Hydrodynamics

potential flow

Outline

1. Intro+kinematics
2. Dynamics
3. Dimensional Analysis
4. Low Reynolds number flow/ Stokes eq.
5. Stokes drag
6. Lubrification-Hele Shaw-Pipe Flows

8. Vorticity conservation and diffusion

9. Boundary layer

10. Invisicd fluid- Bernoulli-potential flow

11. Potential flow, lift

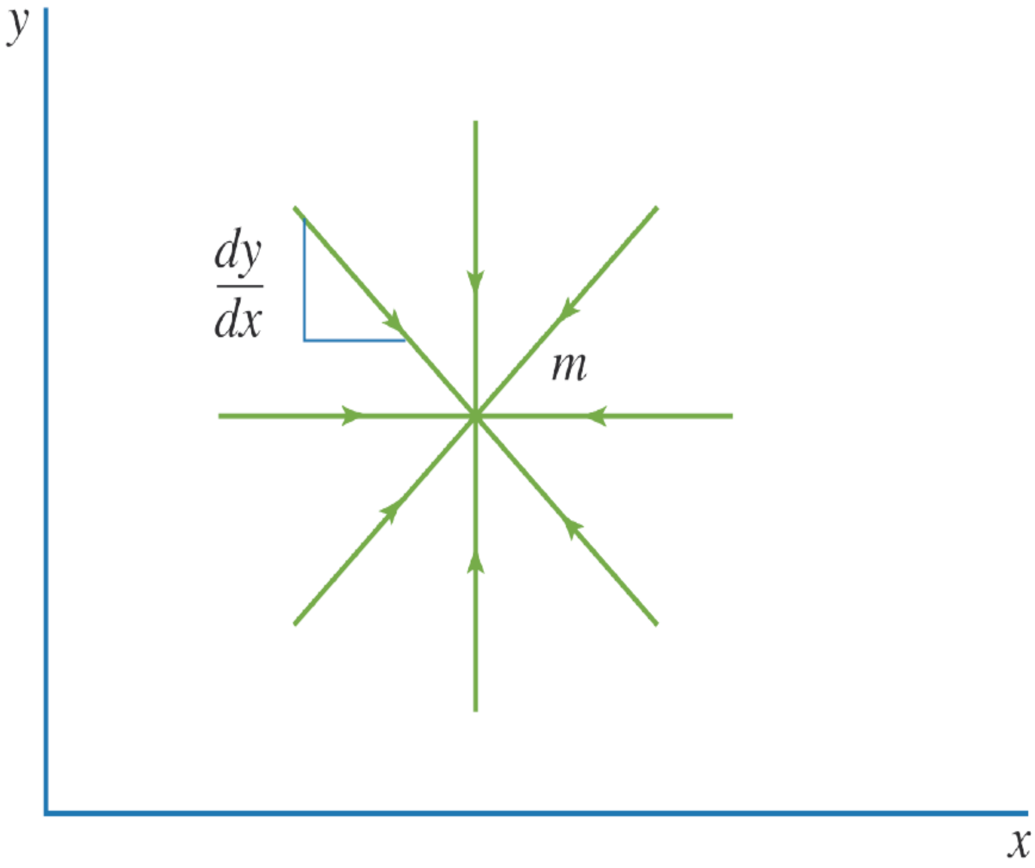
12. Flow separation and detachment

13. Waves

14. Wave drag

Holomorphic functions

$$Z = P(x,y) + iQ(x,y)$$



Holomorphic functions

$$\frac{dZ}{dz} = \frac{\frac{\partial P}{\partial x} + i\frac{\partial Q}{\partial x} + \left(\frac{\partial P}{\partial y} + i\frac{\partial Q}{\partial y}\right)\frac{dy}{dx}}{1 + i\frac{dy}{dx}}$$

$$f'(z) = \lim_{\Delta z \to 0} \frac{f(z+\Delta z) - f(z)}{\Delta z}$$

$$\frac{\partial P}{\partial x} + i\frac{\partial Q}{\partial x} = -i\left(\frac{\partial P}{\partial y} + i\frac{\partial Q}{\partial y}\right)$$

Holomorphic functions

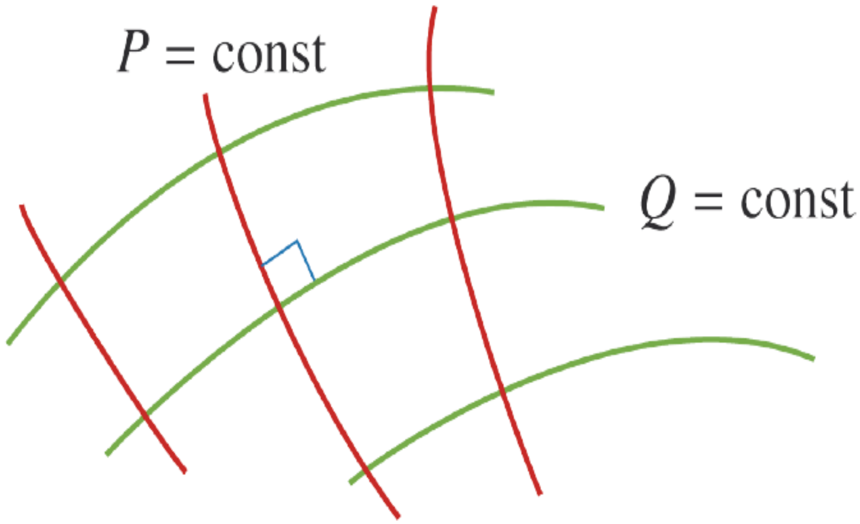
Cauchy-Riemann conditions

$$\frac{\partial P}{\partial x} = \frac{\partial Q}{\partial y} \quad , \quad \frac{\partial P}{\partial y} = -\frac{\partial Q}{\partial x}$$

Conjugate Holomorphic functions

$$\Delta P = 0 \quad , \quad \Delta Q = 0$$

Orthogonal lattice



2D potential flow

$$z = x + iy$$

$$f(z) = \varphi(x, y) + i\psi(x, y)$$

$$w(z) = \frac{df}{dz} = u(x, y) - iv(x, y)$$

Complex Potential

$$f(z) = \phi(x,y) + i\psi(x,y)$$

$$\frac{\partial\phi}{\partial y} = -\frac{\partial\psi}{\partial x}$$

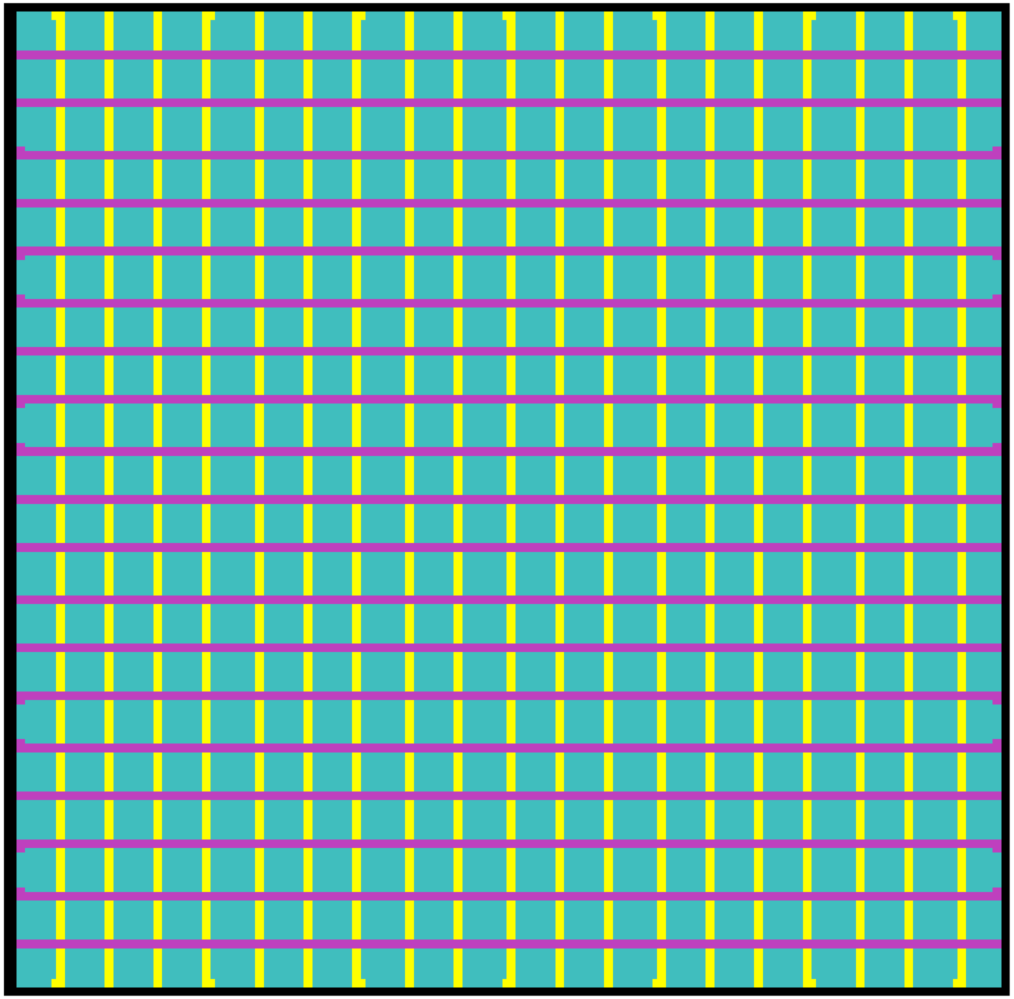
$$\frac{\partial\phi}{\partial x} = \frac{\partial\psi}{\partial y}$$

Complex velocity

$$\frac{df(z)}{dz} = w(z) = u - iv$$

Uniform flow

$$f(z) = z$$



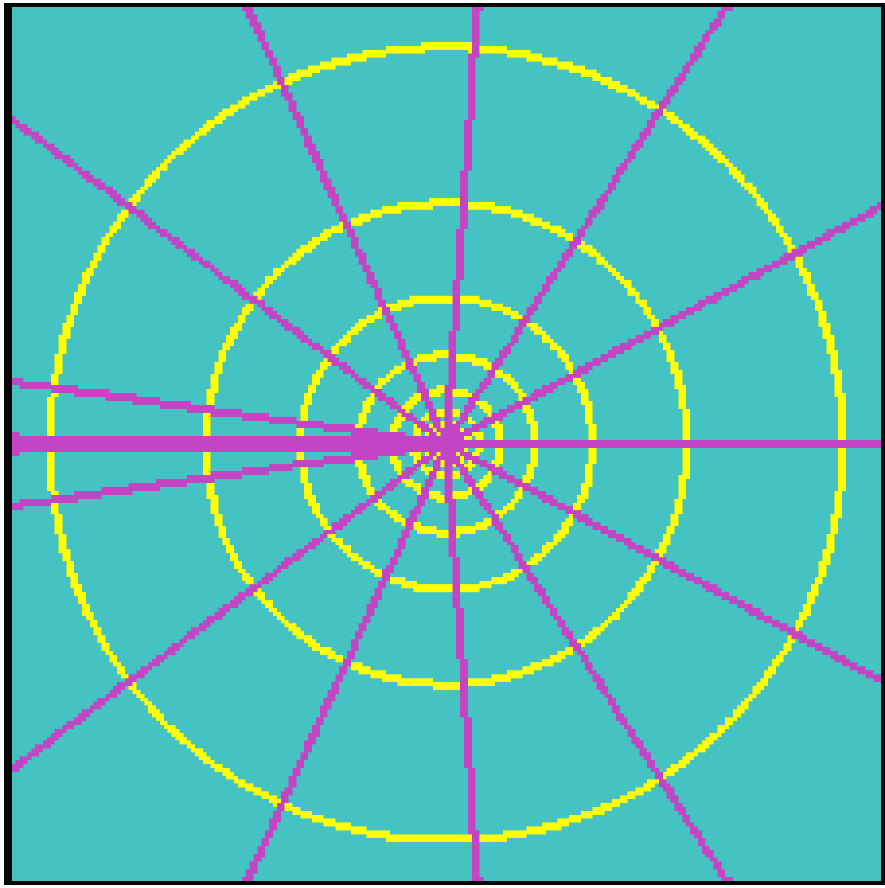
Equipotentielle $\phi(x, y) = x = cte$

Ligne de courant $\psi(x, y) = y = cte$

SOURCE D>0 or SINK D<0

$$f(z) = \frac{D}{2\pi}\,\mathrm{Ln}\,(z)$$

$$\phi(x,y) = \frac{D}{2\pi}\mathrm{Ln}r = \mathrm{cte}$$

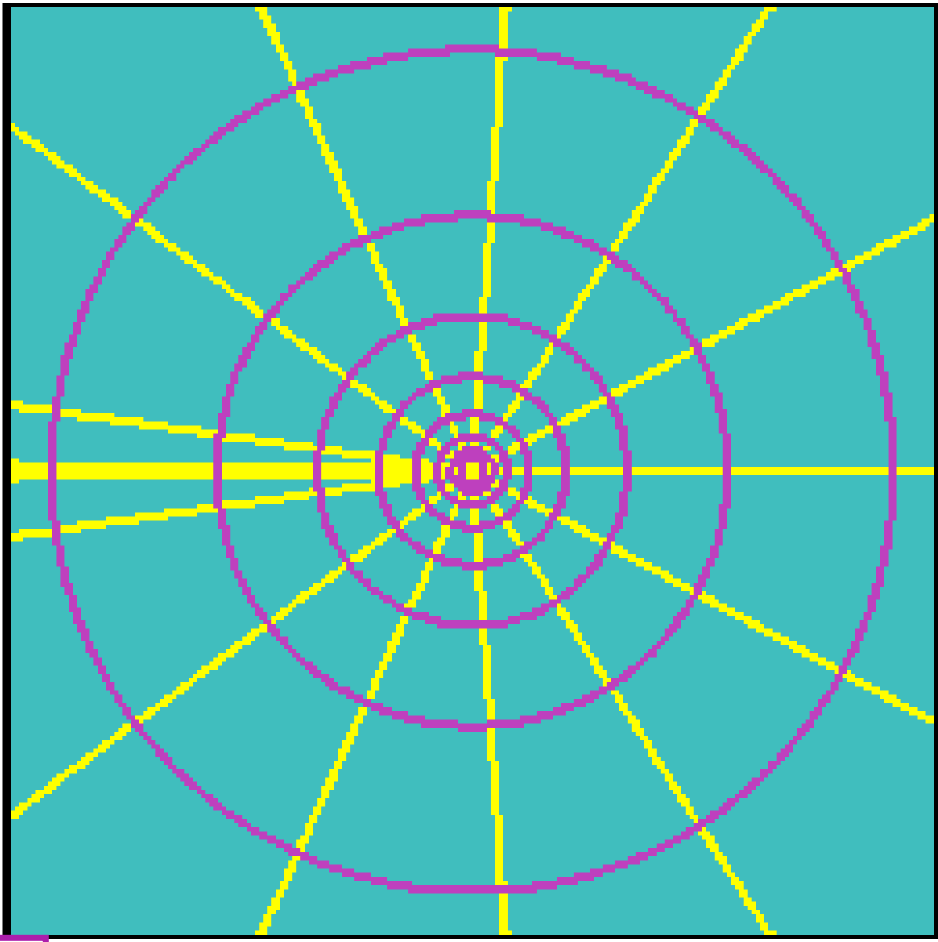


$$\psi(x,y) = \frac{D\theta}{2\pi} = \mathrm{cte}$$

VORTEX

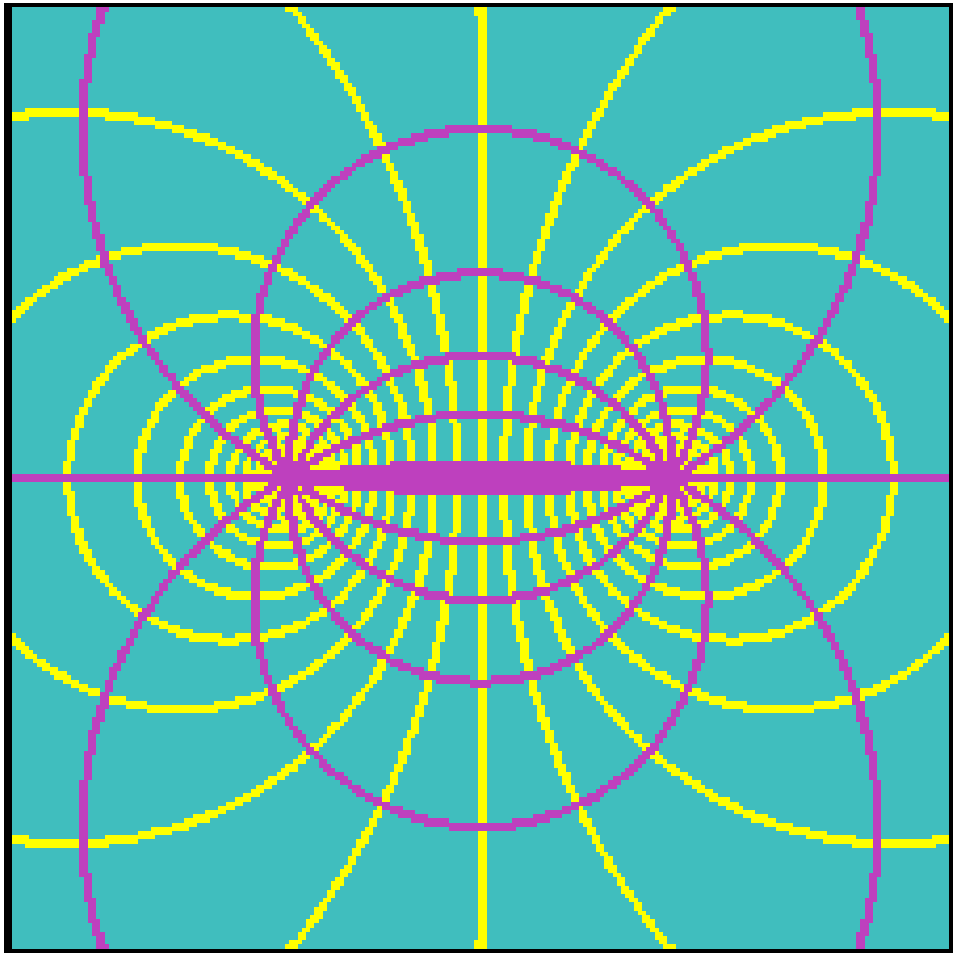
$$f(z) = -\frac{i\Gamma}{2\pi}\,\mathrm{Ln}\,(z)$$

$$\phi(x,y) = \frac{\Gamma\theta}{2\pi} = \mathrm{cte}$$



$$\psi(x,y) = \frac{-\Gamma}{2\pi}\mathrm{Ln}r = \mathrm{cte}$$

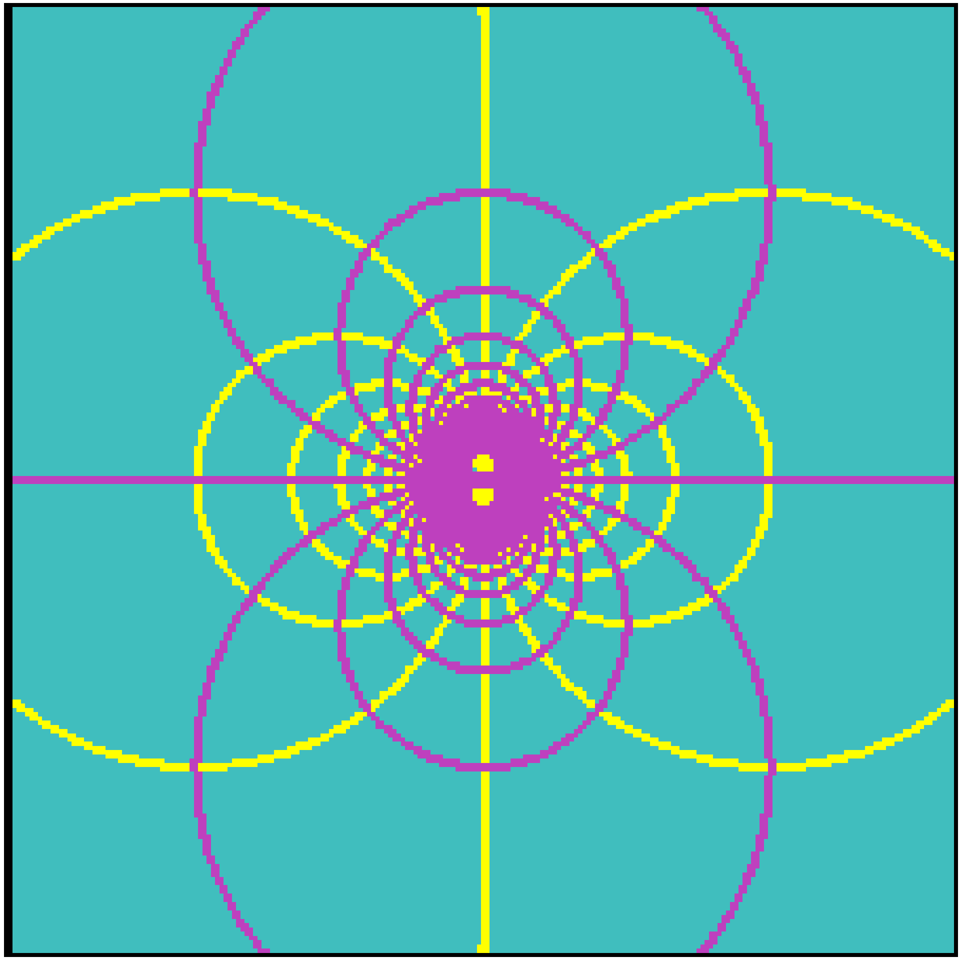
SUPERPOSITION, SOURCE and SINK



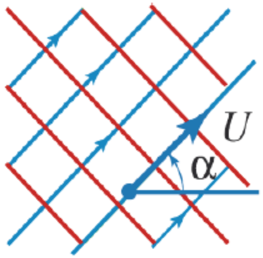
$$f(z) = \frac{D}{2\pi}\,\text{Ln}\,(z-a) - \frac{D}{2\pi}\,\text{Ln}\,(z+a)$$

DOUBLET

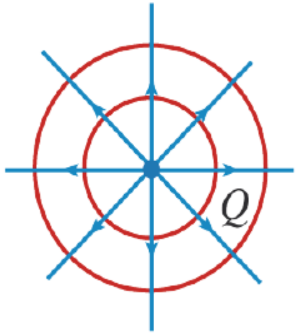
$$f(z) = \frac{k}{z}$$



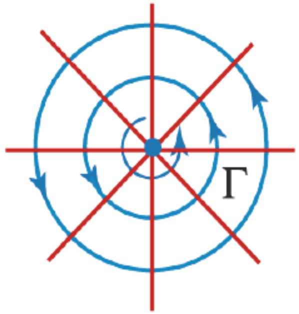
$$f(z) = Ue^{-i\alpha} z$$



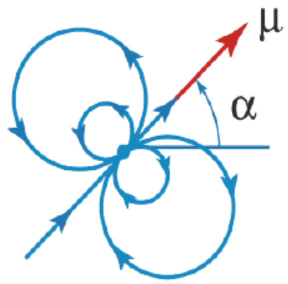
$$f(z) = \frac{Q}{2\pi} \log(z - z_0)$$



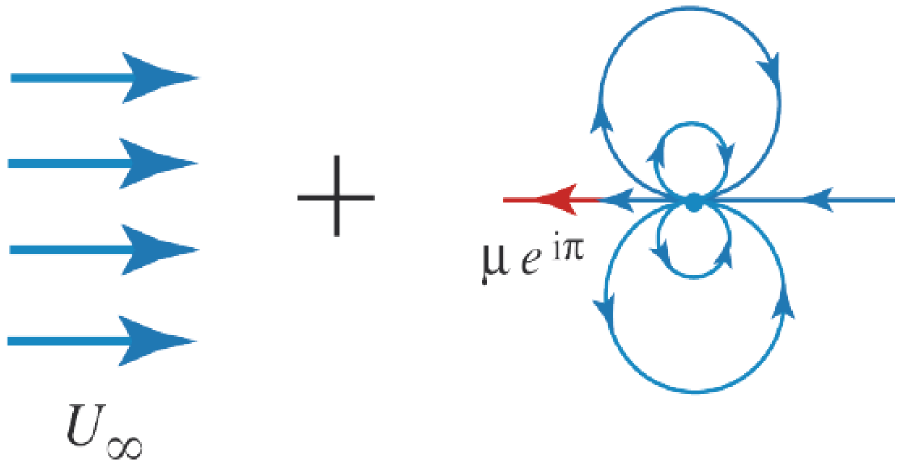
$$f(z) = -\frac{i\Gamma}{2\pi} \log(z - z_0)$$



$$f(z) = -\frac{\mu e^{i\alpha}}{z - z_0}$$



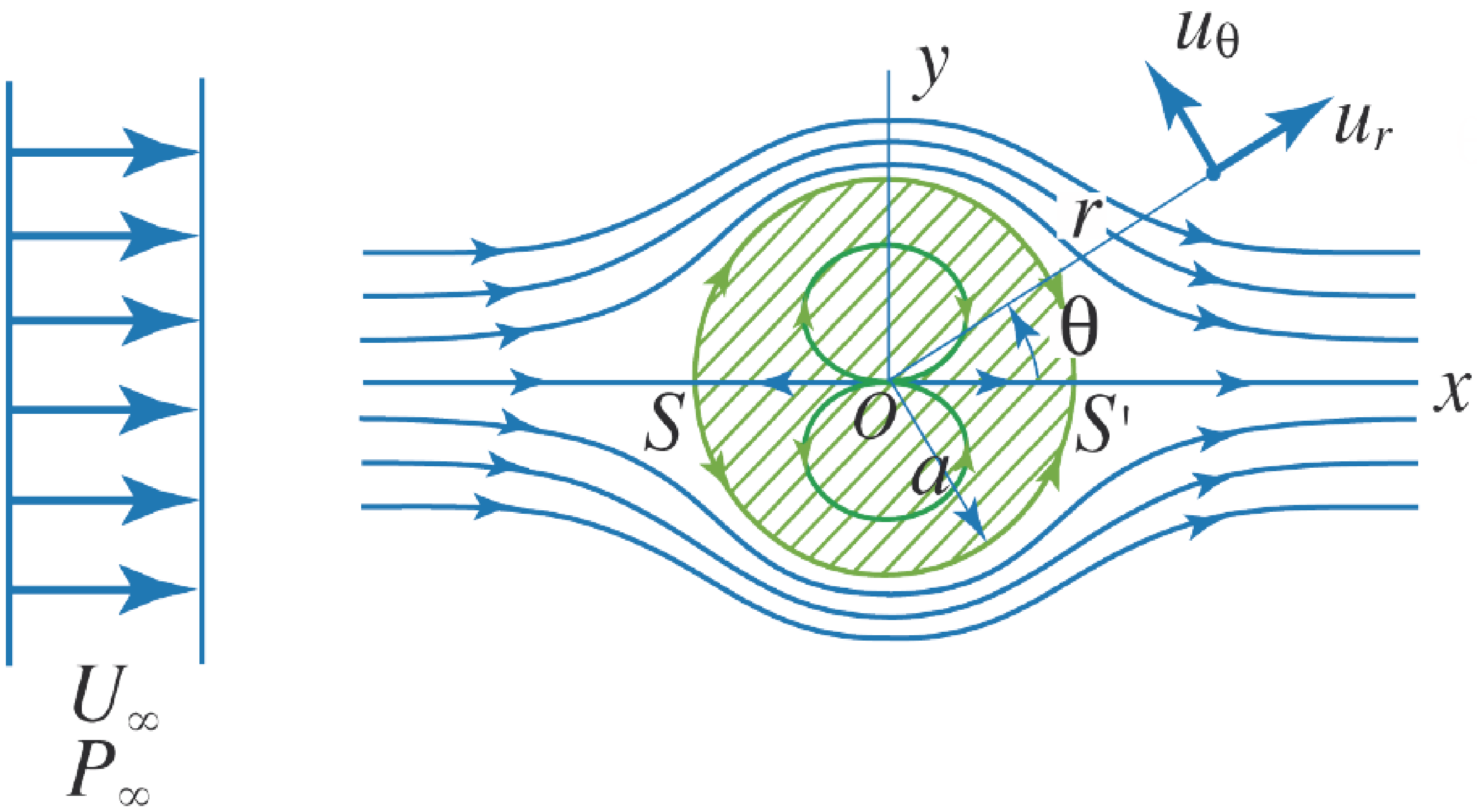
Acyclic flow around cylinder



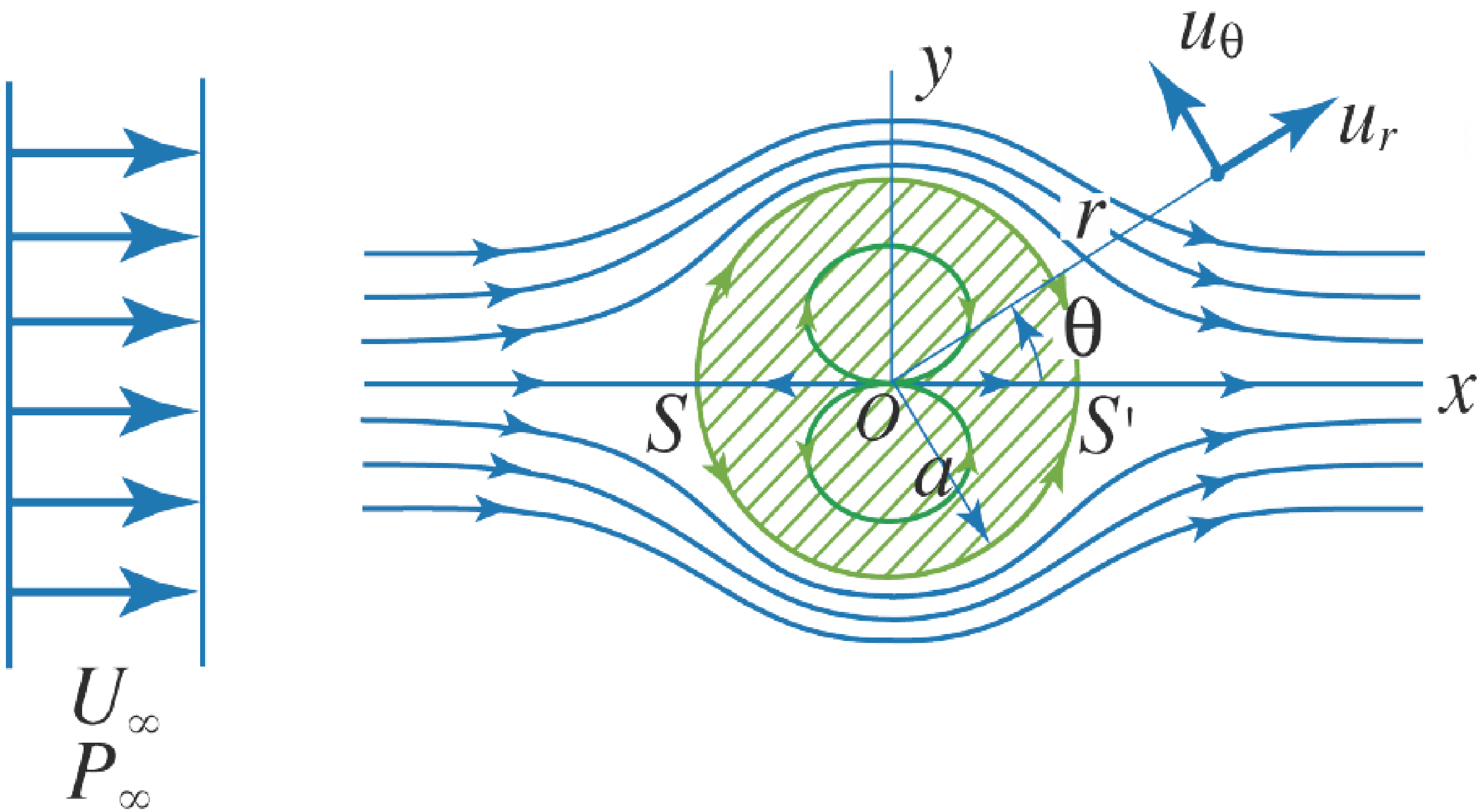
$$f(z) = U_\infty z + \frac{\mu}{z}$$

$$f(z) = U_\infty \left(z + \frac{a^2}{z} \right)$$

Acyclic flow around cylinder



Acyclic flow around cylinder



$$U_r(r,\theta) = U_\infty\left(1 - \frac{a^2}{r^2}\right)\cos\theta \quad , \quad U_\theta(r,\theta) = -\,U_\infty\left(1 + \frac{a^2}{r^2}\right)\sin\theta$$

Bernoulli

$$U_r(r,\theta) = U_\infty\left(1 - \frac{a^2}{r^2}\right)\cos\theta \quad , \quad U_\theta(r,\theta) = -U_\infty\left(1 + \frac{a^2}{r^2}\right)\sin\theta$$

$$p(a,\theta) = \frac{1}{2}\rho U_\infty^2(1 - 4\sin\theta^2)$$

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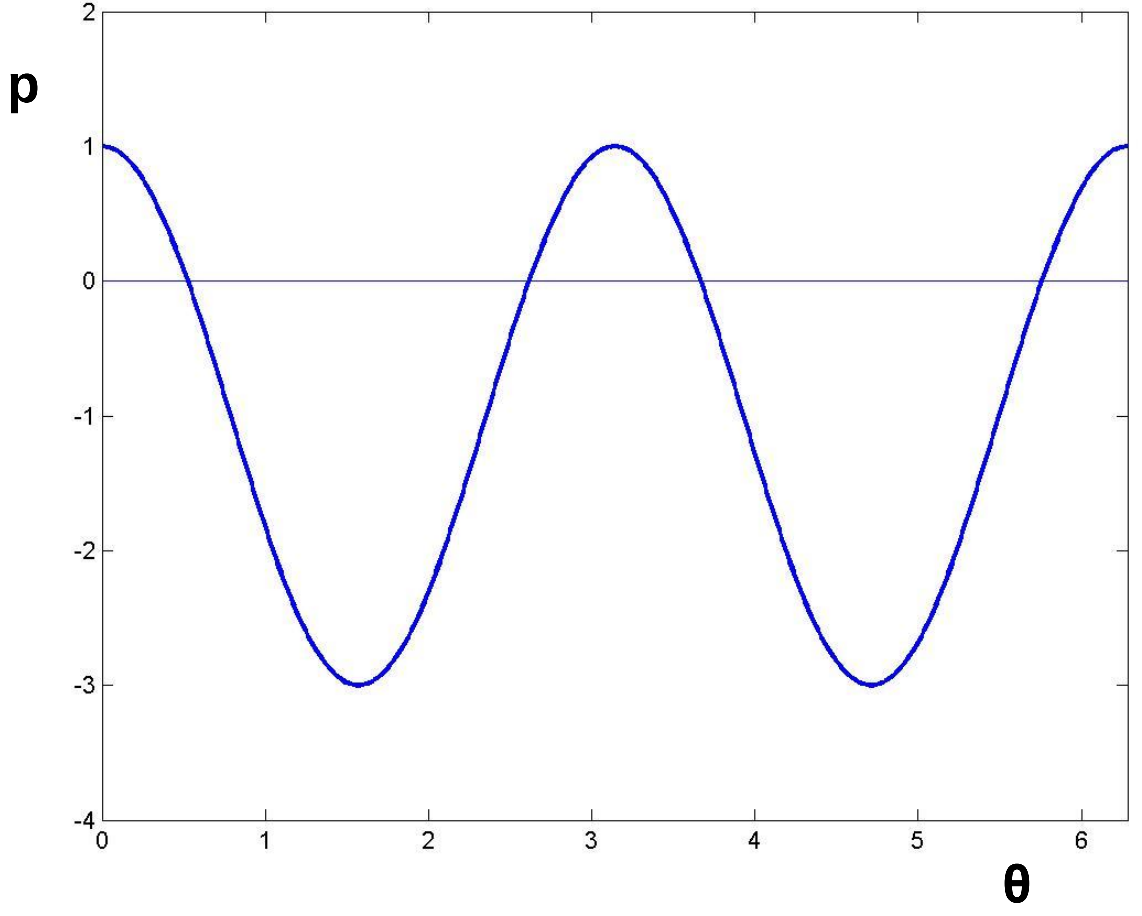

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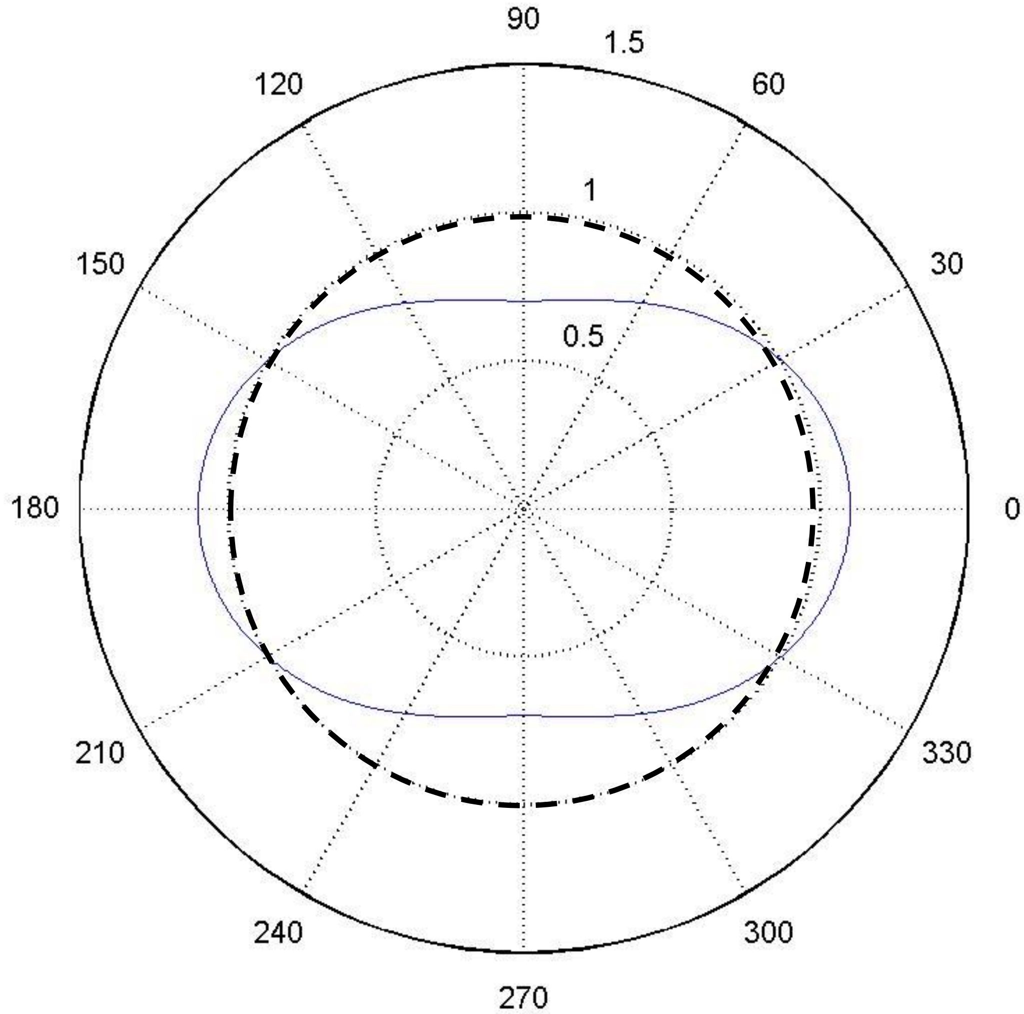

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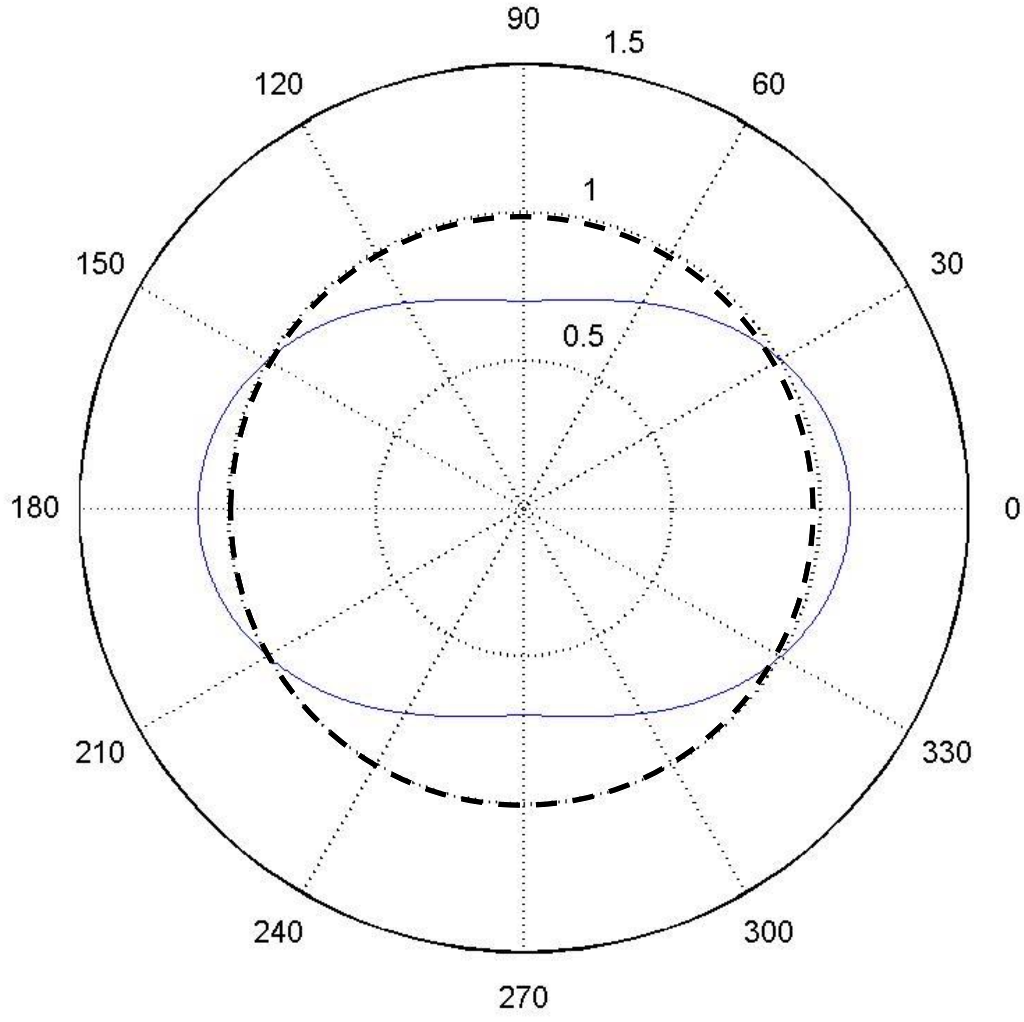


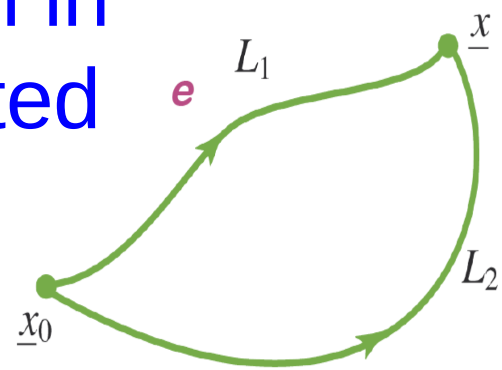
Traînée : F_D=0

Portance: F_L=0

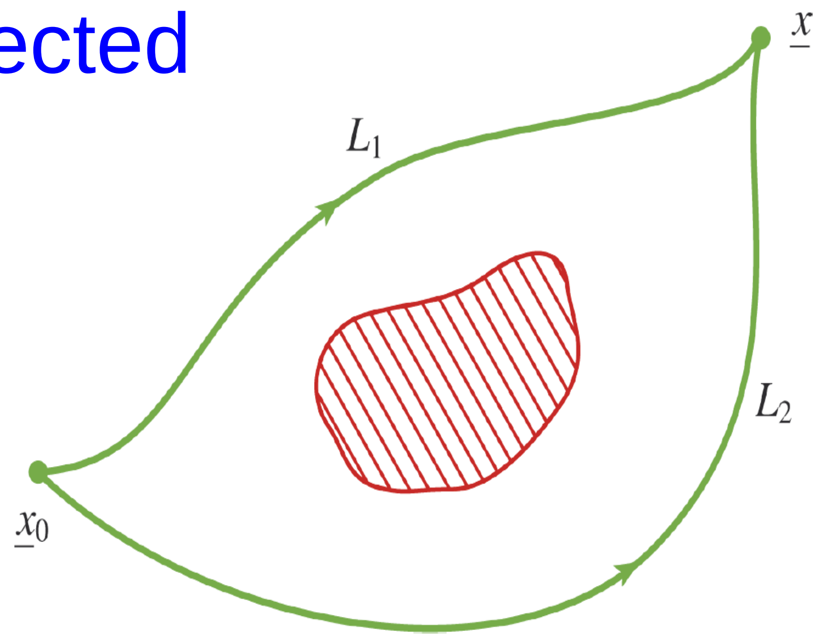
Achtung! Connectivity?

The solution to Laplace equation is unique only in simply connected domains

Unique solution in simply connected domains

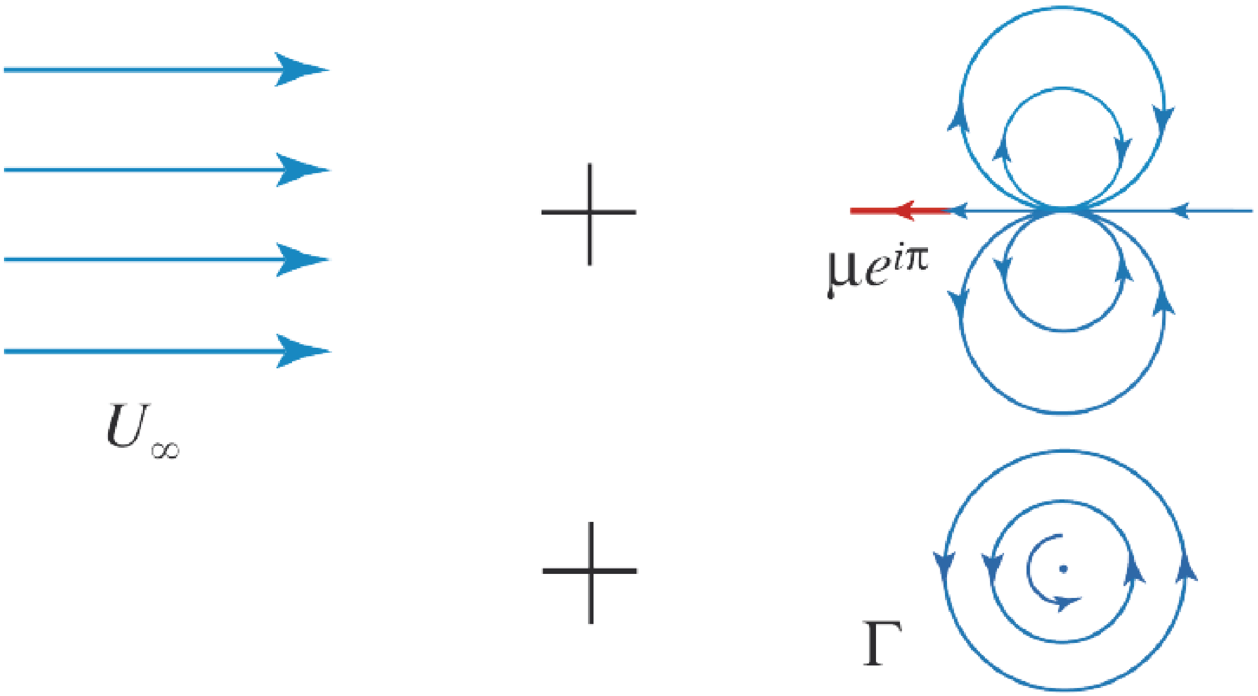


Multiple solutions in multiply connected domains



$$\varphi\left(\underline{x},t\right)=\varphi\left(\underline{x}_0,t\right)+\int_{L_j}\underline{U}\left(\underline{x},t\right).\,\underline{\mathrm{d}\ell}$$

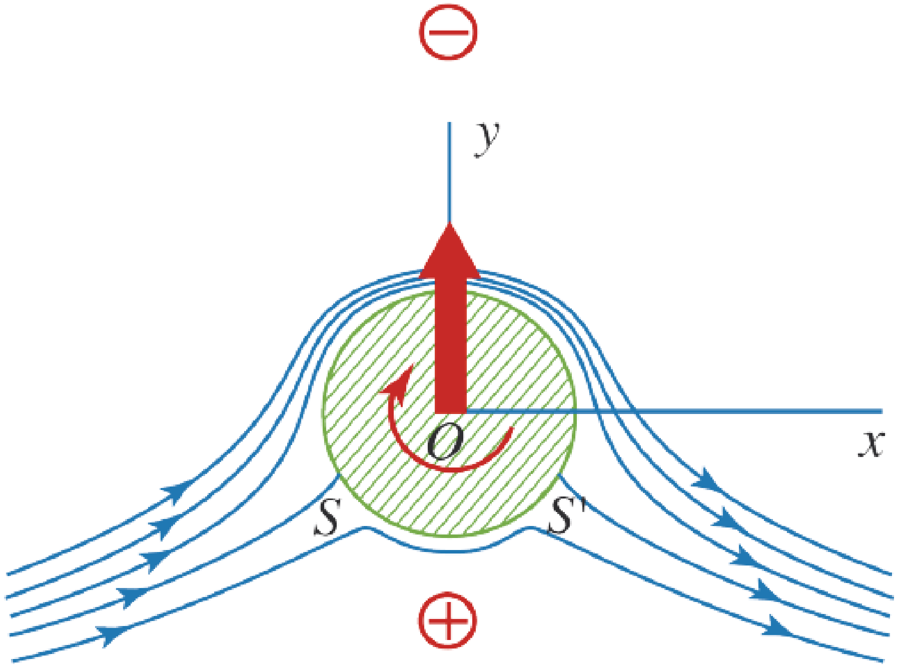
Cyclic flow around cylinder



$$f(z) = U_\infty z + \frac{\mu}{z} - \frac{i\Gamma}{2\pi}\log\left(\frac{z}{a}\right)$$

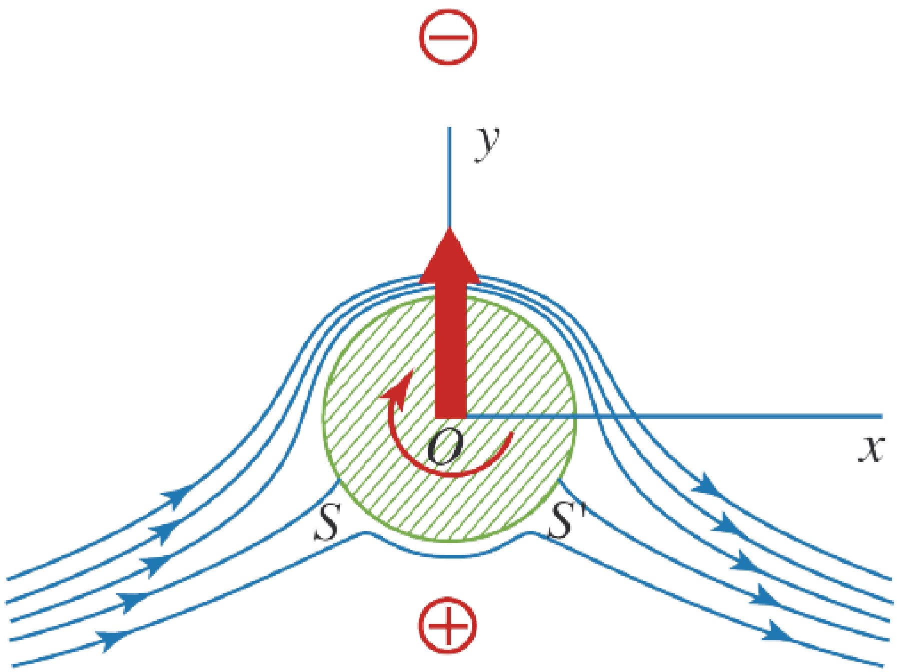
$$f(z) = U_\infty\left(z + \frac{a^2}{z}\right) - \frac{i\Gamma}{2\pi}\log\left(\frac{z}{a}\right)$$

Cyclic flow around cylinder



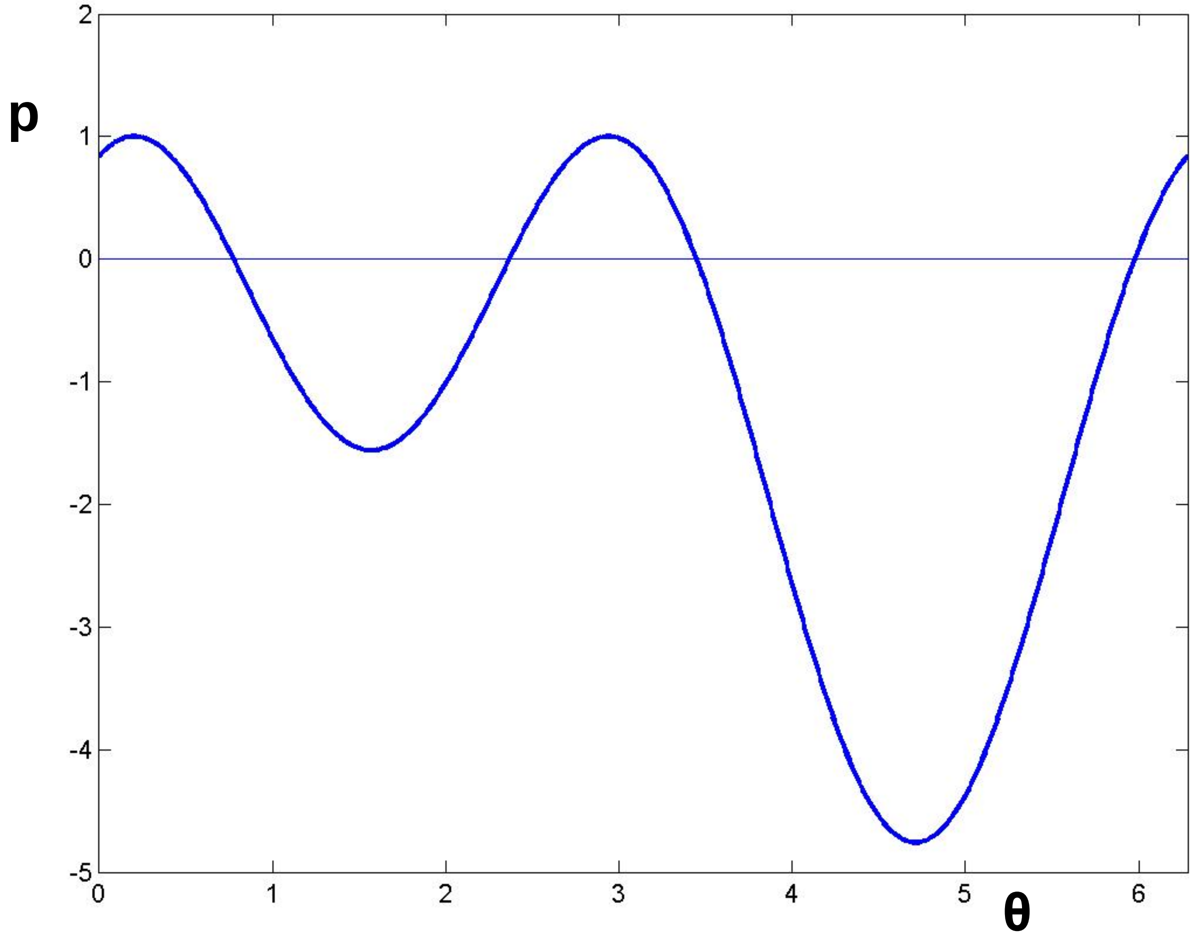
$$-1 < \frac{\Gamma}{4\pi U_\infty a} < 0$$

Effet Magnus et portance

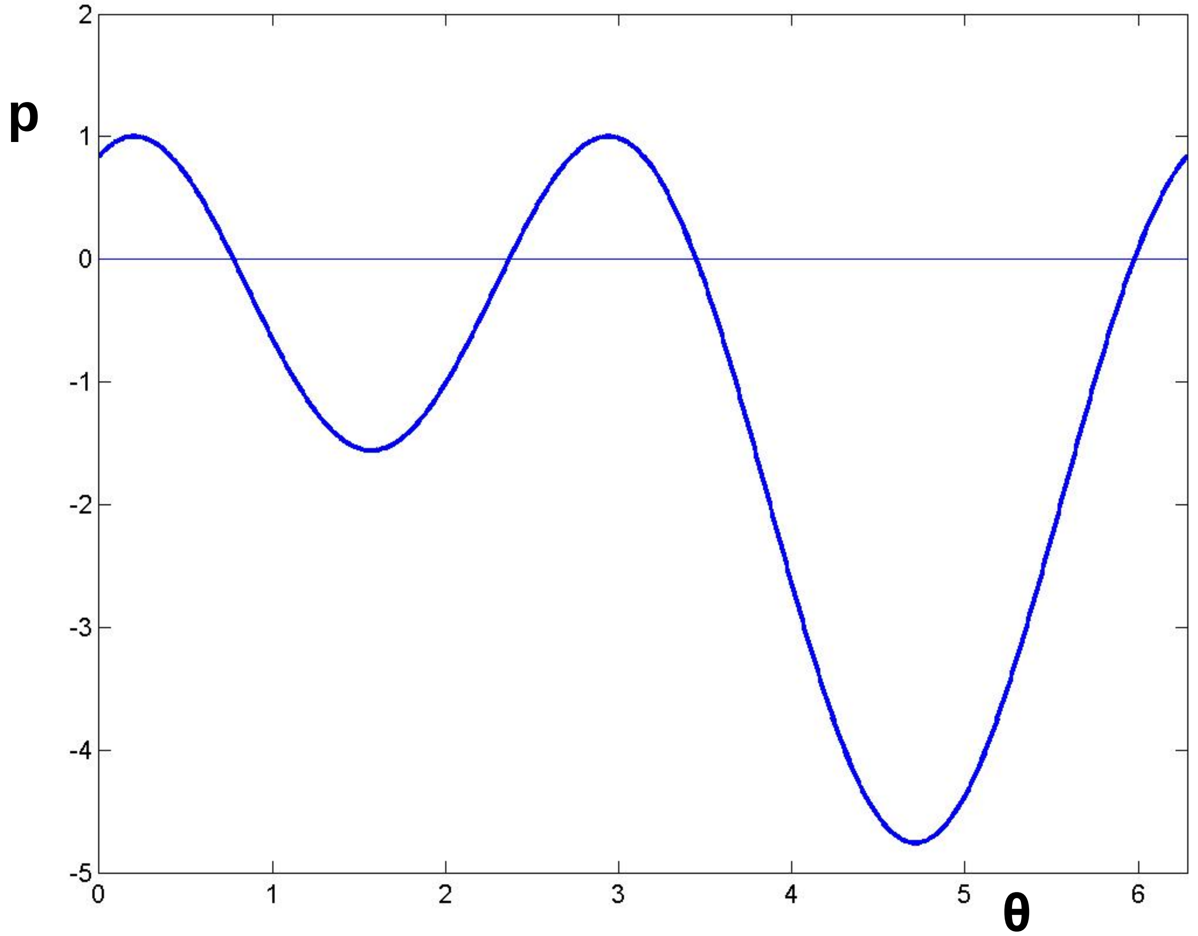


$$U_r(r,\theta) = U_\infty\left(1 - \frac{a^2}{r^2}\right)\cos\theta \quad , \quad U_\theta(r,\theta) = -U_\infty\left(1 + \frac{a^2}{r^2}\right)\sin\theta + \frac{\Gamma}{2\pi r}$$

$$p(a,\theta) = \frac{1}{2}\rho U_\infty^2 \left(1 - 4\left(\sin\theta - \frac{\Gamma}{4\pi U_\infty a}\right)^2\right)$$



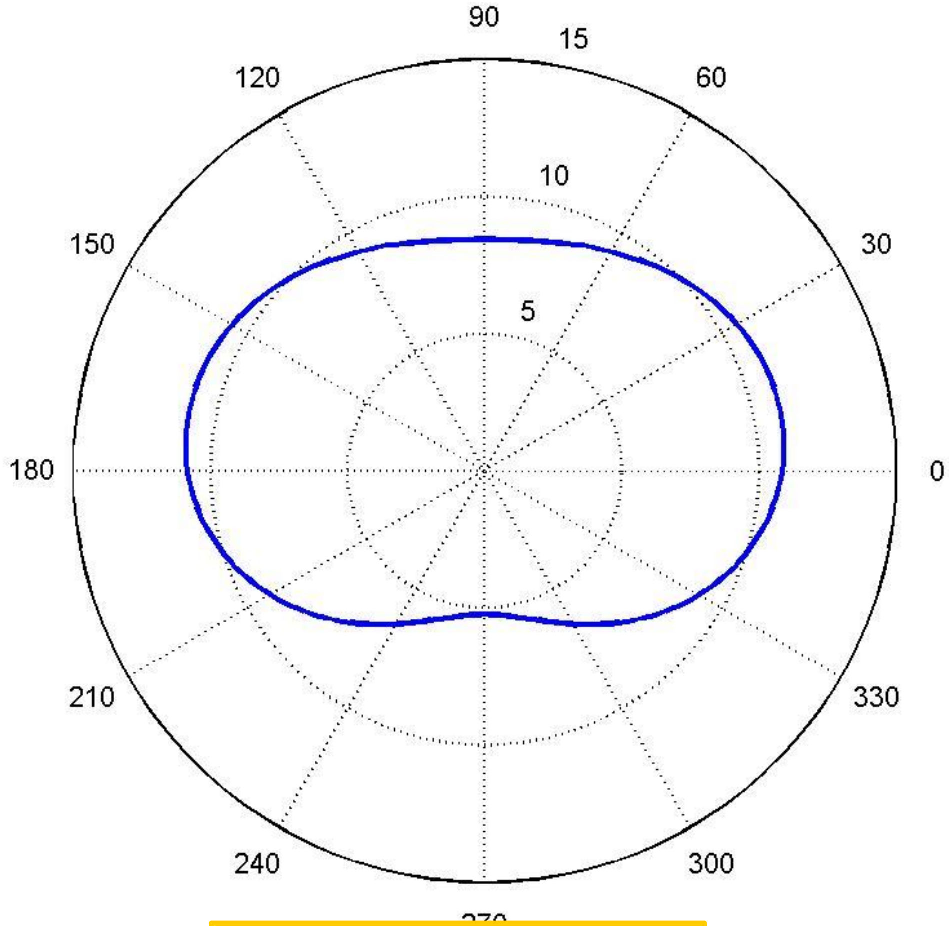
$$p(a,\theta) = \frac{1}{2}\rho U_\infty^2 \left(1 - 4\left(\sin\theta - \frac{\Gamma}{4\pi U_\infty a}\right)^2\right)$$



Traînée : $F_D=0$!

Portance: $F_L \neq 0$

Distribution de pression



- Traînée : $F_D = 0$
- Portance: $F_L \neq 0$

Cyclic flow around cylinder

Drag coefficient

$$C_x = 0$$

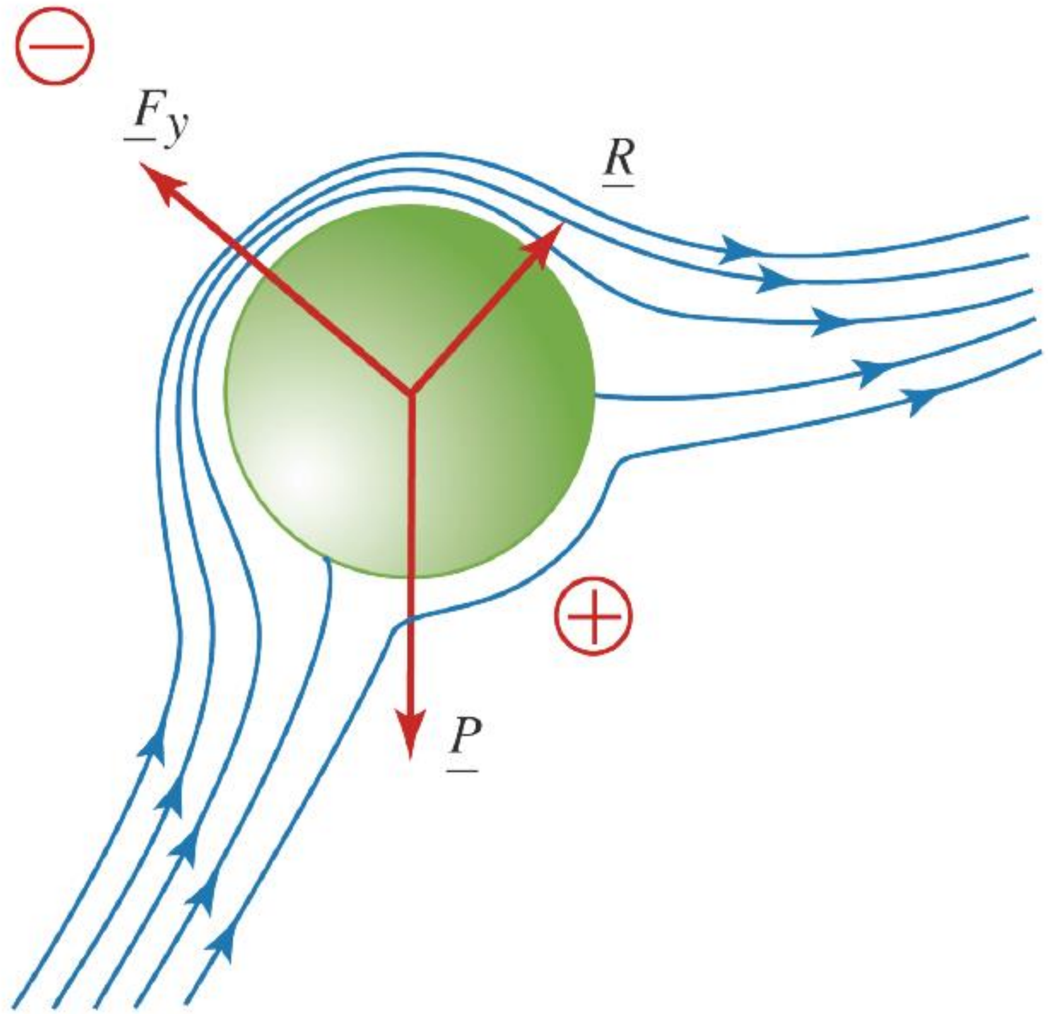
Lift

$$F_y = -\rho U_\infty \Gamma$$

Lift coefficient

$$C_y \neq 0$$

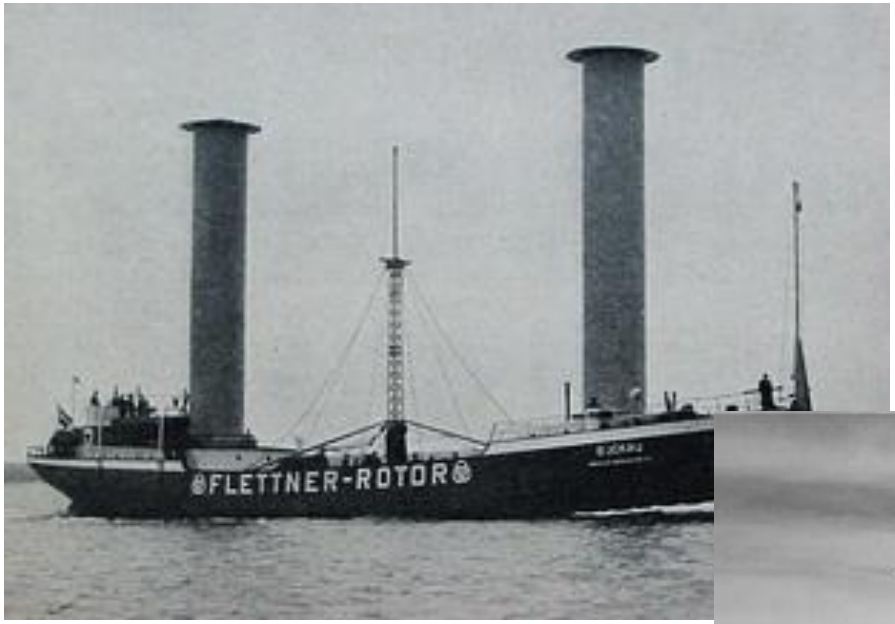
Floating ball



Tangente fluide

Garcia & Chomaz

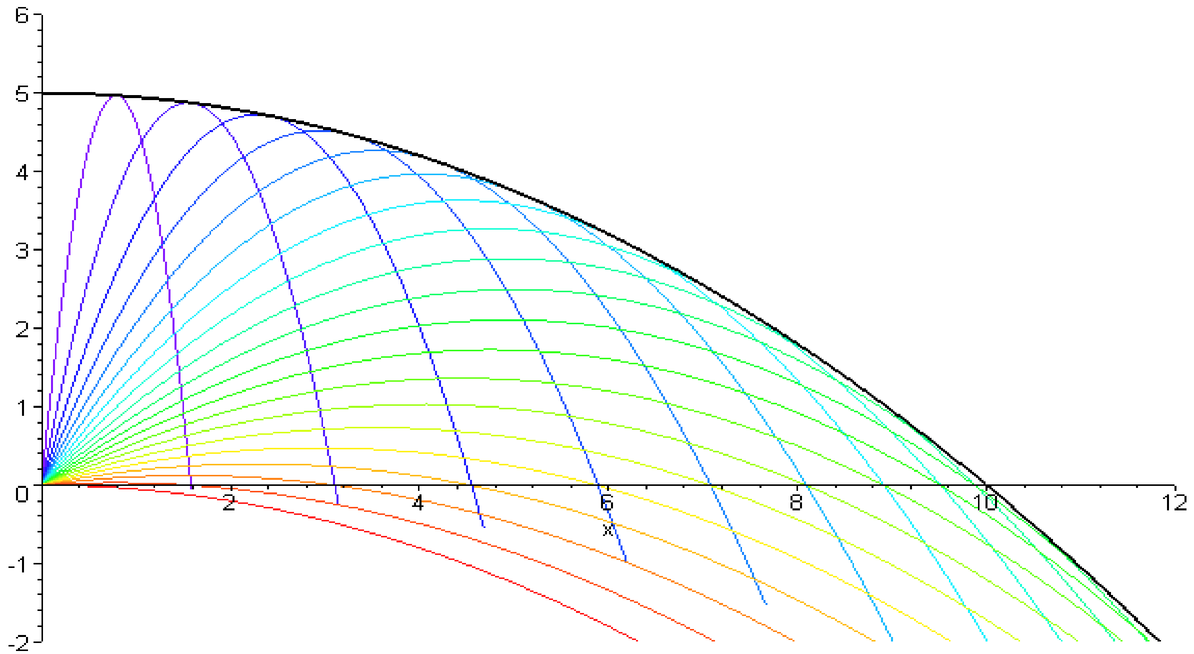
Le Baden Baden



L’Alcyone

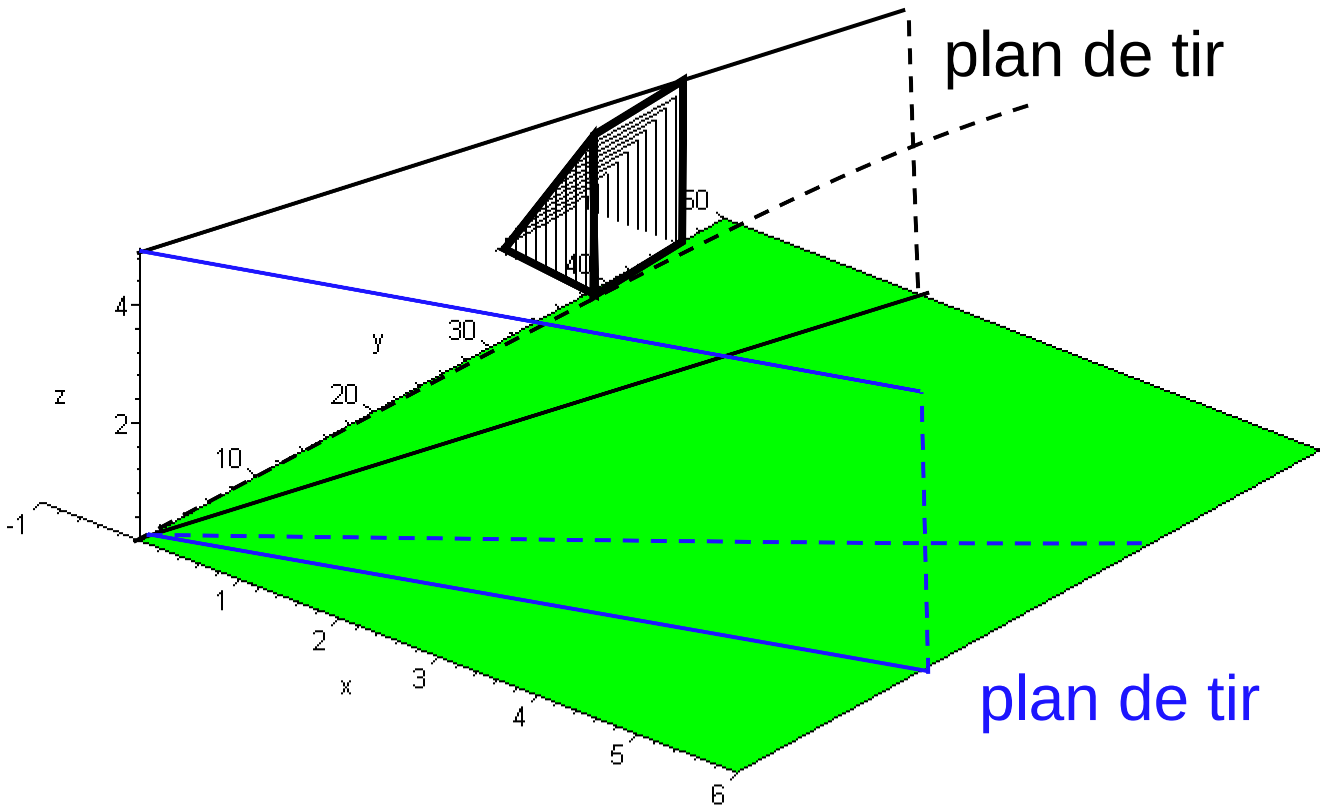
Rotating wings!

Parabolic trajectory

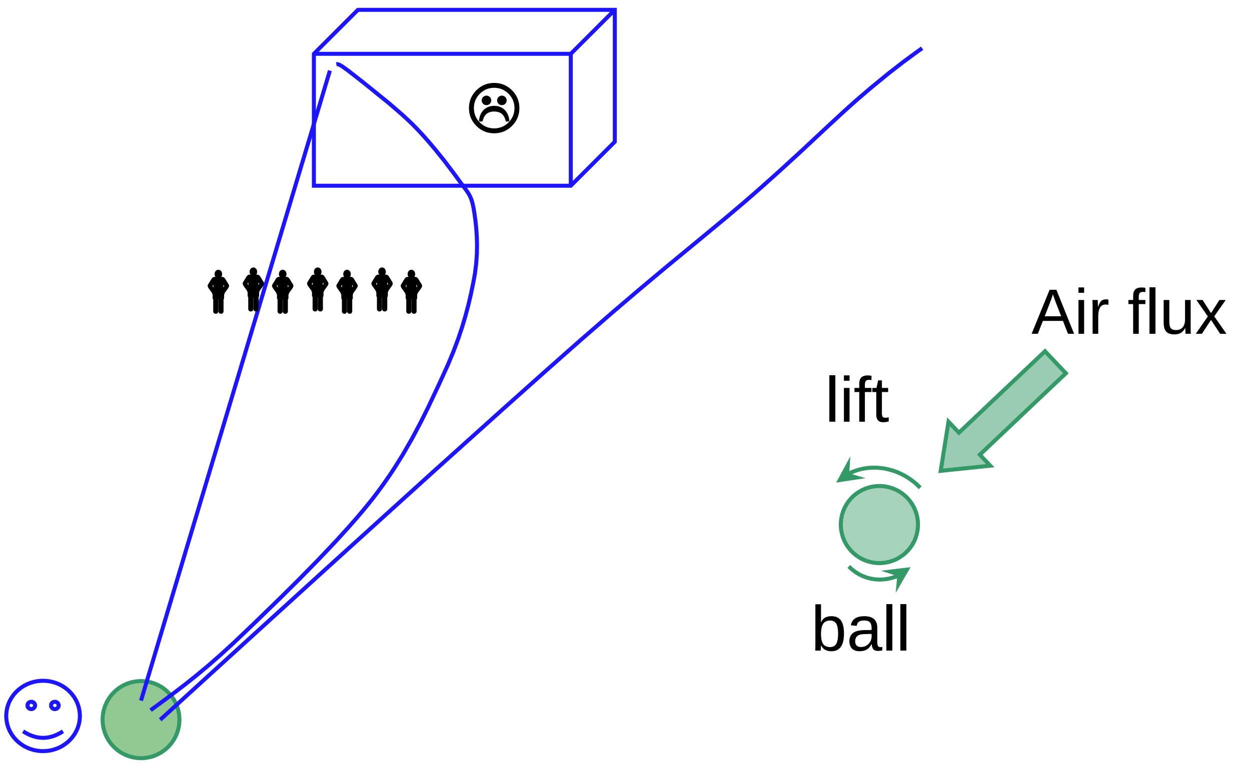


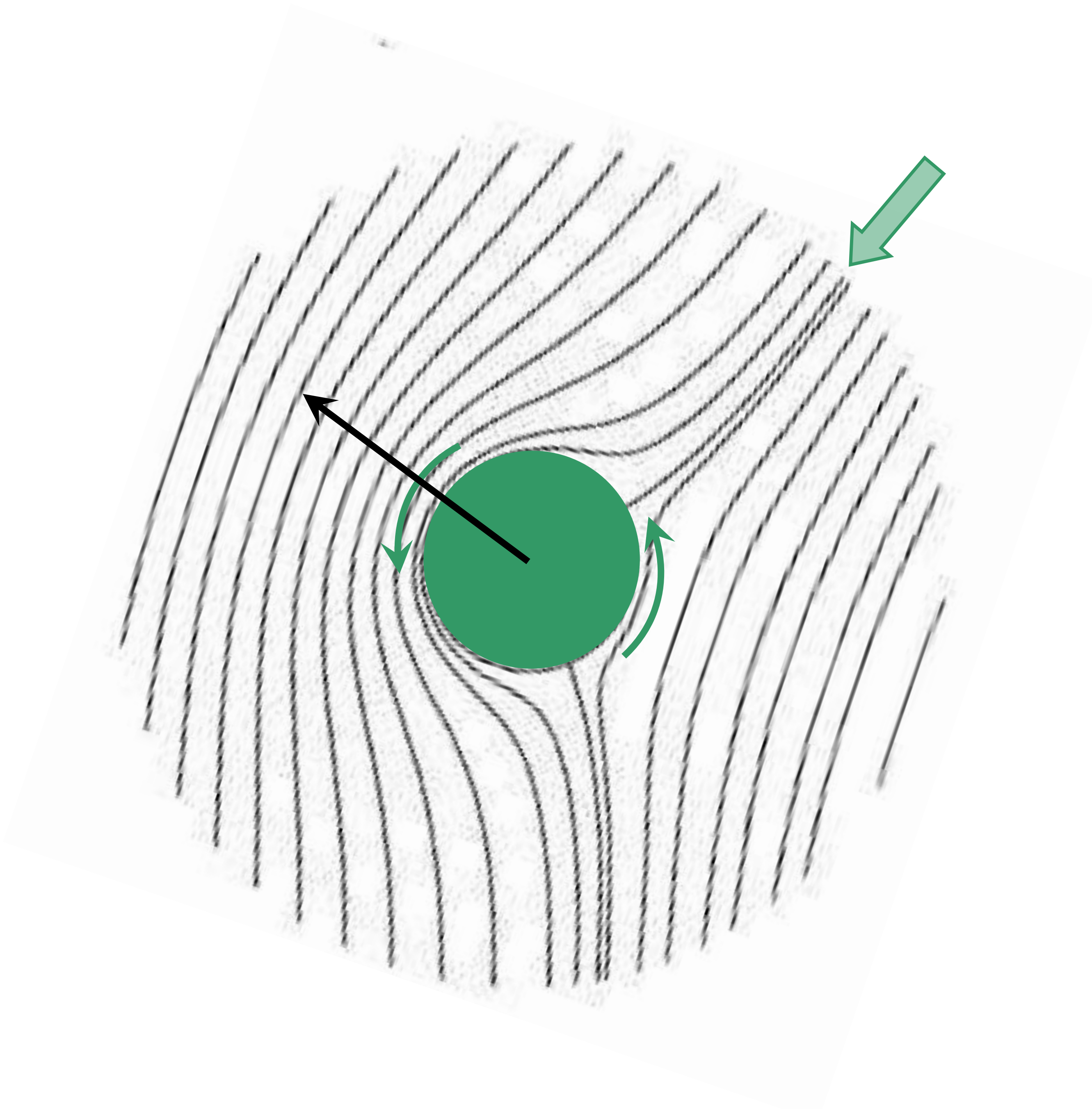
Trajectories with initial velociy10m/s;
the shooting angle varies

Reentrant cornerkick
Brushed kick with vertical rotation of the ball

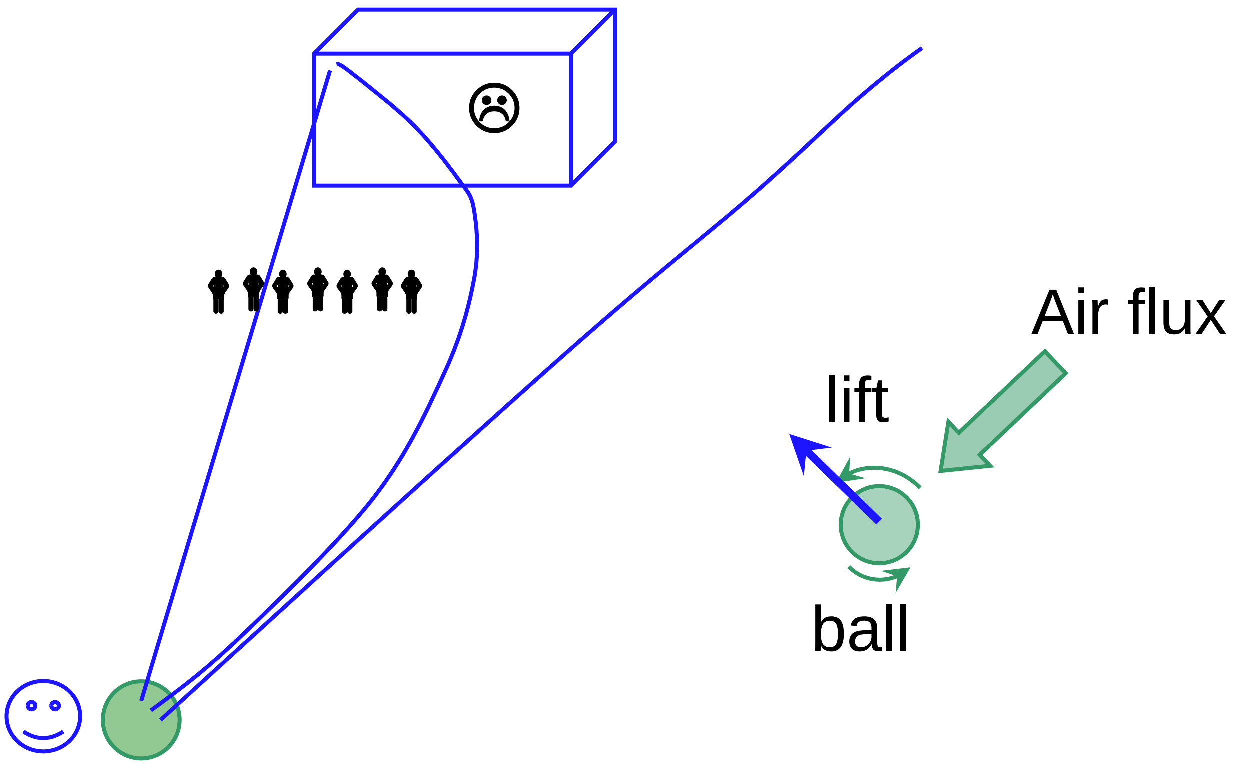


Free kick

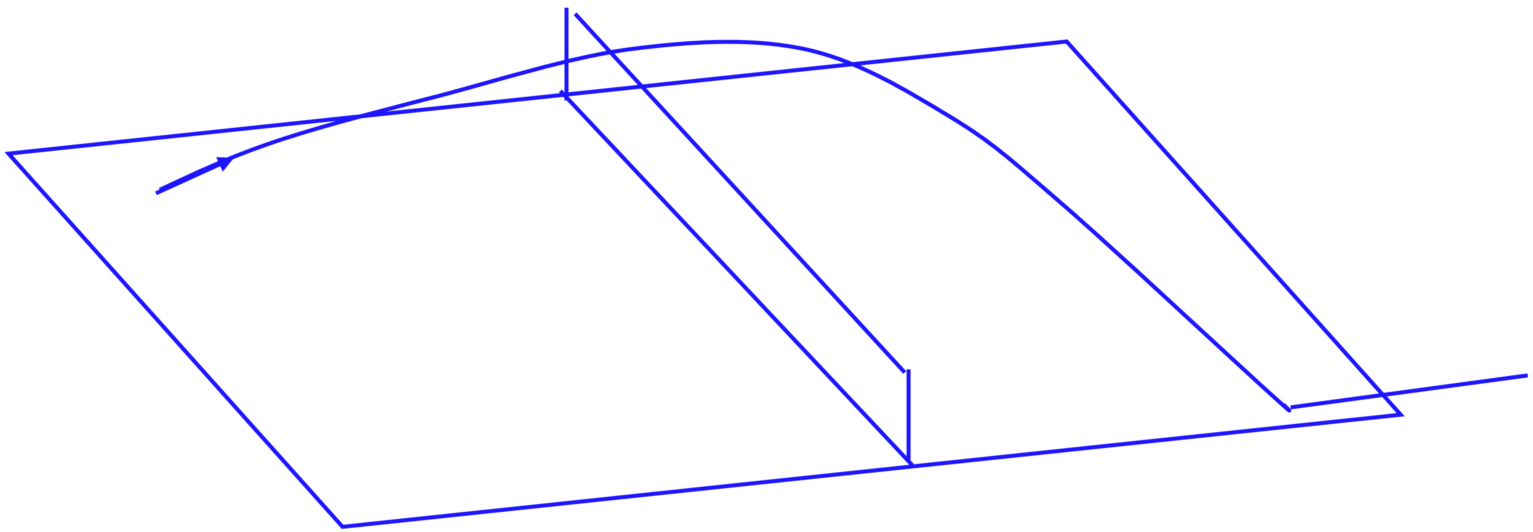


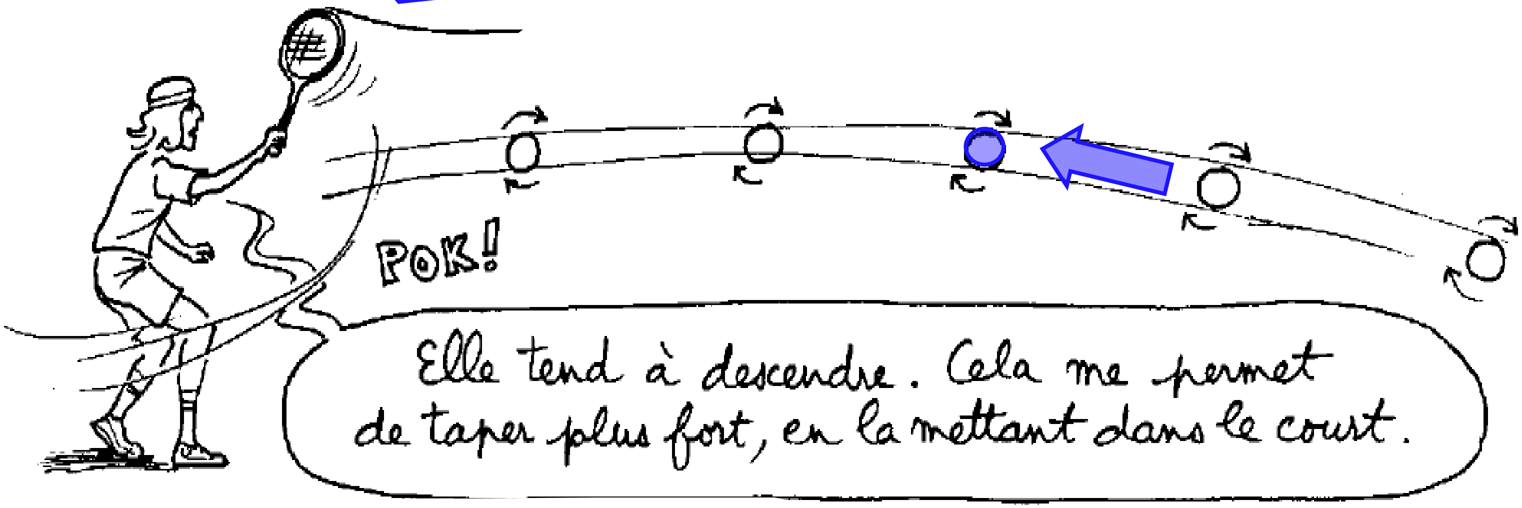


Free kick

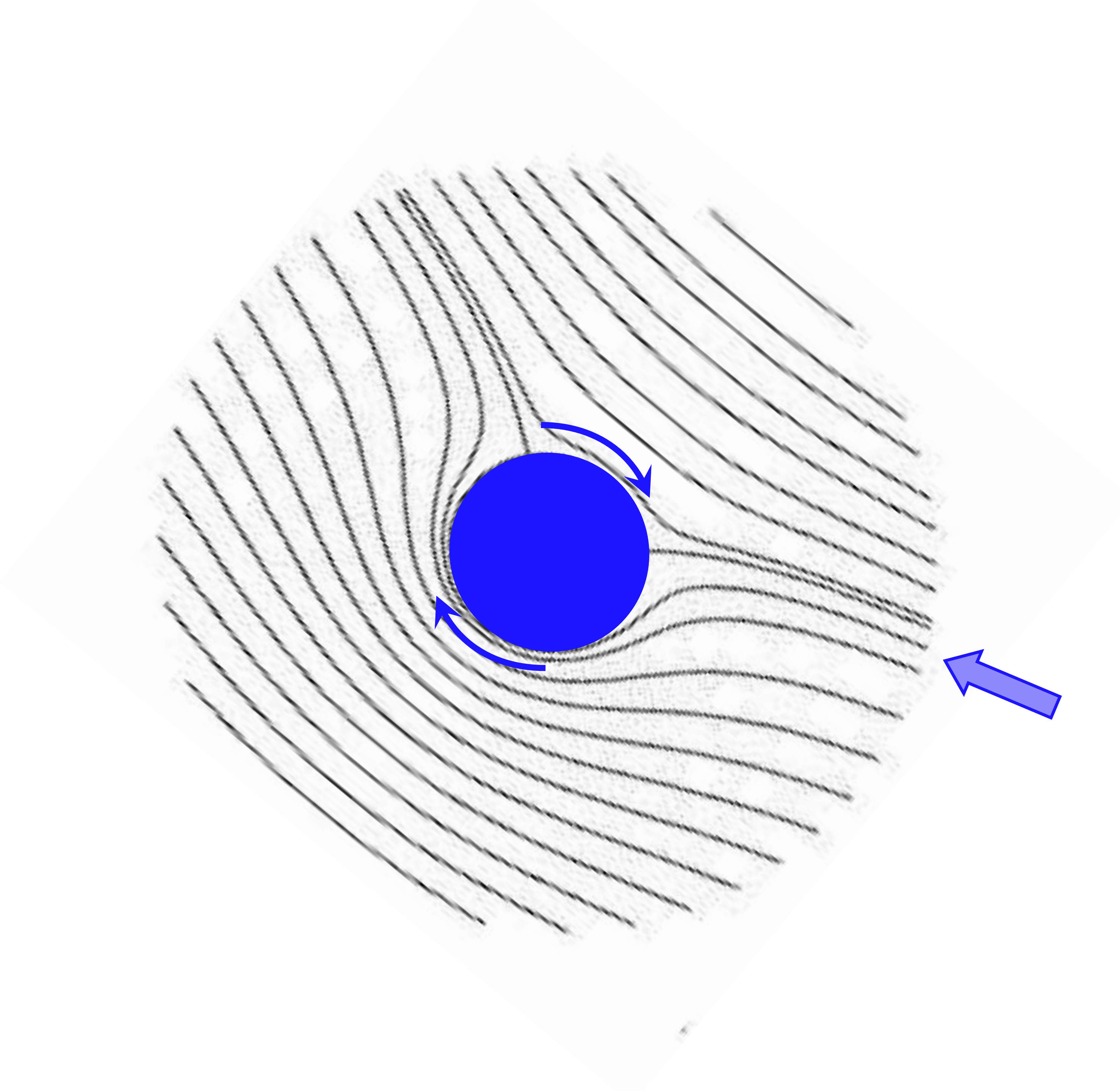


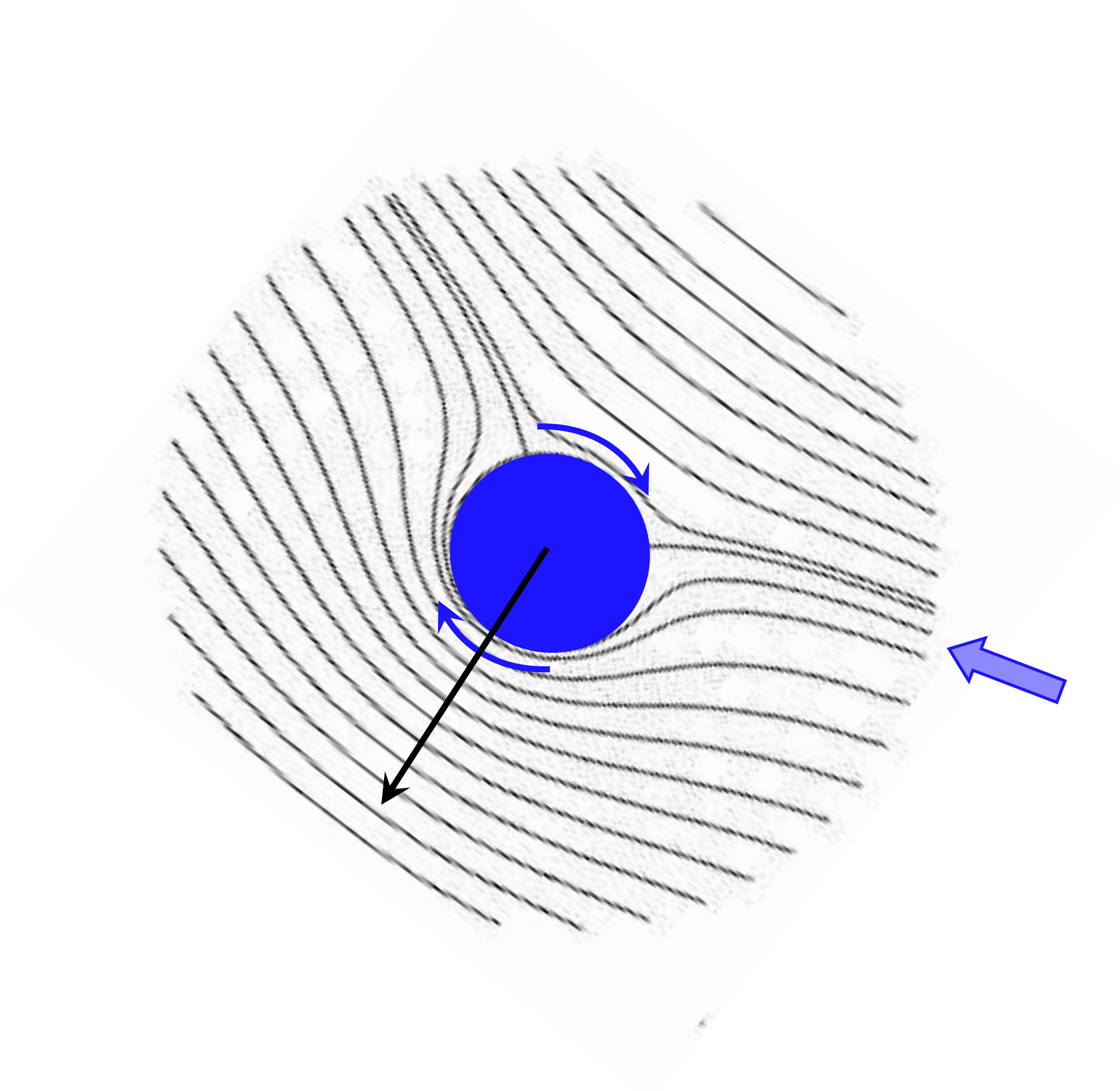
Lift



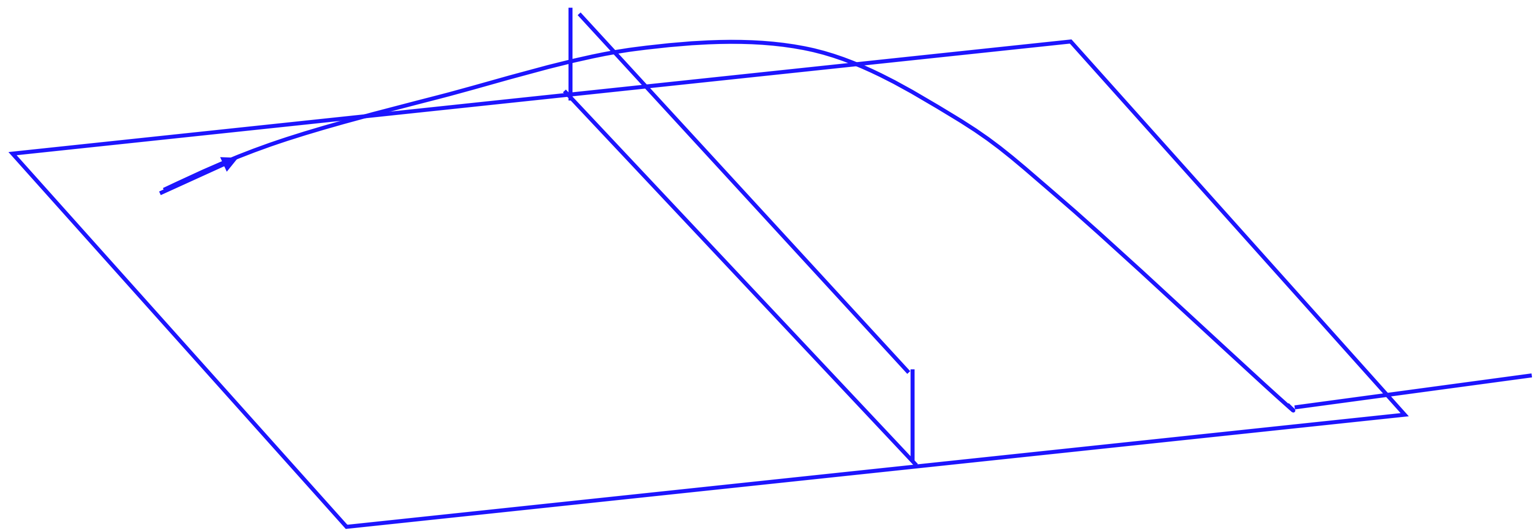


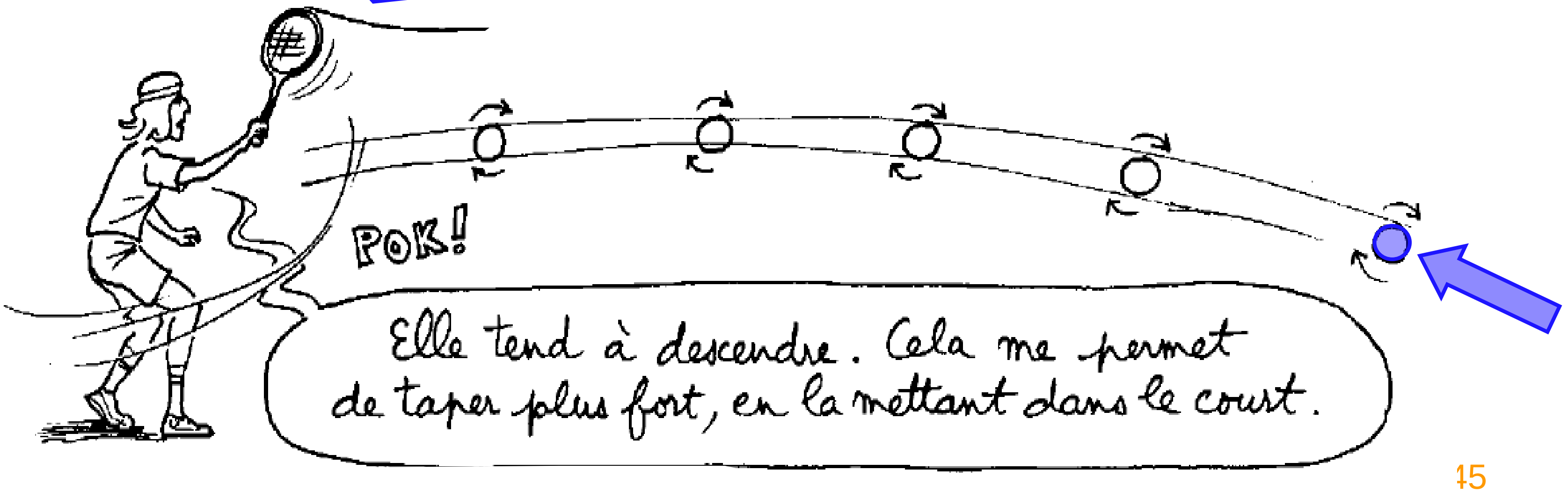
*L'Aspirisouffle, Jean-Pierre Petit





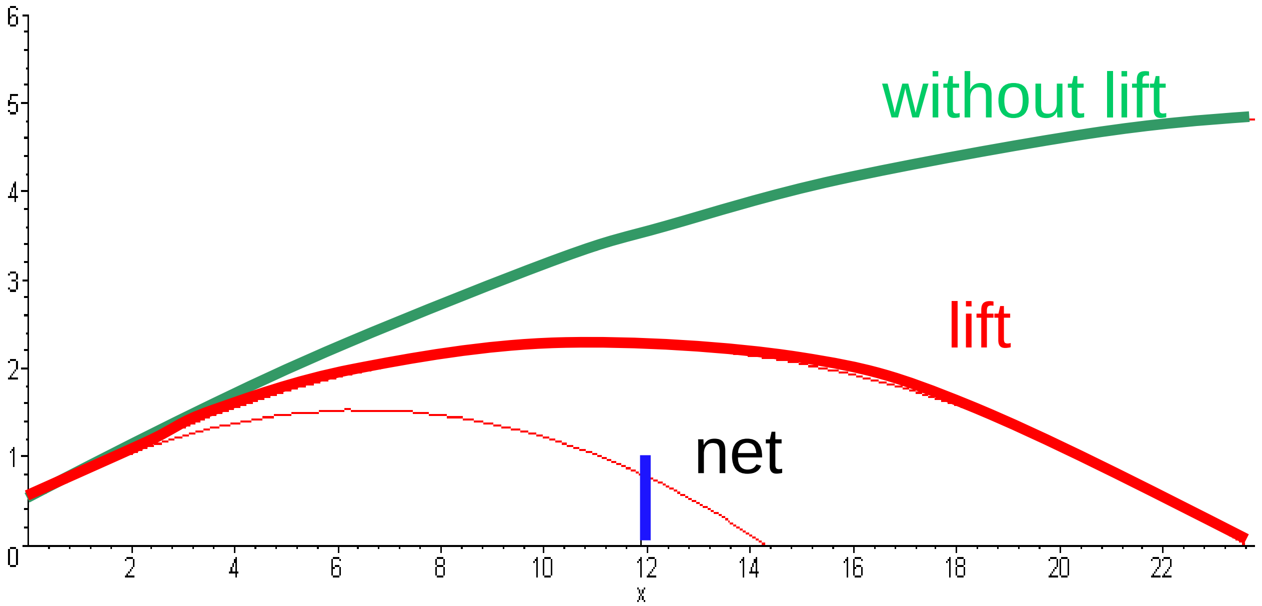
Lifted trajectory





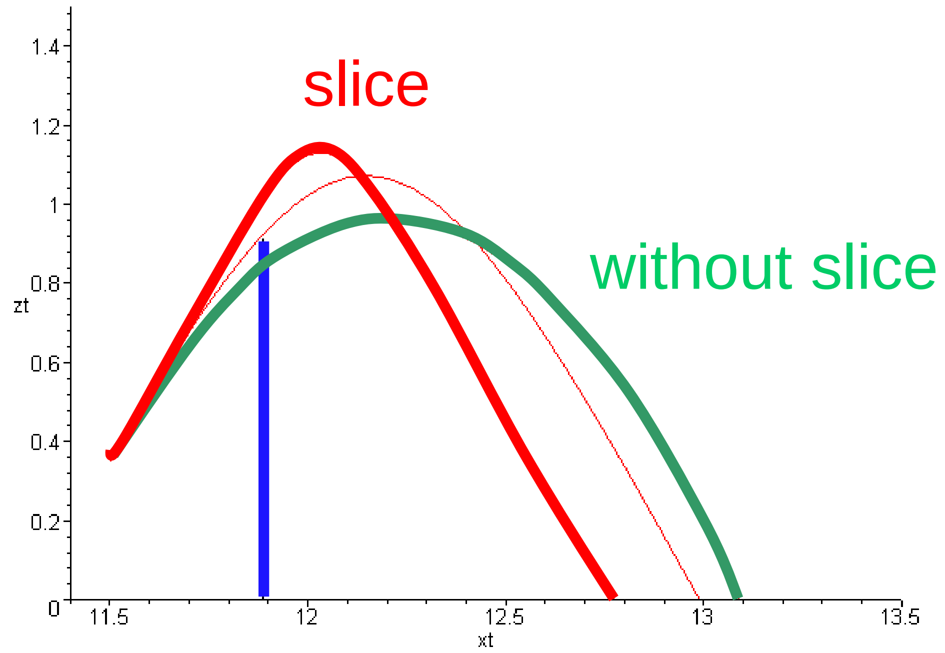
*L'Aspirisouffle, Jean-Pierre Petit

With lift



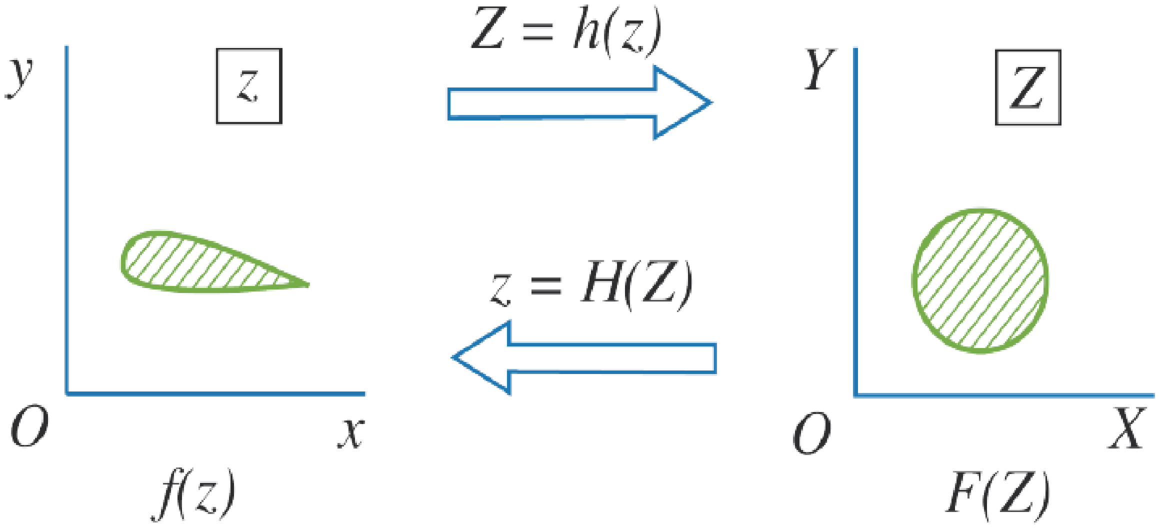
Lift at tennis tennis Vo=30m/s angle18°

Slice



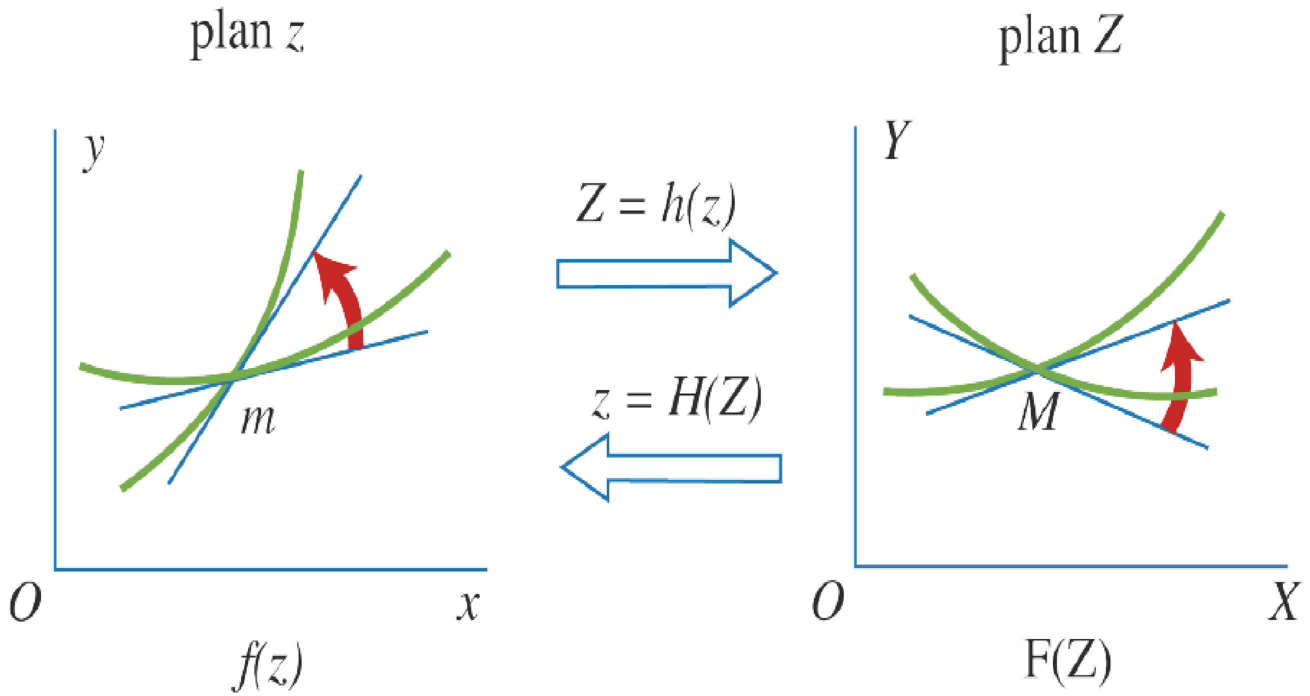
Slice at tennis Vo=5m/s angle 60°

Conformal maps



$$f(z) = F\left[h(z)\right]$$

Conformal maps
Preservation of angles



Conformal maps
Preservation of angles

(except at critical points h'(z)=0)

⇨**The image of an holomorphic function is holomorphic**

Link between z and Z planes

$$F(Z) = f\left[H(Z)\right]$$

Equality of complex potentials

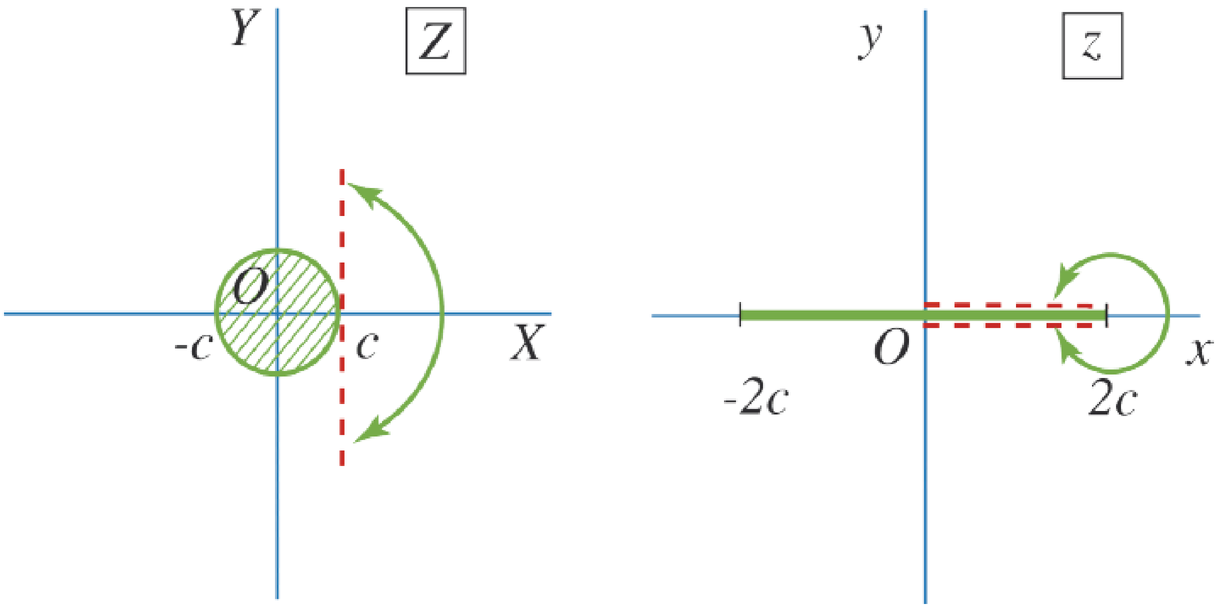
$$W(Z) = w\left[H(Z)\right]H'(Z)$$

Transformation of velocities

⇨**Conservation of streamlines, flow rate, circulation but not stagnation points**

Joukowski's transform

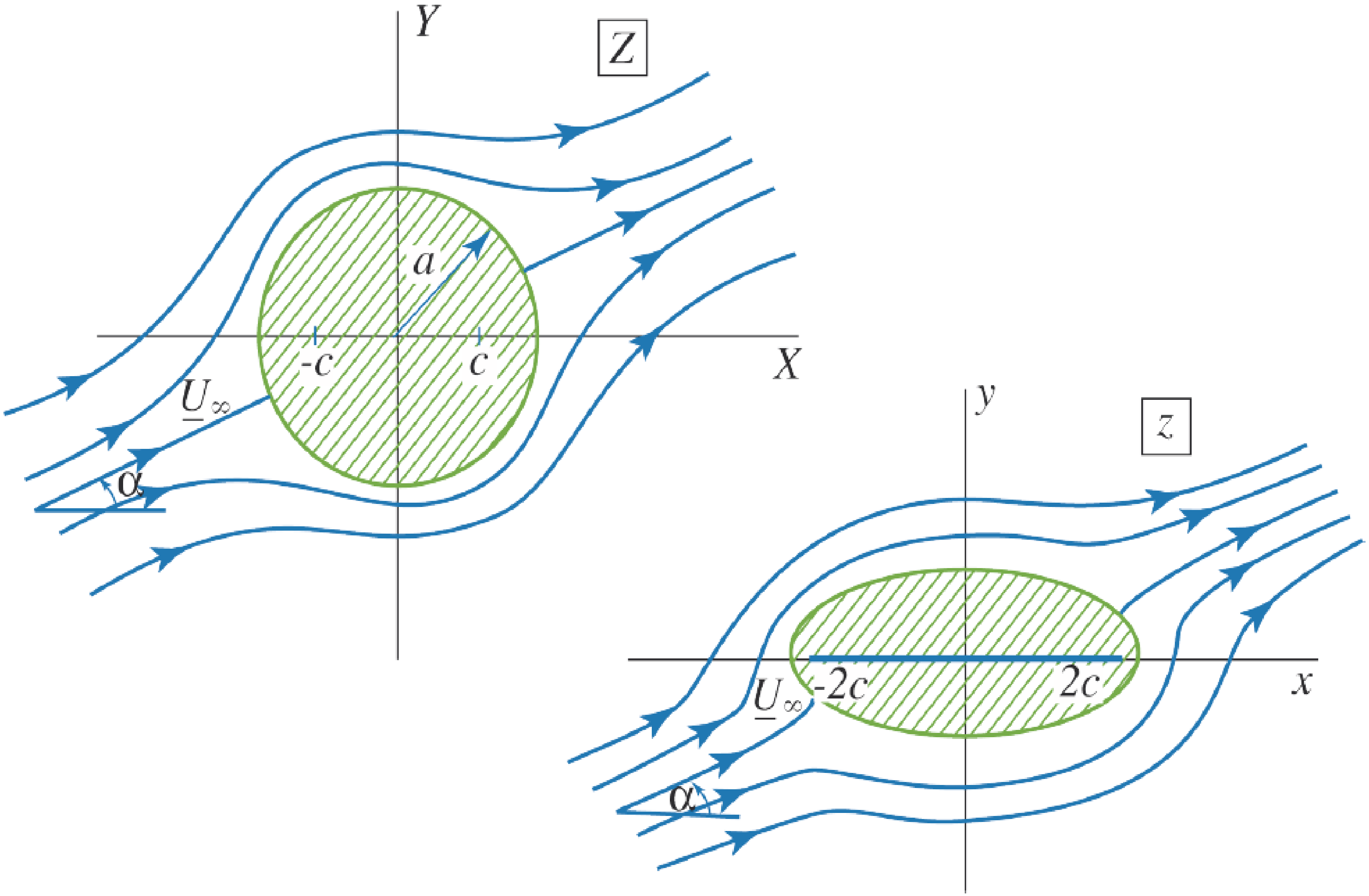
$$z = Z + \frac{c^2}{Z}$$



$$z - 2c \sim \frac{1}{c}(Z - c)^2$$

$$|Z - c| \to 0$$

Joukowski's transform
Flow around ellipse



Joukowski's transform
Flow around ellipse

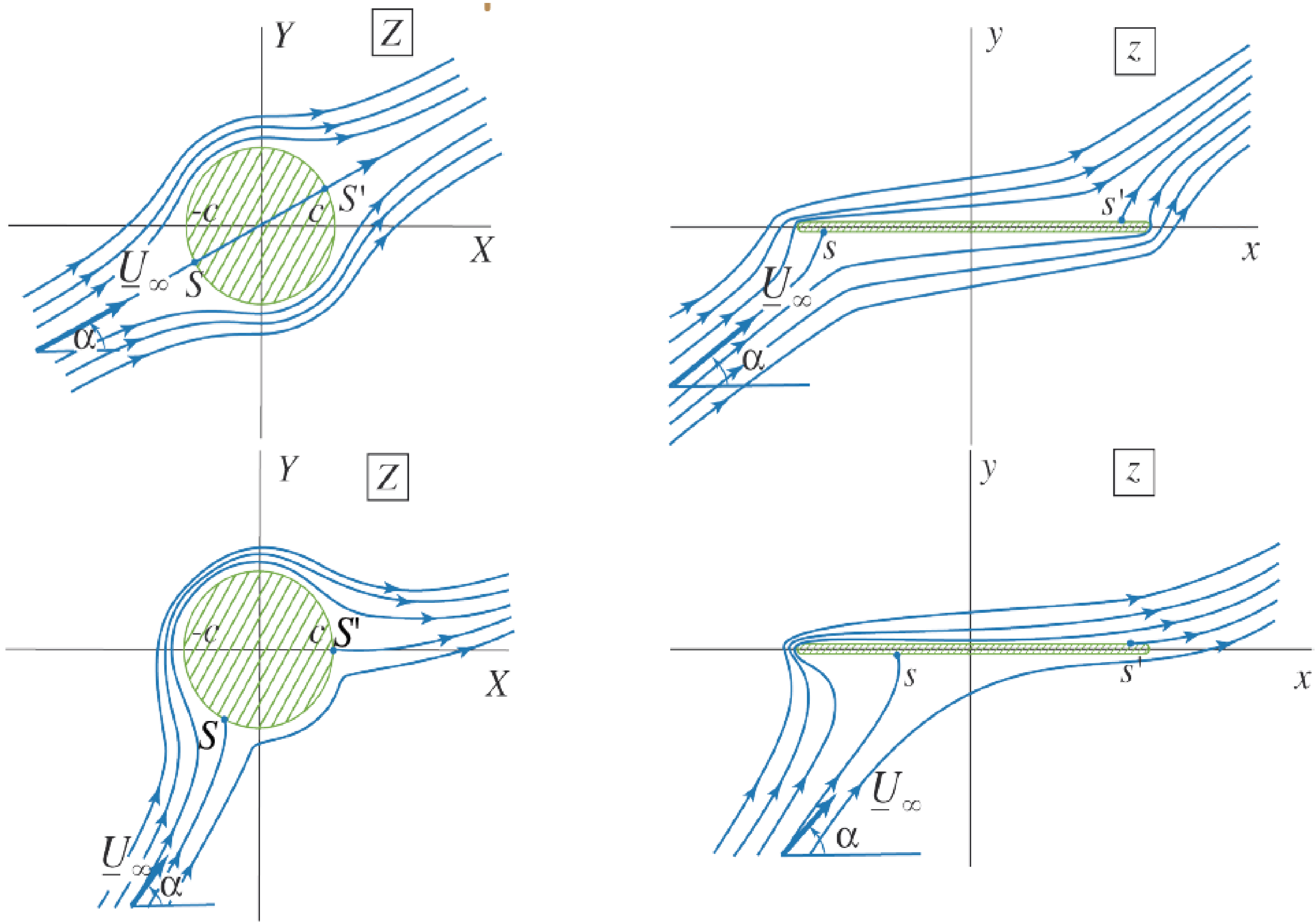
$$F(Z) = U_\infty \left(Z e^{-i\alpha} + \frac{a^2}{Z} e^{i\alpha} \right)$$

$$Z^2 - z\,Z + c^2 = 0$$

$$Z = h(z) = \frac{z}{2} + \sqrt{\left(\frac{z}{2}\right)^2 - c^2}$$

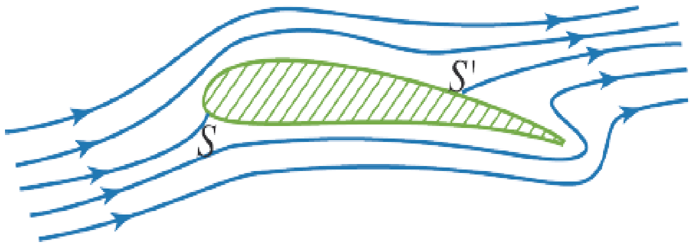
$$f(z) = U_\infty \left[z e^{-i\alpha} + \left(\frac{a^2}{c^2} e^{i\alpha} - e^{-i\alpha} \right) \left(\frac{z}{2} - \sqrt{\left(\frac{z}{2}\right)^2 - c^2} \right) \right]$$

Joukowski's transform
Multiplicity of solutions

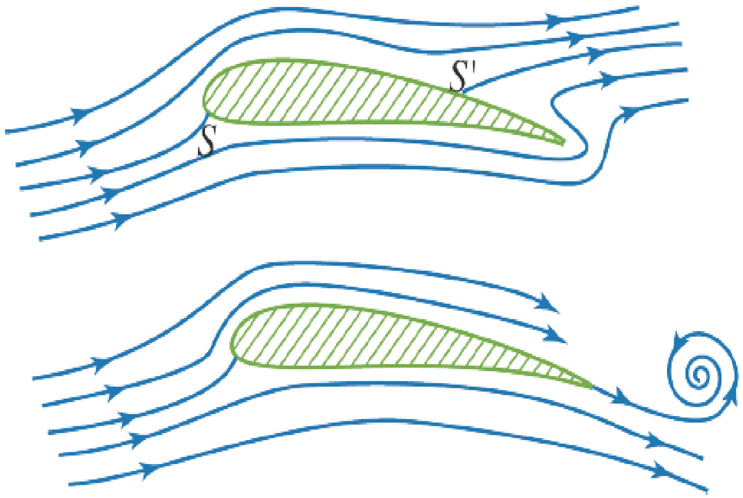


Flow around foil

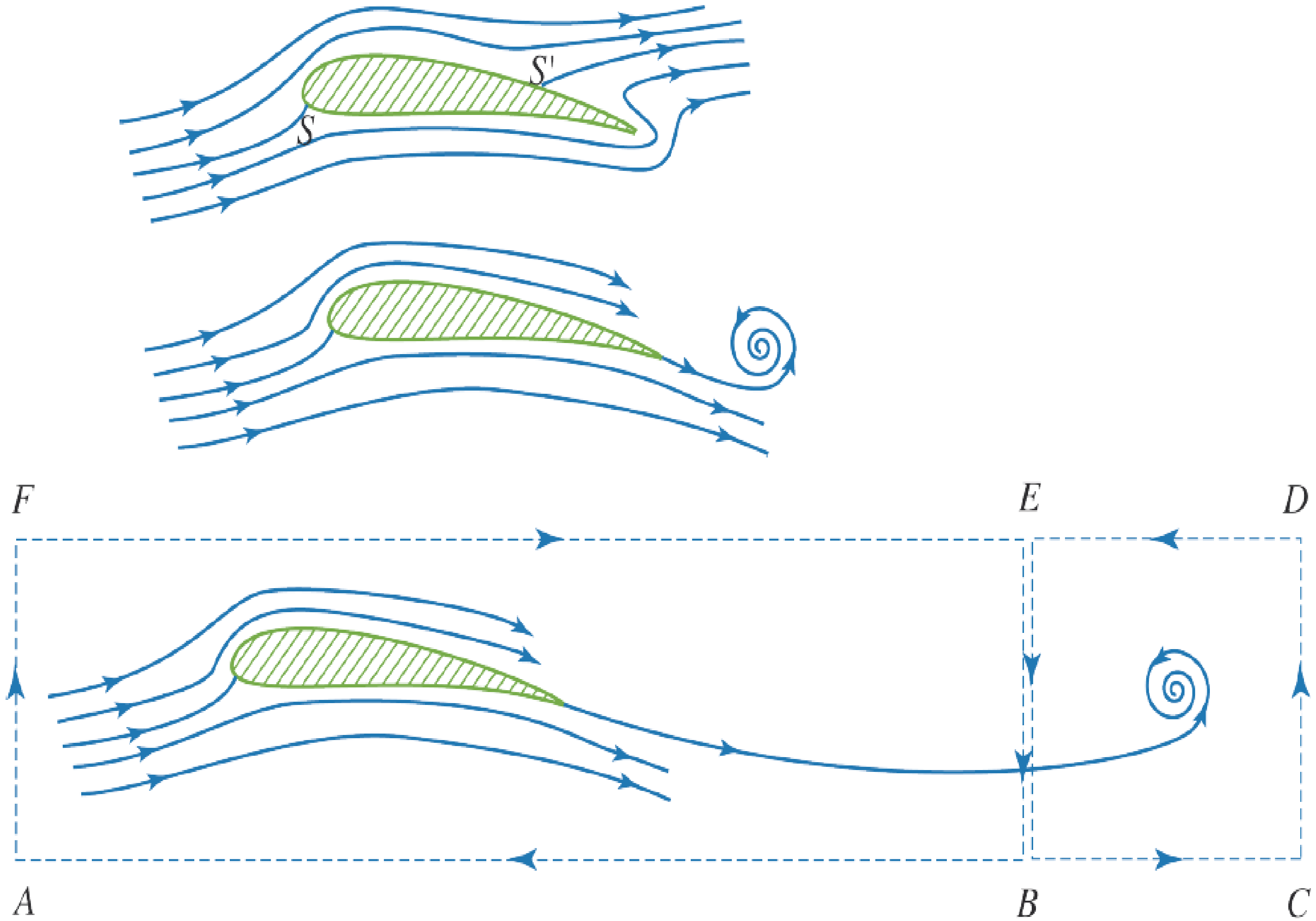
Startup vortex and circulation selection



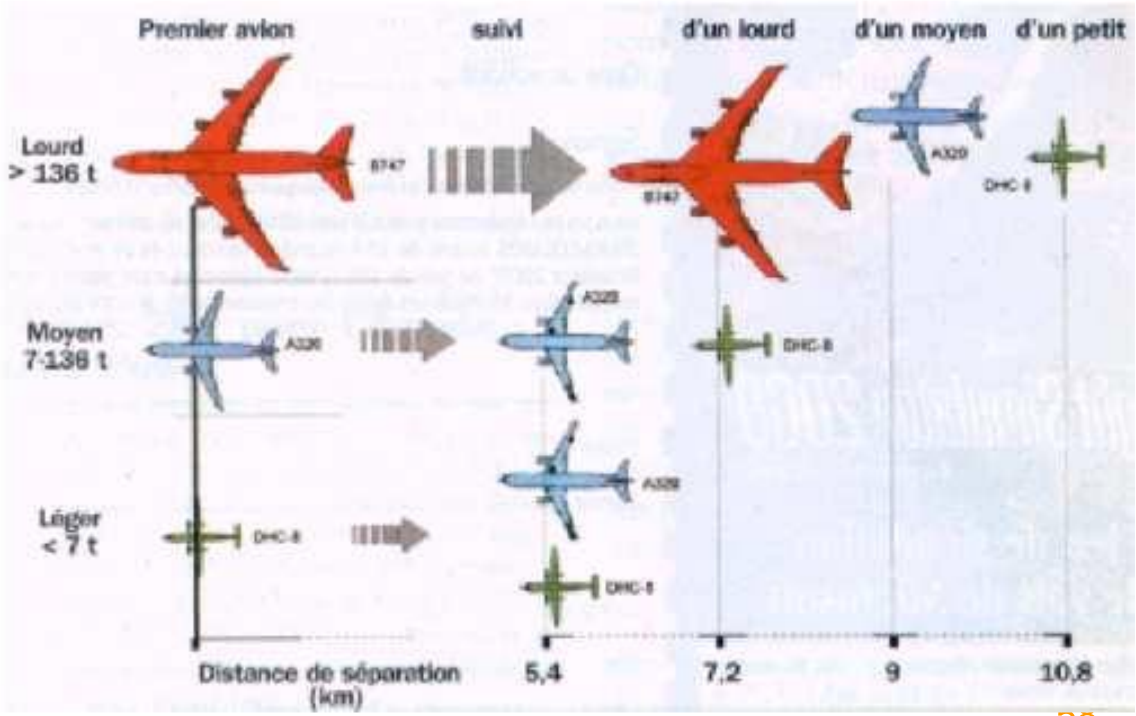
Flow around foil
Startup vortex and circulation selection



Flow around foil
Startup vortex and circulation selection



Real situation



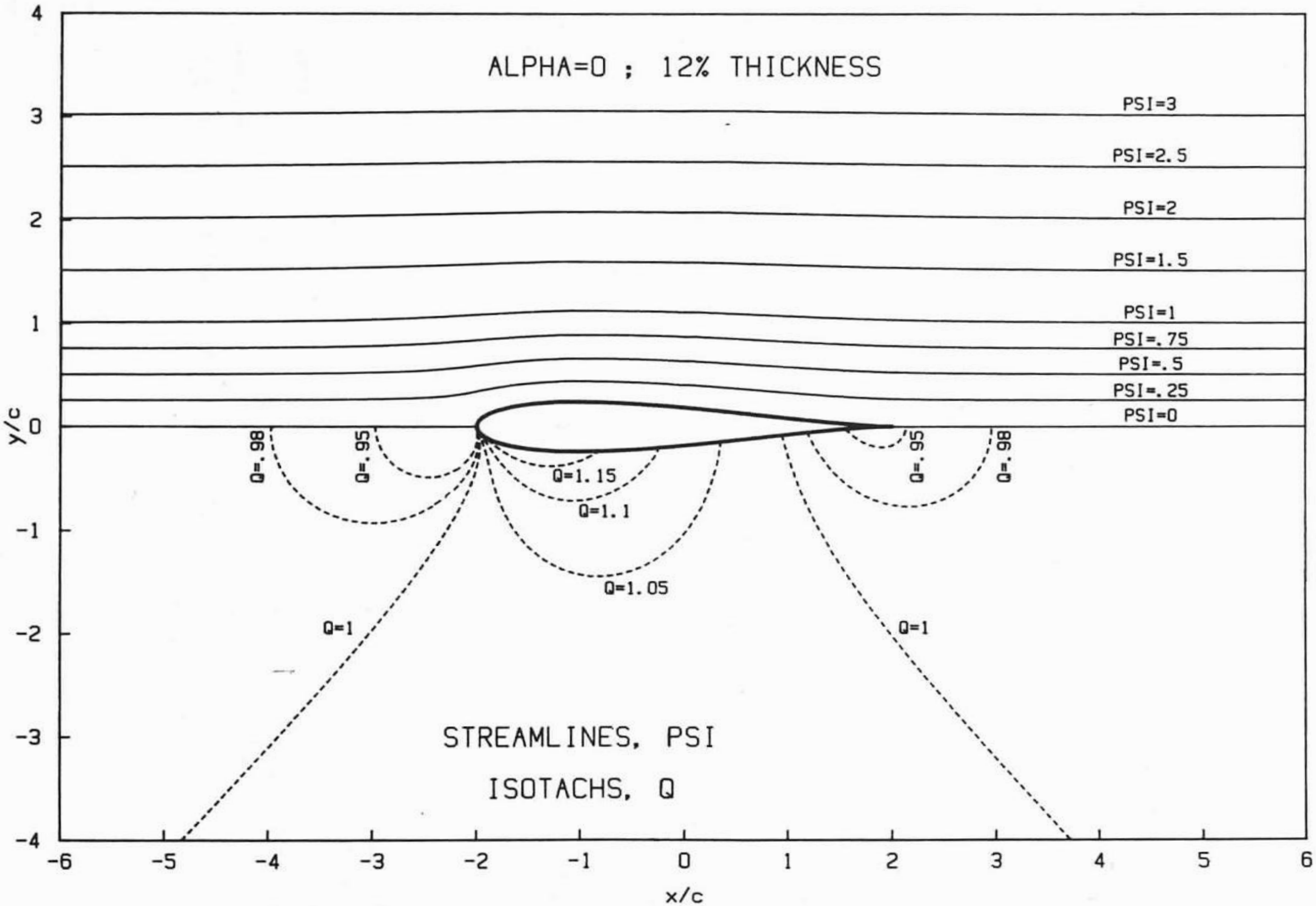


Figure 51 The symmetric Joukowski airfoil, alpha = 0°.

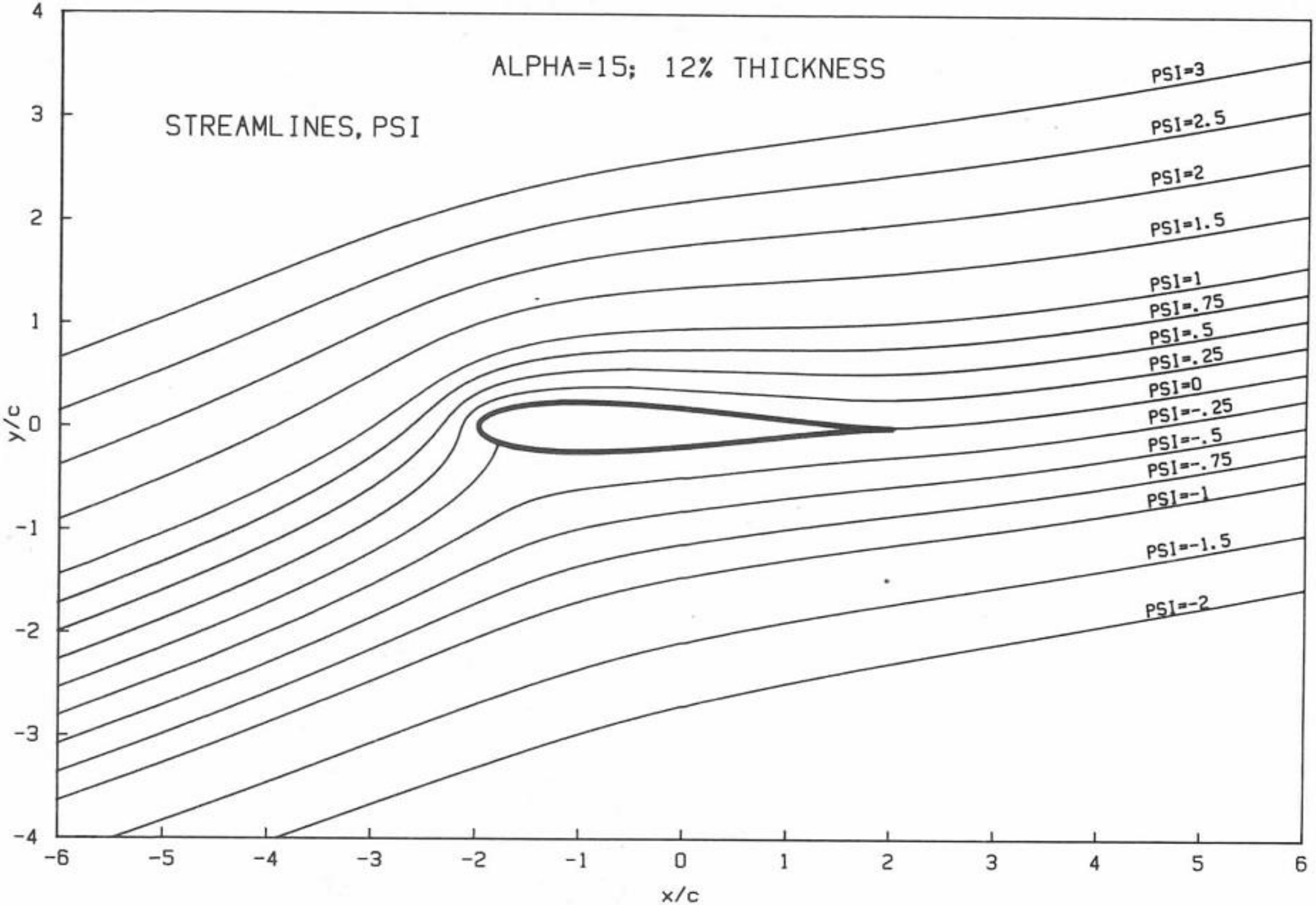
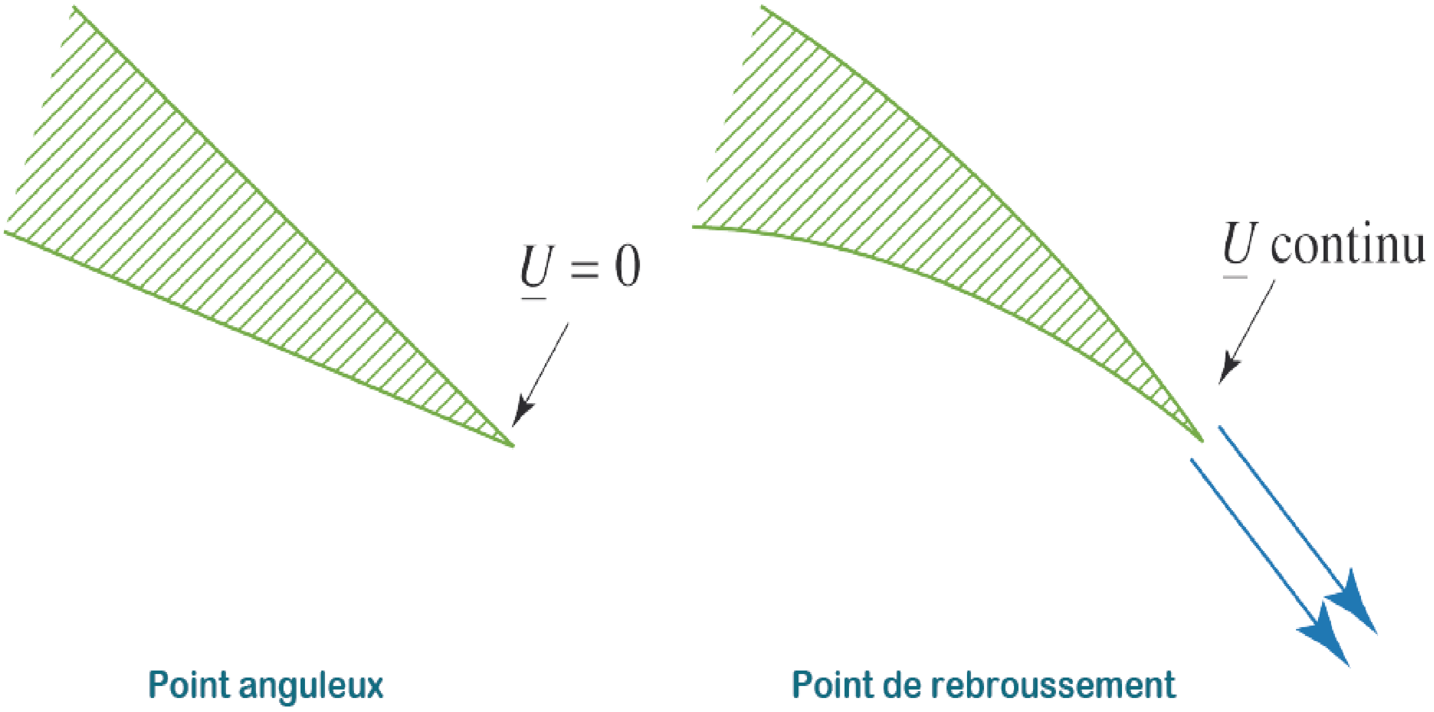


Figure 52 The symmetric Joukowski airfoil, streamlines, alpha = 15°.

Flow around foil
Circulation selection
By Kutta condition at trailing edge



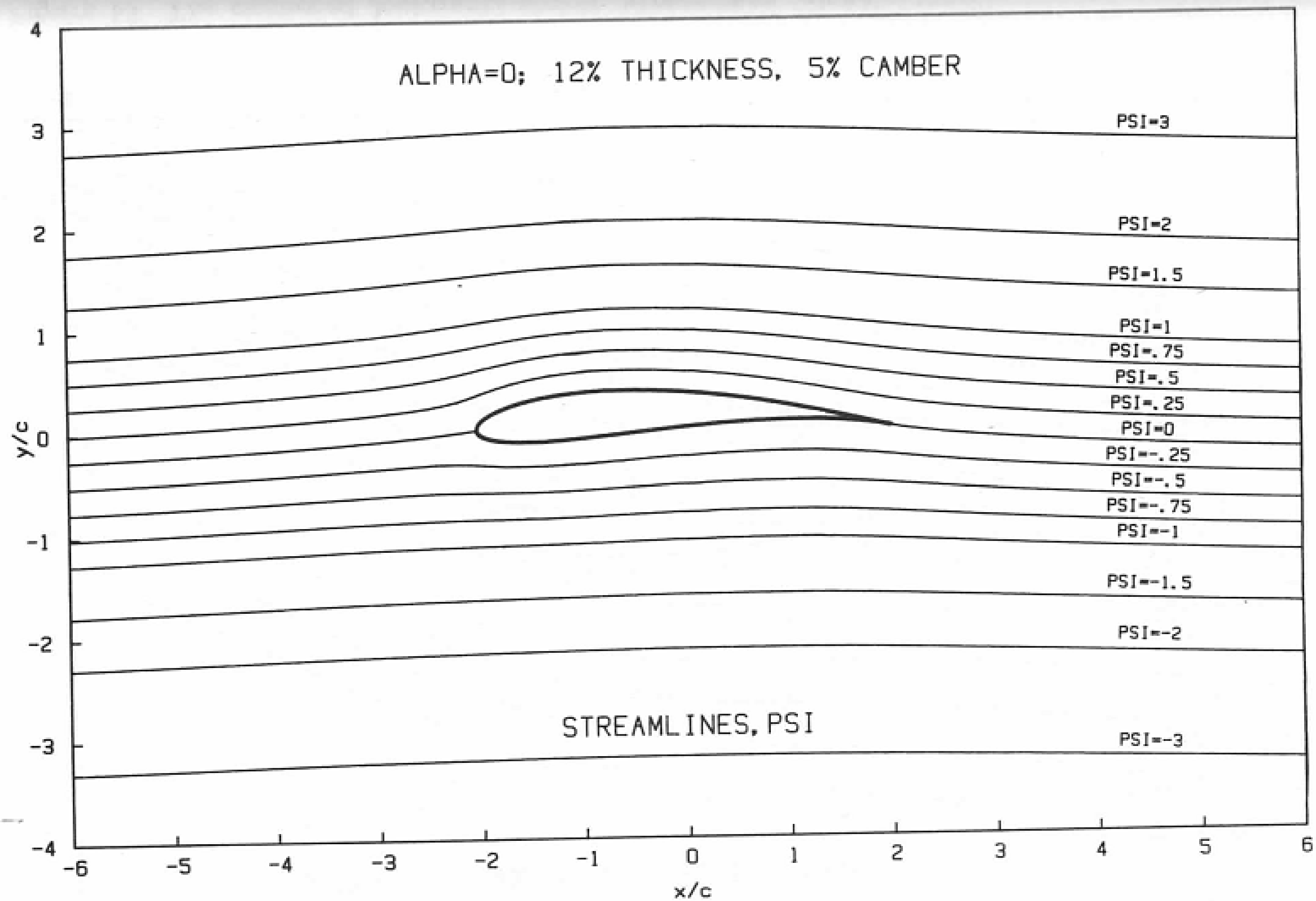


Figure 54 The cambered Joukowski airfoil, streamlines, alpha = 0°.

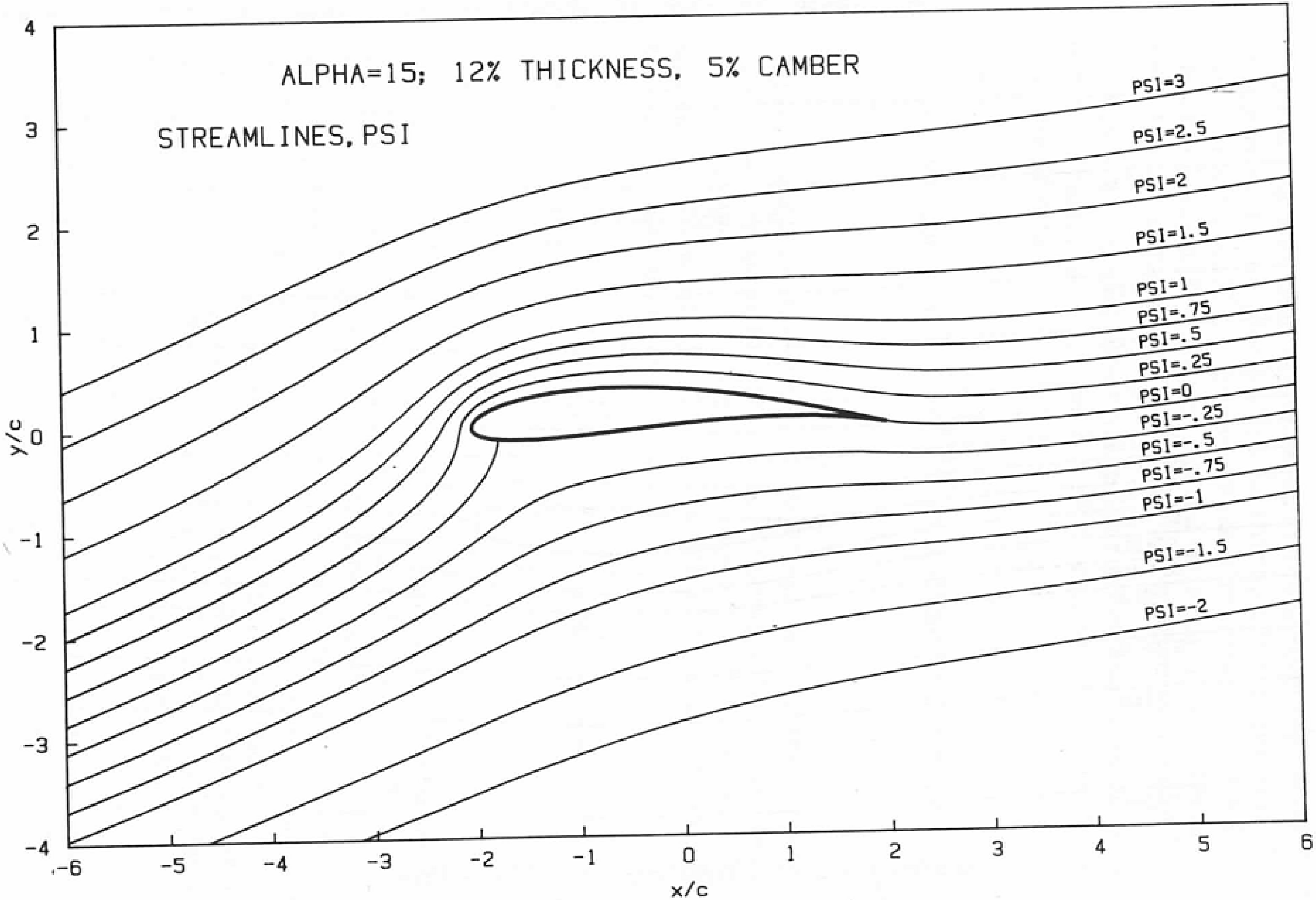


Figure 55 The cambered Joukowski airfoil, streamlines, alpha = 15°.

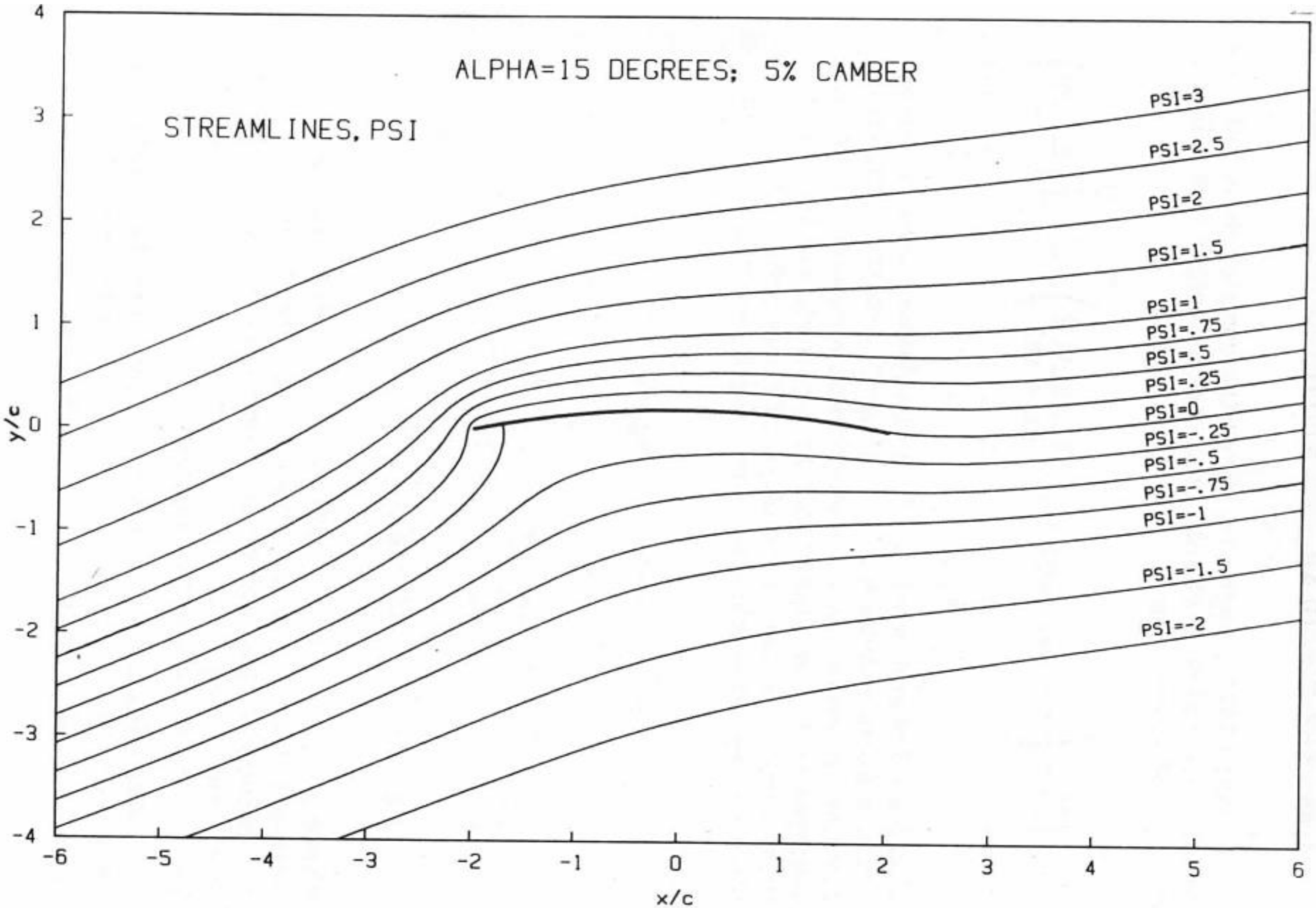


Figure 58 The circular-arc airfoil, streamlines, alpha = 15°.

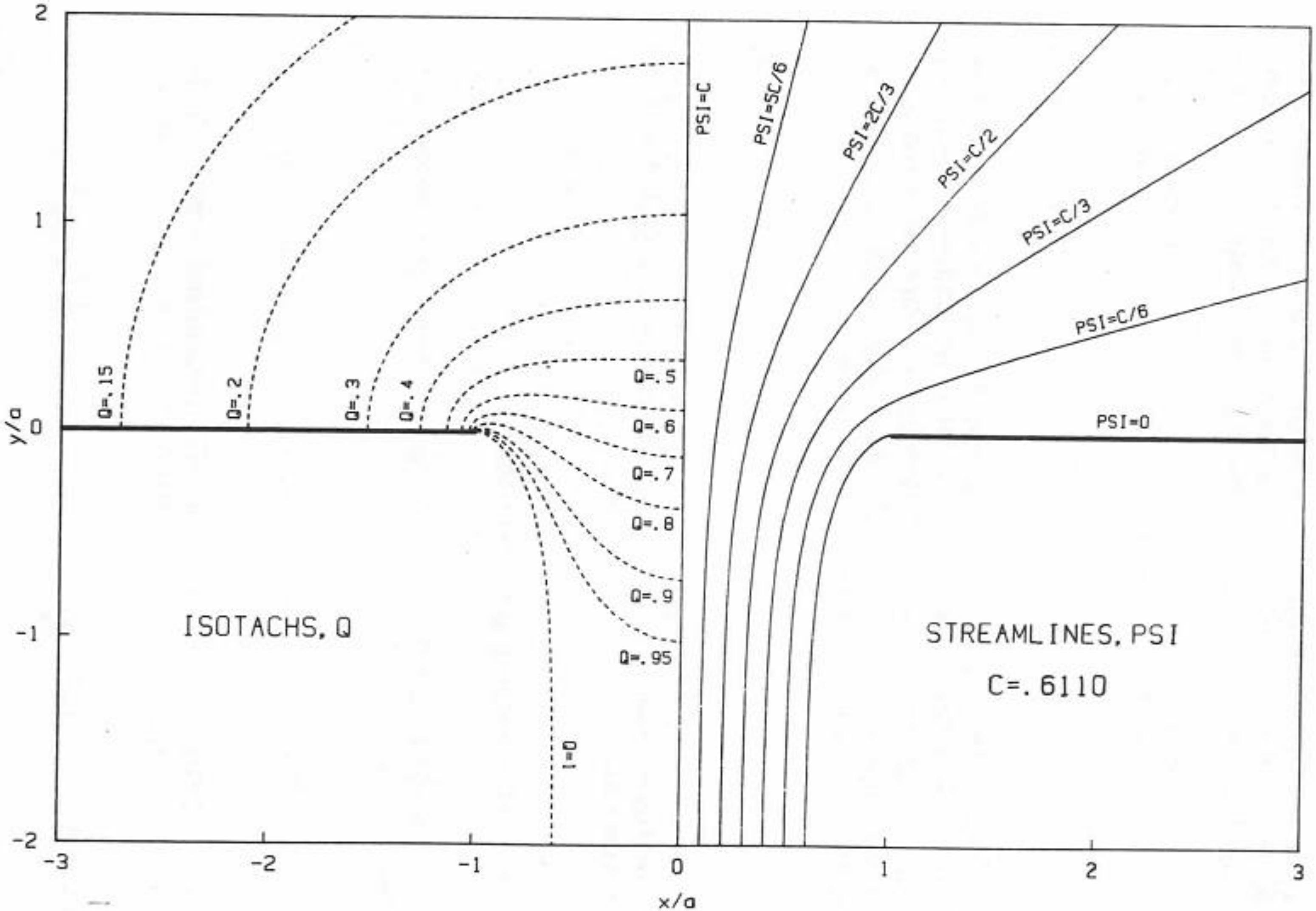


Figure 60 Efflux from an orifice.

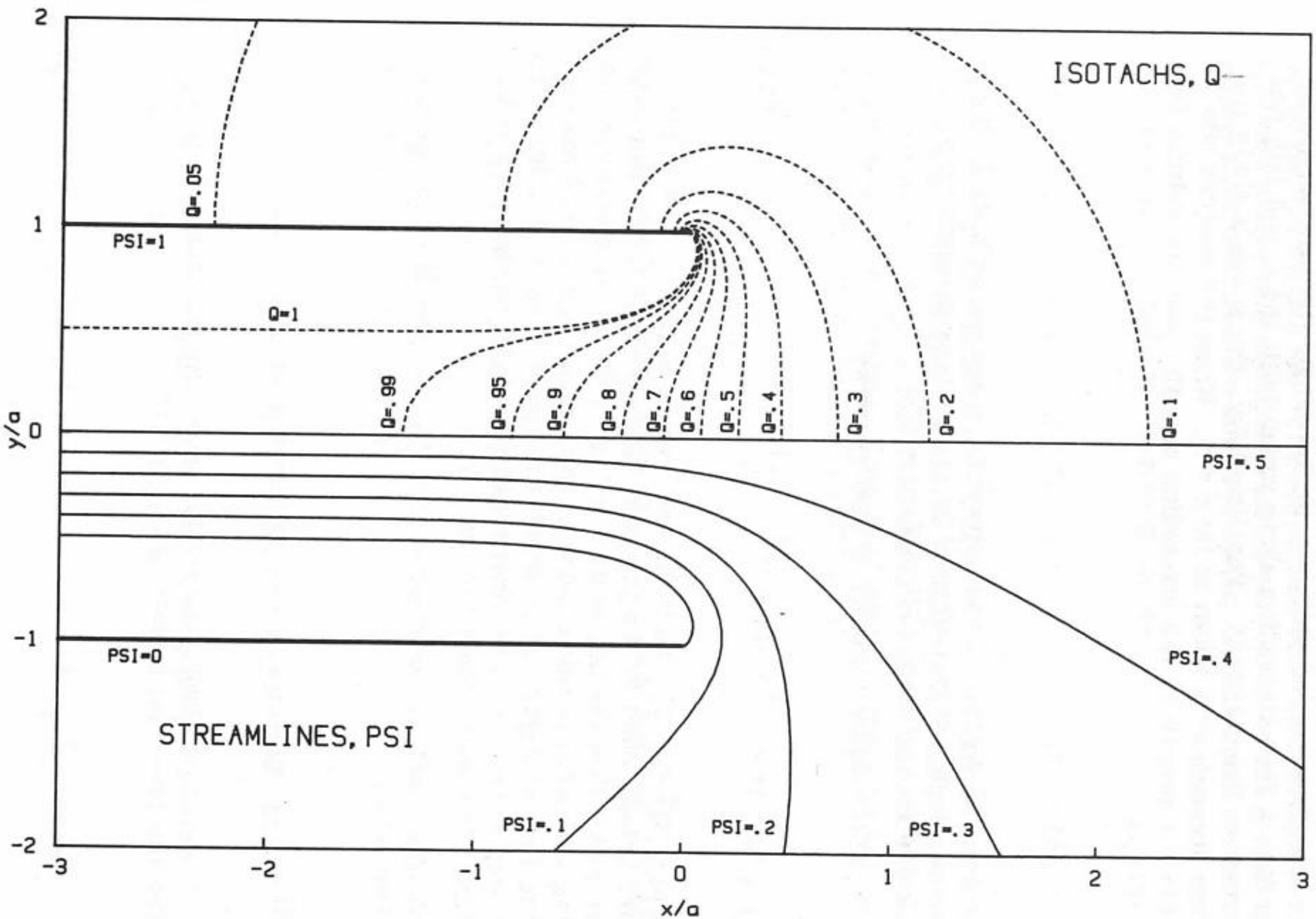
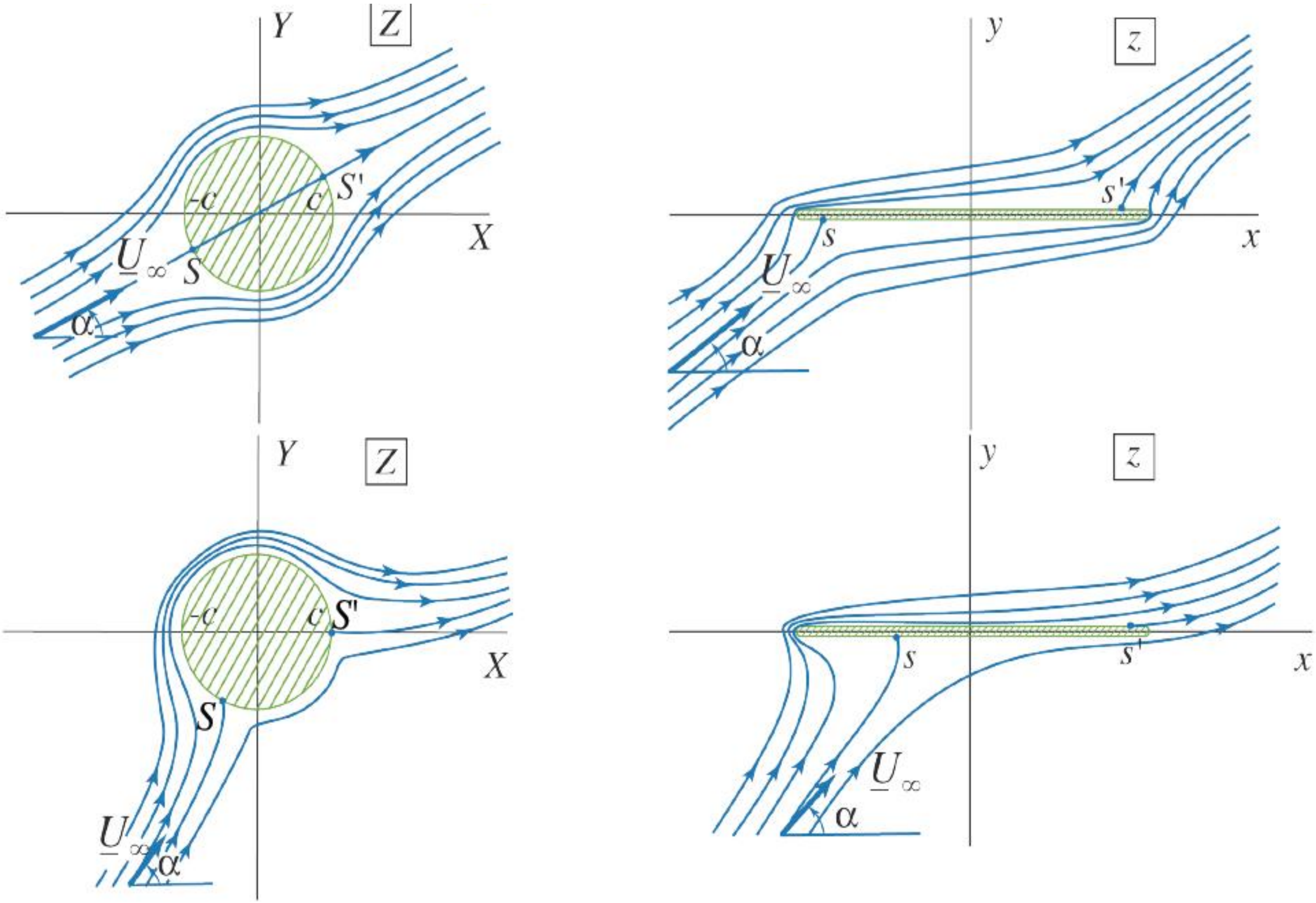
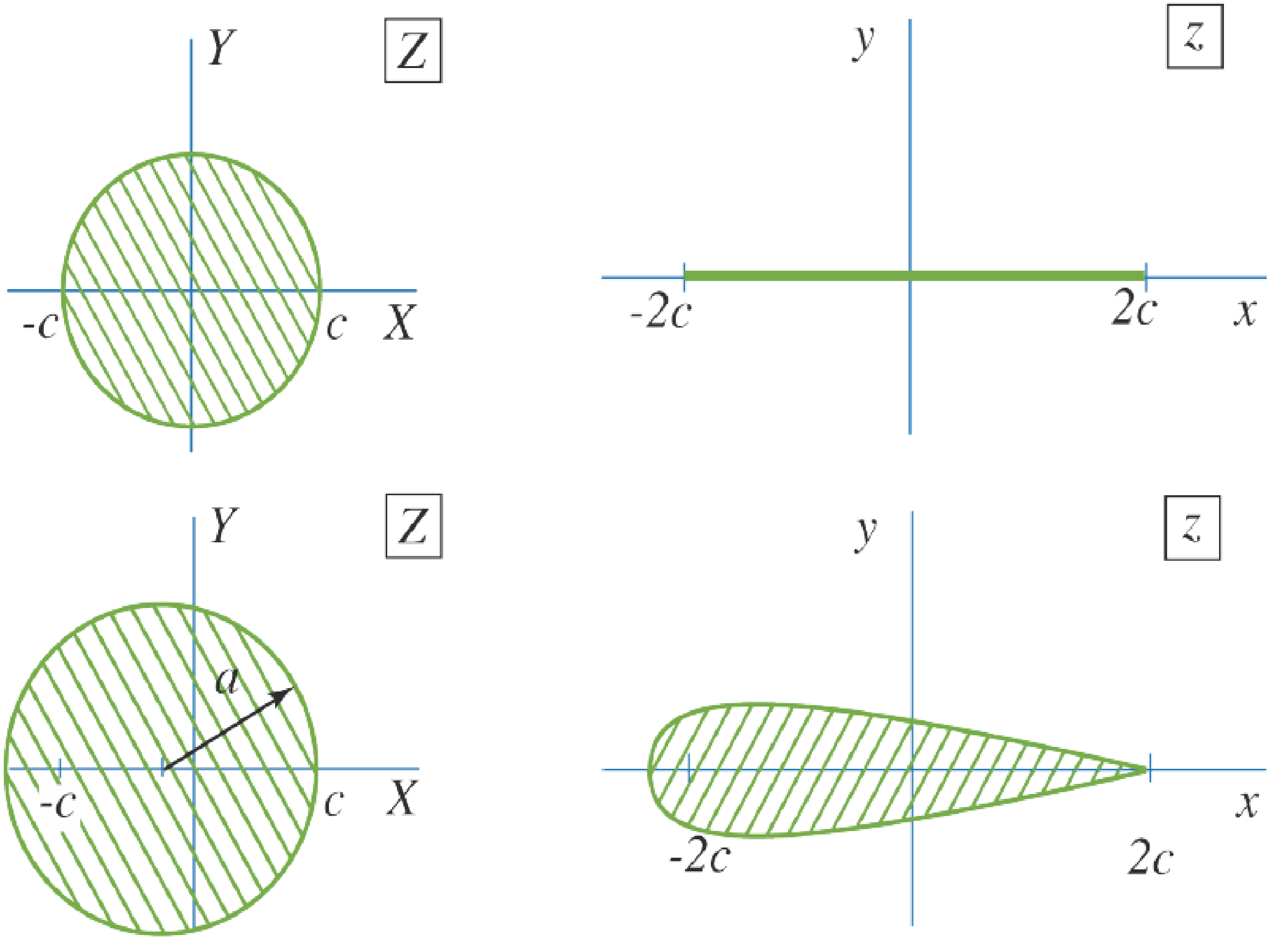


Figure 61 Borda's mouthpiece.

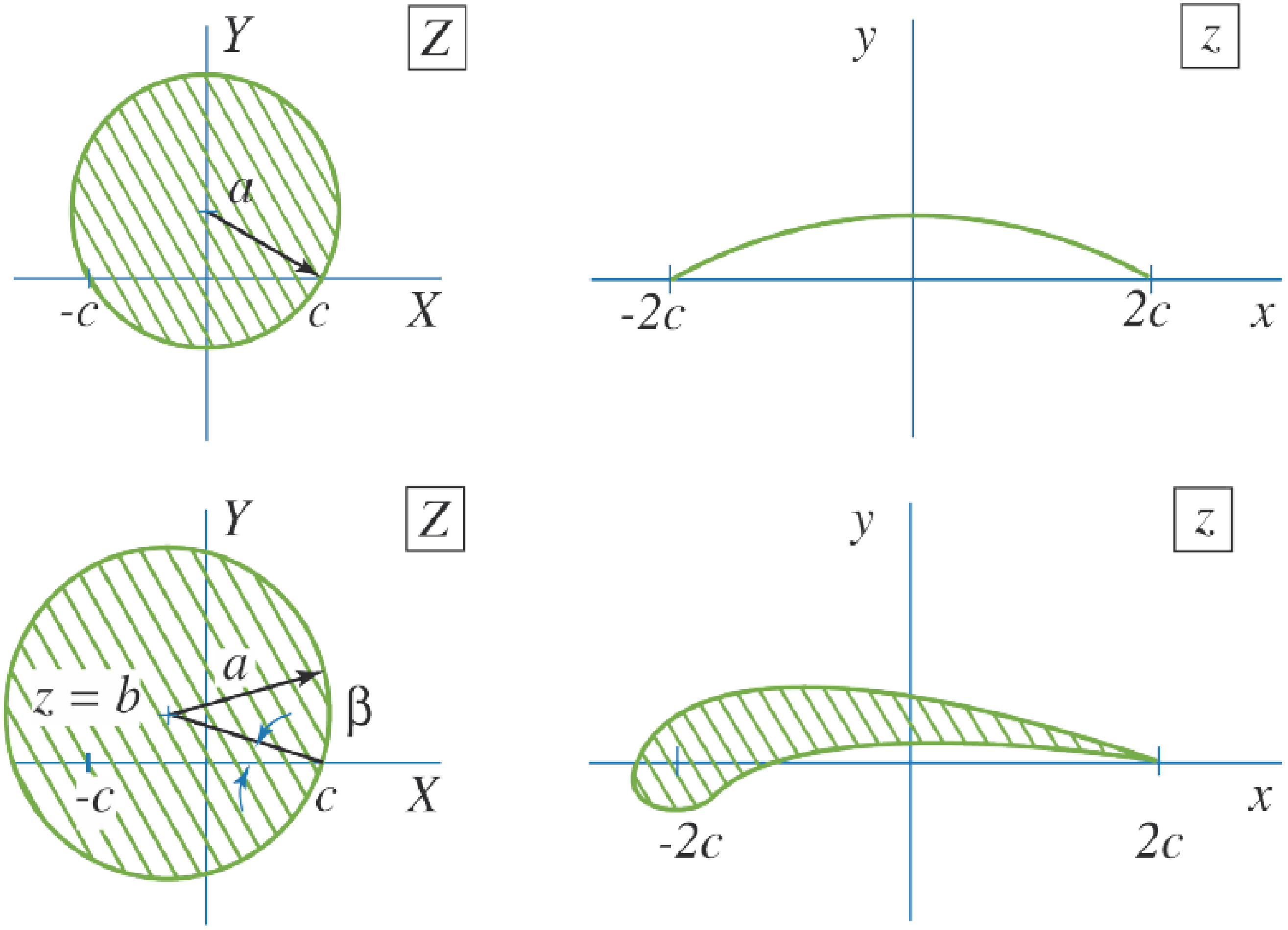
Joukowski's transform
Multiplicity of solutions



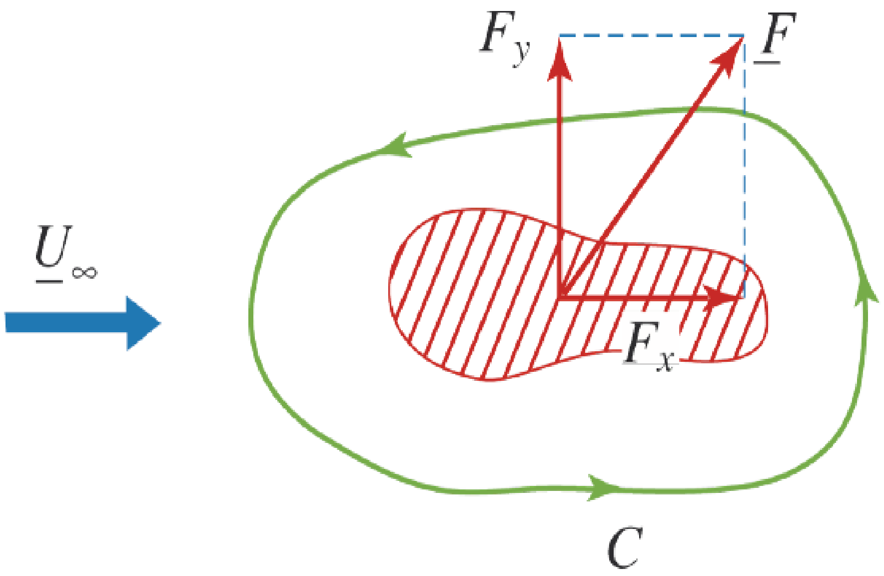
Flow around foil
Thickness effect



Flow around foil
Camber effect



Blasius Formula



$$F_x - iF_y = \frac{i\rho}{2} \int_C w^2(z)dz$$

$$C_0 = -\frac{\rho}{2} \mathcal{R}e \left\{ \int_C zw^2(z)dz \right\}$$

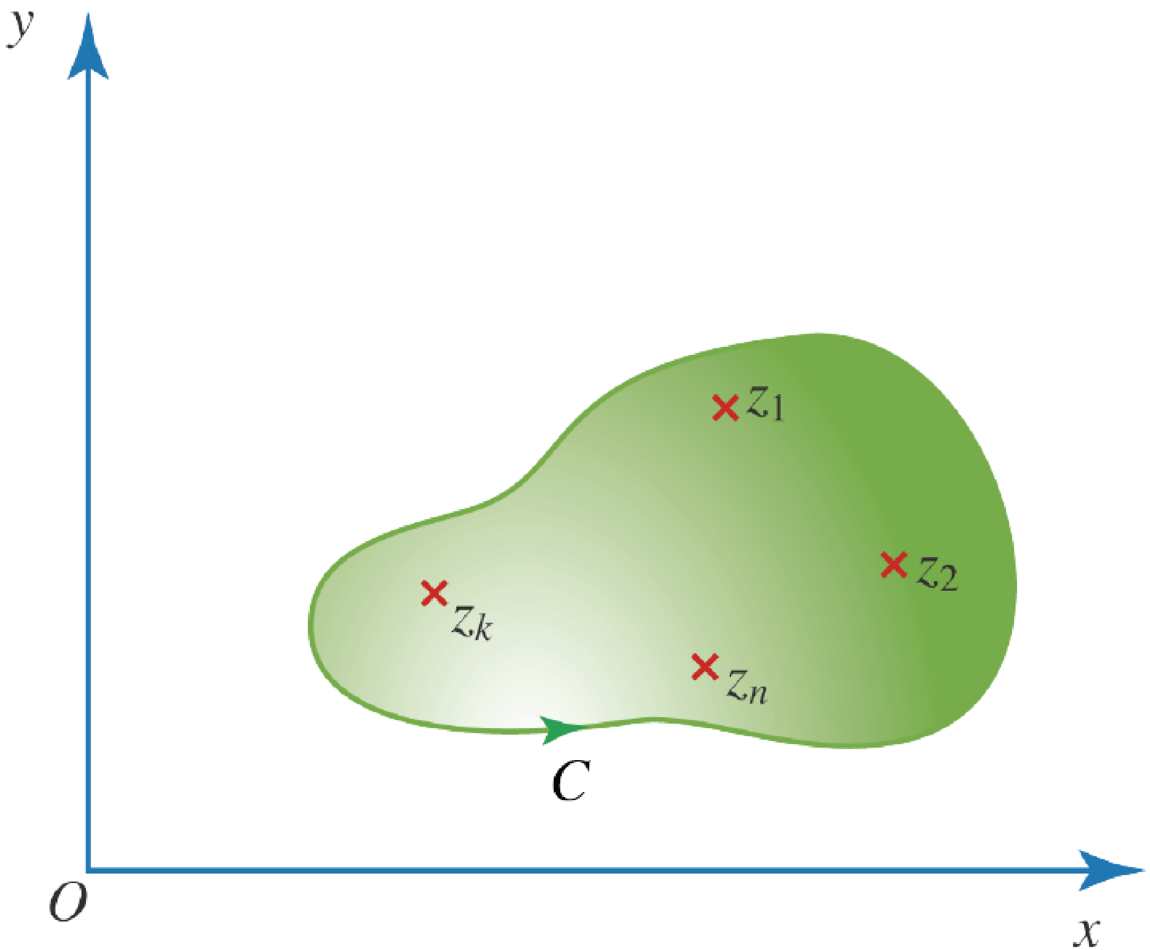
RÉSIDU

$$f(z) = \sum_{n=-\infty}^{-1} a_n (z - z_0)^n + \sum_{n=0}^{\infty} a_n (z - z_0)^n$$

partie principale   partie régulière

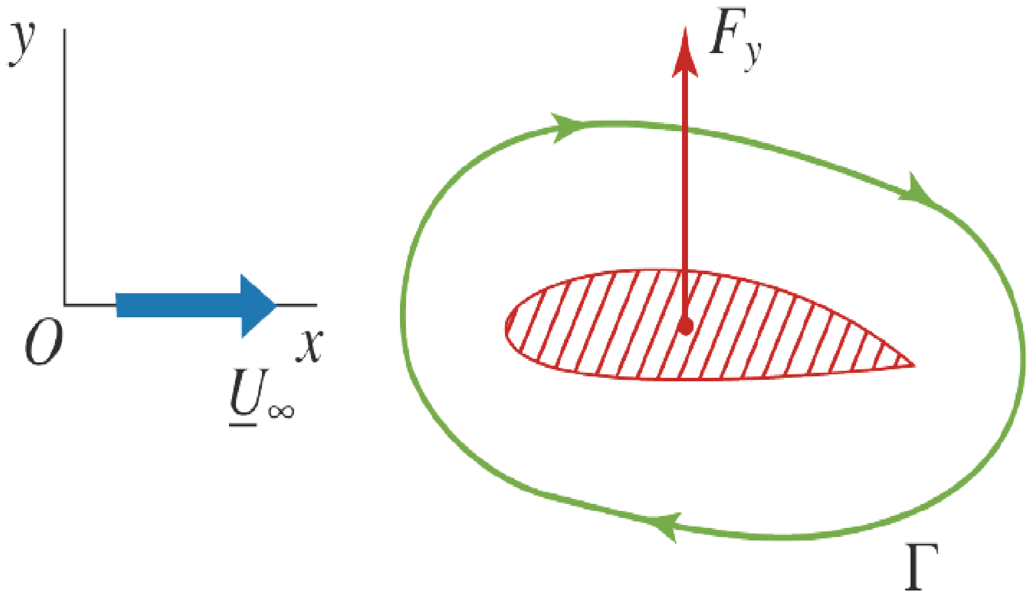
$$a_{-1} = \text{Res}\,(f, z_0) = \frac{1}{2\pi i} \int_C f(z)\,dz$$

THÉORÈME DES RÉSIDUS



$$\int_C f(z)\,dz = 2\pi i \sum_{k=1}^{n} \operatorname{Res}\left(f, z_k\right)$$

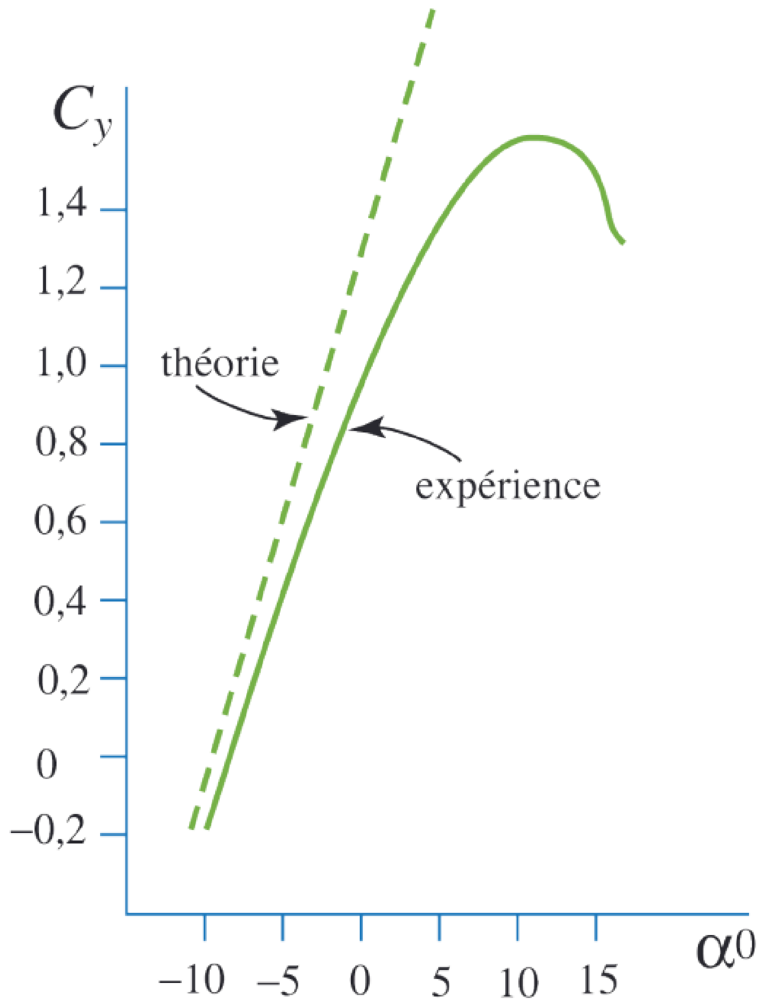
Kutta-Joukowski theorem



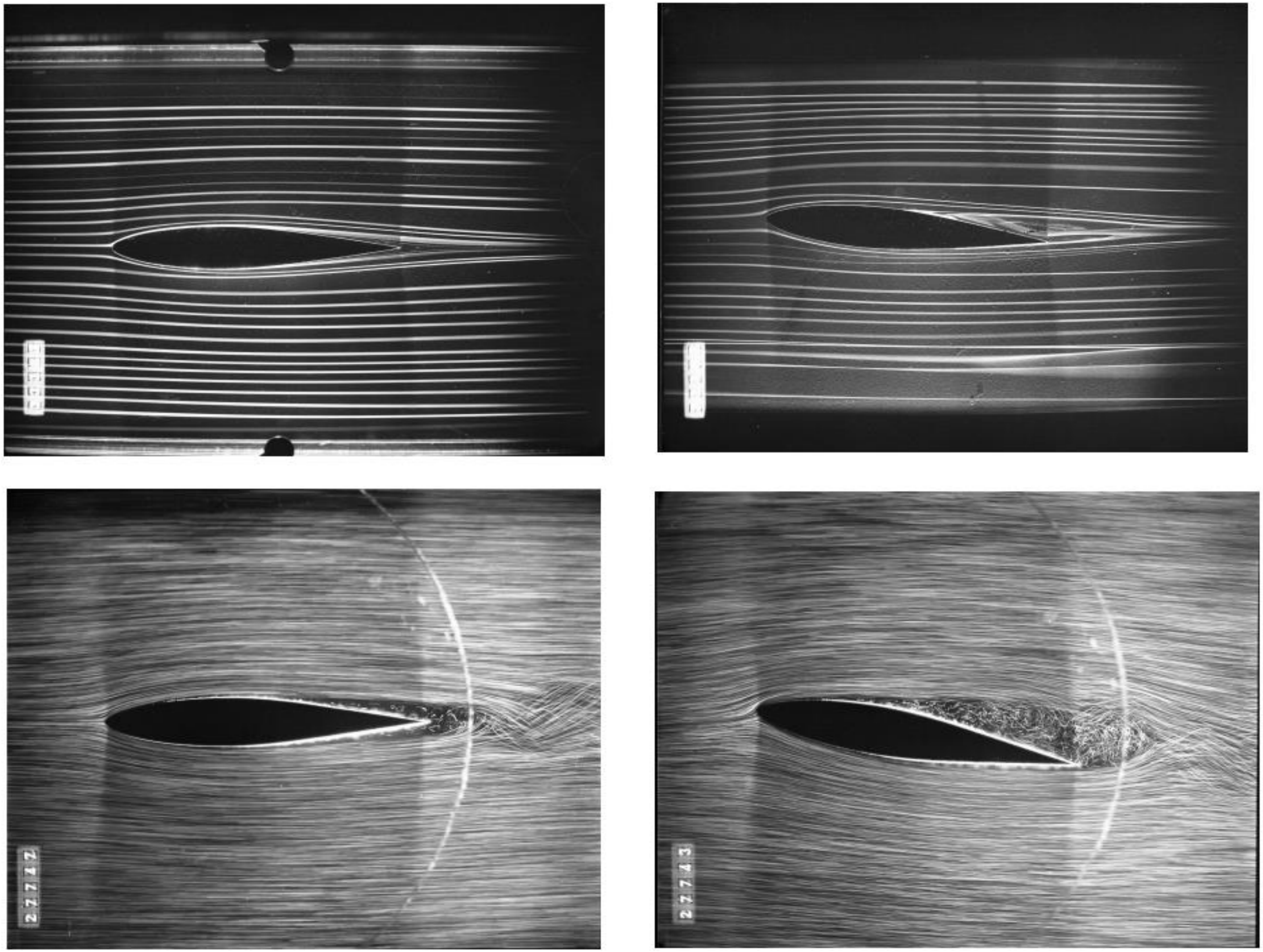
$$F_y = -\rho U_\infty \Gamma$$

Lift of a plate under incidence in incoming flow at angle

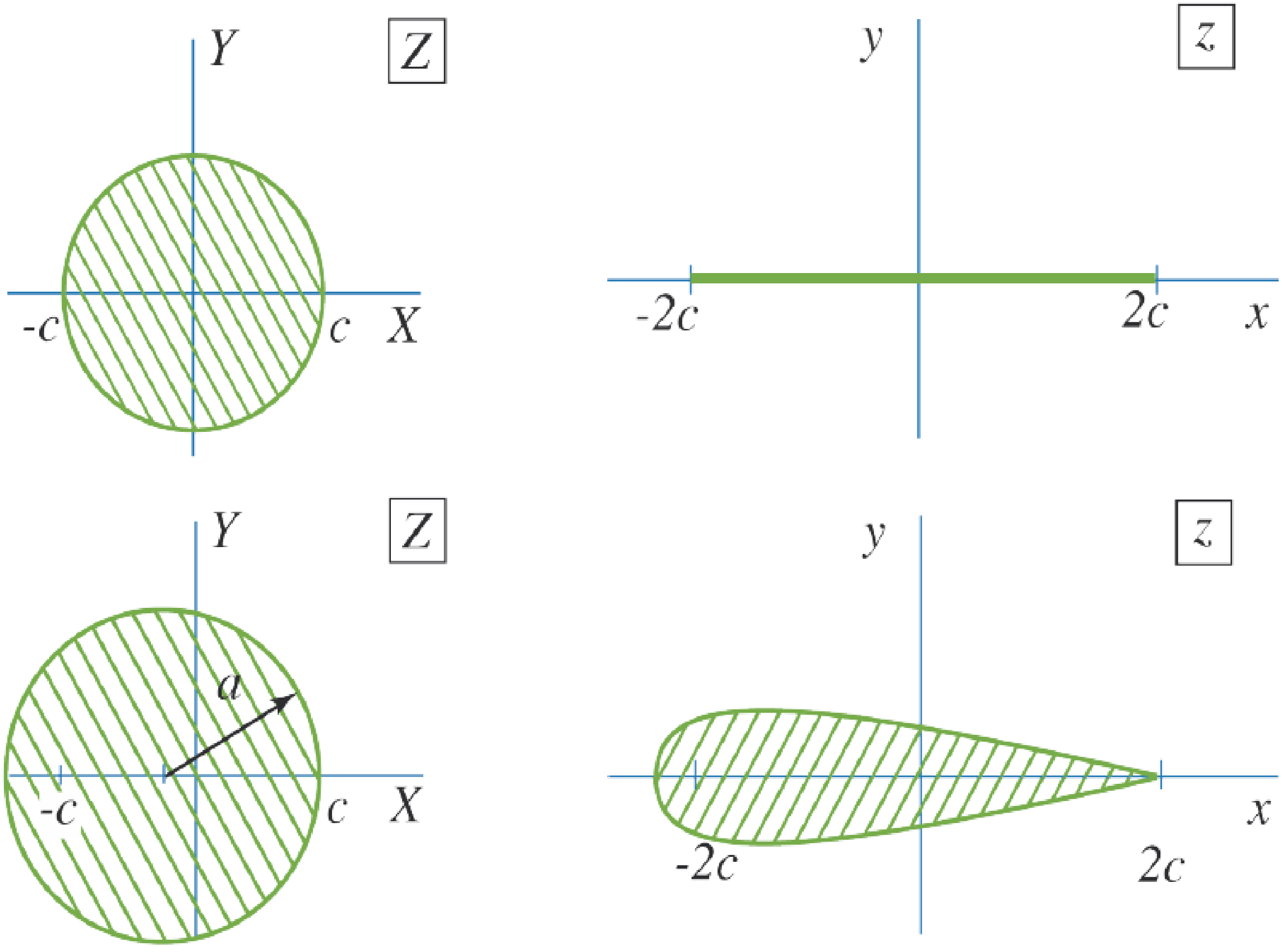
$$C_y = 2\pi(\alpha - \alpha_0)$$



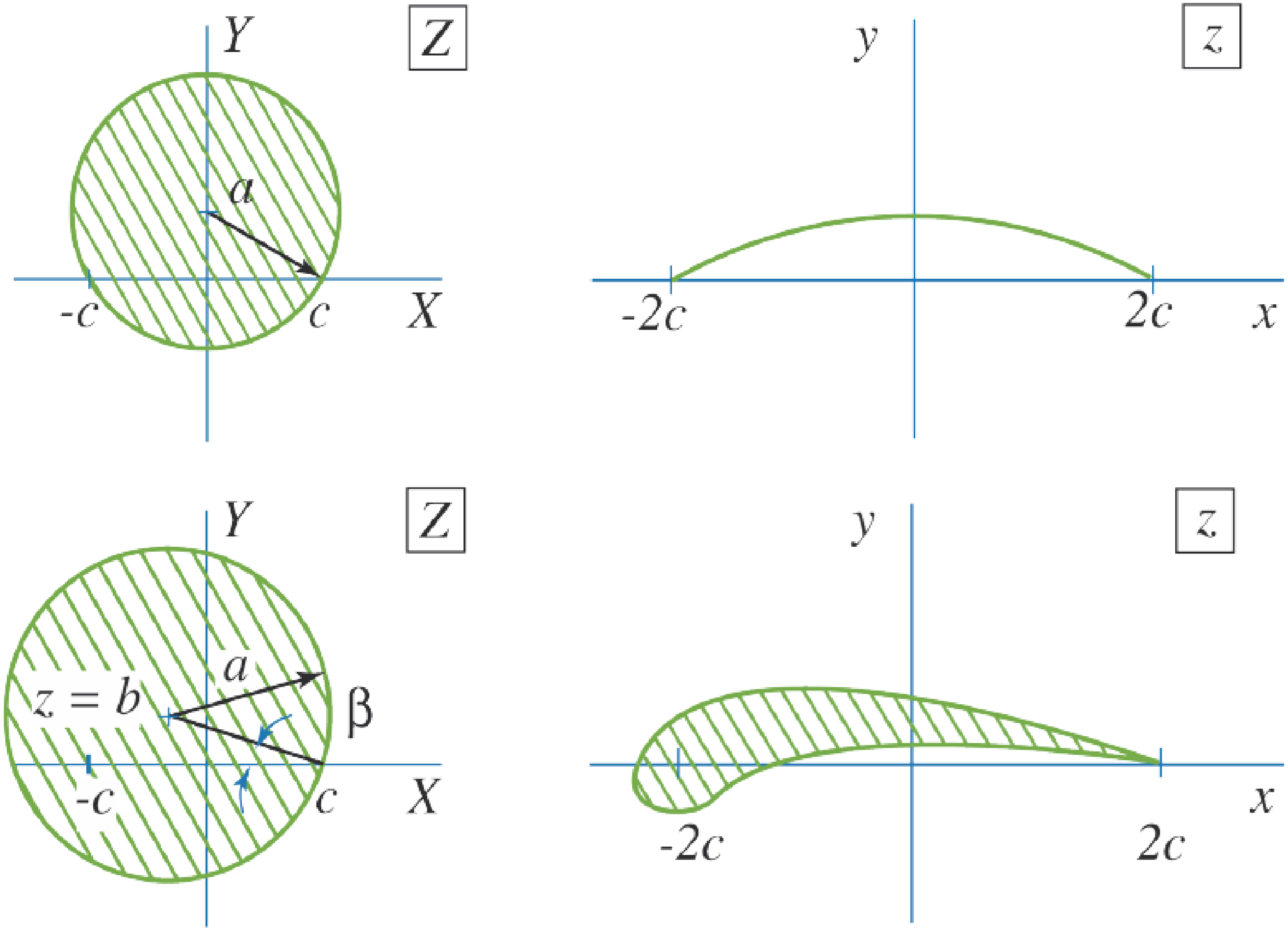
Lift of a wing under incidence



Flow around foil
Thickness effect



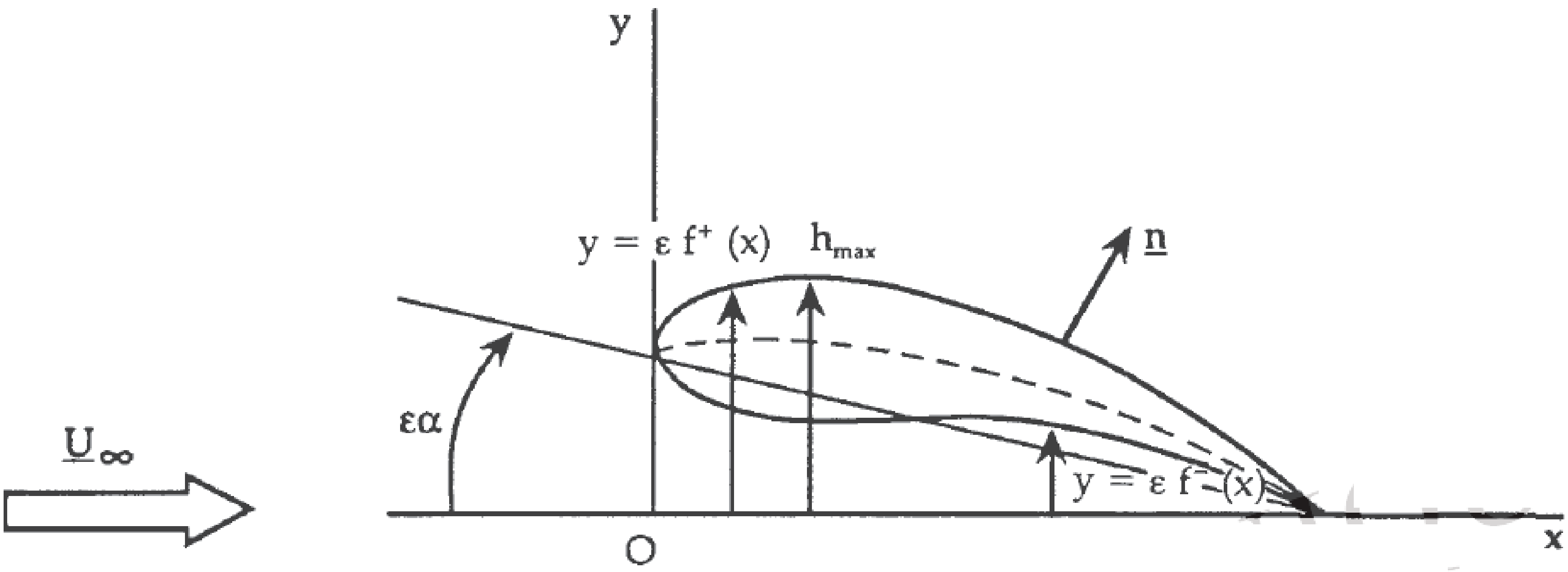
Flow around foil
Camber effect



Linearized profile theory

$$y = \varepsilon f^{+}(x) \qquad 0 \leqslant x \leqslant L \qquad \varepsilon = \frac{h_{max}}{L}$$

$$y = \varepsilon f^{-}(x)$$



Linearized profile theory

$$\Delta\varphi = 0$$

impermeability $\underline{\text{grad}}\ \varphi \cdot \underline{n} = 0$

$$\frac{\partial\varphi}{\partial y}\left[\, x\,,\, \varepsilon\, f^{\pm}(x)\,;\, \varepsilon \,\right] - \varepsilon\, \frac{df^{\pm}}{dx}\, \frac{\partial\varphi}{\partial x}\left[\, x\,,\, \varepsilon\, f^{\pm}(x)\,;\, \varepsilon \,\right] = 0 \quad , 0 \leqslant x \leqslant L$$

far-field $\varphi\,(x\,,\,y\,;\,\varepsilon) \sim U_\infty\, x \quad , |\underline{x}| \to \infty$

Linearized profile theory

$$\varphi(x, y; \varepsilon) \sim U_\infty x + \varepsilon \varphi_1(x, y) + \varepsilon^2 \varphi_2(x, y)$$

At order ε

$$\Delta \varphi_1 = 0 \quad ,$$

$$\frac{\partial \varphi_1}{\partial y}(x, 0^+) = U_\infty \frac{df^+}{dx}(x), \qquad 0 \leqslant x \leqslant L \quad ,$$

$$\frac{\partial \varphi_1}{\partial y}(x, 0^-) = U_\infty \frac{df^-}{dx}(x), \qquad 0 \leqslant x \leqslant L \quad ,$$

$$\varphi_1(x, y) \sim o(|\underline{x}|), \qquad |\underline{x}| \to \infty \quad .$$

Linearized profile theory

$$\varphi(x, y; \varepsilon) \sim U_\infty x + \varepsilon \varphi_1(x, y) + \varepsilon^2 \varphi_2(x, y)$$

$$p(x, y; \varepsilon) \sim p_\infty + \varepsilon p_1(x, y) + \varepsilon p_2(x, y) +$$

$$\underline{U}(x, y; \varepsilon) \sim U_\infty \underline{e}_x + \varepsilon \, \underline{\mathrm{grad}} \, \varphi_1 + \dots$$

$$p + \frac{1}{2} \rho U^2 = p_\infty + \frac{1}{2} \rho U_\infty^2$$

$$p_1 = - \rho U_\infty u_1 = - \rho U_\infty \frac{\partial \varphi_1}{\partial x}$$

$$\boxed{C_p = \frac{p - p_\infty}{\frac{1}{2} \rho U_\infty^2} \sim - \frac{2\varepsilon}{U_\infty} u_1 = - \frac{2\varepsilon}{U_\infty} \frac{\partial \varphi_1}{\partial x} \; .}$$

Linearized profile theory

wing= incidence+ camber+ thickness

Camber $$y = \varepsilon\, f_c(x) = \varepsilon \left[\frac{f^+(x) + f^-(x)}{2} - \alpha (L - x) \right] .$$

Thickness $$y = \varepsilon\, f_e(x) = \varepsilon\, \frac{f^+(x) - f^-(x)}{2}$$

$$\varepsilon\, f^+(x) = \varepsilon \left[\alpha (L - x) + f_c(x) + f_e(x) \right]$$

$$\varepsilon\, f^-(x) = \varepsilon \left[\alpha (L - x) + f_c(x) - f_e(x) \right]$$

Linearized profile theory

$$\varphi_1(x, y) = \varphi_i(x, y) + \varphi_c(x, y) + \varphi_e(x, y)$$

$$\varphi(x, y; \varepsilon) \sim U_\infty x + \varepsilon\left[\varphi_i(x, y) + \varphi_c(x, y) + \varphi_e(x, y)\right]$$

No lift by symmetry

Incidence

Find vorticity distribution $\gamma(x)$

$$f(z) = \varphi_i(x, y) + i\,\psi_i(x, y) = -\frac{i}{2\pi}\int_0^L \gamma(x')\log(z - x')\,dx'$$

such that impermeability

$$\frac{\partial \varphi_i}{\partial y}(x, 0^{\pm}) = -U_\infty \alpha \quad , \qquad 0 \leqslant x \leqslant L \quad ,$$

$$w(z) = \frac{df}{dz} = u_i(x, y) - i\,v_i(x, y) = -\frac{i}{2\pi}\int_0^L \frac{\gamma(x')}{z - x'}\,dx'$$

$$v_i(x, 0^{\pm}) = \frac{1}{2\pi}\,\text{v. p.}\int_0^L \frac{\gamma(x')}{x - x'}\,dx'$$

$$\text{v.p.}\int_0^L \frac{\gamma(x')}{x - x'}\,dx' = \lim_{\delta \to 0}\left[\int_0^{x-\delta} \frac{\gamma(x')}{x - x'}\,dx' + \int_{x+\delta}^L \frac{\gamma(x')}{x - x'}\,dx'\right]$$

$$U_\infty \alpha + \frac{1}{2\pi}\,\text{v.p.}\int_0^L \frac{\gamma(x')}{x - x'}\,dx' = 0$$

Incidence

Kutta!

$$u_i(L, 0^+) = u_i(L, 0^-)$$

$$[\![u_i(x, y)]\!]_{y=0^-}^{y=0^+} = -\gamma(x) \qquad , \quad 0 < x < L$$

$$\boxed{\gamma(L) = 0}$$

On peut démontrer que

$$\boxed{\gamma(x) = -2\,U_\infty\,\alpha\sqrt{\frac{L-x}{x}}}$$

Incidence

Distribution de Cp

$$C_p(x, 0^{\pm}) = \mp 2\,\varepsilon\alpha \sqrt{\frac{L-x}{x}} \quad .$$

Théorème de Kutta-Joukovski

$$F_y = -\rho\, U_\infty\, \Gamma = -\rho\, U_\infty \int_0^L \gamma(x)\, dx = 2\rho\, U_\infty^2\, \alpha \int_0^L \sqrt{\frac{L-x}{x}}\, dx$$

$$F_y = \pi\, \alpha\, \rho\, L\, U_\infty^2 \quad .$$

$$\boxed{C_y = \frac{F_y}{\frac{1}{2}\rho\, U_\infty^2\, L} = 2\,\pi\,\alpha \quad .}$$

Skeleton (camber)

$$\gamma(x) = \frac{1}{\sqrt{x(L-x)}}\left[C + \frac{2U_\infty}{\pi} \text{v.p.} \int_0^L \frac{df_c}{dx} \sqrt{x'(L-x')} \frac{dx'}{x-x'} \right]$$

$$\boxed{\gamma(L) = 0}$$

to be integrated numerically

$$C_y = -2\pi\,\alpha_0$$

Linear superposition

$$\boxed{C_y = 2\pi(\alpha - \alpha_0) \quad ,}$$