Exercise 1

Hydrodynamic forces on an inclined plate

One considers a two-dimensional flow, irrotational, stationary, of a perfect and incompressible fluid, along a plate of length $4a$. The velocity U_∞ of the fluid at infinity is constant is turned with an angle α from the plate. The circulation along the plate is denoted Γ.

1. Show that the use of the Joukowsky transform
$$z = Z + \frac{a^2}{Z}\,,$$
allows to map a circle or radius a in coordinates $Z = X + \mathrm{i}Y$ onto a plate in coordinates $z = x + \mathrm{i}y$.
2. Deduce the complex potential as a function of Z for a cylinder in a flow field $\vec{u} = (U\cos\alpha, U\sin\alpha)$ with an undetermined circulation.
3. In the problem of the circle, show that the leading edge and trailing edge stagnation points are at:
$$\theta = \alpha + \arcsin\left(\frac{\Gamma}{4\pi a V_\infty}\right).$$
Use the fact that the velocity is zero on regular stagnation points. The velocities in polar coordinates can be derived by $\frac{1}{r}\frac{dF}{d\theta}$.
4. Deduce from the potential flow of the cylinder the fluid velocity at the rear tip of the plate at $z = 2a + 0i$. Where is the corresponding point on the cylinder? Remember $w = \frac{df}{dz} = \frac{dF}{dZ}\frac{dZ}{dz} = W\left(\frac{dz}{dZ}\right)^{-1}$.
5. Show that the Kutta condition allows the determination of the circulation Γ, which equivalently leads to a finite velocity at the trailinge edge.
6. Determine the position of the front stagnation point on the plate and the velocity there.
7. Express the complex velocity on the whole plate.
8. Verify that the force acting on the plate is actually the one resulting from Kutta theorem, $F_y = -\rho U \Gamma$.
9. Calculate the couple exerted by the fluid on the plate and show that its point of application is located at the first quarter.

Complex potential flow toolbox

- Uniform flow $f(z) = U\,z\,e^{-i\alpha}$
- Source $f(z) = \frac{Q}{2\pi}\log(z - z_0)$
- Circulation $f(z) = -\frac{i\Gamma}{2\pi}\log(z - z_0)$
- Dipole $f(z) = -\frac{\mu e^{i\alpha}}{z - z_0}$