Dimensional analysis in fluid mechanics

Why dimensional analysis?

All fluid problems have many influential parameters

- fluid properties (μ, K, ρ…)
- upstream fluid conditions (U_∞, p_∞…)
- geometry and scales of the obstacle
- oscillation frequency of moving walls
- …

Why dimensional analysis?

1 – In principle, to study all possible regimes one should investigate each parameter individually

→ Impossible !

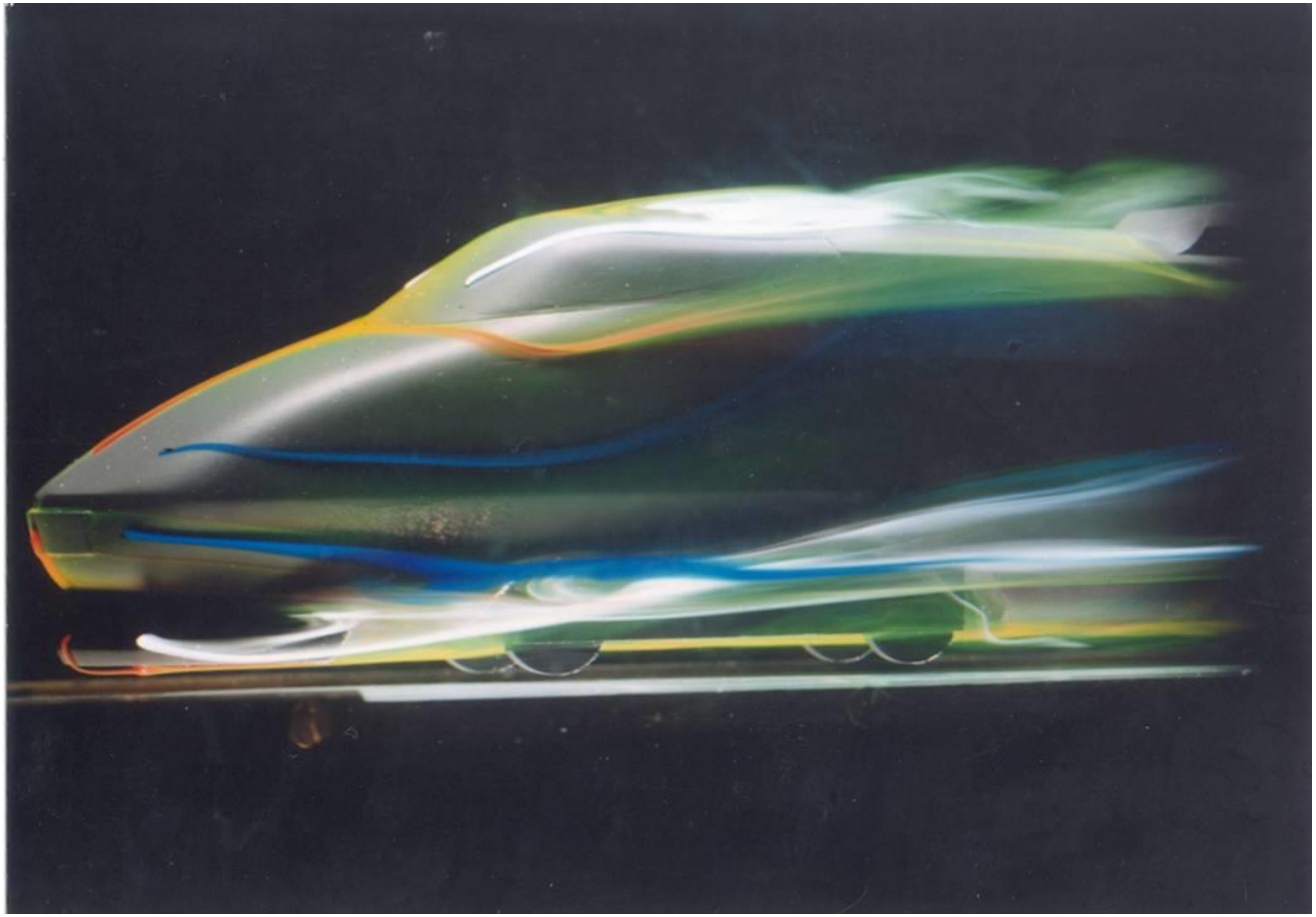
Why dimensional analysis?

2 – It is often very useful to use reduced models to study flow around wings, boats

It is important to know a priori if it will be possible to translate the results obtained on the model to the real flow.

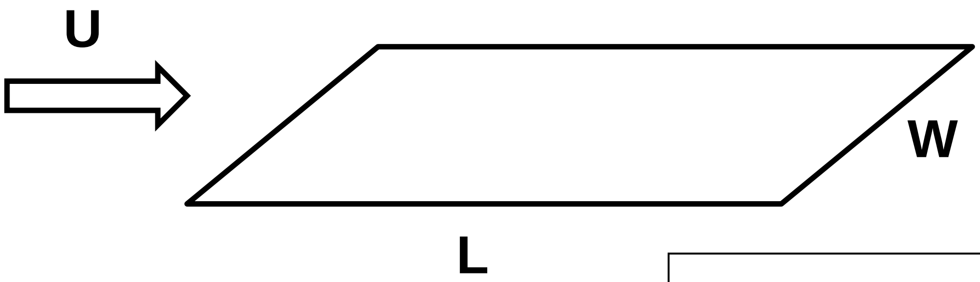
→ The « similarity» of the model parameters is crucial.

Some examples



Other examples

What is the «scaling» of the force acting on a flat plate by an incoming flow?



1. $\mu U^2 L$
2. $\rho U W L$
3. $W(\mu \rho L U^3)^{1/2}$

$W : m \rightarrow [L]$

$L : m \rightarrow [L]$

$U : m/s \rightarrow [L]/[T]$

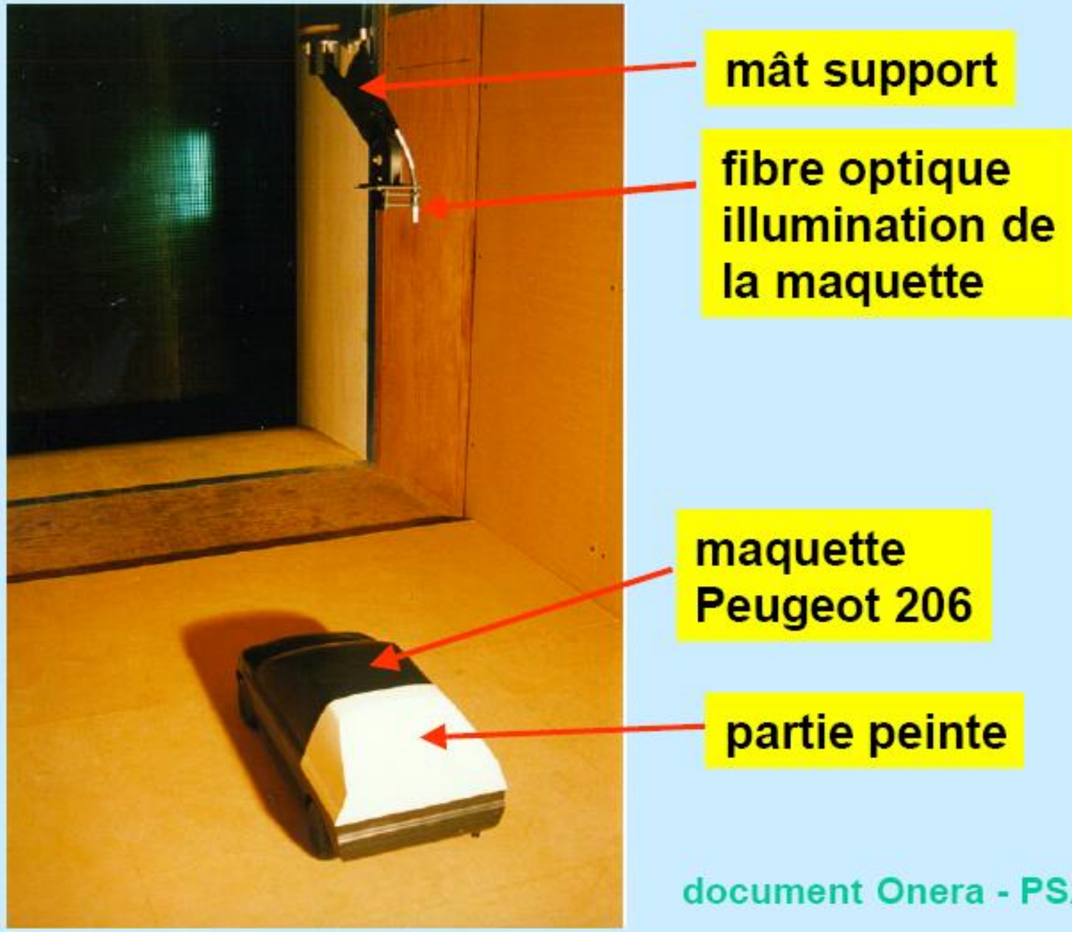
$\rho : kg/m^3 \rightarrow [M]/[L]^3$

$\mu : kg/m/s \rightarrow [M]/[L]/[T]$

More examples

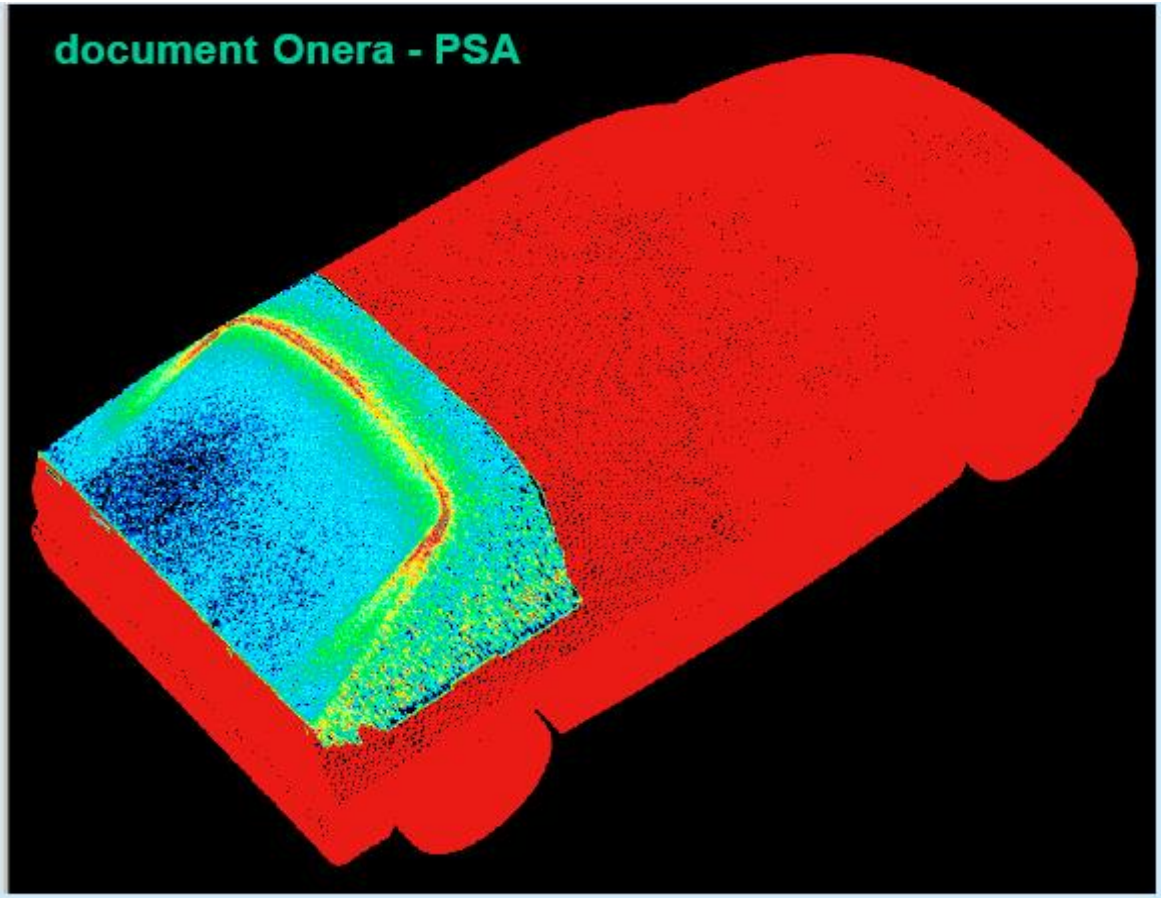
Use of pressure sensitive paint

Peinture sensible à la pression (PSP)



document Onera - PSA

Pressure sensitive paint



répartition de pression sur la la partie arrière de la voiture

Why dimensional analysis?

Objective : reduce the number of parameters to be varied by identifying key nondimensional parameters allowing to regroup experiments that are « similar ».

Experimental similarity

Similarity

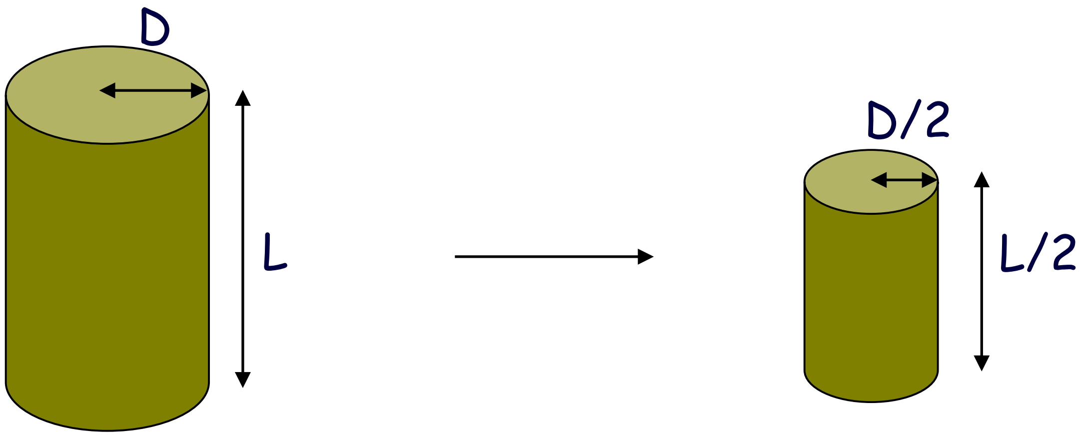
- Geometric
- Dynamic

Geometric similarity

Flow around an obstacle:

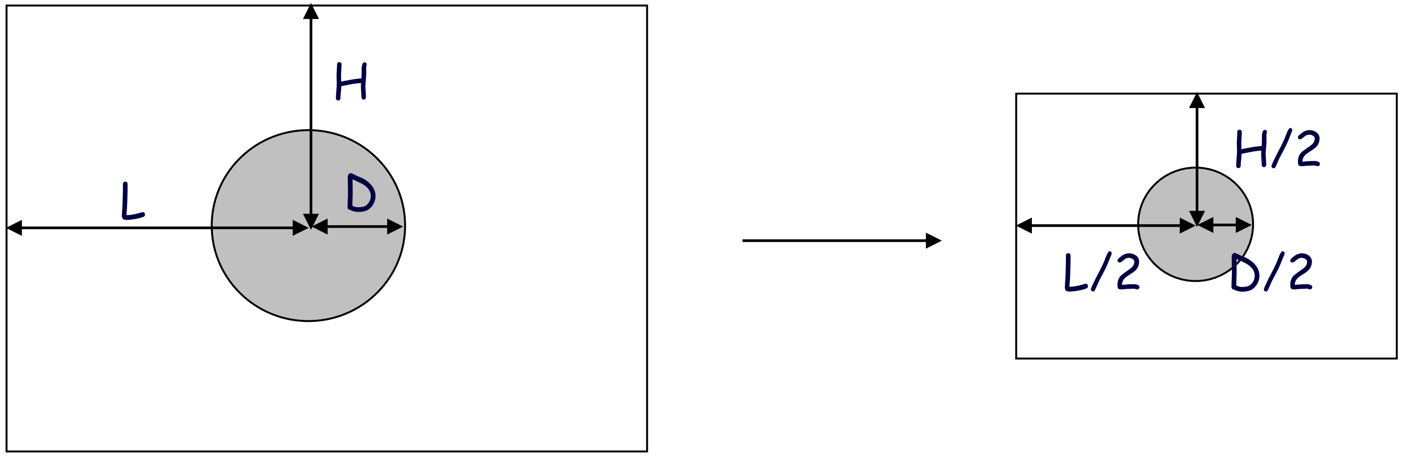
Two obstacles are geometrically similiar if the shape of one can deduced from the other by a uniform dilation.

→ they are at scale



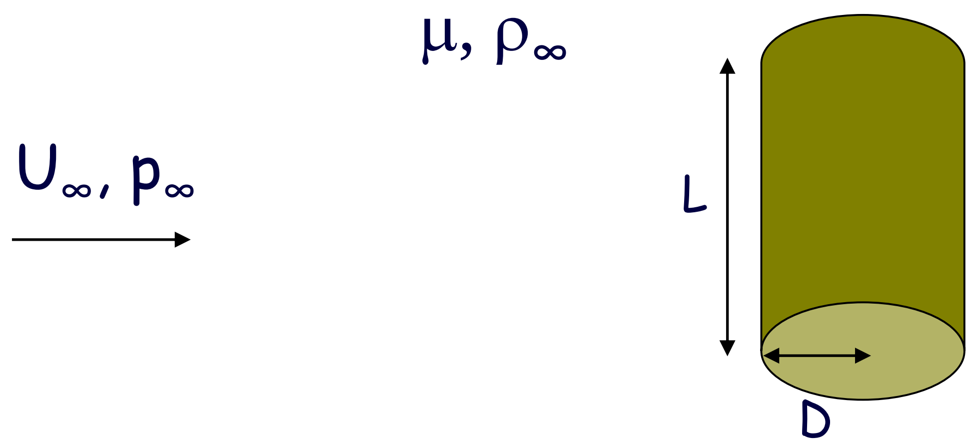
Geomtric similarity

The geometric similarity should also include the domain boundaries



Dynamic similarity

Incompressible flow around obstacle, without volumetric force



Let us try to « calculate » the drag of the flow acting on the body.

Dynamics similarity

Incompressible flow around obstacle, without volumetric force

1 - One should first identify the relevant parameters of the problem

Dynamic similarity

Incompressible flow around obstacle, without volumetric force

$$F=f(D, L, U_\infty, p_\infty, \rho_\infty, \mu)$$

Dynamics similarity

In the case of incompressible flow, the absolute pressure is not a relevant parameter

$$\frac{\partial U_x}{\partial x} + \frac{\partial U_y}{\partial y} = 0$$

$$\rho\left(\frac{\partial U_x}{\partial t} + U_x \frac{\partial U_x}{\partial x} + U_y \frac{\partial U_x}{\partial y}\right) = \boxed{-\frac{\partial p}{\partial x}} + \mu\left(\frac{\partial^2 U_x}{\partial x^2} + \frac{\partial^2 U_x}{\partial y^2}\right)$$

$$\ldots$$

Only the pressure gradient enters the equations

Dynamics similarity

In the case of incompressible flow, the absolute pressure is not a relevant parameter

$$\frac{\partial U_x}{\partial x}+\frac{\partial U_y}{\partial y}=0$$

$$\rho\left(\frac{\partial U_x}{\partial t}+U_x\frac{\partial U_x}{\partial x}+U_y\frac{\partial U_x}{\partial y}\right)=\boxed{-\frac{\partial p}{\partial x}}+\mu\left(\frac{\partial^2 U_x}{\partial x^2}+\frac{\partial^2 U_x}{\partial y^2}\right)$$

...

Only the pressure gradient enters the equations: if (U,p) is solution of the Navier-Stokes equations, than (U,p+p_∞) is also solution : the pressure is defined up to a constant.

Dynamic similarity

Incompressible flow around obstacle, without volumetric force

$$F=f(D, L, U_\infty, \cancel{p_\infty}, \rho_\infty, \mu)$$

Dynamic similarity

Incompressible flow around obstacle, without volumetric force

2 – List the elementary units influencial in the relevant parameters of the problem : [M],[L],[T],[Θ]

Dynamic similarity

Incompressible flow around obstacle, without volumetric force

D :

L :

U_∞ :

ρ_∞ :

μ :

F :

Dynamic similarity

Incompressible flow around obstacle, without volumetric force

$D : m \rightarrow [L]$

$L : m \rightarrow [L]$

$U_\infty :$

$\rho_\infty :$

$\mu :$

$F :$

Dynamic similarity

Incompressible flow around obstacle, without volumetric force

$D : \mathrm{m} \to [L]$

$L : \mathrm{m} \to [L]$

$U_\infty : \mathrm{m/s} \to [L]/[T]$

$\rho_\infty :$

$\mu :$

$F :$

Dynamic similarity

Incompressible flow around obstacle, without volumetric force

$D : m \rightarrow [L]$

$L : m \rightarrow [L]$

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$\rho_\infty : kg/m^3 \rightarrow [M]/[L]^3$

$\mu :$

$F :$

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Incompressible flow around obstacle, without volumetric force

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$F :$

Dynamic similarity

Incompressible flow around obstacle, without volumetric force

$$D : m \rightarrow [L]$$

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$$\mu : kg/m/s \rightarrow [M]/[L]/[T]$$

$$F : kg.m/s^2 \rightarrow [M]\,[L]\,/[T]^2$$

Dynamic similarity

Incompressible flow around obstacle, without volumetric force

$D : m \rightarrow [L]$

$L : m \rightarrow [L]$

$\boxed{\rightarrow \text{3 fondamental units}}$

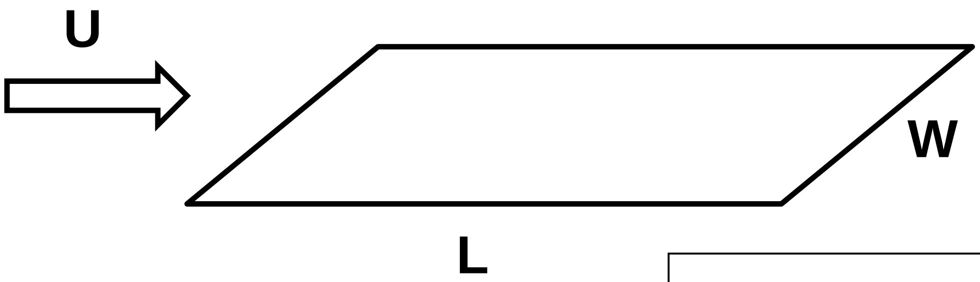
$U_\infty : m/s \rightarrow [L]/[T]$

$\rho_\infty : kg/m^3 \rightarrow [M]/[L]^3$

$\mu : kg/m/s \rightarrow [M]/[L]/[T]$

$F : kg.m/s^2 \rightarrow [M]\,[L]\,/[T]^2$

What is the «scaling» of the force exerted on a plate by an incoming flow?



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$\rho : kg/m^3 \rightarrow [M]/[L]^3$

$\mu : kg/m/s \rightarrow [M]/[L]/[T]$

Dynamic similarity

Incompressible flow around obstacle, without volumetric force

3 – For each fundamental unit, one **choses** a reference scale with use of the relevant parameters

There are often several choices!

Dynamic similarity

Incompressible flow around obstacle, without volumetric force

$[L] \rightarrow D$

$[T] \rightarrow D/U_\infty$

$[M] \rightarrow \rho_\infty D^3$

Dynamic similarity

Incompressible flow around obstacle, without volumetric force

4 – For each relevant parameters of the problem, a nondimensional parameter is constructed using the previously defined reference scales

Dynamic similarity

Incompressible flow around obstacle, without volumetric force

$$\mathrm{D} : \mathrm{m} \rightarrow [\mathrm{L}] \rightarrow \frac{D}{D} = 1$$

Dynamic similarity

Incompressible flow around obstacle, without volumetric force

$$L : m \rightarrow [L] \rightarrow \frac{L}{D}$$

Dynamic similarity

Incompressible flow around obstacle, without volumetric force

$$U_\infty : m/s \rightarrow [L]/[T] \rightarrow \frac{U_\infty}{\left(\dfrac{D}{D/U_\infty}\right)} = \frac{U_\infty}{U_\infty} = 1$$

Dynamic similarity

Incompressible flow around obstacle, without volumetric force

$$\rho_\infty : m/s \rightarrow [M]/[L]^3 \rightarrow \frac{\rho_\infty}{\frac{\rho_\infty D^3}{D^3}} = \frac{\rho_\infty}{\rho_\infty} = 1$$

Dynamic similarity

Incompressible flow around obstacle, without volumetric force

$$\mu : m/s \rightarrow [M]/[L]/[T] \rightarrow \frac{\mu}{\dfrac{\rho_\infty D^3}{D\dfrac{D}{U_\infty}}} = \frac{\mu}{\rho_\infty D U_\infty}$$

Dynamic similarity

Incompressible flow around obstacle, without volumetric force

$$F : kg.m/s^2 \rightarrow [M][L]/[T]^2 \rightarrow \quad \frac{F}{\dfrac{\rho_\infty D^3 D}{\left(\dfrac{D}{U_\infty}\right)^2}} = \frac{F}{\rho_\infty D^2 U_\infty{}^2}$$

Dynamic similarity

The non-dimensional relation becomes (with this particular choice of scales!)

$$\frac{F}{\rho_\infty D^2 U_\infty{}^2} = f\left(1, \frac{L}{D}, 1, 1, \frac{\mu}{\rho_\infty D U_\infty}\right)$$

Dynamic similarity

Finally, the non dimensional problem depends only on two non dimensional parameters

$$\frac{F}{\rho_\infty D^2 U_\infty{}^2} = f\left(\cancel{1}, \frac{L}{D}, \cancel{1}, \cancel{1}, \frac{\mu}{\rho_\infty D U_\infty}\right)$$

Dynamic similarity

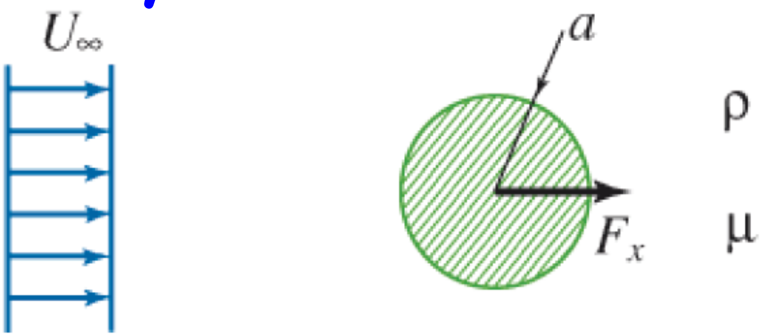
Finally, the non dimensional problem depends only on two non dimensional parameters

$$\frac{F}{\rho_\infty D^2 U_\infty{}^2} = f\left(\frac{L}{D}, \frac{\mu}{\rho_\infty D U_\infty}\right)$$

Aspect ratio of the obstacle

1/Re
(Reynolds number)

Cylinder wake



$$F_x = \mathcal{F}\left(a, U_\infty, \rho, \mu\right)$$

$$\begin{array}{c|ccccc|}
 & [F_x] & [a] & [U_\infty] & [\rho] & [\mu] \\
\hline
L & 0 & 1 & 1 & -3 & -1 \\
M & 1 & 0 & 0 & 1 & 1 \\
T & -2 & 0 & -1 & 0 & -1 \\
\Theta & 0 & 0 & 0 & 0 & 0
\end{array}$$

$$N = 4 \qquad r = 3$$

$$\pi_1 = \frac{F_x}{a^{a_1}\, U_\infty^{a_2}\, \rho^{a_3}}$$

$$\pi = \frac{F_x}{a^{b_1}\, U_\infty^{b_2}\, \rho^{b_3}}$$

$$\pi = \bar{\mathcal{F}}\,(\pi_1)$$

$$\pi = \frac{F_x}{\rho\, U_\infty^2\; a} \qquad \pi_1 = \frac{\mu}{\rho\, U_\infty\; a}$$

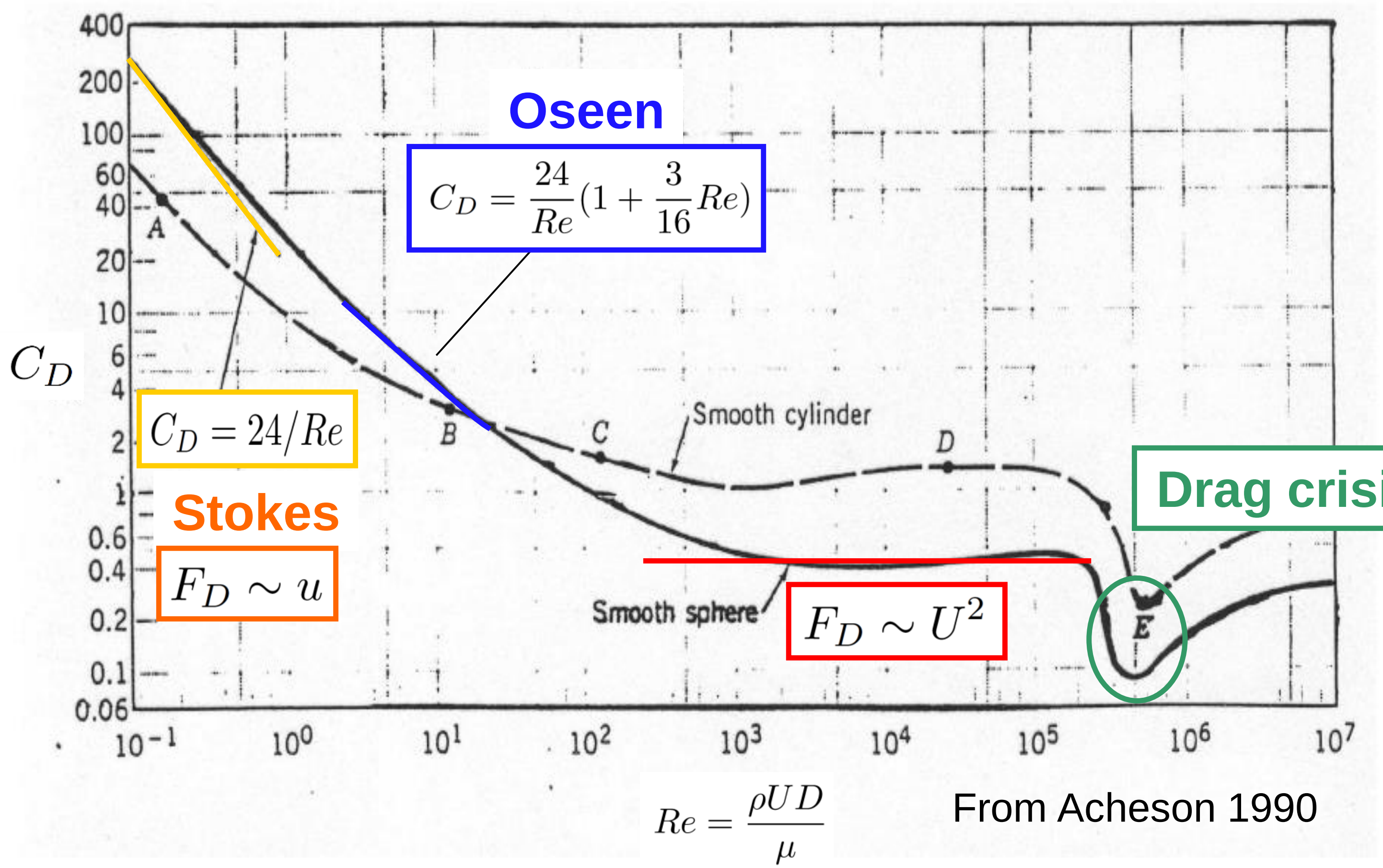
$$C_x = \frac{F_x}{\frac{1}{2}\,\rho\, U_\infty^2\;\; d} \qquad \mathrm{Re} = \frac{\rho\, U_\infty\, d}{\mu}$$

$$C_x = \bar{\mathcal{F}}\,(\mathrm{Re})$$

Sphere wake

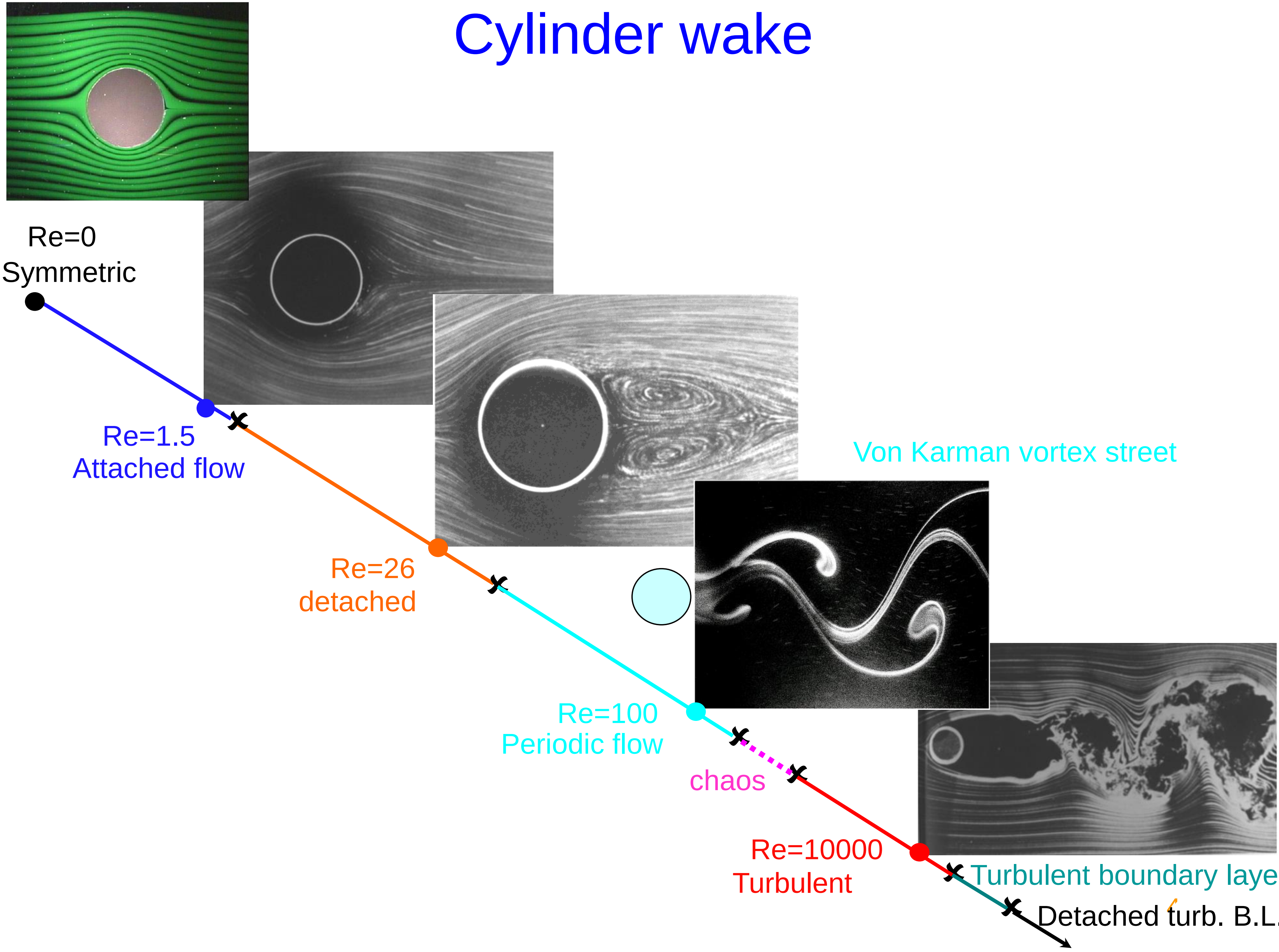
$$F_D = \rho U^2 R^2 F\,(Re)$$

$$C_D = \frac{F_D}{\frac{1}{2}\rho U^2 A} = f(Re)$$

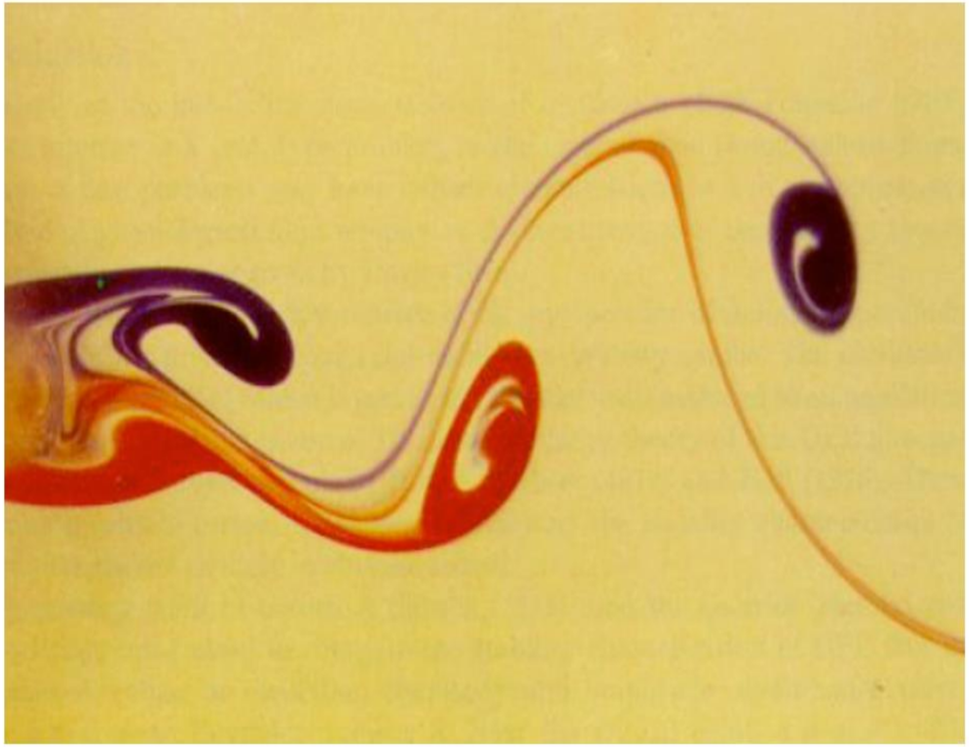


From Acheson 1990

Cylinder wake

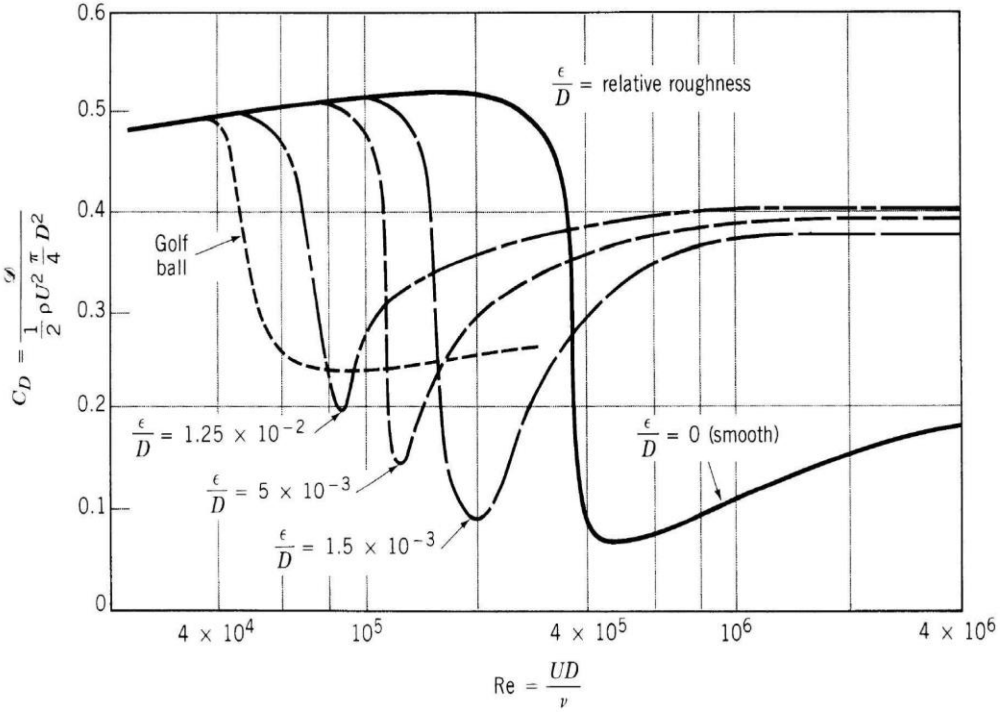


Von Karman vortex street



(Perry, Chong & Lim (1982)

Influence of rugosity on drag



Acheson 1990

Balles de Golf

Plumeuse (1400) | Gutta Percha (1850) | Gutta Martelé–main (1870–80) | Balle moderne (1930–?)

Sac de cuir bourré de plumes. Les pièces de cuir sont cousues, trempées, puis bourrées de plumes d'oie mouillées. Après séchage, les plumes prennent de l'expansion alors que le cuir se rétracte résultant en une masse extrêmement dure et dottée de beaucoup de ressort.

Plus économique, plus durable et plus esthétique. En revanche, elle virevolte de façon imprévisible, volant beaucoup moins loin que le vieux sac de plumes. Mais les tricheurs découvrent rapidement que plus ils `cicatrisent' la balle, plus elle vole droit et loin. Ainsi naît la Gutta martelé-main.

- 330 alvéoles, fossettes, ou dimples
- 336 (balle US)
- 75m/s
- 3500tr/mn de backspin

Drag and athletics

•Increase in drag diue to a temperature decrase.

$$\boxed{F_D = \frac{1}{2}\rho U^2 A\, C_D}$$

ρ(10°C)= ρ(30°C) +10 %

⇨ $F_D(10°C) = F_D(10°C) + 10\%$

•Records can be officially registered only if the wind is less than 2m/s.

$$\boxed{F_D = \frac{1}{2}\rho U^2 A\, C_D}$$

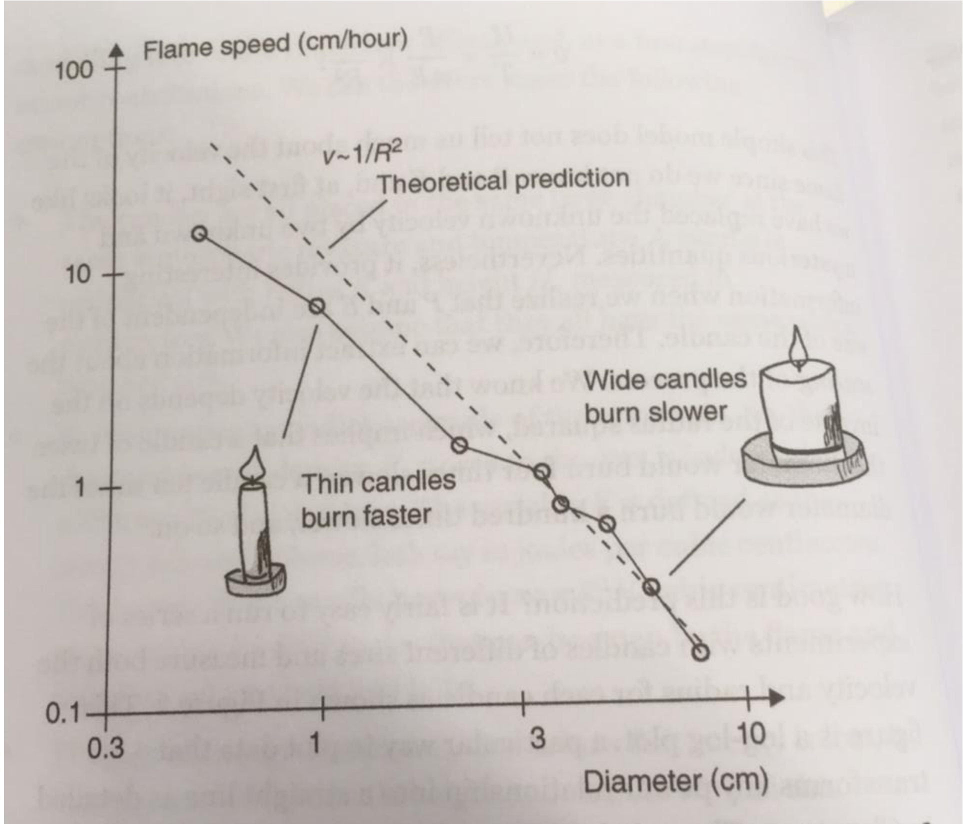
100 m in 10s ⇨u=100 m/ 10 s = 10 m/s

⇨relative velocity = 8,10 ou 12 m/s.

⇨$F_D(+2\ m/s) = F_D(0 m/s) - 36\%$

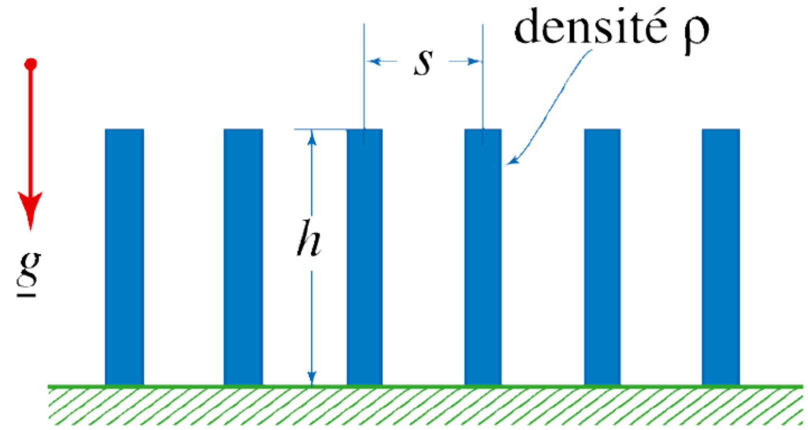
⇨$F_D(-2\ m/s) = F_D(0 m/s) + 44\%$

Candle burning



2. Experiments on candles: if we take the logarithm of the velocity and plot it against the logarithm of the radius, a relationship of the form $v \sim R^{-2}$ is, in a log-log plot, the dashed line of gradient -2. Therefore, if our prediction is correct, the data should sit on a line of gradient -2. Comparing data with prediction, we see that, indeed, the burning velocity of large candles is proportional to $1/R^2$ as predicted from the model.

Celerity of domino waves



$$c = \mathcal{F}\left(h, g, \rho, s\right)$$

$$\begin{array}{c c}
 & \begin{array}{ccccc} [c] & [h] & [g] & [\rho] & [s] \end{array} \\
\begin{array}{c} L \\ M \\ T \\ \Theta \end{array} & \begin{bmatrix} 1 & 1 & 1 & -3 & 1 \\ 0 & 0 & 0 & 1 & 0 \\ -1 & 0 & -2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}
\end{array}$$

$$N = 4 \qquad r = 3$$

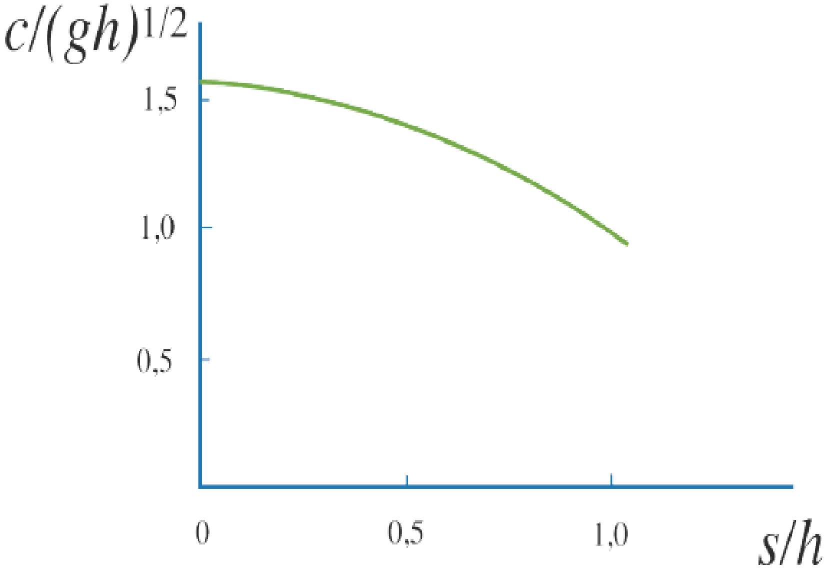
One should have the intuition not to take into account the thickness l!

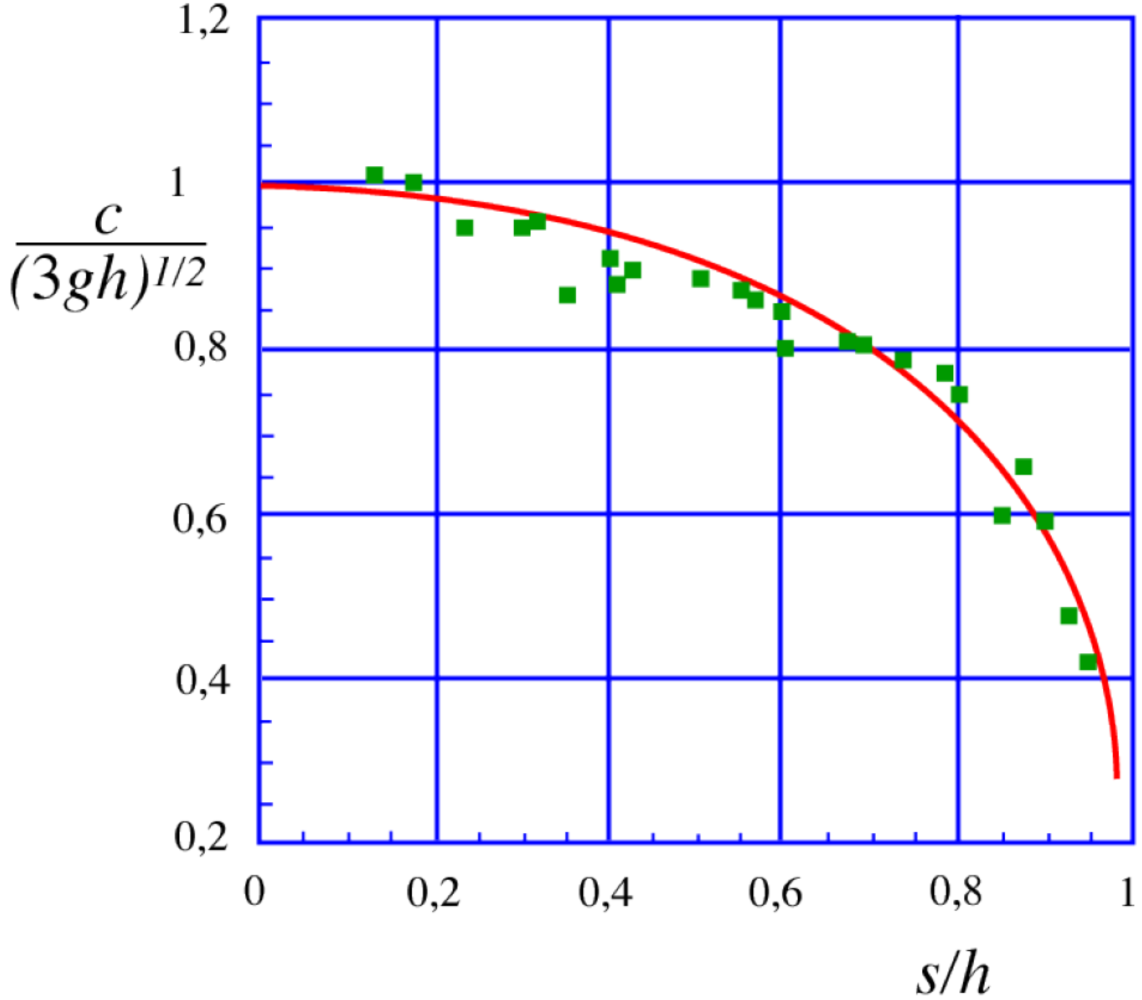
$$\pi = \frac{c}{h^{a_1}\, g^{a_2}\, \rho^{a_3}} = \frac{c}{(gh)^{1/2}}$$

$$\pi_1 = \frac{s}{h^{b_1}\, g^{b_2}\, \rho^{b_3}} = \frac{s}{h}$$

$$\pi = \overline{\mathcal{F}}(\pi_1)$$

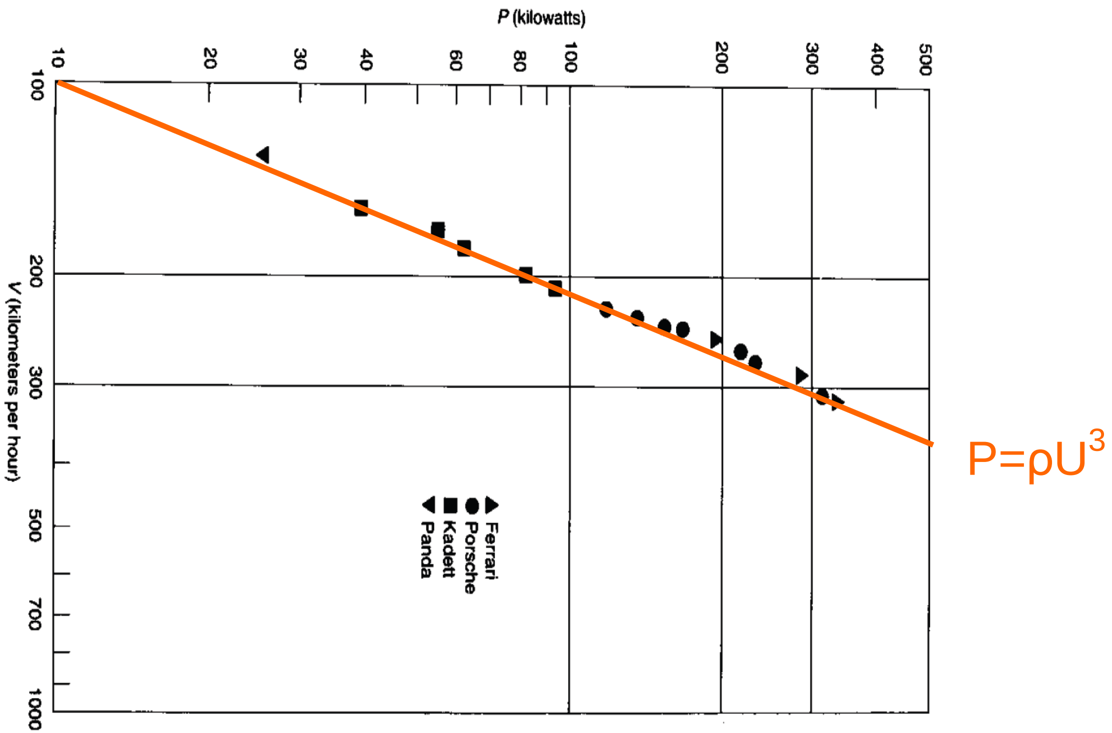
$$\frac{c}{(gh)^{1/2}} = \overline{\mathcal{F}}\left(\frac{s}{h}\right)$$





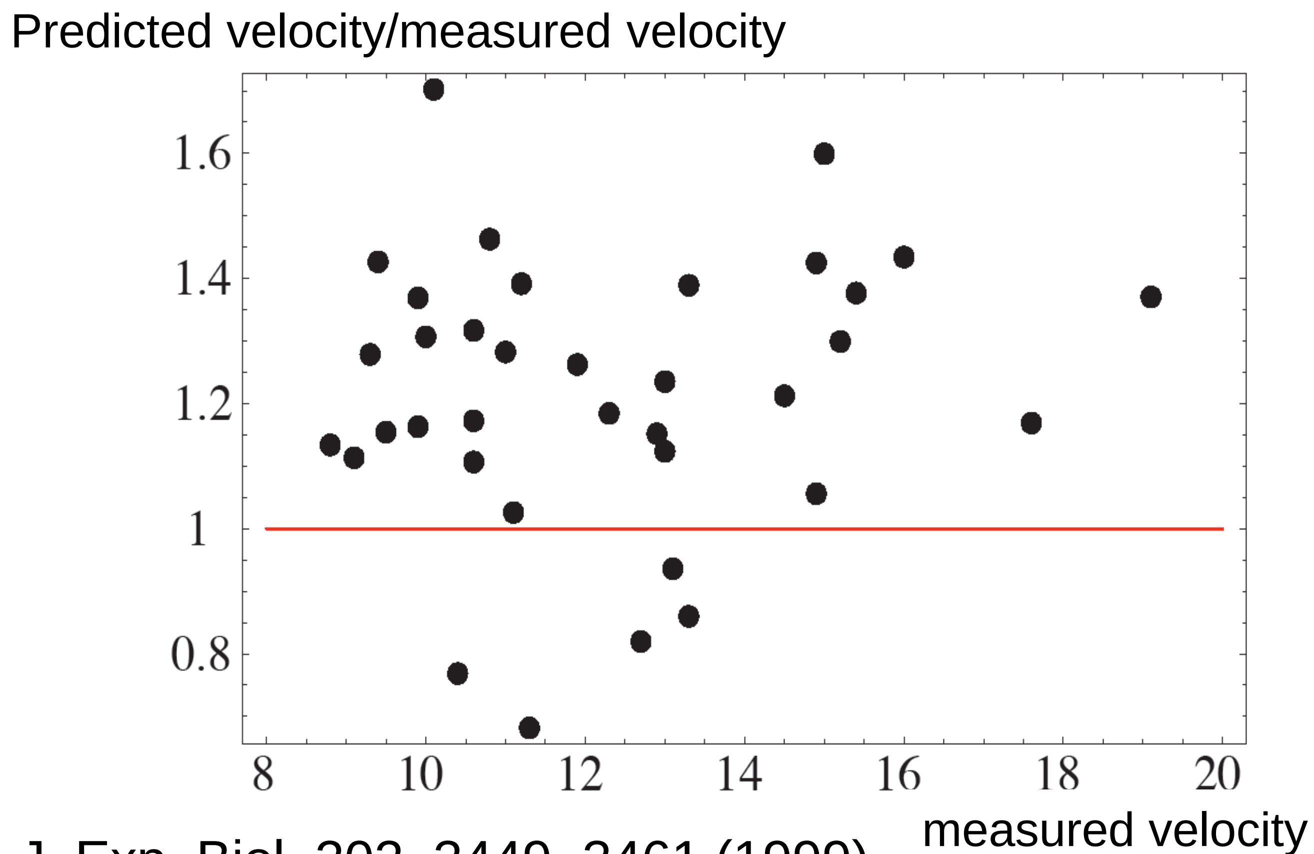
Clanet (2001)

Power of car as a function of velocity



Tennekes, H. (1996) The Simple Science of Flight.

Predicted bird velocity

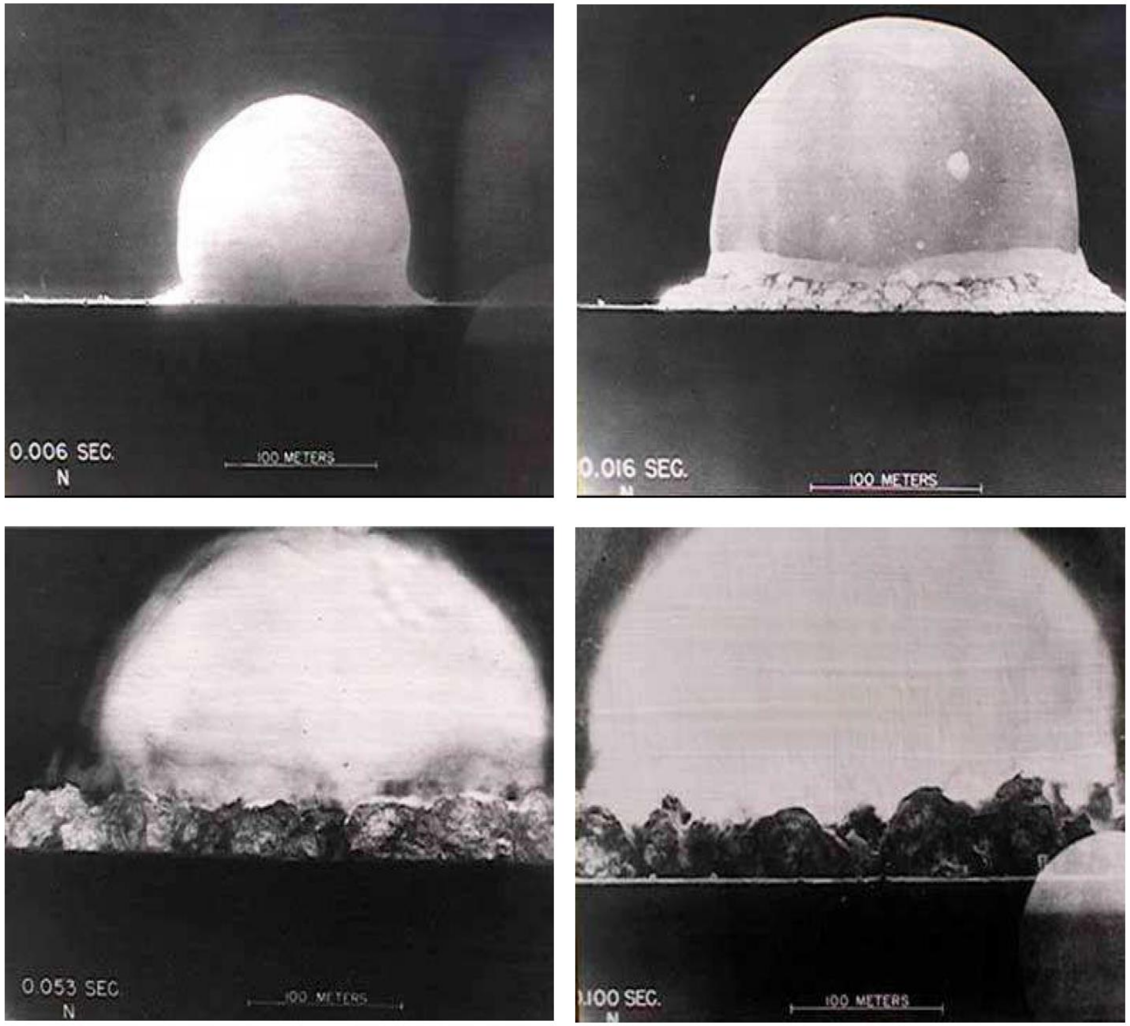


Rayner, J. Exp. Biol. 202, 3449–3461 (1999)

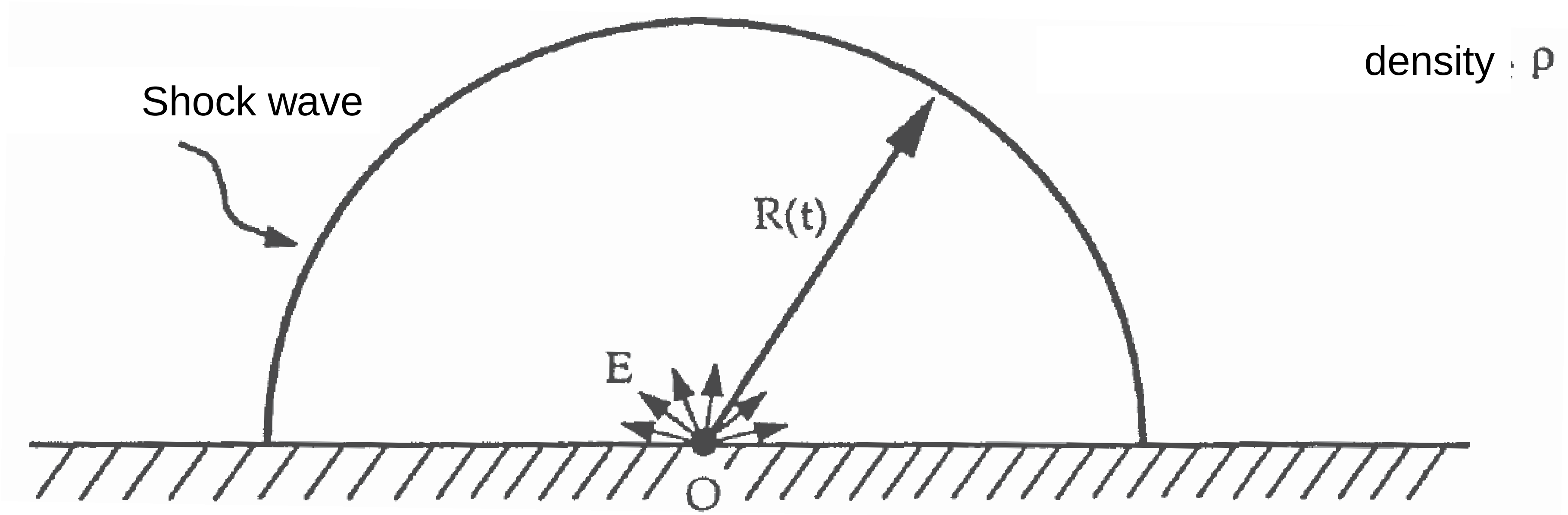
Lift=$\rho U^2 S$=Weight=mg

$U=(mg/\rho S)^{1/2}$

G.I. Taylor’s nuclear explosion



G.I. Taylor's nuclear explosion



$$R=f(E,\rho,t)$$

Dynamic similarity

$R : m \rightarrow [L]$

$t : s \rightarrow [T]$

$E : kg.m^2/s^2 \rightarrow [M]\,[L]^2\,/[T]^2$

$\rho : kg/m^3 \rightarrow [M]/[L]^3$

Dynamic similarity

$R : m \rightarrow [L]$

$t : s \rightarrow [T]$

$E : kg.m^2/s^2 \rightarrow [M]\,[L]^2\,/[T]^2$

$\rho : kg/m^3 \rightarrow [M]/[L]^3$

For every fundamental unit, one **picks up** a reference scale à using the relevant parameters

Dynamic similarity

Exponent equation

$$[\,t\,]^{a_1}[\,\rho\,]^{a_2}[\,E\,]^{a_3} = [\,R\,]$$

Dynamic similarity

Exponent equation

$$
\begin{aligned}
-3a_2 + 2a_3 &= 1 \\
a_2 + a_3 &= 0 \\
a_1 - 2a_3 &= 0
\end{aligned}
$$

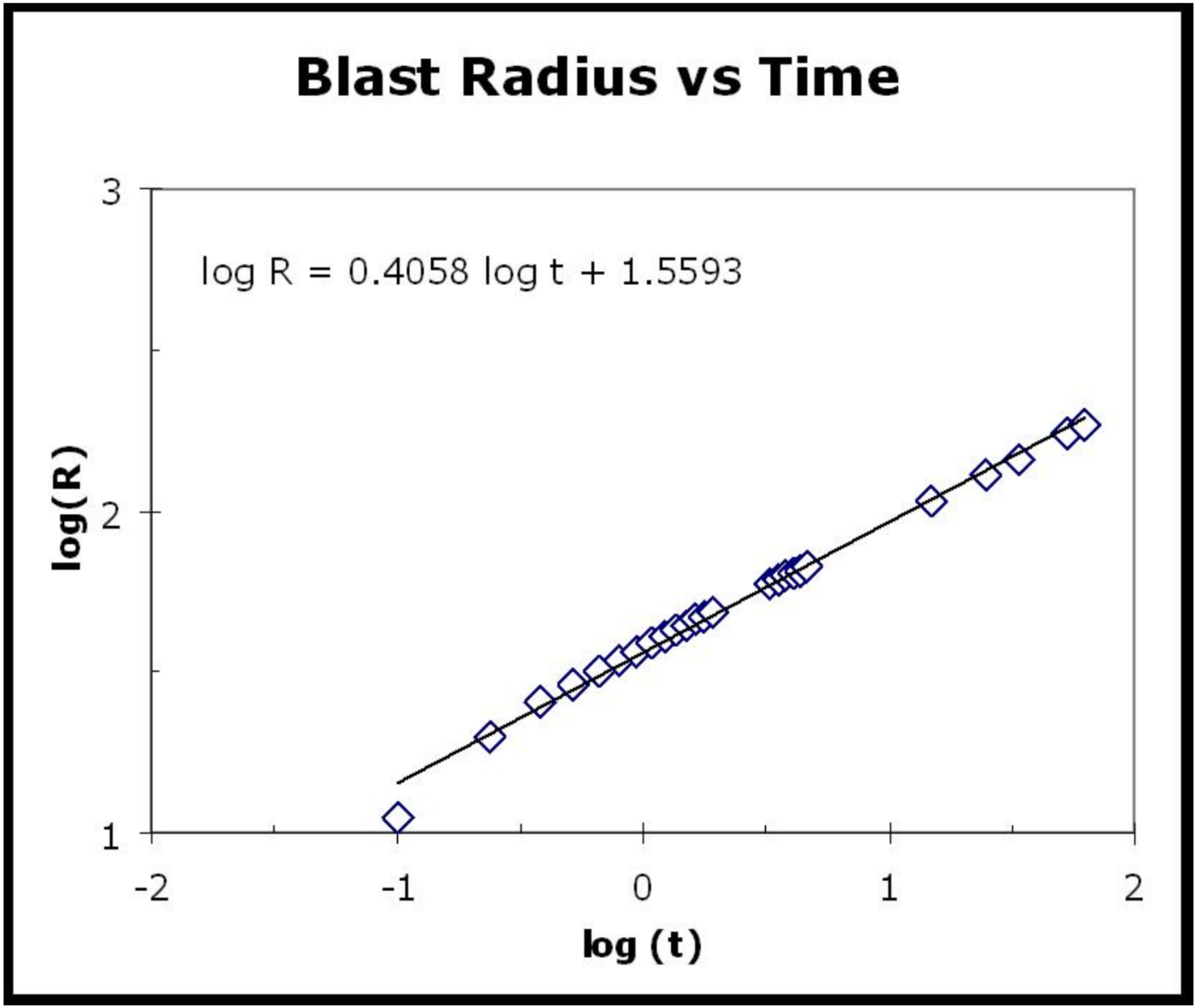
Dynamic similarity

Exponent equation

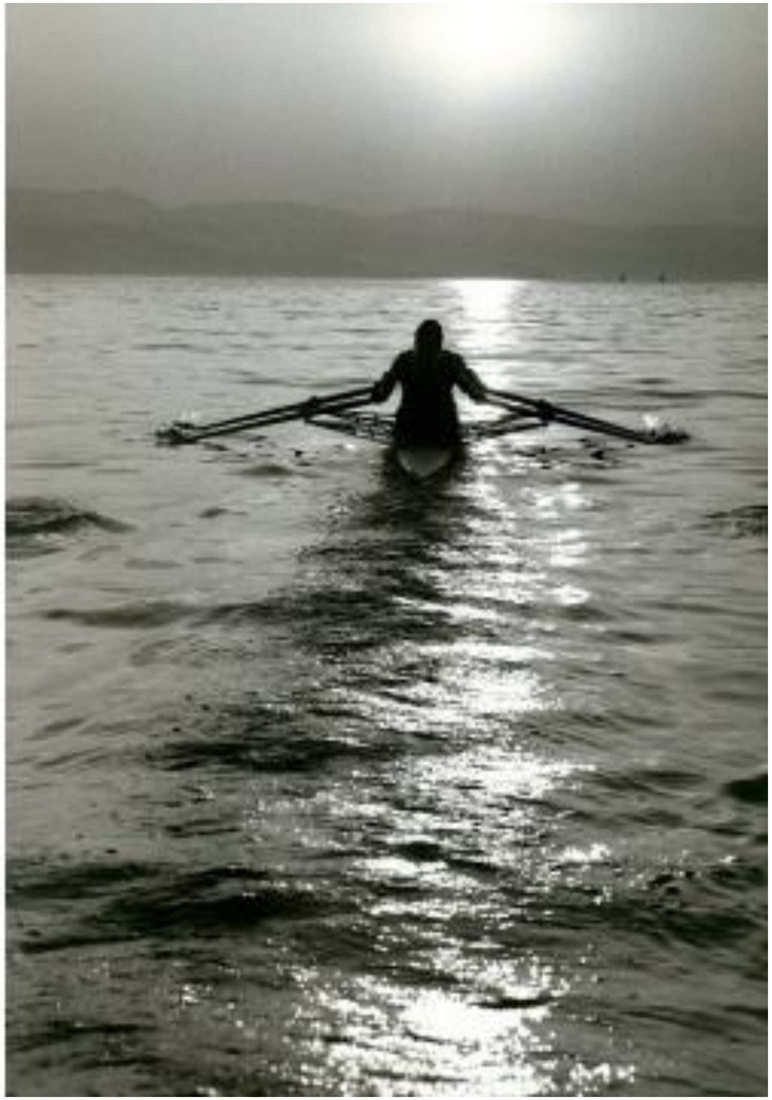
$$
\begin{aligned}
-3a_2 + 2a_3 &= 1 \\
a_2 + a_3 &= 0 \\
a_1 - 2a_3 &= 0
\end{aligned}
$$

$$\frac{R}{t^{2/5}\,\rho^{-1/5}\,E^{1/5}} = \textit{constant}$$

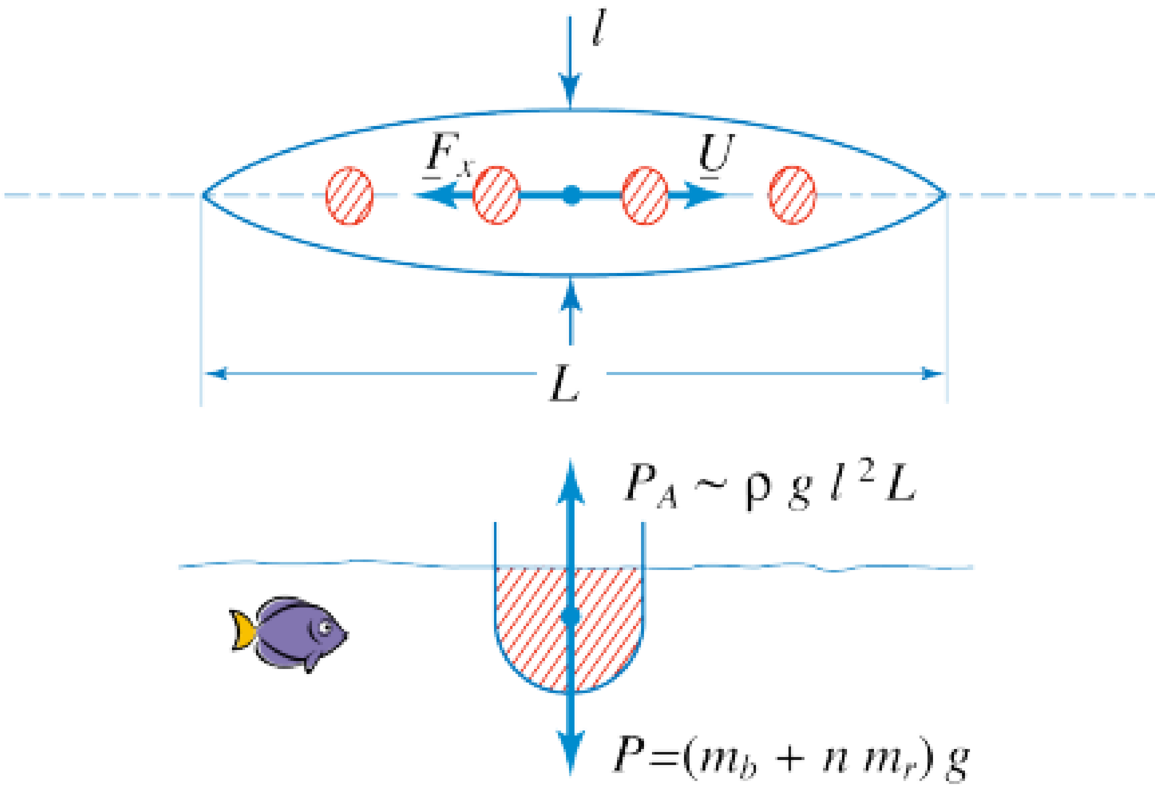
G.I. Taylor's analysis of a nuclear explosion



Rowing



n rameurs de masse m_r et de puissance P_r



Hypothèses :

- similitude géométrique $L/\ell = \text{const.}$
- $m_b/n = \delta m = \text{const.}$
- $F_x \big/ \left(\rho L^2 U^2\right) = C_x = \text{const.}$

ÉQUILIBRE DES FORCES SELON LA VERTICALE

$$P_A = P$$

$$\rho\, g\, L^3 \sim (m_b + n\, m_r)\, g$$

$$\rho\, g\, L^3 \sim n\,(\delta m + m_r)\, g$$

$$\boxed{L \sim \left(\frac{\delta m + m_r}{\rho}\right)^{1/3} n^{1/3}}$$

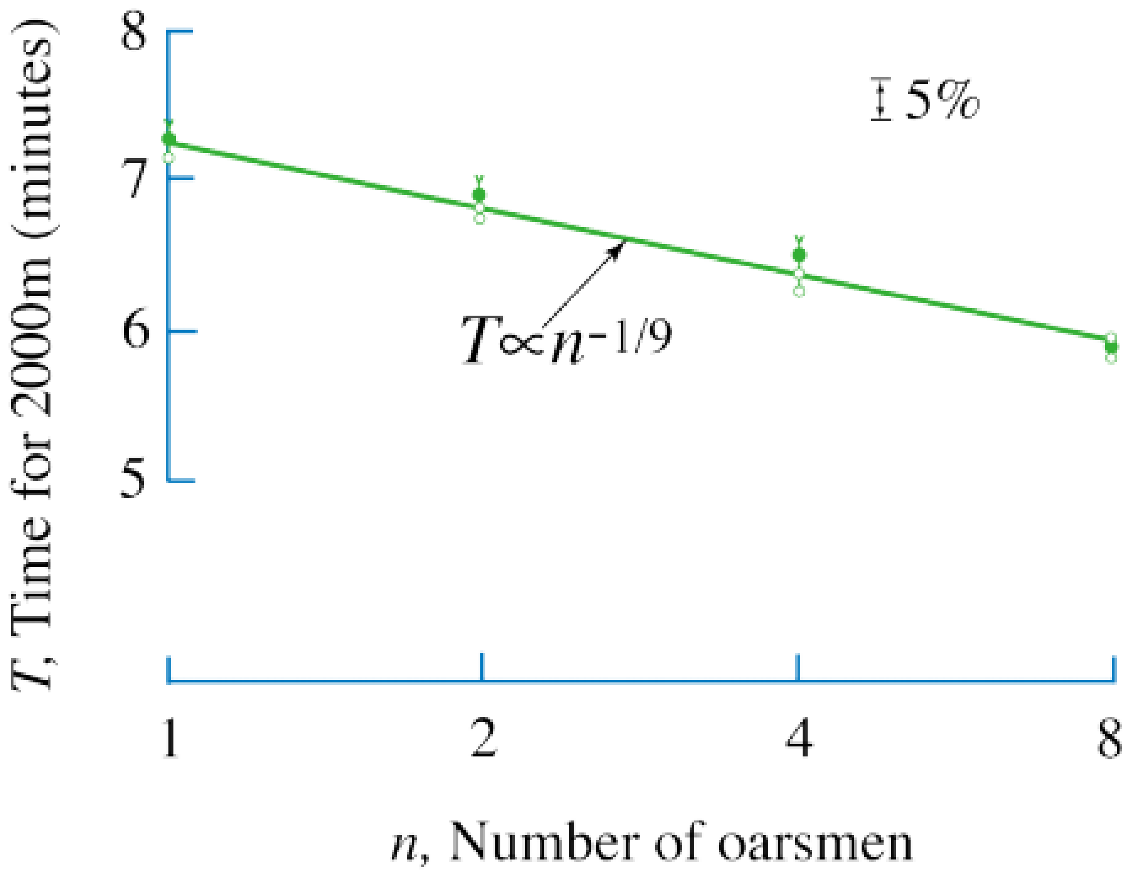
PUISSANCE DISSIPÉE = PUISSANCE FOURNIE

$$F_x\, U = n\, P_r$$

$$\rho\, L^2 U^3 \sim n\, P_r$$

$$\boxed{U \sim \left(\frac{P_r}{\rho}\right)^{1/3} L^{-2/3} n^{1/3}}$$

$$U \sim \left(\frac{P_r}{\rho}\right)^{1/3} \left(\frac{\delta m + m_r}{\rho}\right)^{-2/9} n^{1/9}$$



Mc Mahon (1972)

Dynamic similarity

Non dimensional Navier-Stokes equations

reference scales are used to write non-dimensional equations

$$\tilde{t}, \tilde{x}, \tilde{y}, \tilde{U}_x, \tilde{U}_y, \tilde{p}, \tilde{\rho}$$

Non dimensional Navier-Stokes equations

reference scales are used to write non-dimensional equations

$$\tilde{t} = tU_\infty / D$$

$$\tilde{x} = x / D, \quad \tilde{y} = y / D$$

$$\tilde{U}_x = U_x / U_\infty, \quad \tilde{U}_y = U_y / U_\infty$$

$$\tilde{p} = p / \rho_\infty U_\infty^{\ 2}$$

$$\tilde{\rho} = \rho / \rho_\infty = 1!$$

Non dimensional Navier-Stokes equations

- Continuity equation/ mass conservation

$$\frac{\partial U_x}{\partial x} + \frac{\partial U_y}{\partial y} = 0$$

$$\frac{\partial\left(U_\infty \tilde{U}_x\right)}{\partial\left(L\tilde{x}\right)} + \frac{\partial\left(U_\infty \tilde{U}_y\right)}{\partial\left(L\tilde{y}\right)} = 0$$

$$\frac{U_\infty}{D}\left(\frac{\partial \tilde{U}_x}{\partial \tilde{x}} + \frac{\partial \tilde{U}_y}{\partial \tilde{y}}\right) = 0$$

$$\boxed{\frac{\partial \tilde{U}_x}{\partial \tilde{x}} + \frac{\partial \tilde{U}_y}{\partial \tilde{y}} = 0}$$

Non dimensional Navier-Stokes equations

- Momentum equation

$$\rho\left(\frac{\partial U_x}{\partial t}+U_x\frac{\partial U_x}{\partial x}+U_y\frac{\partial U_x}{\partial y}\right)=-\frac{\partial p}{\partial x}+\mu\left(\frac{\partial^2 U_x}{\partial x^2}+\frac{\partial^2 U_x}{\partial y^2}\right)$$

$$\frac{\rho_\infty U_\infty{}^2}{D}\tilde{\rho}\left(\frac{\partial \tilde{U}_x}{\partial \tilde{t}}+\tilde{U}_x\frac{\partial \tilde{U}_x}{\partial \tilde{x}}+\tilde{U}_y\frac{\partial \tilde{U}_x}{\partial \tilde{y}}\right)=-\frac{\rho_\infty U_\infty{}^2}{D}\frac{\partial \tilde{p}}{\partial \tilde{x}}+\frac{\mu U_\infty}{D^2}\left(\frac{\partial^2 \tilde{U}_x}{\partial \tilde{x}^2}+\frac{\partial^2 \tilde{U}_x}{\partial \tilde{y}^2}\right)$$

$$\boxed{\frac{\partial \tilde{U}_x}{\partial \tilde{t}}+\tilde{U}_x\frac{\partial \tilde{U}_x}{\partial \tilde{x}}+\tilde{U}_y\frac{\partial \tilde{U}_x}{\partial \tilde{y}}=-\frac{\partial \tilde{p}}{\partial \tilde{x}}+\frac{\mu}{\rho_\infty U_\infty D}\left(\frac{\partial^2 \tilde{U}_x}{\partial \tilde{x}^2}+\frac{\partial^2 \tilde{U}_x}{\partial \tilde{y}^2}\right)}$$

Non dimensional Navier-Stokes equations

$$\frac{\partial \tilde{U}_x}{\partial \tilde{x}} + \frac{\partial \tilde{U}_y}{\partial \tilde{y}} = 0$$

$$\frac{\partial \tilde{U}_x}{\partial \tilde{t}} + \tilde{U}_x \frac{\partial \tilde{U}_x}{\partial \tilde{x}} + \tilde{U}_y \frac{\partial \tilde{U}_x}{\partial \tilde{y}} = -\frac{\partial \tilde{p}}{\partial \tilde{x}} + \frac{\mu}{\rho_\infty U_\infty D}\left(\frac{\partial^2 \tilde{U}_x}{\partial \tilde{x}^2} + \frac{\partial^2 \tilde{U}_x}{\partial \tilde{y}^2}\right)$$

$$\frac{\partial \tilde{U}_y}{\partial \tilde{t}} + \tilde{U}_x \frac{\partial \tilde{U}_y}{\partial \tilde{x}} + \tilde{U}_y \frac{\partial \tilde{U}_y}{\partial \tilde{y}} = -\frac{\partial \tilde{p}}{\partial \tilde{y}} + \frac{\mu}{\rho_\infty U_\infty D}\left(\frac{\partial^2 \tilde{U}_y}{\partial \tilde{x}^2} + \frac{\partial^2 \tilde{U}_y}{\partial \tilde{y}^2}\right)$$

Non dimensional Navier-Stokes equations

$$\frac{\partial \tilde{U}_x}{\partial \tilde{x}} + \frac{\partial \tilde{U}_y}{\partial \tilde{y}} = 0$$

$$\frac{\partial \tilde{U}_x}{\partial \tilde{t}} + \tilde{U}_x \frac{\partial \tilde{U}_x}{\partial \tilde{x}} + \tilde{U}_y \frac{\partial \tilde{U}_x}{\partial \tilde{y}} = -\frac{\partial \tilde{p}}{\partial \tilde{x}} + \boxed{\frac{\mu}{\rho_\infty U_\infty D}} \left(\frac{\partial^2 \tilde{U}_x}{\partial \tilde{x}^2} + \frac{\partial^2 \tilde{U}_x}{\partial \tilde{y}^2} \right)$$

$$\frac{\partial \tilde{U}_y}{\partial \tilde{t}} + \tilde{U}_x \frac{\partial \tilde{U}_y}{\partial \tilde{x}} + \tilde{U}_y \frac{\partial \tilde{U}_y}{\partial \tilde{y}} = -\frac{\partial \tilde{p}}{\partial \tilde{y}} + \boxed{\frac{\mu}{\rho_\infty U_\infty D}} \left(\frac{\partial^2 \tilde{U}_y}{\partial \tilde{x}^2} + \frac{\partial^2 \tilde{U}_y}{\partial \tilde{y}^2} \right)$$

Theses equations have only 1 nondimensional parameter: 1/Re

Reynolds number

$$\mathrm{Re} = \frac{\rho U L}{\mu}$$

Reynolds = inertial forces / viscous forces

Re>>1 → inertia dominates(inviscid fluid limit)
Re<<1 → viscosity dominates

Mach number

$$M = \frac{U}{a}$$

Sound velocity

Mach = measure of the compressibility of a flow

Froude number

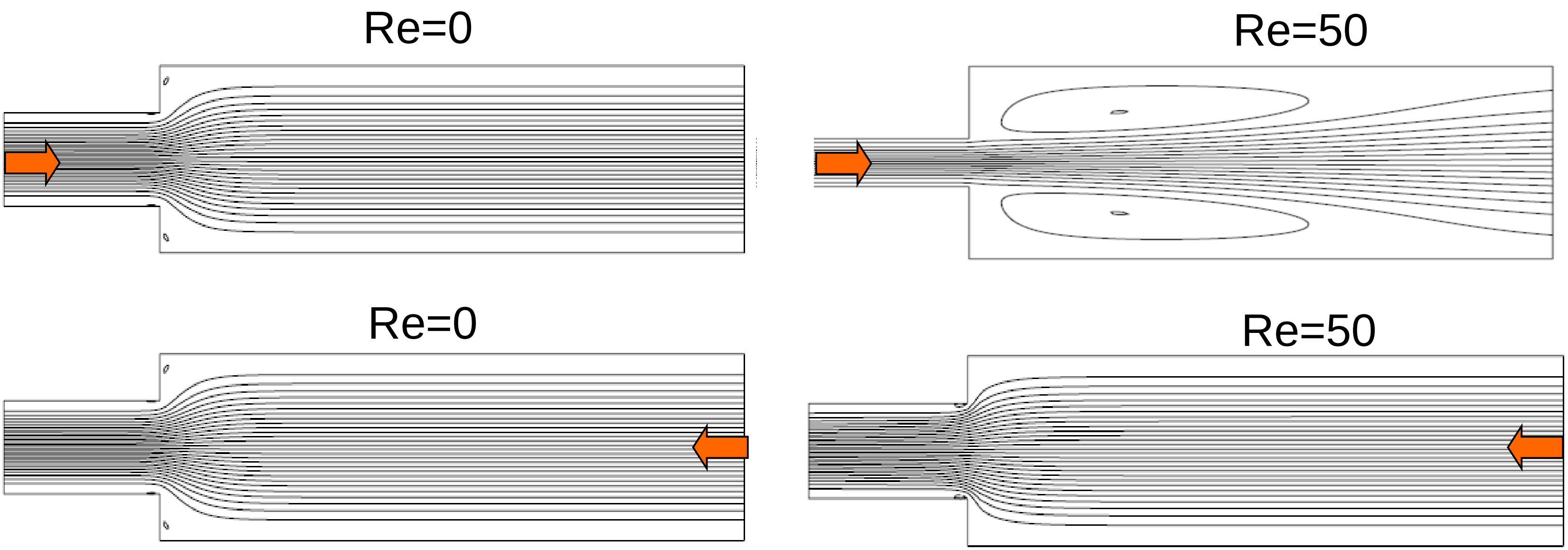
$$Fr = \frac{U}{\sqrt{gL}}$$

Froude = inertia/ gravity

Many more non dimensional numbers

More non dimensional numbers characterizing heat transfer (Péclet, Eckhert, Prandtl...), or surface tension effets (Capillary, Weber, Ohnesorge,...)

Reversibility of the Stokes equations



Simulations Marc Fermigier