Exercise 1

The humming bird (*fr. colibri*) is a bird of the american continent, capable of stationary flight, backwards flight and vertical ascent. These properties make it distinct from other birds. More than 300 species have been identified. The Helene hummingbird (*Mellisuga Helenae*) is a species that lives in Cuba and is the worlds smallest vertebrate and endotherm animal: its weight being $2g$, its wing spread $6cm$ and with its wings flapping 60 times per second when in stationary flight. The giant hummingbird of the Andes (*Patagonia gigas*) is the biggest hummingbird species with a weight of $20g$, wing spread $21cm$: for a stationary flight it needs only to flap 15 times per second. In Northern America, the most common is the ruby throated hummingbird (*Archilocus colibris*) (figure 1) : he measures in wing spread $10cm$, in weight $5g$ and his wing flap 25 times per second to compensate gravity.

Figure 1

In order to gain a better understanding of the characteristics of these fascinating and endangered bird, one is proposed to use dimensional analysis. This exercises objective is to determine the similarity rules, which relate flapping frequency f, wing spread l the birds mass M. Throughout the exercise we will make the following assumptions:

- The birds body, which contains the vital organs and muscles that actuate the wings, is considered to constitute most of the weight. As a simplification we will consider the birds entire mass to be concentrated in the body. The wings mass will therefore be neglected. Its wing spread is l, its body size (length) is c and the distance swept by its extremities b.
- The surrounding air is supposed to be incompressible, possessing the density ρ with a neglegible viscosity. These assumptions will be justified lateron.

1. Determine by dimensional analysis the similarity rules that relate mean lifting force F_y that is produced by the flapping of the wings to the properties of the ambient air and the characteristics of the moving wings.
2. Assuming the bird to be in stationary flight, deduce from the answer to the preceding question a first relation between frequency f, the mass M and the characteristics of the air and the flapping wings. Herein g denotes accaleration due to gravity.
3. Determine by dimensional analysis the similarity rule that relates the power P, necessary for the flapping of the wings from the characteristics of the air and flapping wings.

Stipulating that 40% of the mass M consists of muscles of mass m. For all the other flying species, insects, mammals or birds, the mean is 30%, whereas the hummingbird has a higher muscular mass due to the stationary flight, which is more exhaustive. He needs to feed himself every $6mn$. In the aerobe regime its muscles deliver a power P' proportional to the muscular mass m: $P' = Km$, where K is a constant of order $50W/kg$ of muscles. This constant is, in first order approximation, the same for all species. One shall assume furthermore, that a stationary flight needs all the muscular power of the colibri.

4. Deduce from the answer to question 3 a second relation between frequency f, mass M, constant K and the characteristics of air and the flapping wings.

Let us suppose now that the flapping of all hummingsbird species are geometrically similar. In other words the wings flap c/l and the amplitude relative b/l are identical for all species.

5. Deduce from the relations found in part 2 and 4 the power law of frequence f as function of the mass M. Deduce from the same relations the power law of the wing spread l as a function of mass M. Using an appropriate graphical representation, compare the power laws to the precise data given in the begin of the exercise. Comment.
6. Explain why it was legitimate to neglect the viscosity of air by estimating the order of magnitude of the appropriate non-dimensial parameter.

Exercise 2

A ships propeller of diameter D is moving along its axis in an incompressible fluid at rest. This fluid is limited in its vertical by a free surface, above which exists the atmosphere at pressure p_a. The under water depth of the propellers axis h is such as $h > D/2$. The ships velocity is $\boldsymbol{V_0} = V_0 \boldsymbol{e_z}$, and its rotational velocity is defined by the vector $\boldsymbol{\Omega_0} = \Omega_0 \boldsymbol{e_z}$, being colinear with $\boldsymbol{V_0}$. One designates G as the ensemble of non-dimensional parameters that characterise the propellers geometry.

Figure 2

1. Show that the driving force T (pousse) and the resistive moment Q (couple) of the propeller can be writter as:

$$
\begin{aligned}
T &= \rho \Omega_0^2 D^4 \mathcal{F}\left(Re, Fr, \lambda, \frac{h}{D}, G\right) , \\
Q &= \rho \Omega_0^2 D^5 \mathcal{G}\left(Re, Fr, \lambda, \frac{h}{D}, G\right) ,
\end{aligned}
$$

where Re is the Reynolds number, Fr is the Froude number and λ is the driving parameter

$$\lambda = \frac{V_0}{\Omega_0 D}\,.$$

2. Give an expression for the degree of efficiency η of the propeller as a function of the dimensionless parameter λ, K_T et K_Q, where

$$K_T = \frac{T}{\rho\Omega_0^2 D^4}\,, \qquad K_Q = \frac{Q}{\rho\Omega_0^2 D^5}\,.$$

One desires to know the advancing force of a propeller with a diameter of $4m$, rotating at $180rpm$, for an advancing velocity of $10m/s$ and an immersion of $5m$. Therefore one builds a geometrical similar model of diameter $0.25m$.

3. Show that, if the model and real propeller move in the same liquid, it is not possible to attain a complete similarity. One is therefore pushed to neglect the influence of the viscosity, because a big part of the drag is due to surface waves (formed by gravity). Determine the experimental conditions for the model (advancing velocity, rotational velocity, immersion of the axis).

4. One measures then a propulsion force of the model of $8.6kg\,f$ for an applied couple of $0.39m\,kg\,f$. Determine from these values the advancing force, couple and degree of efficiency at real scale ($1kg\,f = 9.8N$).

If a ships propeller rotates to fast, it could cause locally vaporisation of the liquid on the blades (cavitation, see 3) that alter the performance of the propeller. In fact, the flow around the blade becomes a two-phase flow and the advancing force becomes a function of additional parameters, the margin of the static pressure $p_a + \rho gh - p_v$ between the pressure of the immersed propeller and the vaporisation pressure of the fluid p_v , supposing its constant.

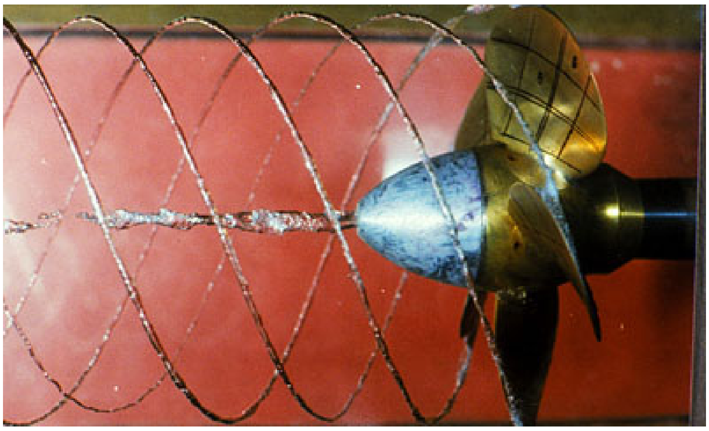

Figure 3

5. Show that the relations tabulated in question 1 need to include a further parameter denoted:

$$\sigma = \frac{p_a + \rho gh - p_v}{\rho\Omega_0^2 D^2}\,. \tag{2}$$

6. The model propeller is tested in an installation that allows to regulate the value of the atmospheric pressure p_a. If it is supposed to be equal to $10^5 Pa$ for a real propeller, determine the pressure under which the model needs to be tested in order to get a reliable estimate for the risk of cavitation (one uses $p_v = 1.8 \times 10^3 Pa$).

Materials	Young's modulus [in GPa]	Density [in g/cm^3]
Steel	220	7.8
Inox	200	7.8
Copper	100	8.9
Cast iron	100	7.2
Titanium	100	4.5
Aluminium	70	2.8
Bronze	100	8.4
Concrete	20	1.9
Epoxy glue	5	1.15

Under the hydrodynamique forces acting on the blades only, the propeller is subject to elastic deformations, resulting in a further modifications of its performance.

7. Show that measured force and couple are thus subject to a further non-dimensional parameter

$$\beta = \frac{E}{\rho \Omega_0^2 D^2} \,, \tag{3}$$

   where E is the Youngs modulus of the material, which is suppose to be sufficient to characterise the deformations (real propeller being in bronze).

8. The experimental conditions as determined in the preceding part, chose the material that has to be used for the model in order to account correctly for the deformations.