Exercise 1

A ships propeller of diameter D is moving along its axis in an incompressible fluid at rest. This fluid is limited in its vertical by a free surface, above which exists the atmosphere at pressure p_a. The under water depth of the propellers axis h is such as $h > D/2$. The ships velocity is $\boldsymbol{V_0} = V_0 \boldsymbol{e_z}$, and its rotational velocity is defined by the vector $\boldsymbol{\Omega_0} = \Omega_0 \boldsymbol{e_z}$, being colinear with $\boldsymbol{V_0}$. One designates G as the ensemble of non-dimensional parameters that characterise the propellers geometry.

Figure 2

1. Show that the driving force T (pousse) and the resistive moment Q (couple) of the propeller can be writter as:

$$
\begin{aligned}
T &= \rho \Omega_0^2 D^4 \mathcal{F}\left(Re, Fr, \lambda, \frac{h}{D}, G\right) , \\
Q &= \rho \Omega_0^2 D^5 \mathcal{G}\left(Re, Fr, \lambda, \frac{h}{D}, G\right) ,
\end{aligned}
$$

where Re is the Reynolds number, Fr is the Froude number and λ is the driving parameter

$$\lambda = \frac{V_0}{\Omega_0 D} \, .$$

2. Give an expression for the degree of efficiency η of the propeller as a function of the dimensionless parameter λ, K_T et K_Q, where

$$K_T = \frac{T}{\rho \Omega_0^2 D^4} \, , \qquad K_Q = \frac{Q}{\rho \Omega_0^2 D^5} \, .$$

One desires to know the advancing force of a propeller with a diameter of $4m$, rotating at $180rpm$, for an advancing velocity of $10m/s$ and an immersion of $5m$. Therefore one builds a geometrical similar model of diameter $0.25m$.

3. Show that, if the model and real propeller move in the same liquid, it is not possible to attain a complete similarity. One is therefore pushed to neglect the influence of the viscosity, because a big part of the drag is due to surface waves (formed by gravity). Determine the experimental conditions for the model (advancing velocity, rotational velocity, immersion of the axis).

4. One measures then a propulsion force of the model of $8.6kg\,f$ for an applied couple of $0.39m\,kg\,f$. Determine from these values the advancing force, couple and degree of efficiency at real scale ($1kg\,f = 9.8N$).

If a ships propeller rotates to fast, it could cause locally vaporisation of the liquid on the blades (cavitation, see 3) that alter the performance of the propeller. In fact, the flow around the blade becomes a two-phase flow and the advancing force becomes a function of additional parameters, the margin of the static pressure $p_a + \rho g h - p_v$ between the pressure of the immersed propeller and the vaporisation pressure of the fluid p_v , supposing its constant.

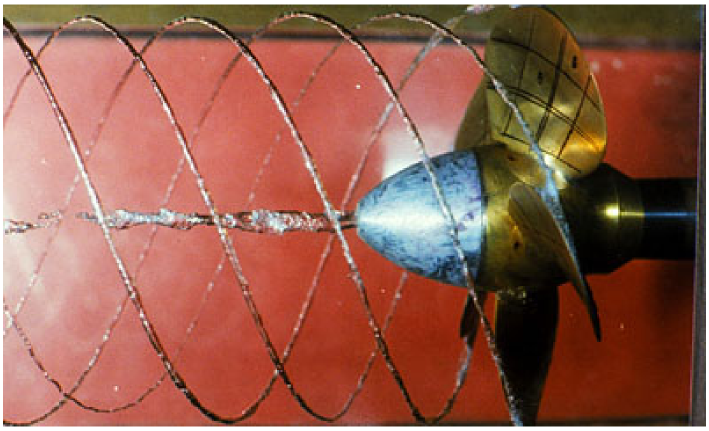

Figure 3

5. Show that the relations tabulated in question 1 need to include a further parameter denoted:

$$\sigma = \frac{p_a + \rho g h - p_v}{\rho \Omega_0^2 D^2} \, . \tag{2}$$

6. The model propeller is tested in an installation that allows to regulate the value of the atmospheric pressure p_a. If it is supposed to be equal to $10^5 Pa$ for a real propeller, determine the pressure under which the model needs to be tested in order to get a reliable estimate for the risk of cavitation (one uses $p_v = 1.8 \times 10^3 Pa$).

Materials	Young's modulus [in GPa]	Density [in g/cm^3]
Steel	220	7.8
Inox	200	7.8
Copper	100	8.9
Cast iron	100	7.2
Titanium	100	4.5
Aluminium	70	2.8
Bronze	100	8.4
Concrete	20	1.9
Epoxy glue	5	1.15

Under the hydrodynamique forces acting on the blades only, the propeller is subject to elastic deformations, resulting in a further modifications of its performance.

7. Show that measured force and couple are thus subject to a further non-dimensional parameter

$$\beta = \frac{E}{\rho \Omega_0^2 D^2}\,, \tag{3}$$

   where E is the Youngs modulus of the material, which is suppose to be sufficient to characterise the deformations (real propeller being in bronze).

8. The experimental conditions as determined in the preceding part, chose the material that has to be used for the model in order to account correctly for the deformations.

Exercise 2: 2D Stokes flow around a circle

We consider the 2D Stokes flow around a circle (with far field velocity U in the $\theta = 0$ direction and we introduce a polar coordinate system (θ, r) and a Stokes streamfunction ψ such that

$$u_r = \frac{1}{r}\frac{\partial \psi}{\partial \theta}; u_\theta = -\frac{\partial \psi}{\partial r}. \tag{1}$$

1. Check that with these expressions of the r and θ velocity components, the flow is indeed incompressible

2. Denoting by ω the z-component of the vorticity, show that $\Delta\omega = 0$, where

$$\Delta = \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2}{\partial \theta^2} \tag{2}$$

   is the Laplacian operator.

3. Show also that

$$\omega = -\Delta\psi \tag{3}$$

   and deduce that

$$\Delta^2\psi = 0 \tag{4}$$

4. Write the boundary conditions in $r = R$ and for $r + \infty$

5. Using the Ansatz $\psi = f(r)\sin(\theta)$ show that

$$\frac{d^4 f}{dr^4} + \frac{2}{r}\frac{d^3 f}{dr^3} - \frac{3}{r^2}\frac{d^2 f}{dr^2} + \frac{3}{r^3}\frac{df}{dr} - \frac{3}{r^4}f = 0 \tag{5}$$

6. Show that there are only 3 fundamental solutions for f with the form r^n with n a signed integer.

7. Is the last fundamental solution $\ln(r)$ or $r\ln(r)$?

8. Can the boundary conditions be satisfied?