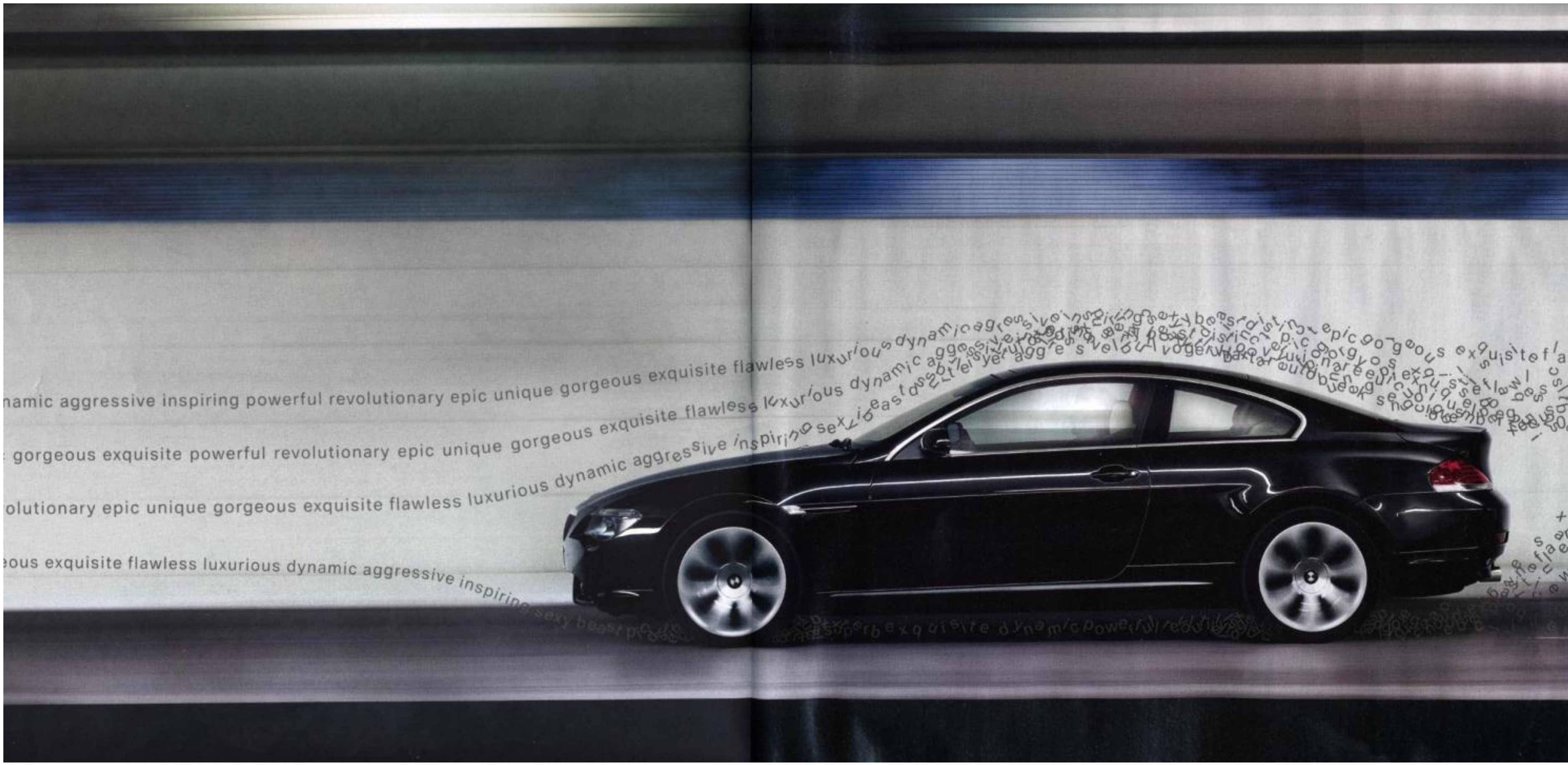


Figure 1:
BMW advertising

Detachment on... les modern cars

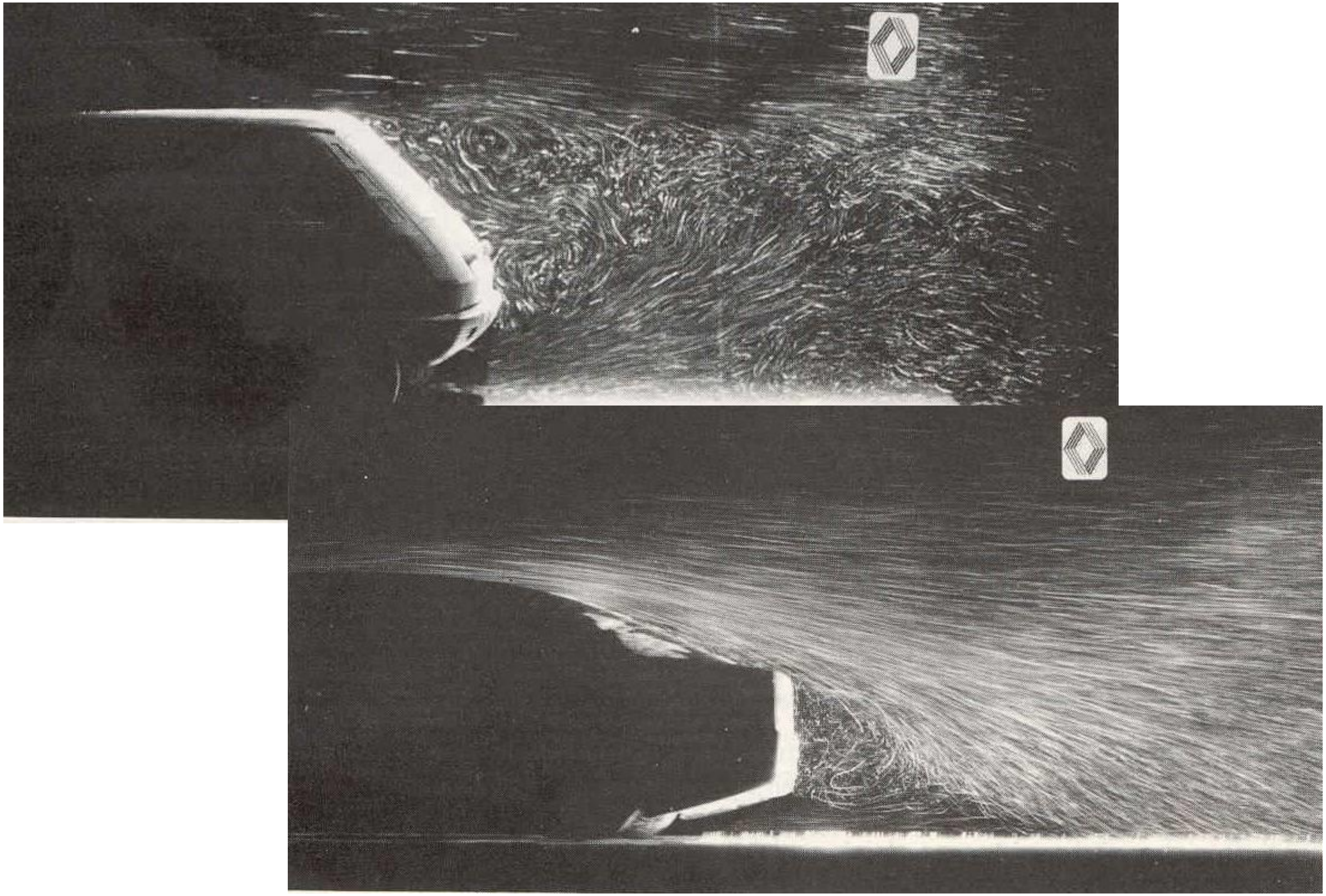


Figure 2: PIV experiment on Renault cars

Introduction: Naval hydrodynamics

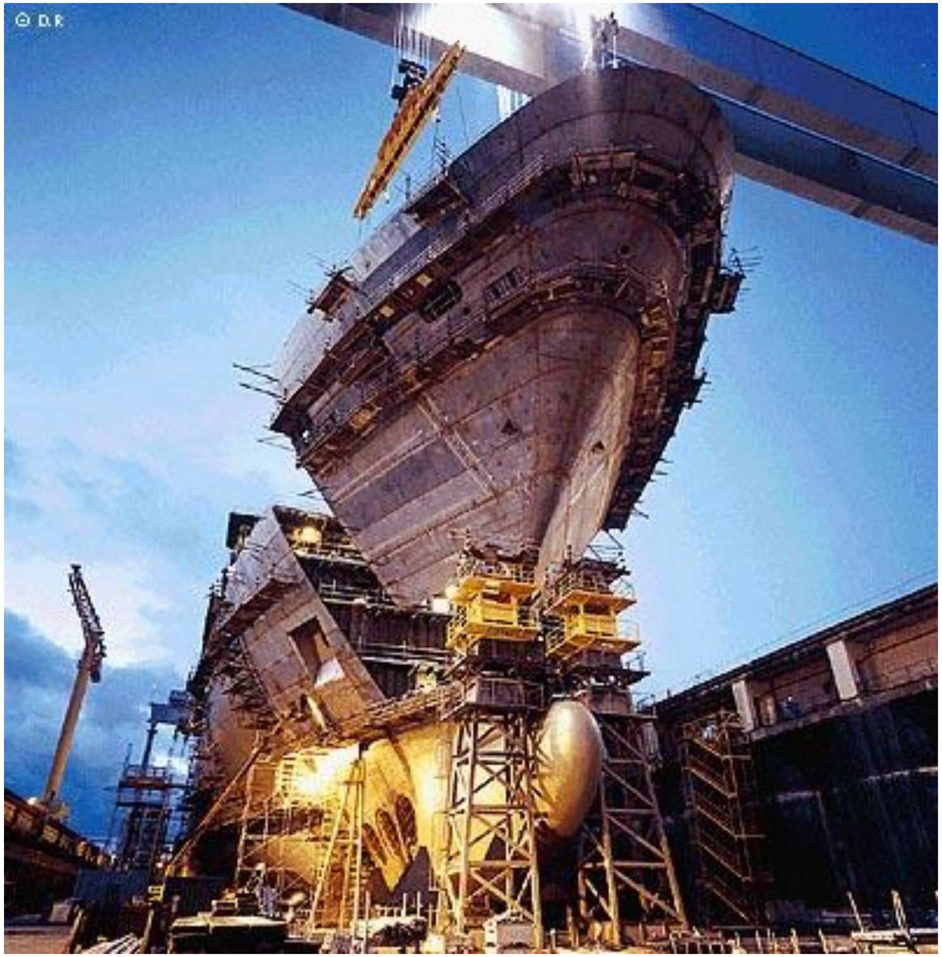


Figure 3:
Boat under construction

Introduction

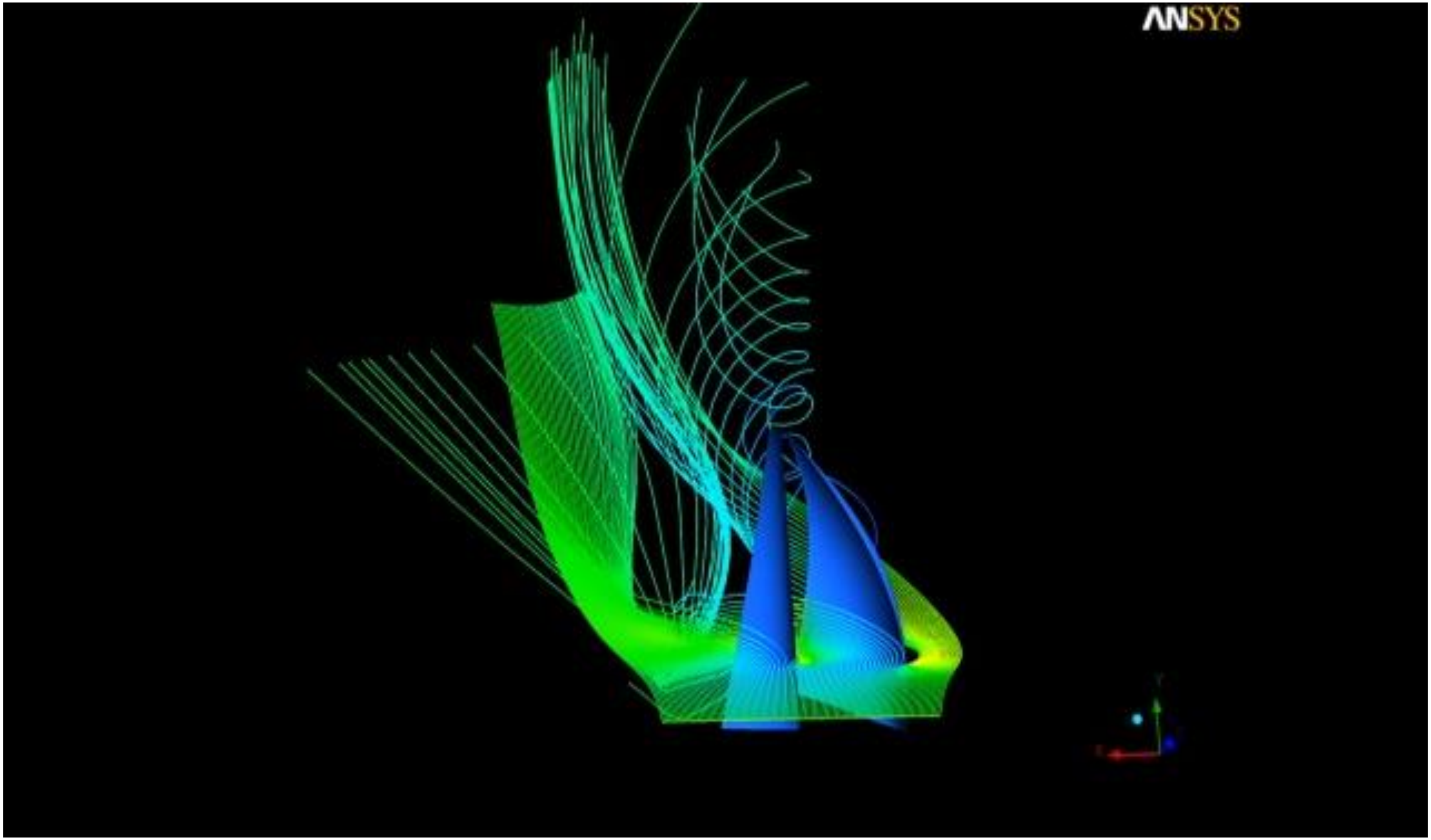


Figure 4:
Alinghi CFD model, EPFL

Introduction: Drag reduction

Figure 5:
Rowing team

Introduction: Turbines, cavitation

Figure 6:
Cavitation erosion on turbine blades

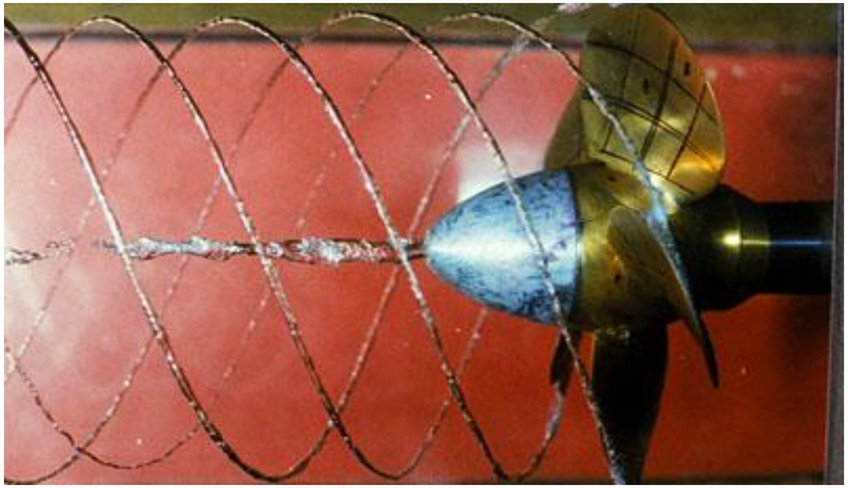


Figure 7:
Tip vortices and cavitation on turbine

Introduction: Geophysics

Figure 8: Kelvin-Helmholtz instability over mountain

Figure 9:
Rio Negro (slow and clean) meets amazon
(quick and dirty)

Introduction: Geophysics

Figure 10: Satellite image of Hurricane Luis above Guadeloupe (1995), Meteo France

Introduction: Geophysics

Figure 11: Waterspout

Introduction: Aeronautics/Aerospace

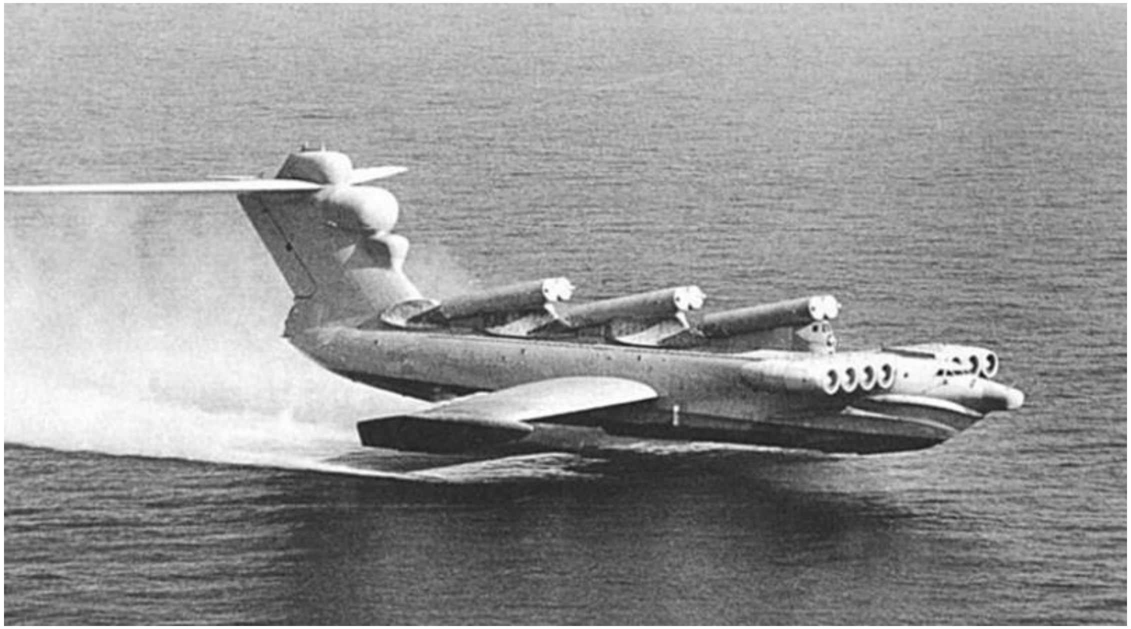


Figure 12: Military seaplane

Introduction: Oil

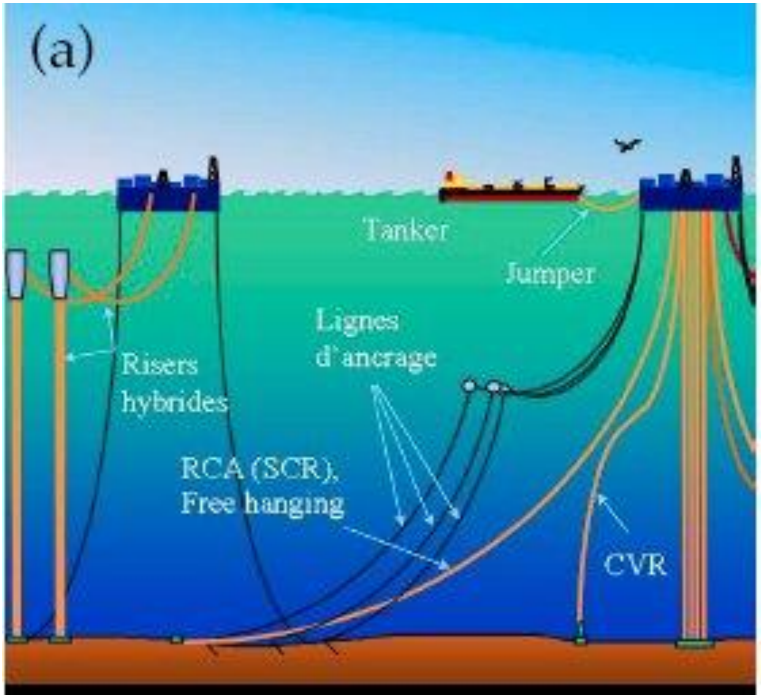


Figure 13: Offshore oil rig

Introduction:

Tidal and ocean waves energy harvesting

Figure 14: Pelamis snake

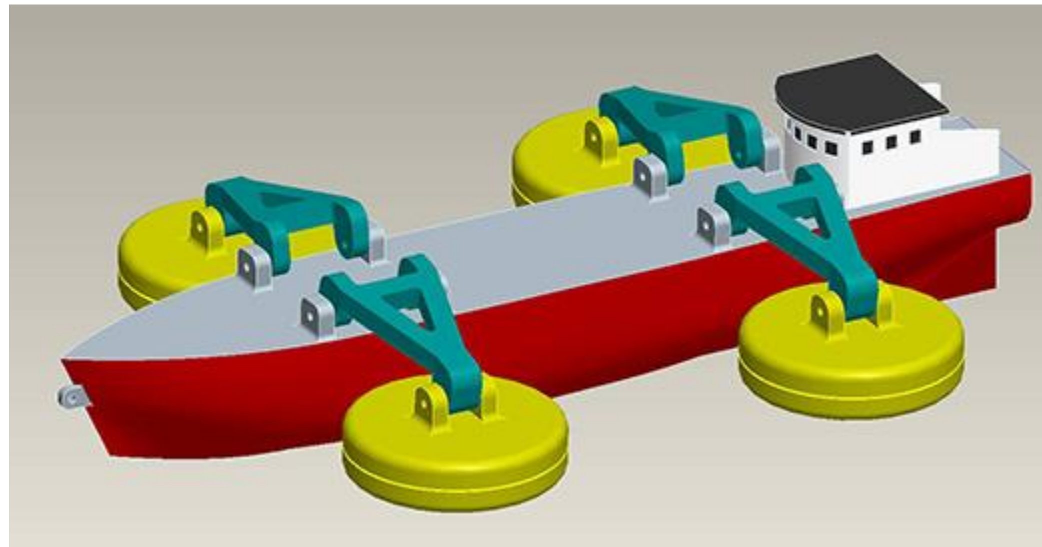


Figure 15: Wave energy harvesting boat concept

Introduction: Construction

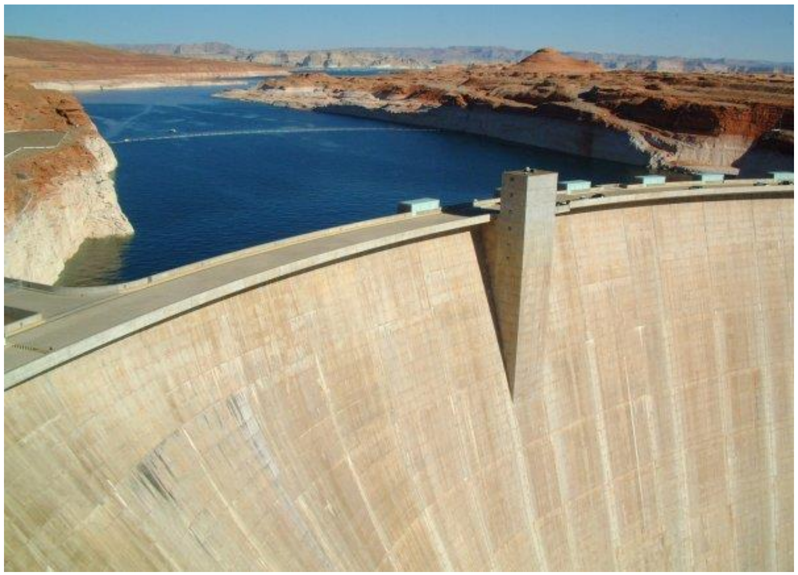


Figure 16:
Glen Canyon Dam

Figure 17:
Jiaozhou Bay bridge (26.4 miles)

Introduction: Sports

Introduction: Agriculture

Size of the droplets?

Figure 18:
Irrigation sprinklers, Eggers and Villermaux (2008)

Introduction

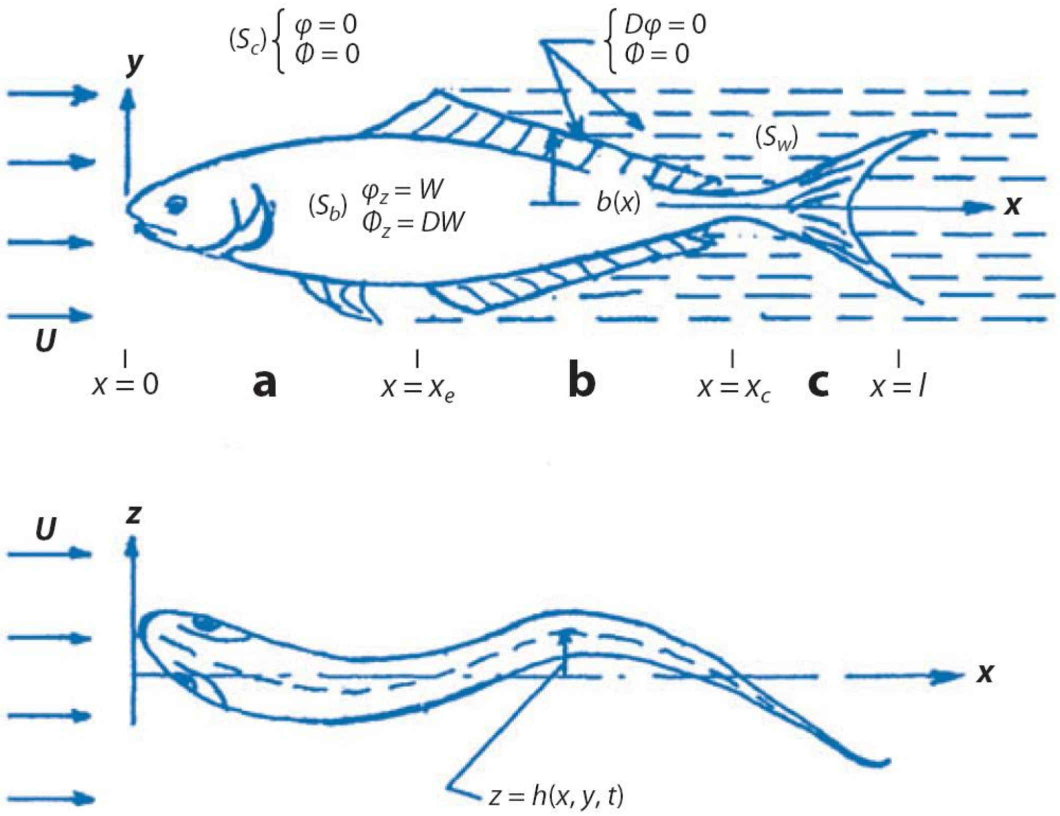
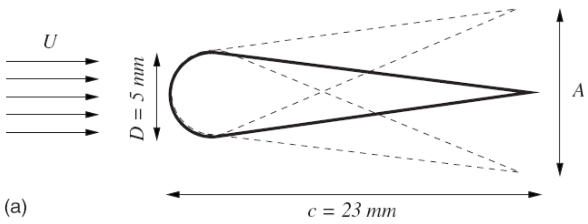
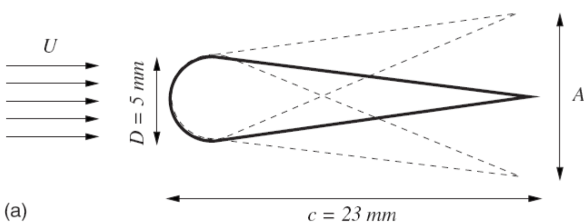


Figure 19:
Flow regions for analyzing fish propulsion: a) Anterior leading-edge section, b) Trailing side-edge section, c) Caudal-fin section

Introduction





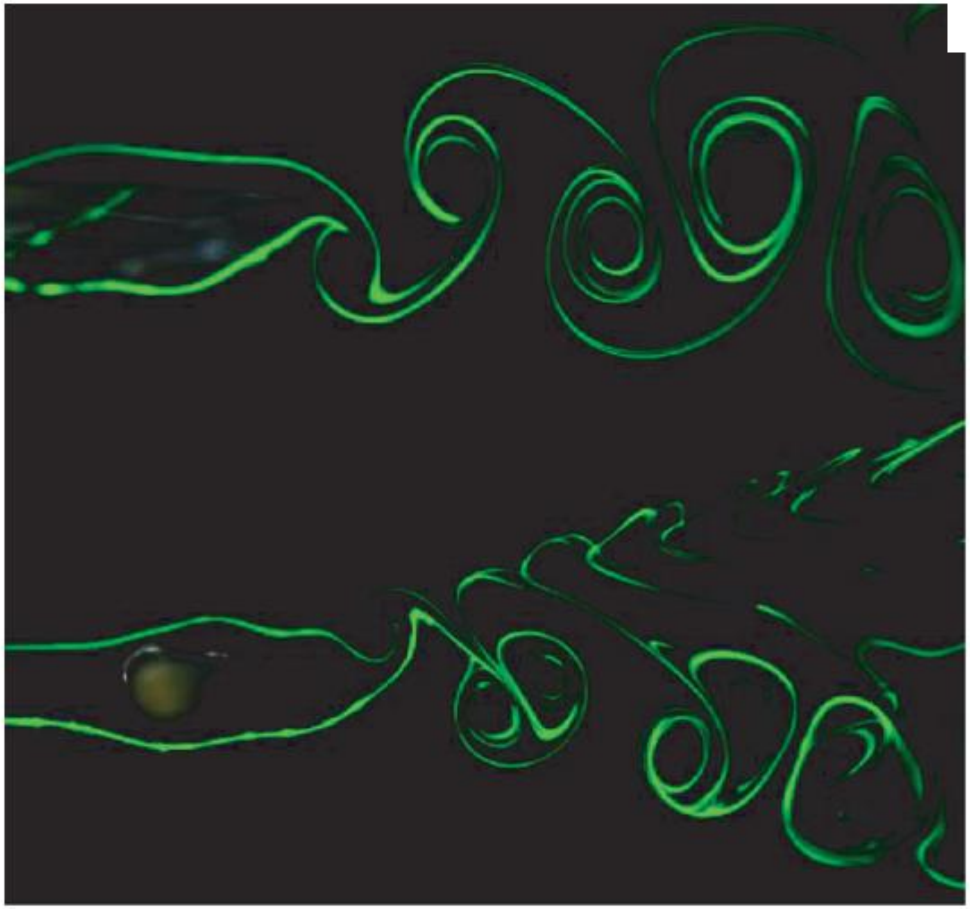


FIGURE 2. Fluorescein dye visualization of the typical reverse BvK vortex street that characterizes flapping-based propulsion (top), and an asymmetric wake (bottom) that is produced by some flapping configurations even when the flap motion is symmetric.

Figure 20: Symmetry breaking of the reverse Bénard-Von Karman vortex street (PMMH-EPSCI-Paris)

Introduction: Rayleigh-Taylor instability

Figure 21: Rayleigh-Taylor instability in a glass

Introduction: But also…

Figure 22: Pint of Guinness and beer head

Flow models

- Continuous model
- Newtonian fluid
- Creeping flow
- Inviscid fluid
- Incompressible flow
- Potential flow
- Boundary layer
- Turbulent flow

Flow models

- Integral relations of conservation laws
- Partial differential equations
- Unidirectional flows
- Harmonic fields
- Similarity analysis/ nondimensional numbers
- Boundary layers
- Matched Asymptotic expansions
- Self-similar solutions

Beware!

All the flows tackled in this class, although quite far from hydrodynamic applications, will hopefully help you to develop the required intuition to avoid falling into the engineer's most frequent **pitfall**:

Using CFD software without thinking and simplifying

Example

Are you really going to implement a 3D-fluid structure coupling CFD code before:

1. You determine the relevant nondimensional parameters?
2. You estimate the boundary layer thickness and evaluate the feasibility of a correct CFD computation?
3. You model the exact shape by a simplified one where literature might be abundant?

Hydrodynamics

Course: Monday 14h15-16h
Exercises: Tuesday 8h15-10h
with Shaha Eghbali and Isha Shukla

Grade:

Homework (20%)

1. exercise
2. article study

Exam: Written

Books:
- Guyon Hulin & Petit, Physical hydrodynamics [Electronic version on NEBIS in french]
- Kundu
- Ryhming PPUR
- Multimedia Fluid Dynamics

Outline

1. Introduction
2. Fluid: Definition and models
3. Fluid Kinematics

What is a fluid? Some definitions

- Dictionary : not solid nor thick, flows easily. Takes the form of its container.
- Physicist : in a fluid, the spatial organization is not that of a solid (crystal) nor the free agitation of molecules of a low pressure gaz.
- Mechanists : a solid is weakly deformable. A fluid is very deformable. Fluids can take any form when they are subjected to forces, regardless of how strong these forces are. Deformation continues until the strain stop (no memory of the reference configuration).

Limits between solid/fluid rather fuzzy

What is a fluid? Some definitions

« **FLUIDE,** *adj. pris subst. (Phys. & Hydrodyn.)* est un corps dont les parties cèdent à la moindre force, & en lui cédant sont aisément mûes entr'elles. Il faut donc pour constituer la fluidité, que les parties se séparent les unes des autres, & cèdent à une impression si petite, qu'elle soit insensible à nos sens ; c'est ce que font l'eau, l'huile, le vin, l'air, le mercure… »

Figure 23:
Definition of a fluid from *l'Encyclopédie Diderot,* d'Alembert.

A Fluid is a body, the constituent parts of which break to the least force, and by breaking are easily moved by one another. In order to constitute fluidity, the parts thus need to separate and break at such a negligible effort that it is unperceivable to our senses; which is what water, oil, air or mercury do…

What is a fluid? Some definitions

- A fluid is a continuum medium that cannot be maintained at rest when stressed.

- In general, this definition is sufficient.

- There exist materials which behave closer to a solid or a fluid, depending on the applied forces, as the so called visco-elastic materials for instance.

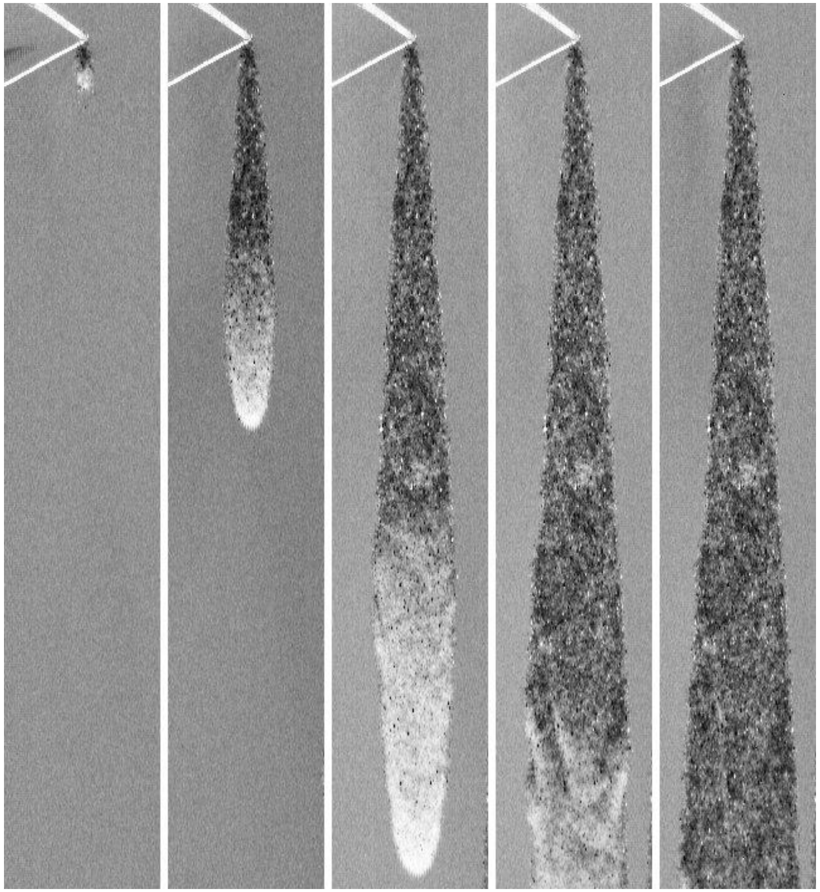
Fluid or solid?

Figure 24: Aletsch Glacier

Fluid or not fluid?

Figure 25: Granular avalanche

Fluid or not fluid?



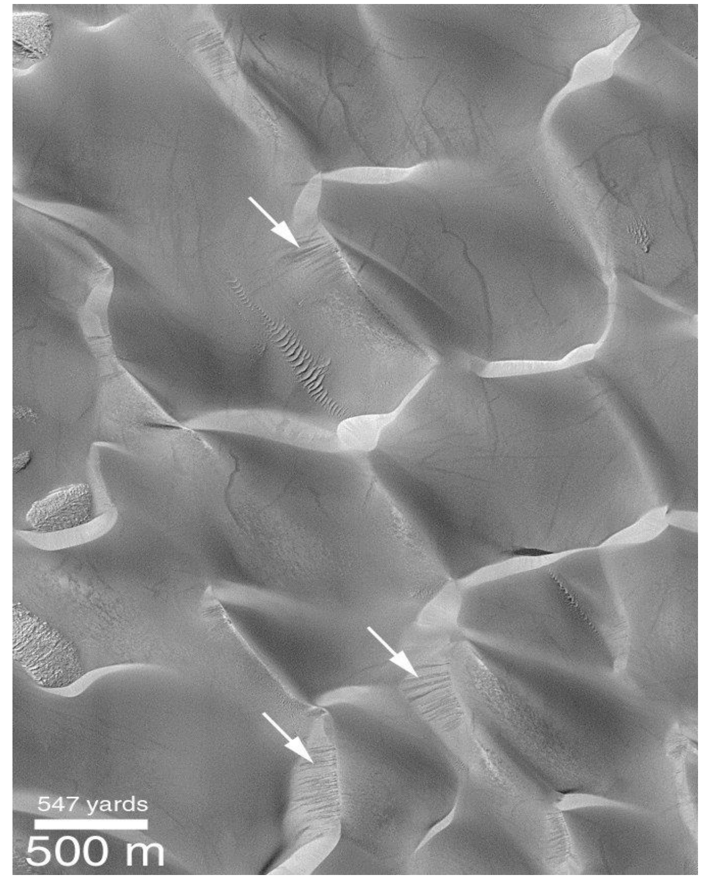


Figure 26: Granular avalanche (PMMH/ESPCI)

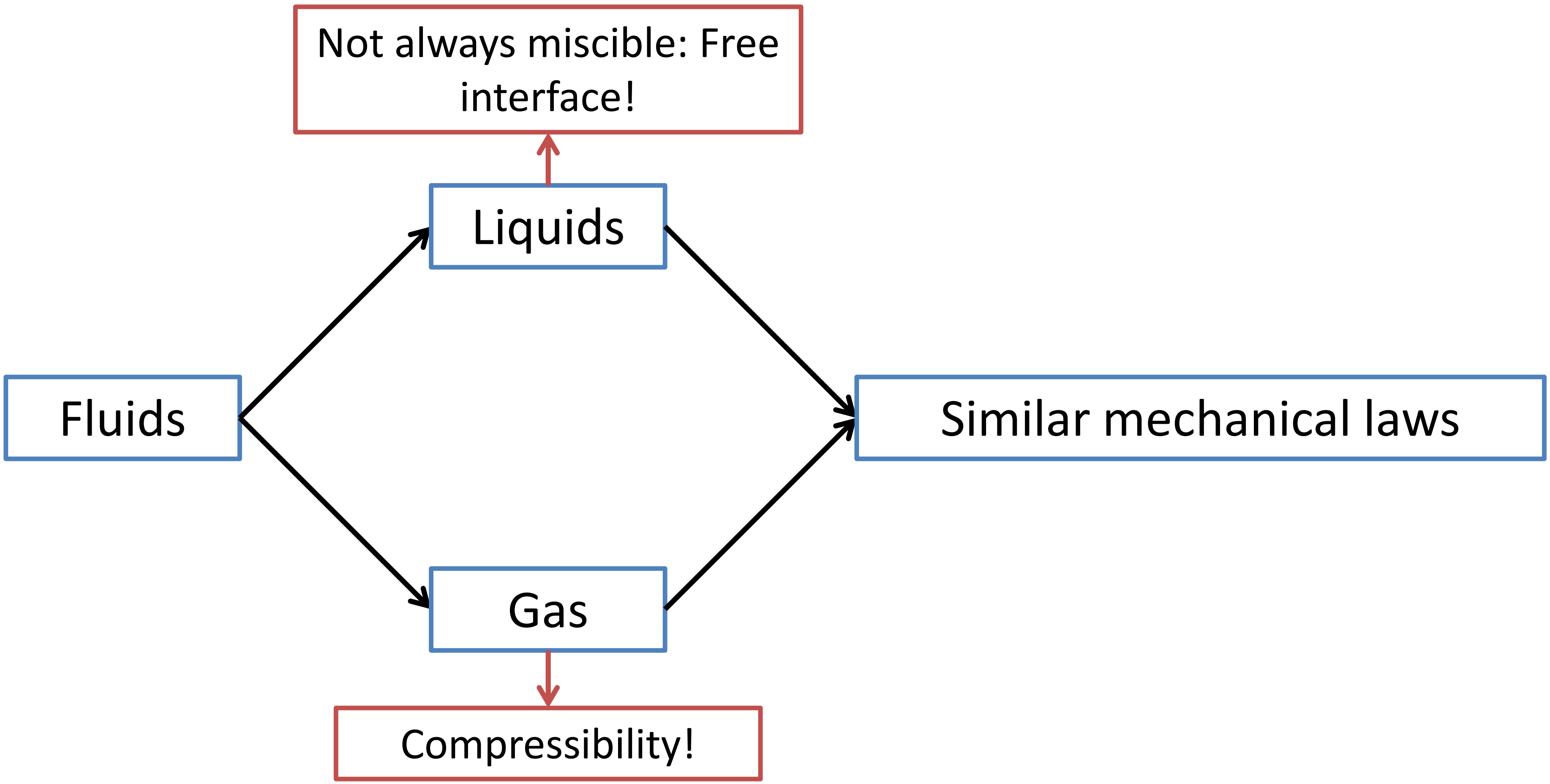
Fluid properties

- 3 scalar quantities : p, ρ, T
- 1 vector quantity : **u**
- All these quantities depend on position and time

  → p(x,y,z,t)…

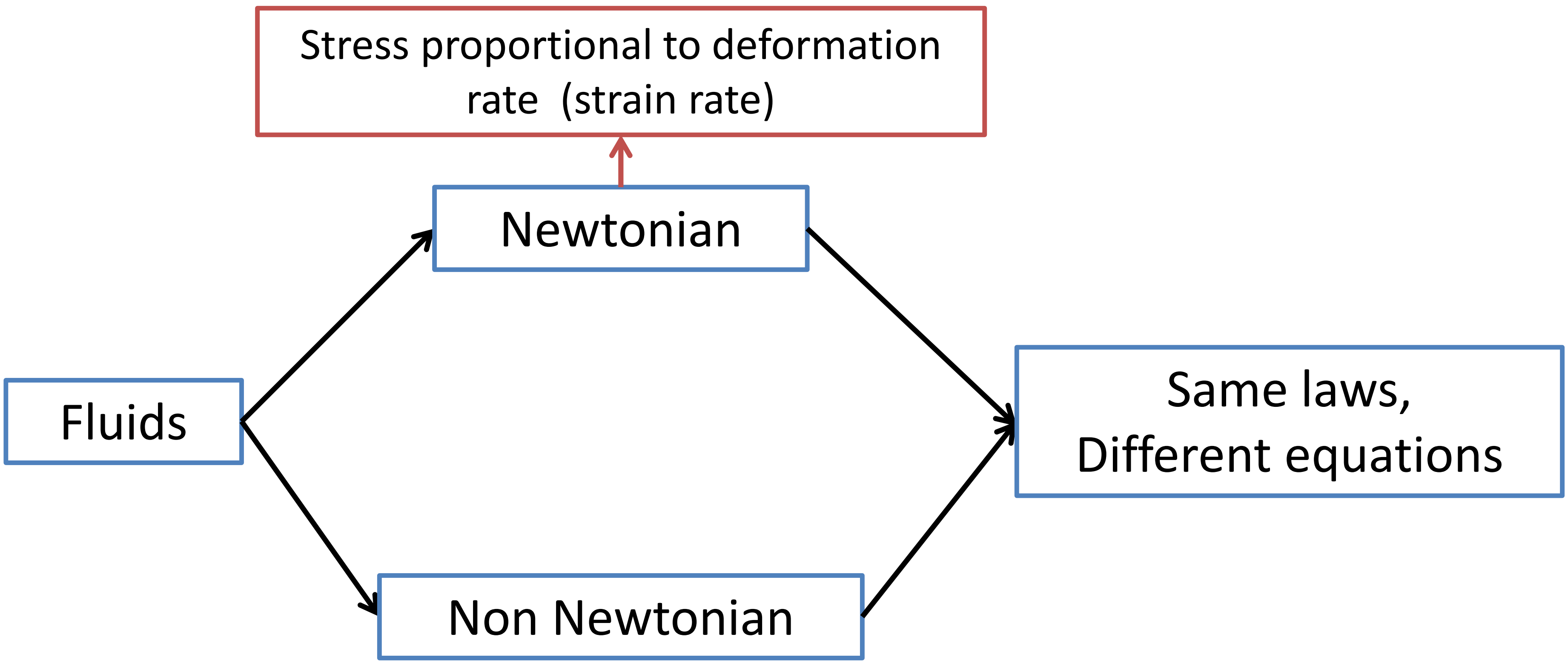
Homogeneous flow : these quantities are independent of the location
p(t)…

Fluid properties



Is there a situation where water is seen to be compressible?

Fluid models: How to relate the deformation of a fluid to the applied stress?



Classification: Several types of flows

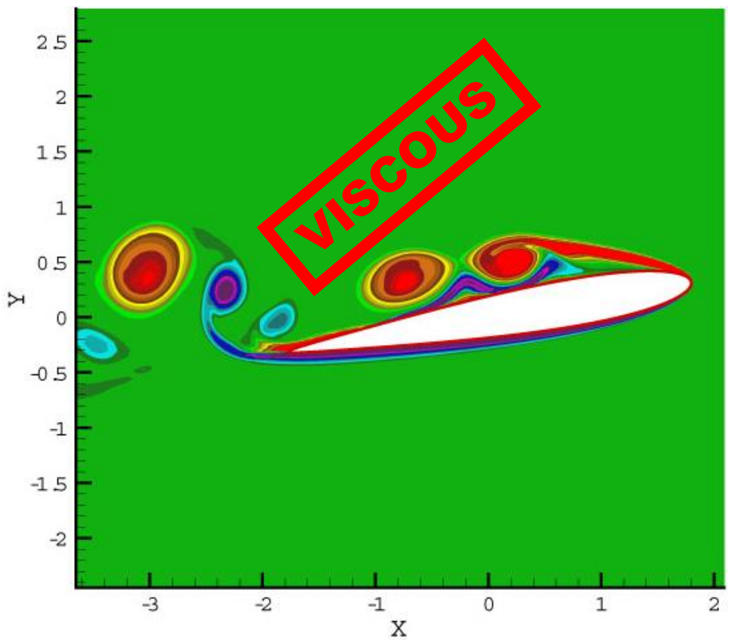
- Compressible/incompressible

Mach > 0.3
« high velocity »
(discontinuities, choc waves…)

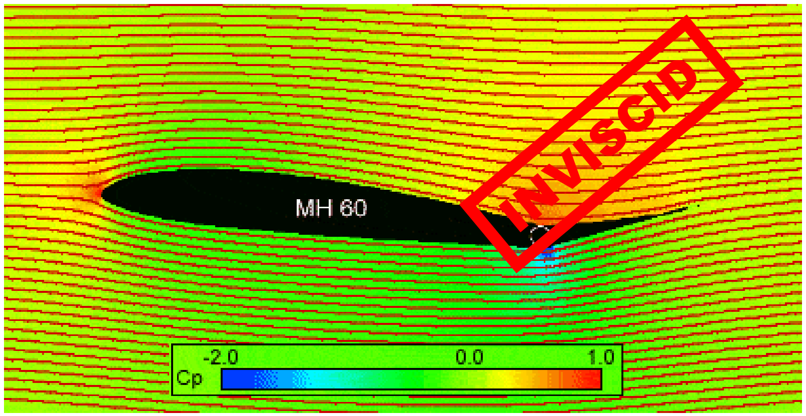
Mach < 0.3
« low velocity »

Classification: Several types of flows

- Viscous/Inviscid



The fluid sticks to the wall, which originates in a boundary layer



The fluid slips at the wall

Instabilities and turbulences

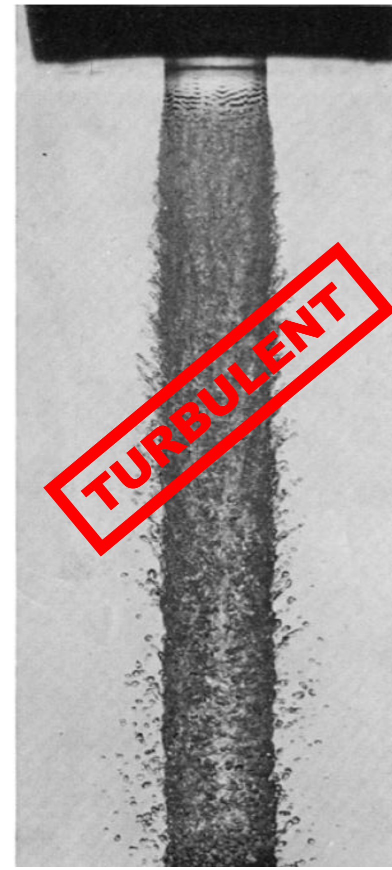
Laminar ⇨ Instability ⇨ Disorder/Pattern/Chaos ⇨ Turbulence

Transition

Marmottant and Villermaux (2004)

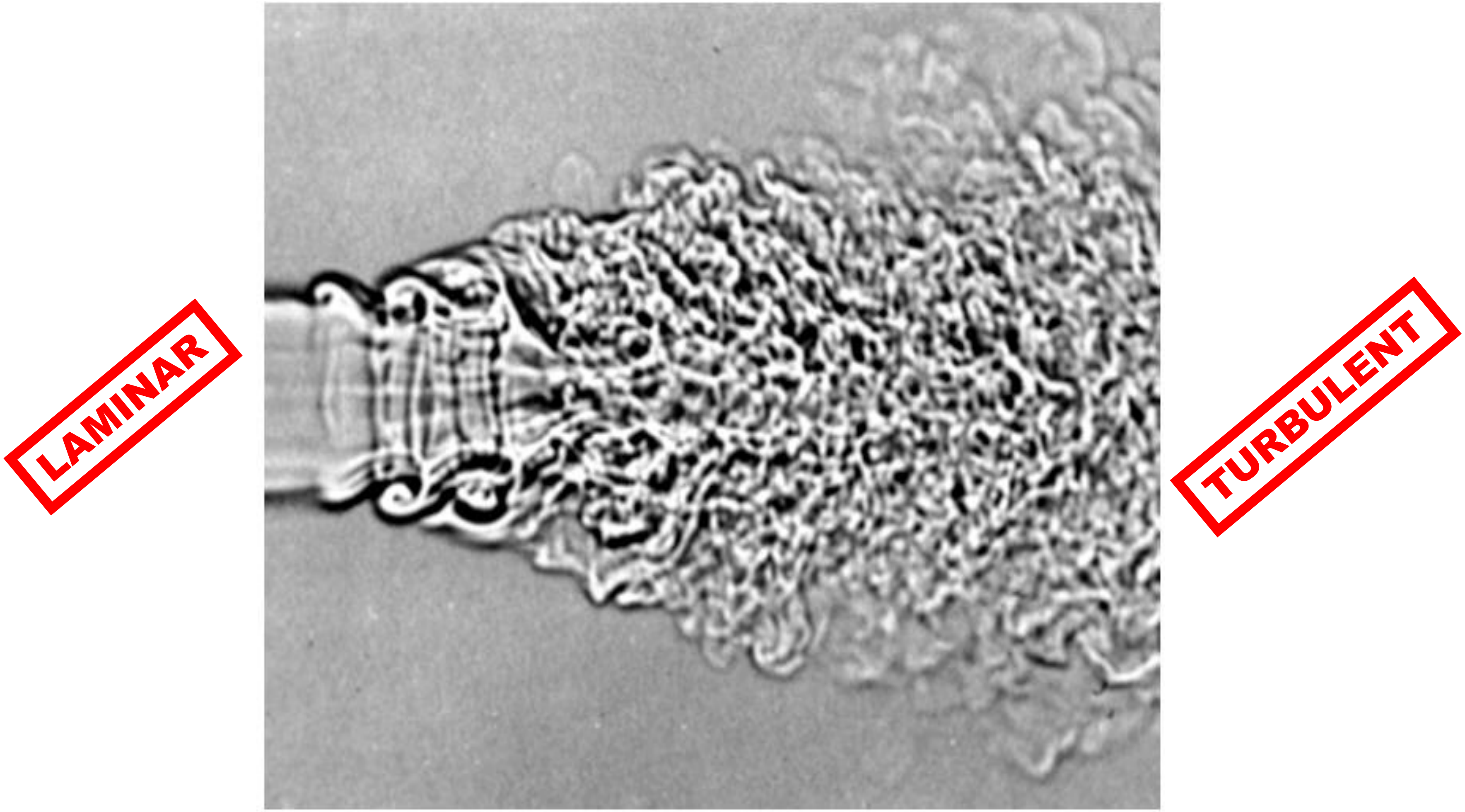
Rayleigh (1891)

Marmottant and Villermaux (2004)



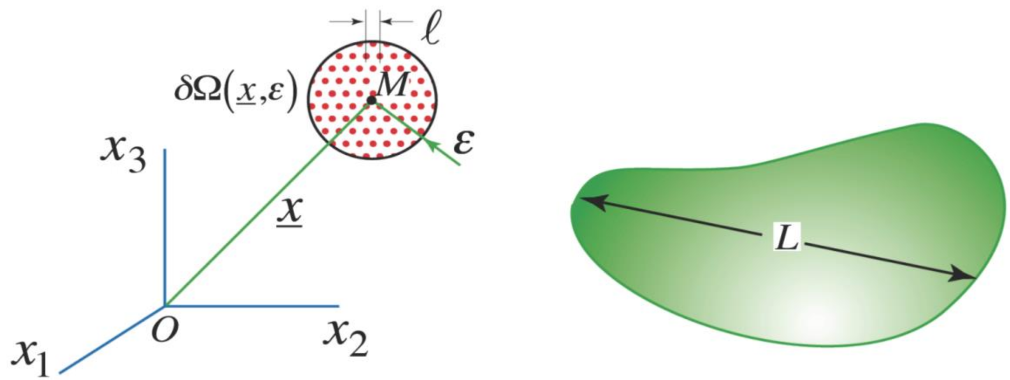
Hoyt and Taylor (1977)

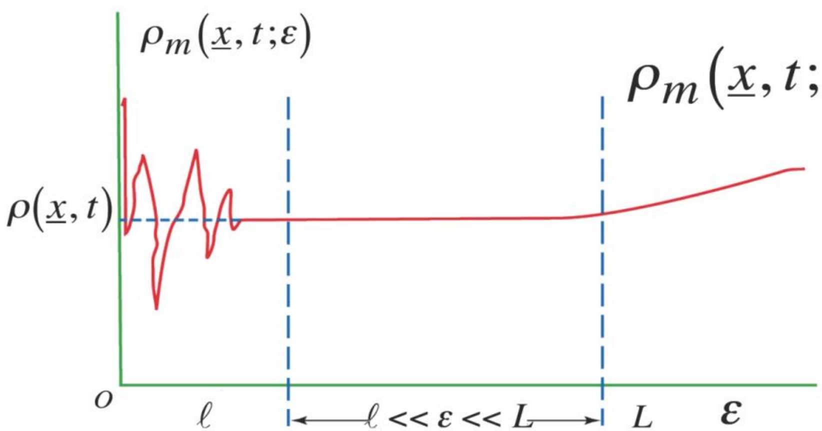
Transition to turbulence



Unsteady, intermittent, no predictability, random

Continuum hypothesis





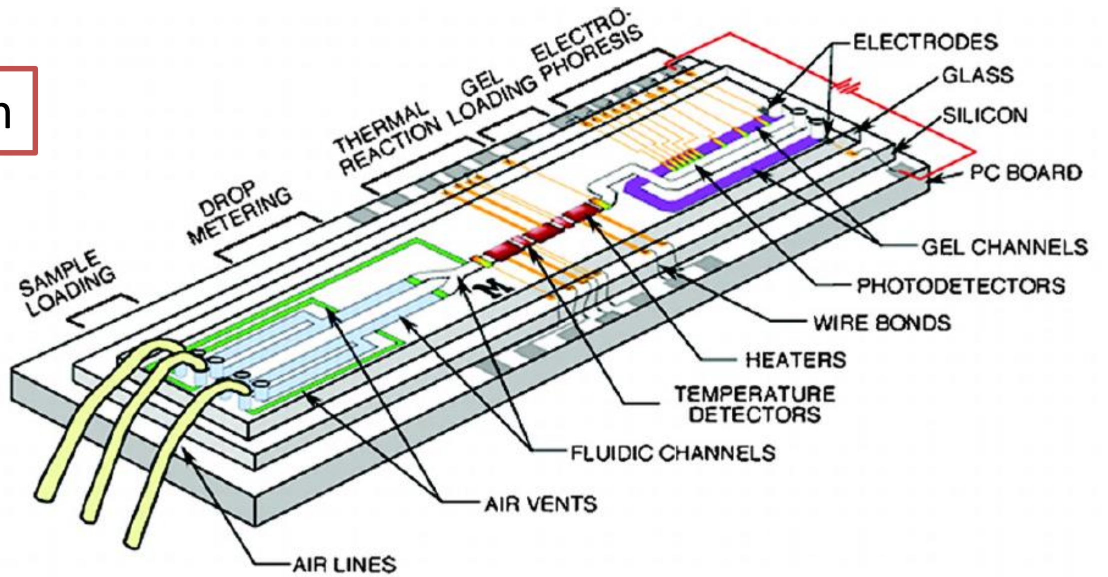
$$\rho_m(\underline{x}, t; \varepsilon) = \frac{\delta m(\underline{x}, t; \varepsilon)}{\delta\Omega(\underline{x}; \varepsilon)}$$

Knudsen number: $$Kn = \frac{l}{L} \ll 1$$

Continuum hypothesis:

Micro-Electro-Mechanical systems

L~100 nm



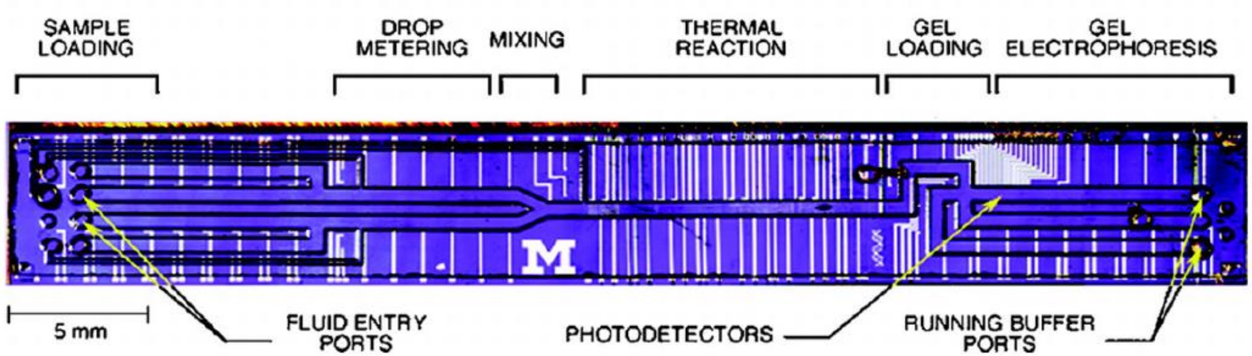


Figure 27: “Lab on a chip” Burns & al (1998)

Outline

1. Introduction
2. Fluid: Definition, models and classifications
3. Fluid Kinematics

Fluid kinematics: Two approaches

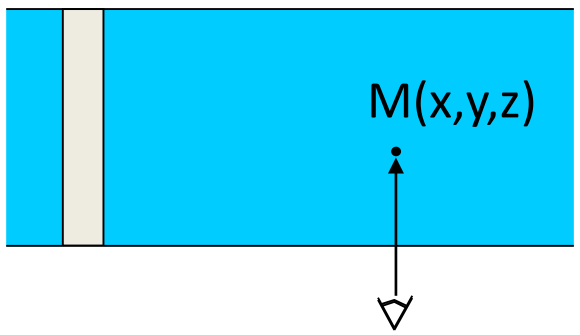
- Fluid kinematics is the study of fluid motion without taking into account of the forces at their origin.

- Two possible approaches:

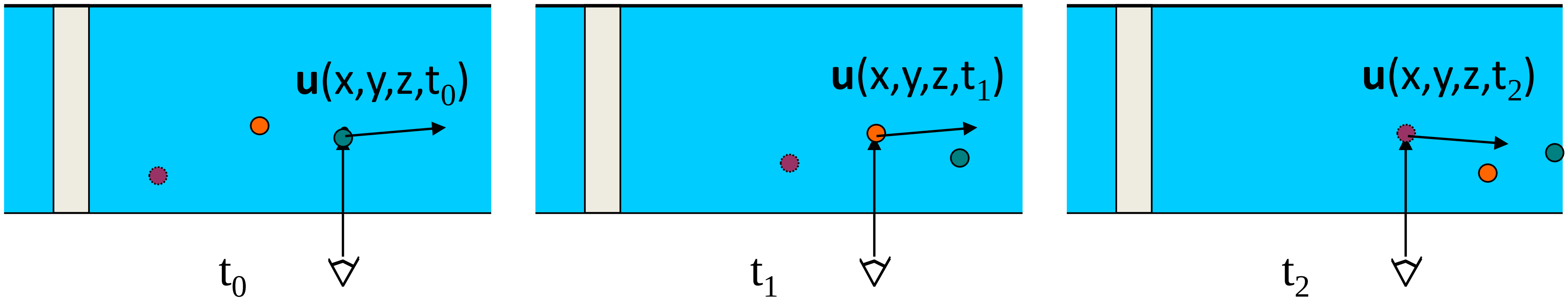
  - Eulerian description
  - Lagrangian description

Eulerian description

- One considers the velocity $\mathbf{u}(x,y,z)$ at a given fixed location M(x,y,z)

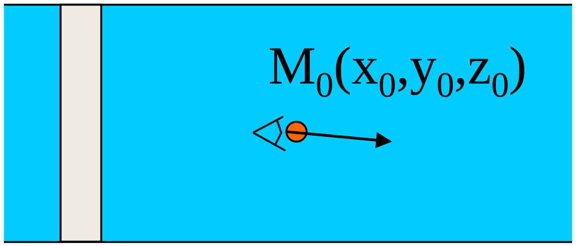


At each time-instant, we consider the velocity of a different fluid parcel



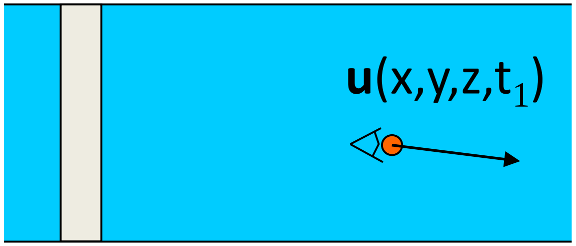
Lagrangian description

- One considers the velocity **u**(x,y,z,t) of a fluid parcel in its motion, by specifying its position $M_0(x_0,y_0,z_0)$ at time t_0.

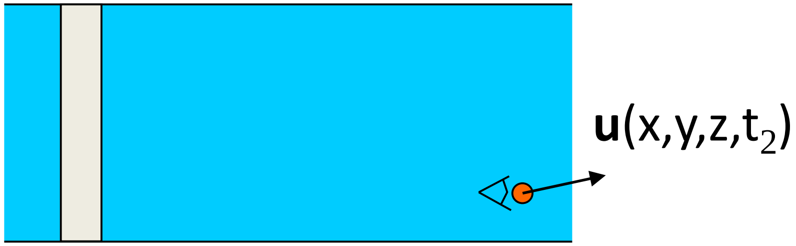


t_0

At each time instant, one considers the same fluid parcel

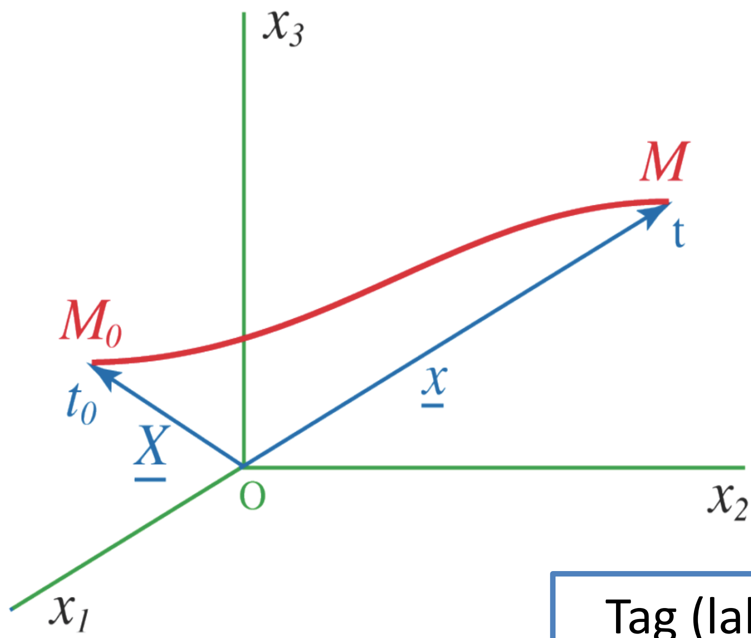


t_1



t_2

Lagrangian description



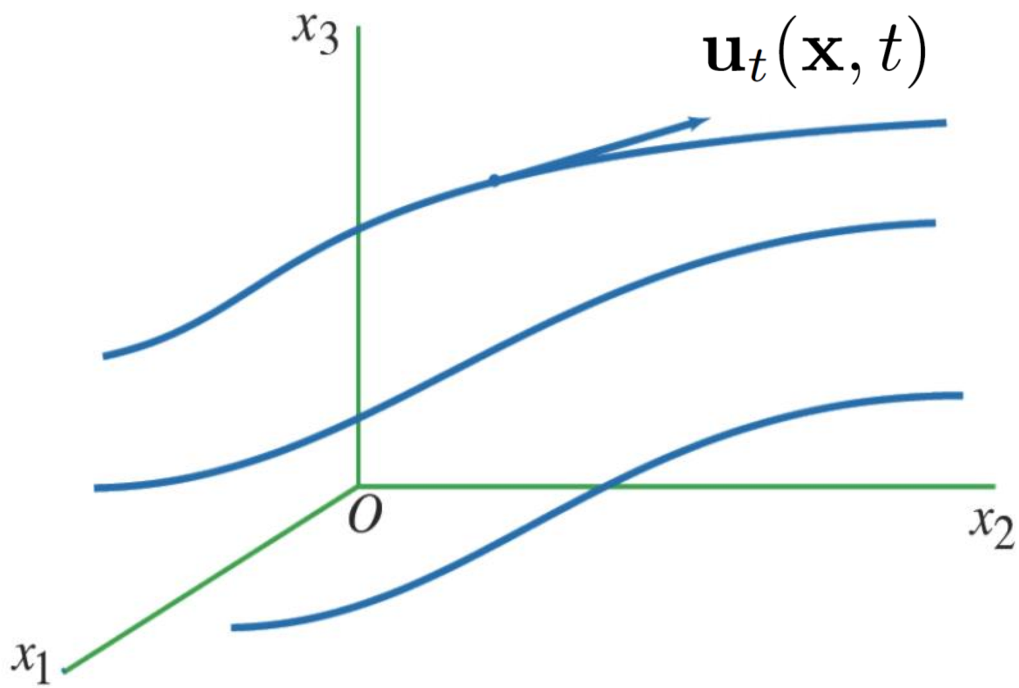
Tag (label)

Trajectory: $$\mathbf{x} = \mathbf{\Phi}(\mathbf{X}, t)$$

Field: $$B = B(\mathbf{X}, t)$$

Velocity: $$\mathbf{U}(\mathbf{X}, t) = \frac{\partial \mathbf{\Phi}}{\partial t}(\mathbf{X}, t)$$

Eulerian description



Trajectory: $$\frac{d\mathbf{x}}{dt} = \mathbf{u}\ (\mathbf{x}, t)$$

$$\mathbf{x}(t = 0) = \mathbf{X}$$

Field: $$B = b(\mathbf{x}, t)$$

↑ Location

Total derivative

$$B(\mathbf{X}, t) = b(\mathbf{x}, t) = b[\mathbf{\Phi}(\mathbf{X}, t), t]$$

$$\dot{B} = \frac{\partial B}{\partial t} = \frac{\partial b}{\partial t} + \nabla b \cdot \frac{\partial \mathbf{\Phi}}{\partial t}$$

$$\boxed{\dot{B} = \frac{db}{dt} = \frac{\partial b}{\partial t} + \nabla b \cdot \mathbf{u}}$$

Total derivative	Local derivative	Convective derivative
$\frac{db}{dt}$	$\frac{\partial b}{\partial t}$	$\nabla b \cdot \mathbf{u}$

Special cases

Uniform flow

$$\nabla \mathbf{u} = \begin{pmatrix} \frac{\partial u_x}{\partial x} & \frac{\partial u_x}{\partial y} & \frac{\partial u_x}{\partial z} \\ \frac{\partial u_y}{\partial x} & \frac{\partial u_y}{\partial y} & \frac{\partial u_y}{\partial z} \\ \frac{\partial u_z}{\partial x} & \frac{\partial u_z}{\partial y} & \frac{\partial u_z}{\partial z} \end{pmatrix} = 0$$

Stationary flow

$$\frac{\partial \mathbf{u}}{\partial t} = 0$$

Total derivative (material derivative)

- In the Eulerian description, one aims at quantifying the temporal variations of a quantity associated to a fluid parcel

$$\frac{db}{dt} = \frac{\partial b}{\partial t} + u_x \frac{\partial b}{\partial x} + u_y \frac{\partial b}{\partial y} + u_z \frac{\partial b}{\partial z}$$

Total derivative (material derivative)

$$\frac{db}{dt} = \frac{\partial b}{\partial t} + u_x \frac{\partial b}{\partial x} + u_y \frac{\partial b}{\partial y} + u_z \frac{\partial b}{\partial z}$$

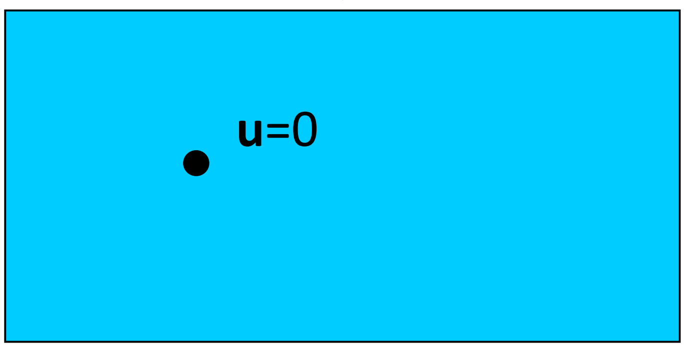
Convective derivative, i.e. temporal variations of b in the fluid parcel due to the transport (advection) of the inhomogeneous field b at the velocity U into from the fluid parcel

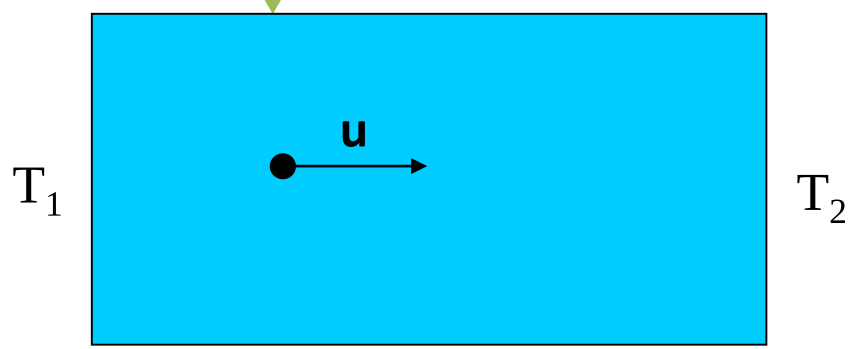
Local derivative, i.e. temporal variation of b at the location of the fluid parcel, i.e. at a geometric fixed location M0

Material derivative, i.e. temporal variation of b inside a fluid parcel

Total derivative (material derivative)

$$\frac{db}{dt} = \boxed{\frac{\partial b}{\partial t}} + \boxed{u_x \frac{\partial b}{\partial x} + u_y \frac{\partial b}{\partial y} + u_z \frac{\partial b}{\partial z}}$$





Example: I am floating in a heated pool i.e. T(t)

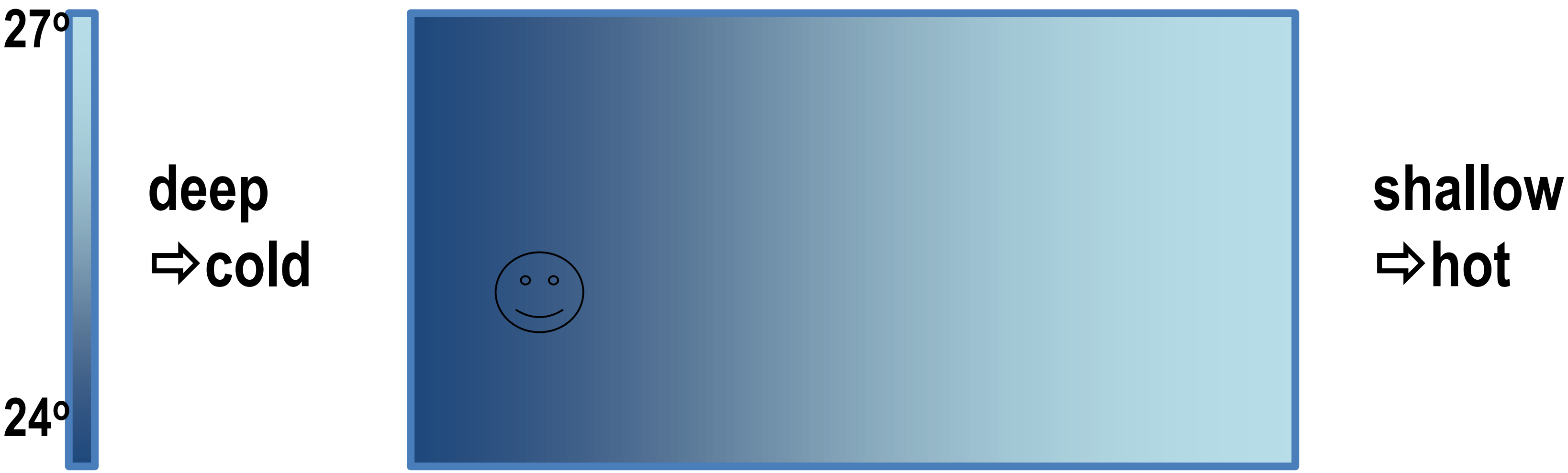
$$\frac{\partial T}{\partial t} \neq 0$$

Example: I am floating in pool where T=T(x,y,z)

$$\frac{\partial T}{\partial t} = 0 \quad \text{but} \quad \frac{dT}{dt} \neq 0$$

Lagrange/Euler?

Ex: felt temperature by a swimmer in a swimming pool with varying depth and therefore temperature

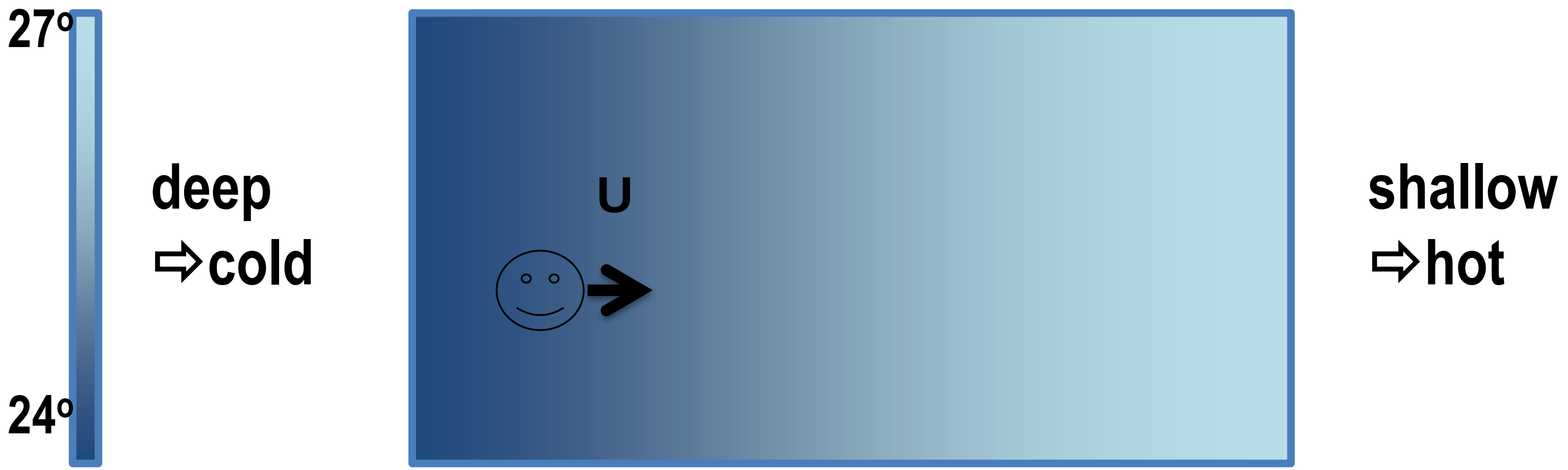


The swimmer is immobile. The temperature does not change with time

$$\frac{DT}{Dt} = 0$$

Lagrange/Euler?

The swimmer now swims at U



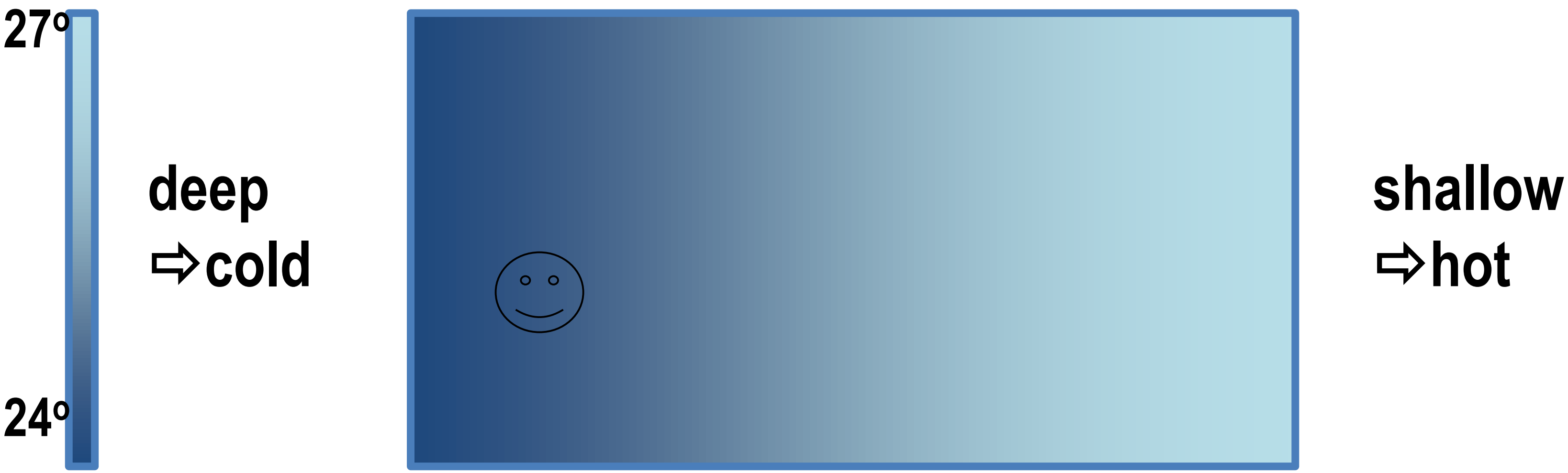
The temperature felt by the swimmer increases with time $\frac{DT}{Dt} > 0$

despite the fact that from an Eulerian point of view $\frac{\partial T}{\partial t} = 0$

$$\frac{DT}{Dt} = U\frac{\partial T}{\partial x}$$

Lagrange/Euler?

The swimmer is at rest again, but the sun shines hard



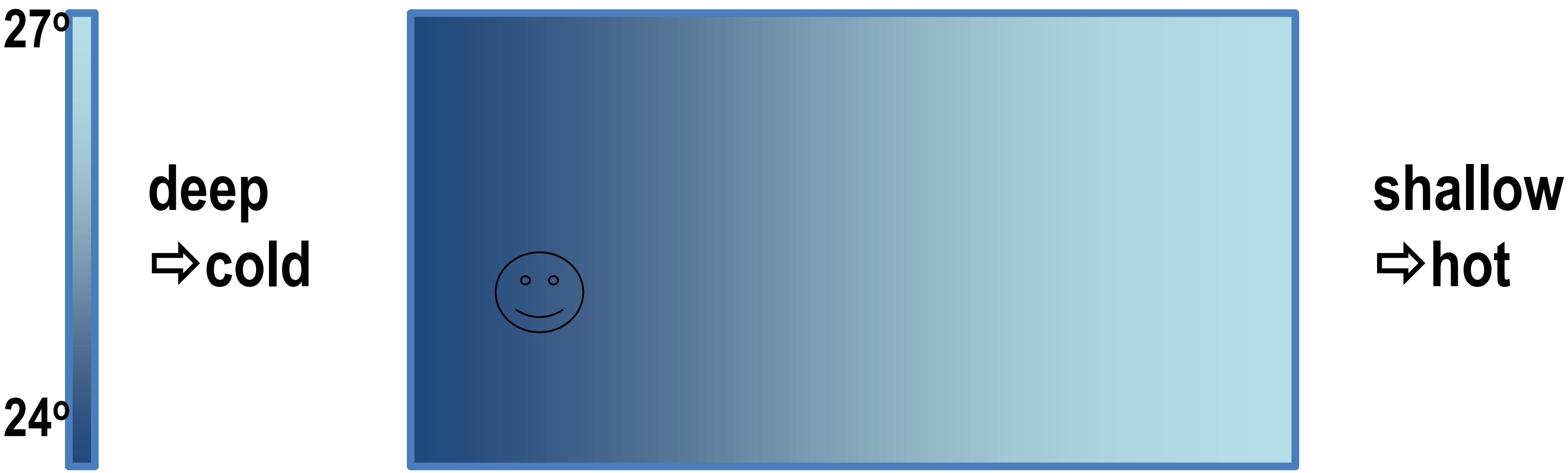
The temperature felt by the swimmer increases with time because it increases point wise. There is no motion, so that Euler and Lagrange have the same point of view.

$$\frac{DT}{Dt} > 0$$

$$\boxed{\frac{DT}{Dt} = \frac{\partial T}{\partial t}}$$

Lagrange/Euler?

The swimmer is at rest again, but the sun shines hard



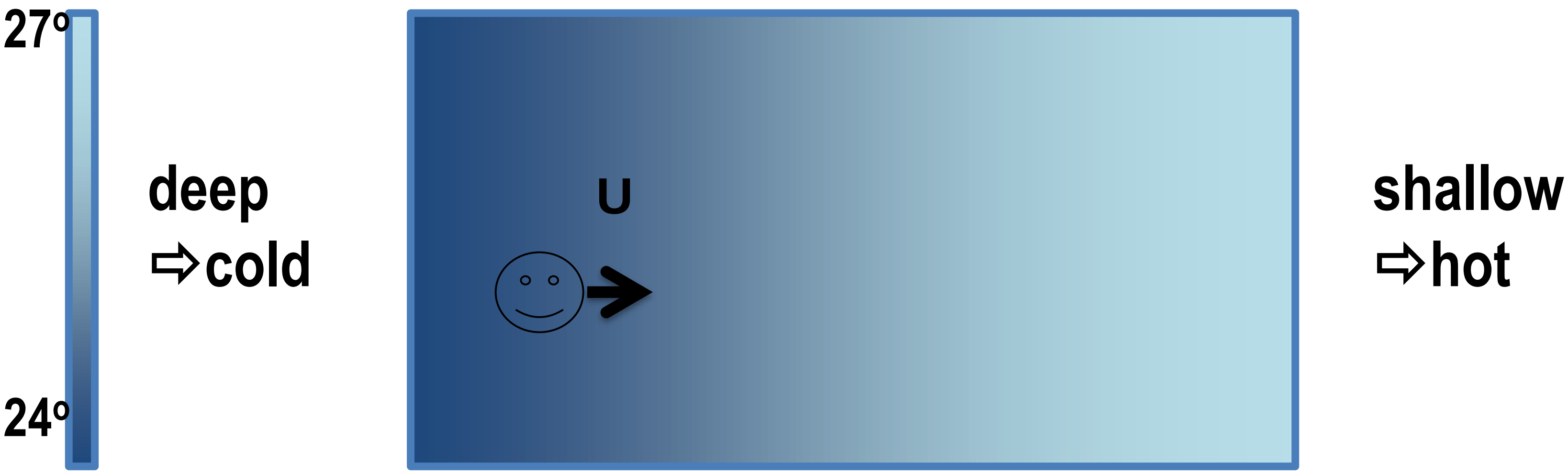
The temperature felt by the swimmer increases with time because it increases point wise. There is no motion, so that Euler and Lagrange have the same point of view.

$$\frac{DT}{Dt} > 0$$

$$\boxed{\frac{DT}{Dt} = \frac{\partial T}{\partial t}}$$

Lagrange/Euler?

The swimmer starts swimming again and clouds arrive…



$$\boxed{\frac{DT}{Dt}} = \boxed{\frac{\partial T}{\partial t}} + \boxed{u_x\frac{\partial T}{\partial x} + u_y\frac{\partial T}{\partial y} + u_z\frac{\partial T}{\partial z}}$$

Lagrangienne derivative
Total derivative

Advective derivative

Eulerian derivative

Acceleration

The acceleration is the particular derivative of the velocity

Local acceleration

Convective acceleration

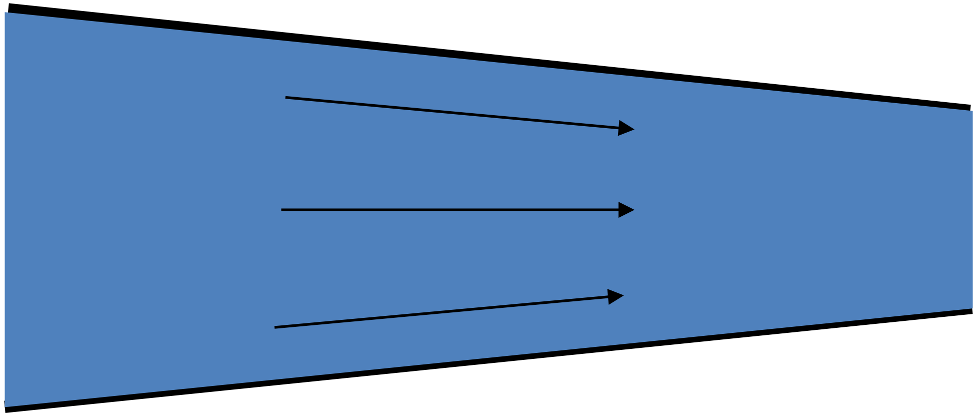
$$a_x = \frac{\partial u_x}{\partial t} + u_x \frac{\partial u_x}{\partial x} + u_y \frac{\partial u_x}{\partial y} + u_z \frac{\partial u_x}{\partial z}$$

$$a_y = \dots$$

$$a_z = \dots$$

Acceleration

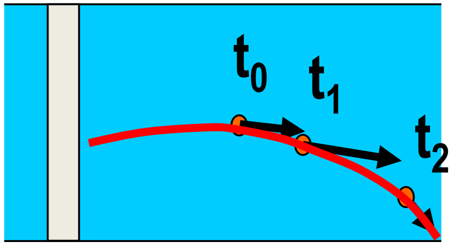
- **Stationary** flow in a **convergent** pipe



The acceleration is not zero (= convective acceleration)

Trajectory

A trajectory is the path of a particle



ODE

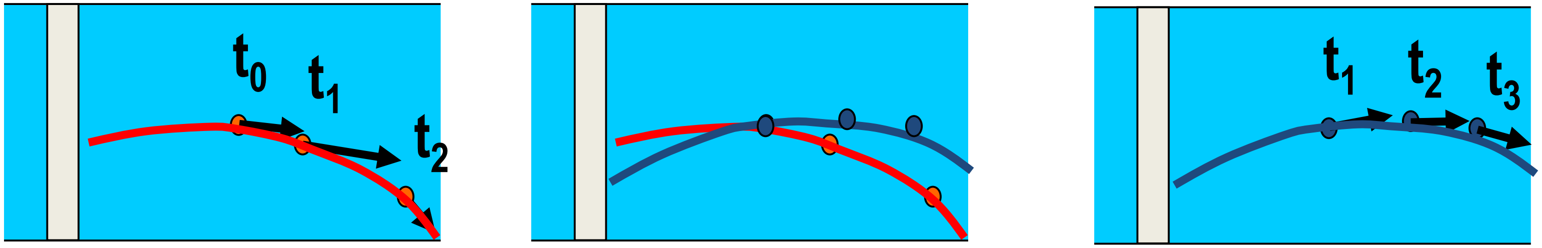
$$\frac{d\mathbf{X}}{dt} = \mathbf{u}(\mathbf{X}, t)$$

Initial condition

$$\mathbf{X}(t_0) = \mathbf{X}_0$$

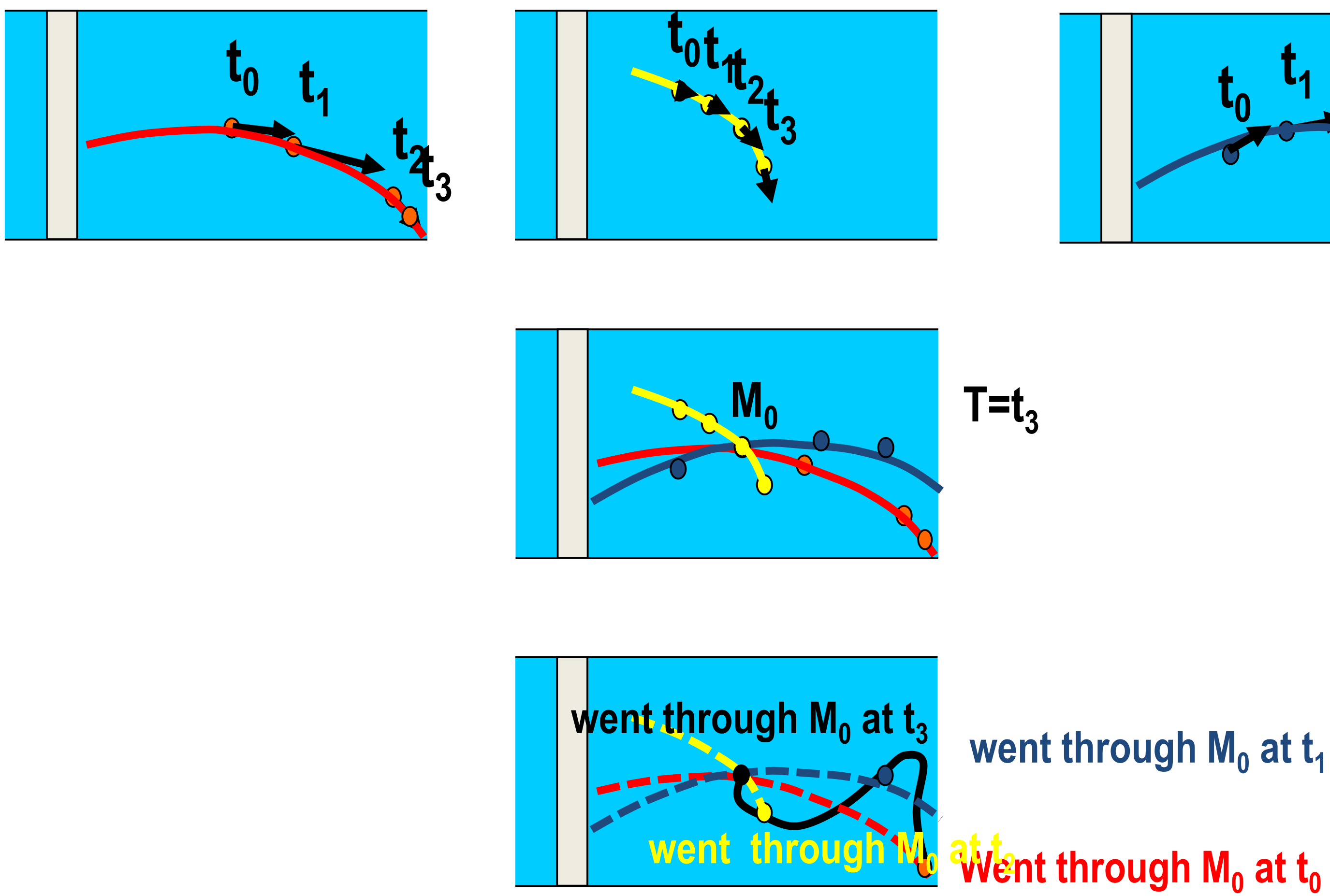
Trajectories can cross

In an usteady flow, trajectories can cross



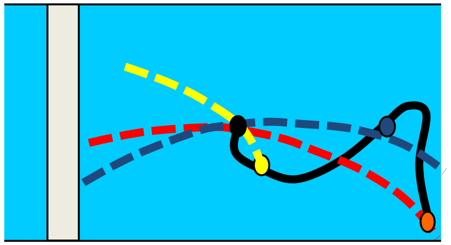
Path lines

Collection of locations of particles at t=T, that went through M_0 at t<T



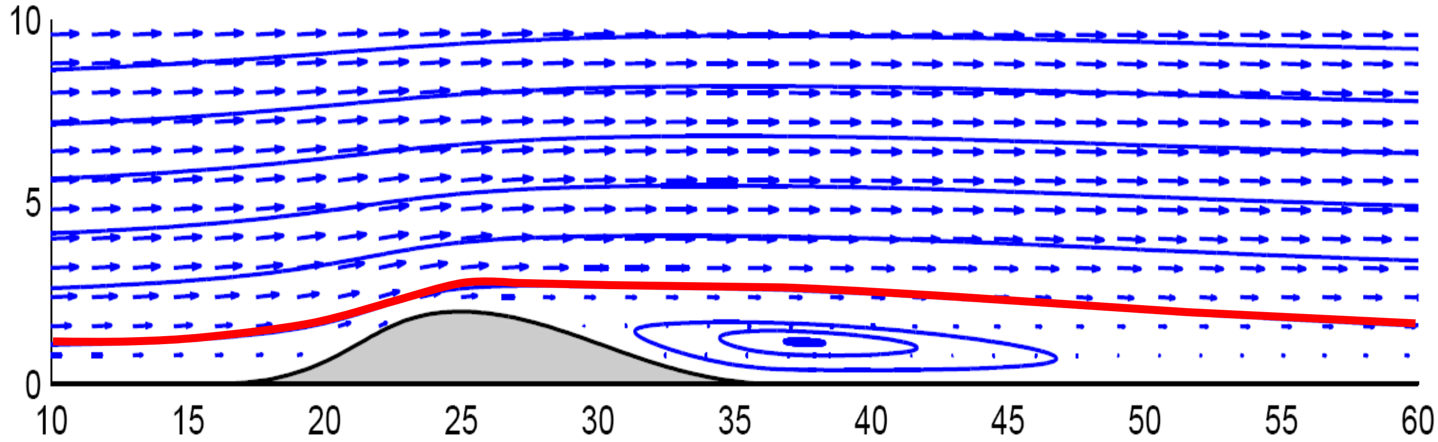
Trajectories and path lines

In an unsteady flow, trajectories and path lines are not superimposed



Streamlines

Eulerian concept : curve everywhere tangent to the velocity field

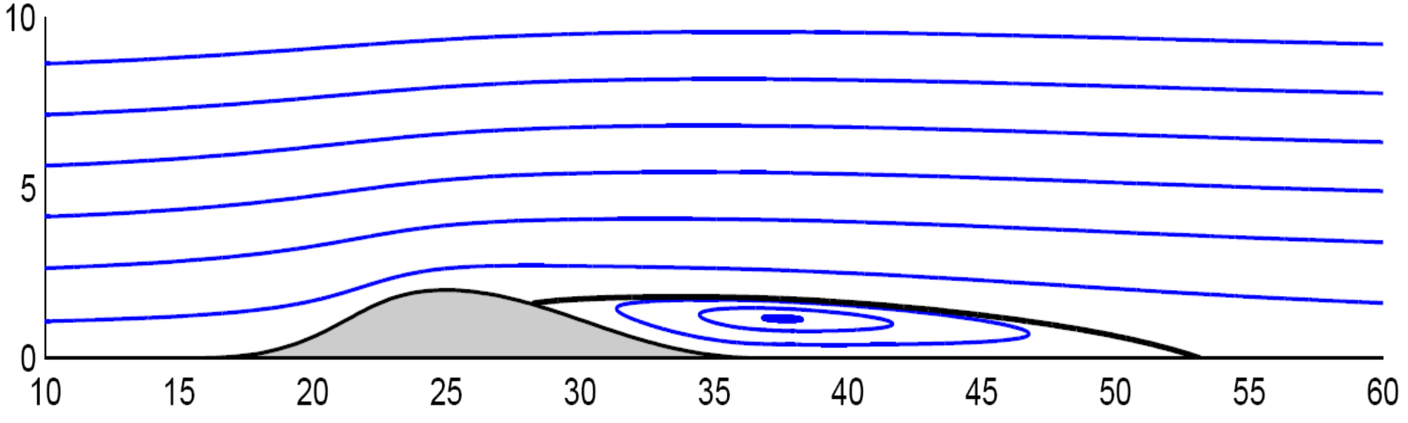


This is a geometric property at a given time t

Streamlines

A streamline does not touch walls

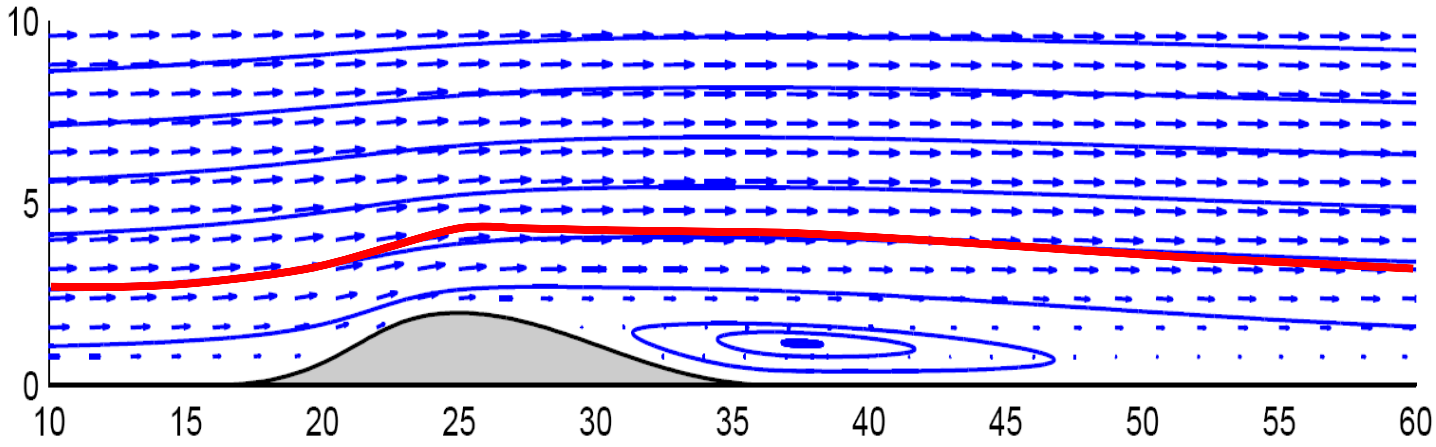
Unless at a stagnation point*, where a separatrix emanates



***where the wall shear stress is zero**

Streamline equation

Curve everywhere tangent to the flow field

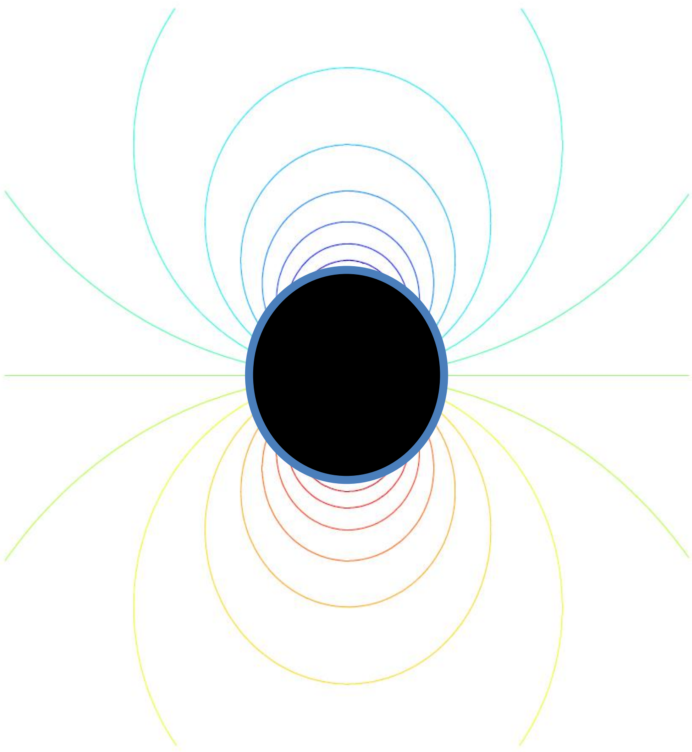


Differential equation

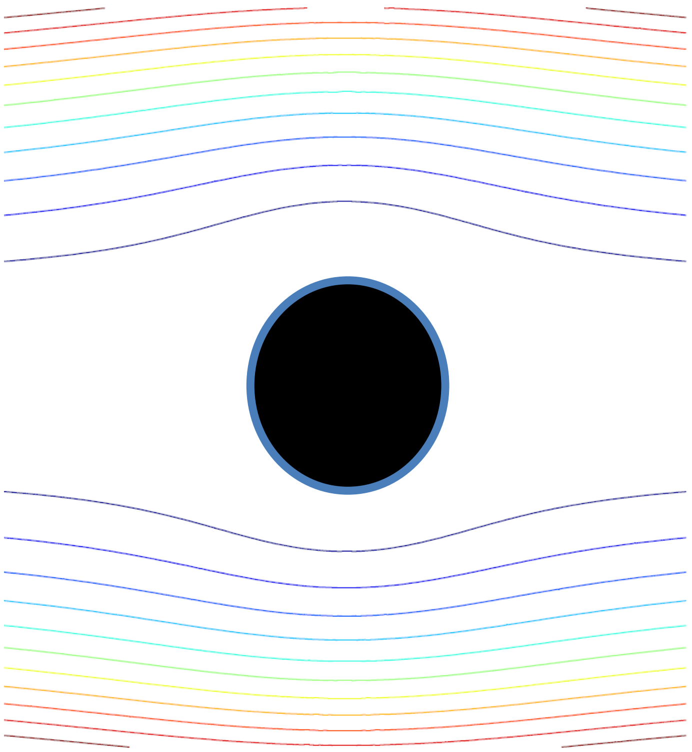
$$\mathbf{u} \wedge d\mathbf{x} = 0$$

Beware of the reference frame!

A cylinder moves at constant velocity in a very viscous fluid

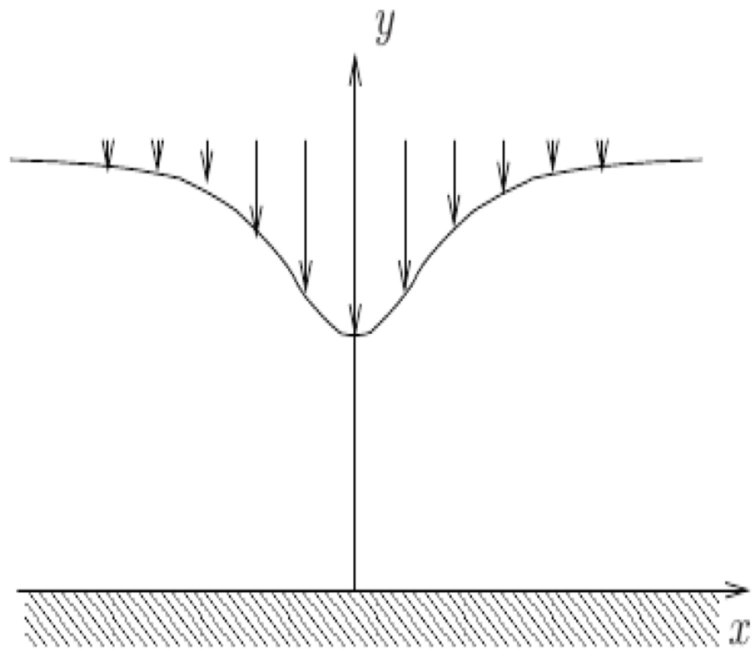


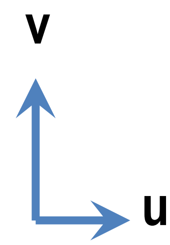
Lab reference frame



Cylinder reference frame

Impacting jet



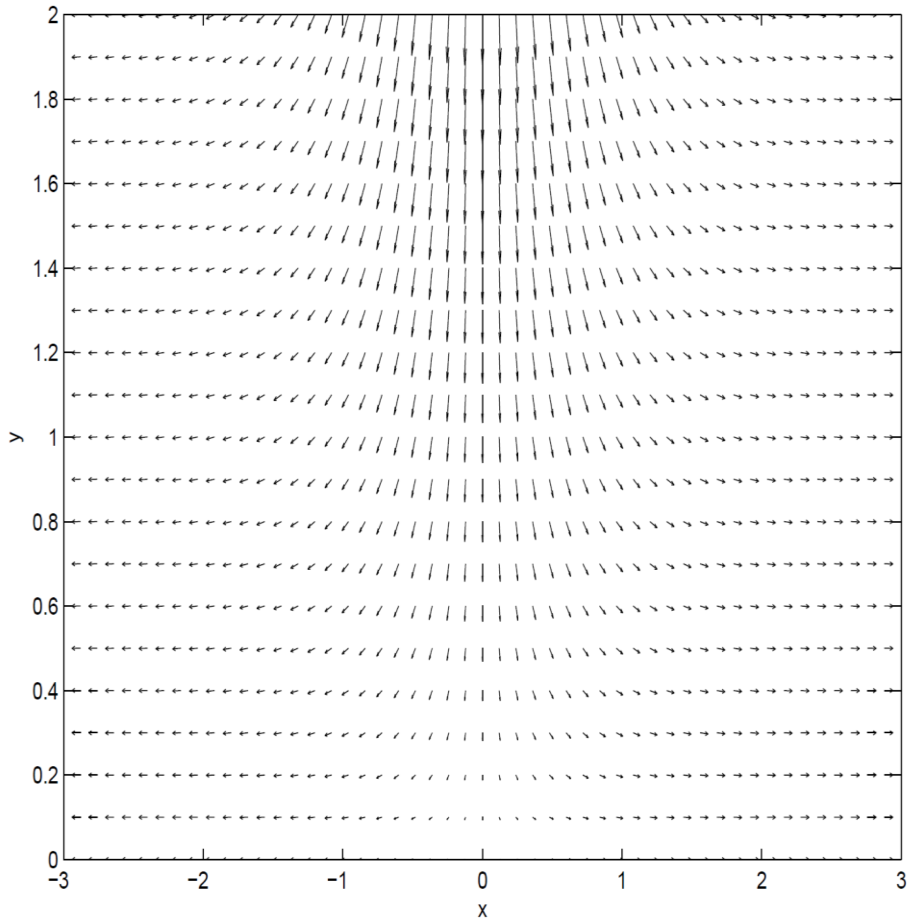


$$v = -V_0 y \cosh^{-2}(x - x_0)$$

$$u = V_0 \tanh(x - x_0)$$

Stationary jet

Flow field



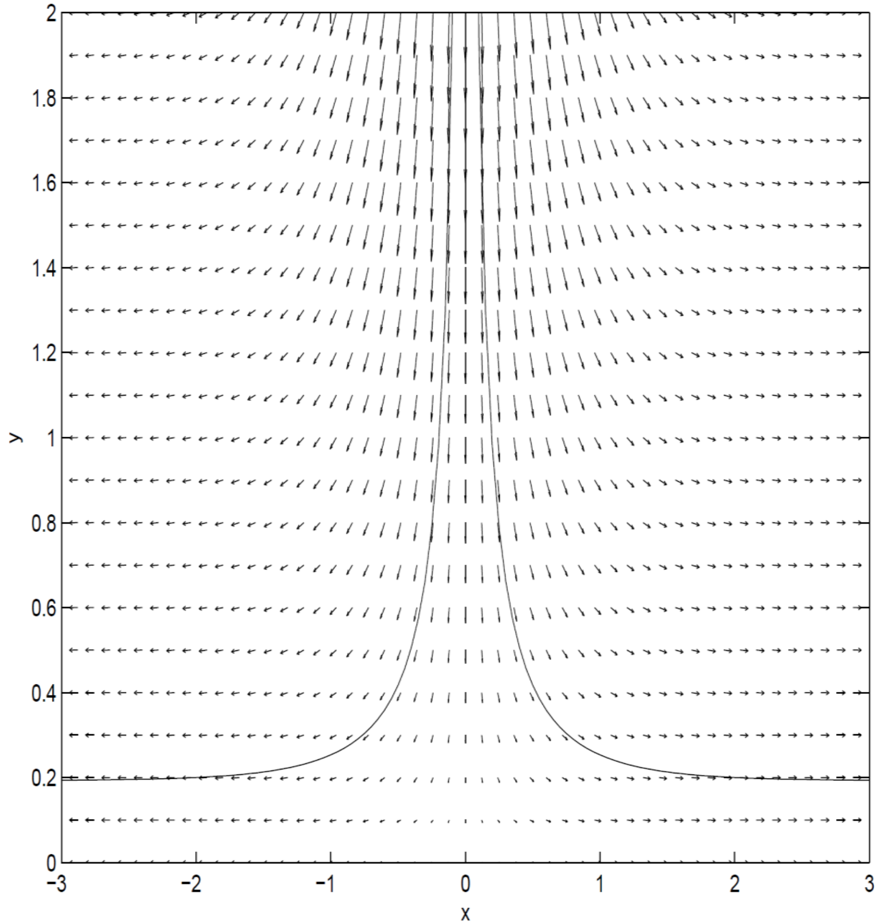
$$v = -V_0 y \cosh^{-2}(x - x_0)$$

$$u = V_0 \tanh(x - x_0)$$

$$x_0 = 0$$

$$V_0 = 1$$

Streamline



$$\frac{dx}{dy} = \frac{u}{v}$$

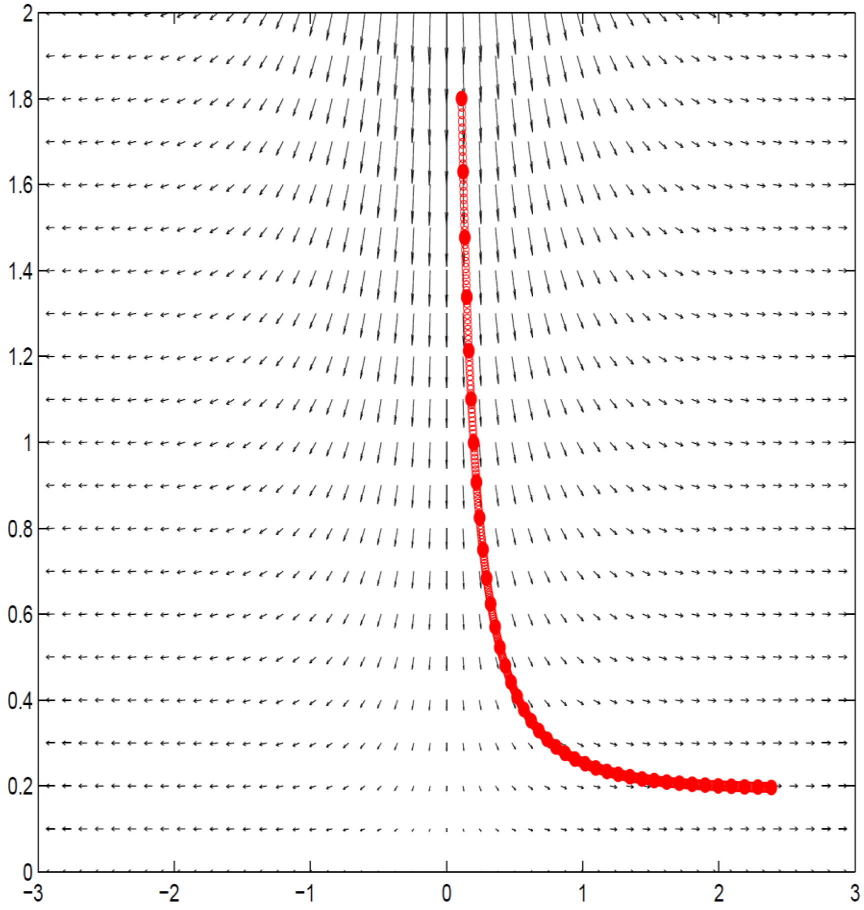
$$u = V_0 \tanh(x - x_0)$$

$$v = -V_0 y \cosh^{-2}(x - x_0)$$

$$\frac{dx}{\cosh(x - x_0)\sinh(x - x_0)} = \frac{dy}{y}$$

$$y = \left| \frac{k}{\tanh(x - x_0)} \right|$$

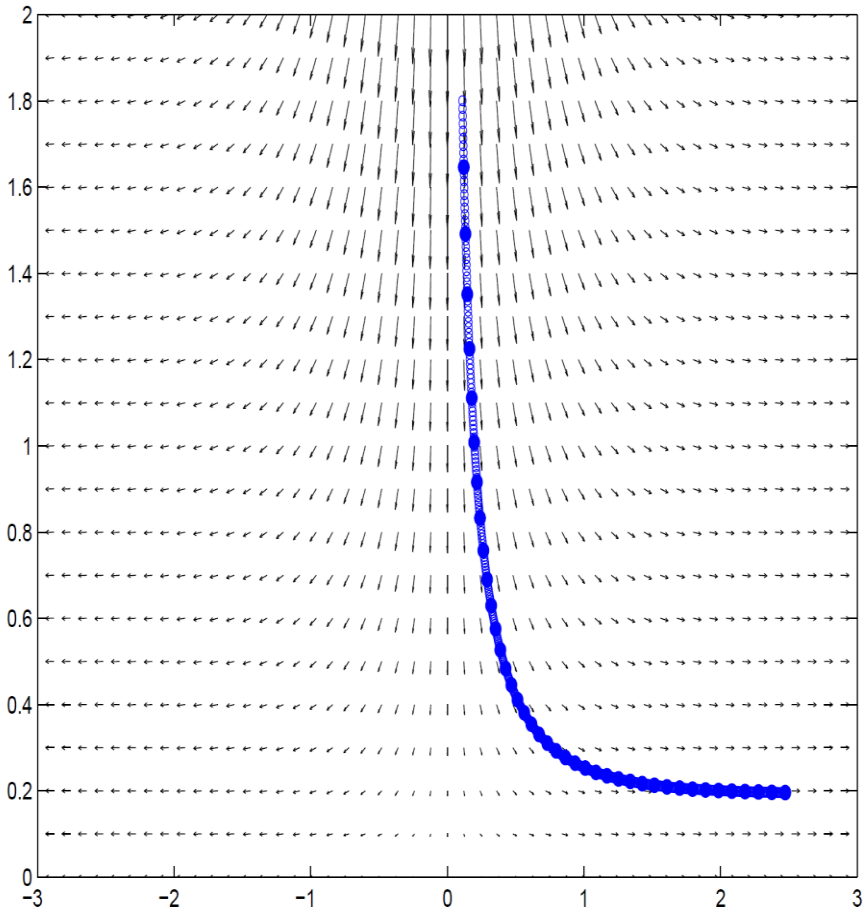
Trajectory (till T=4)



Position de la particule tous les dt

Position de la particule tous les 10dt

Streakline



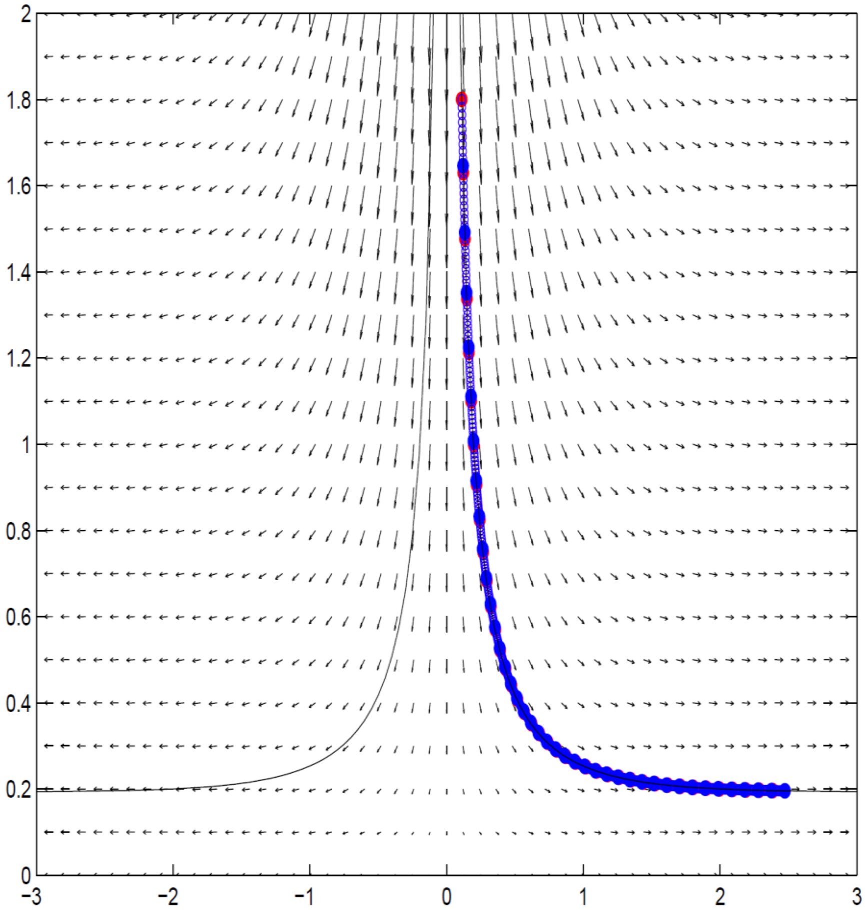
Particule émise tous les dt ○

Particule émise tous les 10dt ●

$$u = V_0 \tanh(x - x_0)$$

$$v = -V_0 y \cosh^{-2}(x - x_0)$$

Trajectory = Streakline = Streamline



Position de la particule tous les dt

Position de la particule tous les 10dt

Particule émise tous les dt

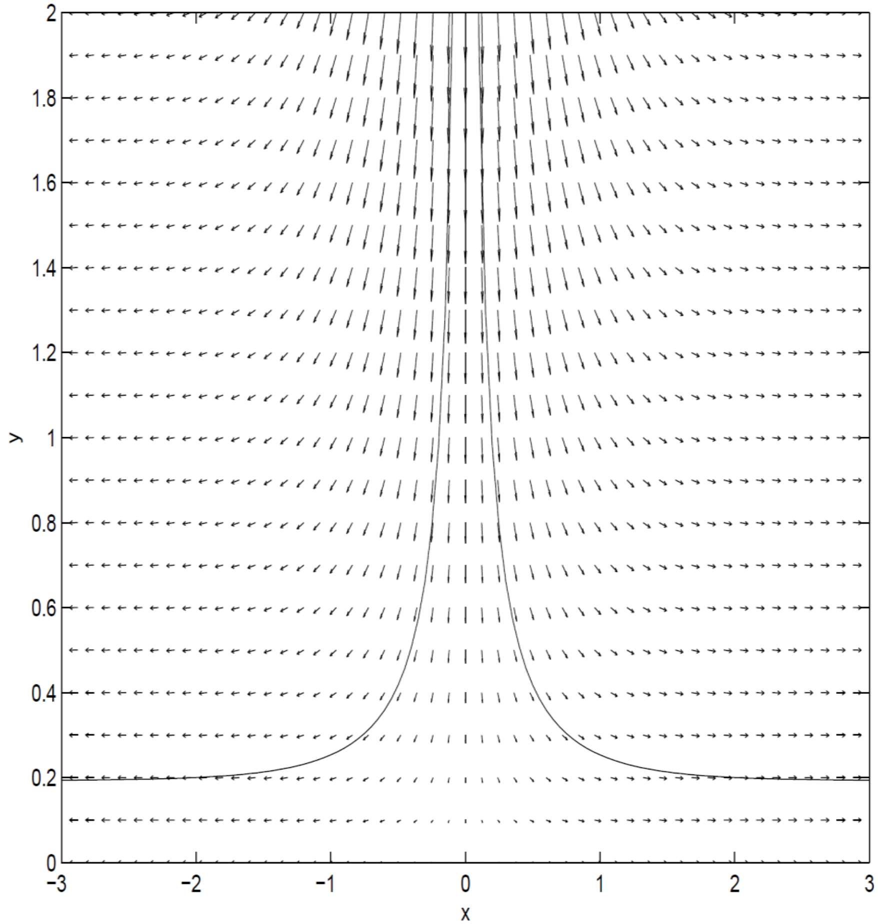
Particule émise tous les 10dt

$$u = V_0 \tanh(x - x_0)$$

$$v = -V_0 y \cosh^{-2}(x - x_0)$$

$$y = \left| \frac{k}{\tanh(x - x_0)} \right|$$

Oscillating jet : instantaneous streamline



$$x_0 = \sin(2t) \;\; V_0 = 1$$

$$\frac{dx}{dy} = \frac{u}{v}$$

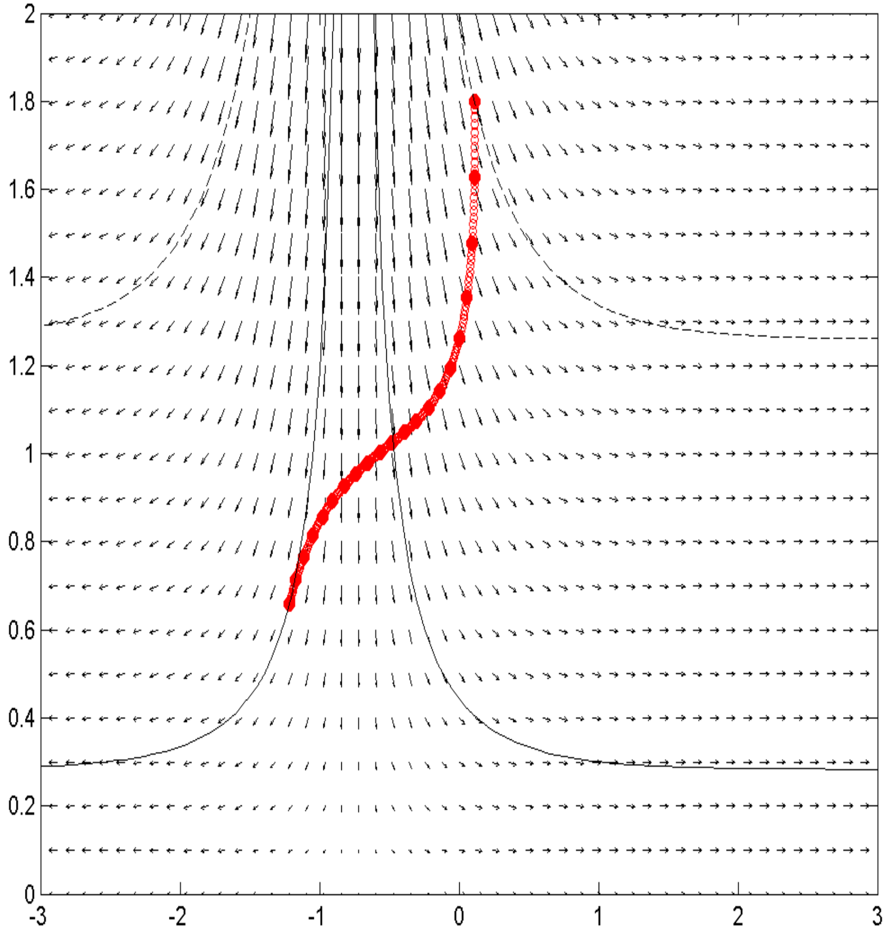
$$u = V_0 \tanh(x - x_0)$$

$$v = -V_0 y \cosh^{-2}(x - x_0)$$

$$\frac{dx}{\cosh(x - x_0)\sinh(x - x_0)} = \frac{dy}{y}$$

$$y = \left| \frac{k}{\tanh(x - x_0)} \right|$$

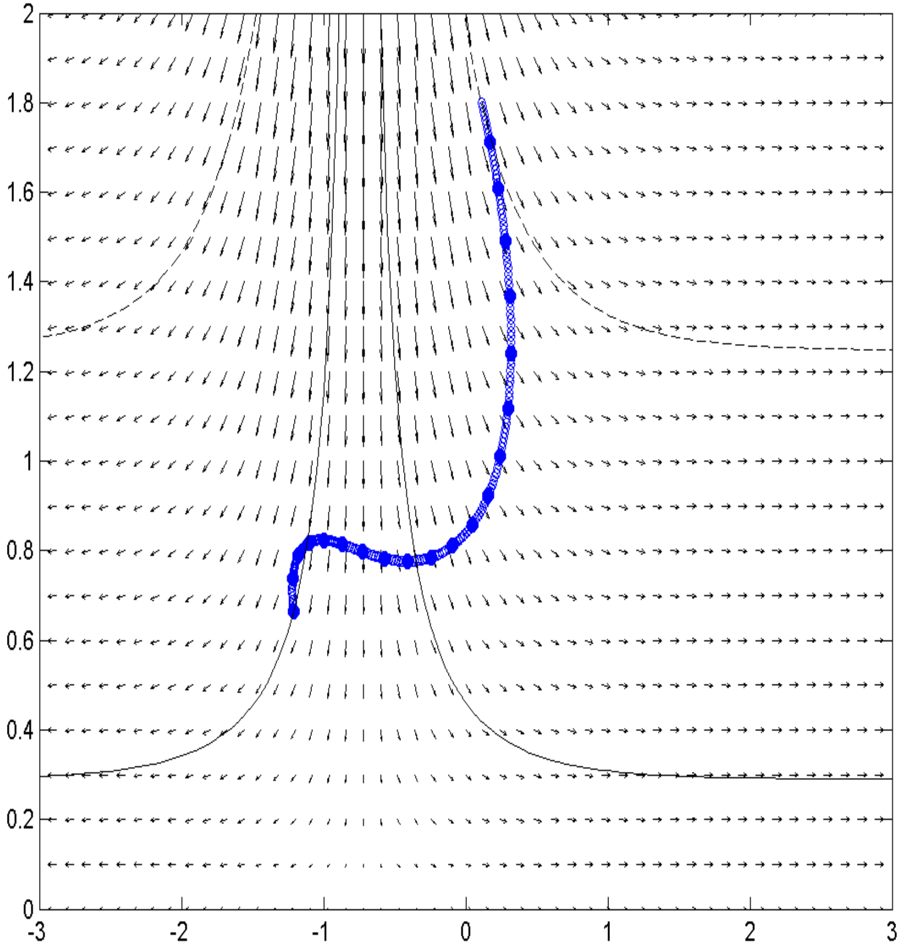
Trajectory (till T=2)



osition de la particule tous les dt

osition de la particule tous les 10dt

Streakline



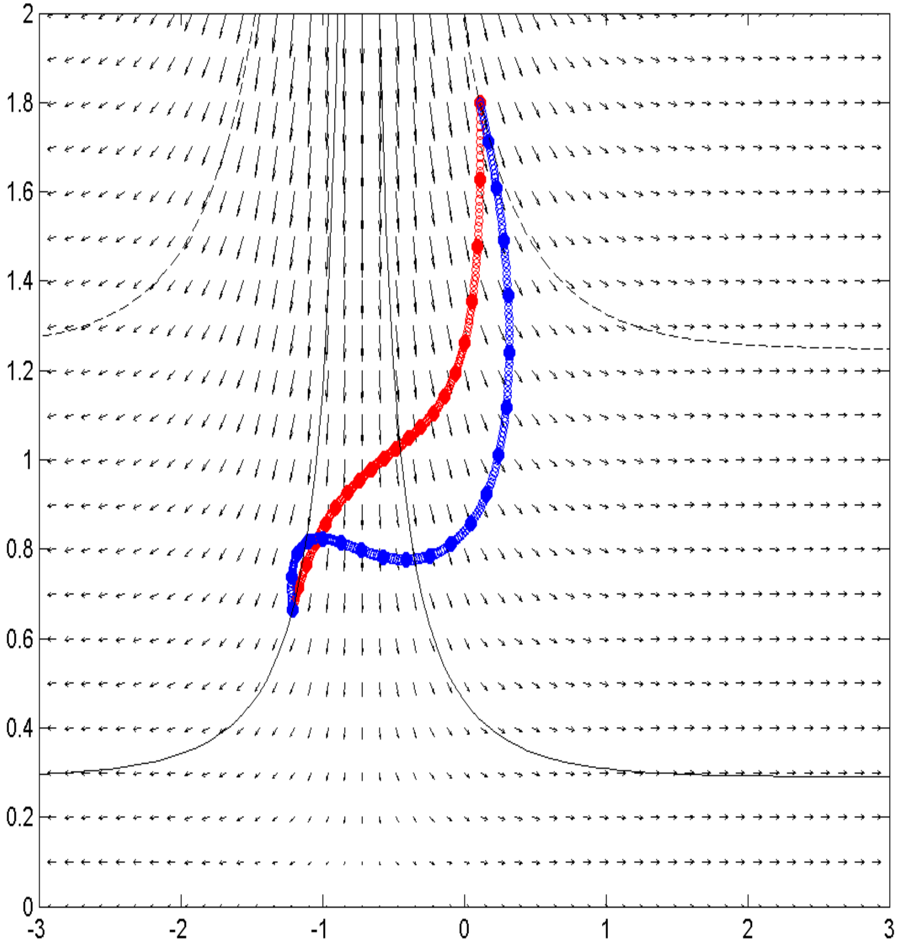
Particule émise tous les dt ○

Particule émise tous les 10dt ●

$$u = V_0 \tanh(x - x_0)$$

$$v = -V_0 y \cosh^{-2}(x - x_0)$$

Trajectory ≠ Streakline ≠ Streamline



Position de la particule tous les dt (open red circle)

Position de la particule tous les 10dt (filled red circle)

Particule émise tous les dt (open blue circle)

Particule émise tous les 10dt (filled blue circle)

$$u = V_0 \tanh(x - x_0)$$

$$v = -V_0 y \cosh^{-2}(x - x_0)$$

$$y = \left| \frac{k}{\tanh(x - x_0)} \right|$$