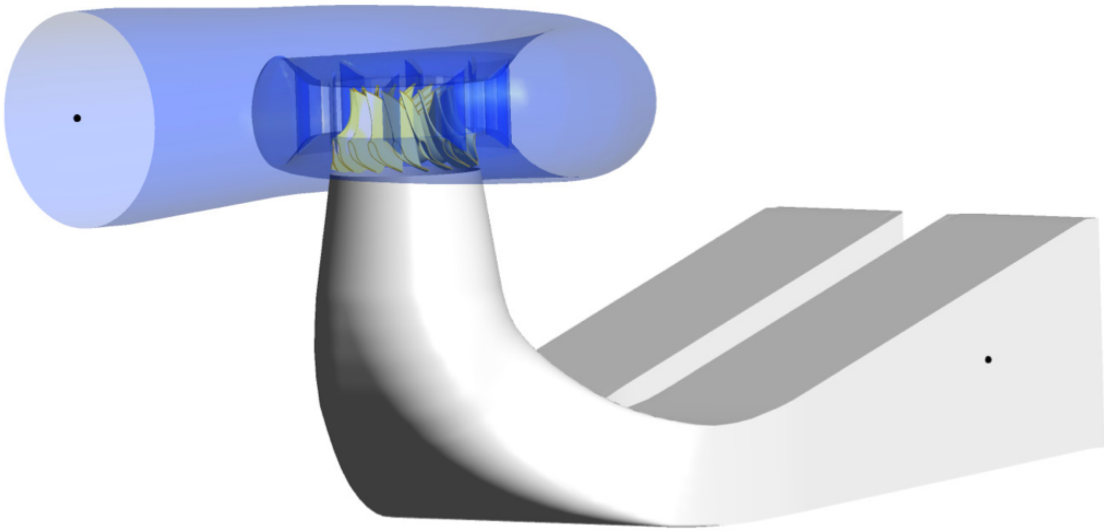
LMH Laboratoire de machines hydrauliques

ÉCOLE POLYTECHNIQUE FÉDÉRALE DE LAUSANNE

HYDRAULIC MACHINE FORMULARY



Laboratory for Hydraulic Machines

Institute of Mechanical Engineering

École Polytechnique Fédérale de Lausanne

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Contents

I Nomenclature

LATIN

Symbol	Unit [SI]	Description
g	$\left[\mathrm{m\ s^{-2}}\right]$	Gravity
k	$\left[-\right]$	Distribution coefficient
k	$\left[\mathrm{m}\right]$	Roughness
n	$\left[-\right]$	Normal vector
n	$\left[\mathrm{rot\ s^{-1}}\right]$	Rotational speed
p	$\left[\mathrm{Pa}\right]$	Static pressure
t	$\left[\mathrm{s}\right]$	Time
A	$\left[\mathrm{m^2}\right]$	Area
A	$\left[-\right]$	Churchill formula coefficient
B	$\left[-\right]$	Churchill formula coefficient
C	$\left[\mathrm{m\ s^{-1}}\right]$	Absolute velocity
Cu	$\left[\mathrm{m\ s^{-1}}\right]$	Peripheral absolute velocity component
Cm	$\left[\mathrm{m\ s^{-1}}\right]$	Meridional absolute velocity
D	$\left[\mathrm{m}\right]$	Diameter
D	$\left[\mathrm{s^{-1}}\right]$	Deformation tensor
D	$\left[-\right]$	Material derivative operator
E	$\left[\mathrm{J\ kg^{-1}}\right]$	Available or supplied specific energy
H	$\left[\mathrm{m}\right]$	Head
I	$\left[-\right]$	Identity tensor
K	$\left[-\right]$	Specific energy losses coefficient
L	$\left[\mathrm{m}\right]$	Pipe length

P	$\left[\mathrm{W}\right]$	Power
P	$\left[\mathrm{J\ kg^{-1}}\right]$	Total stress tensor
Q	$\left[\mathrm{m^3\ s^{-1}}\right]$	Discharge
R	$\left[\mathrm{m}\right]$	Radius
T	$\left[\mathrm{N\ m}\right]$	Torque
U	$\left[\mathrm{m\ s^{-1}}\right]$	Rotating velocity
V	$\left[\mathrm{m^3}\right]$	Volume
W	$\left[\mathrm{m\ s^{-1}}\right]$	Relative velocity
X	$\left[\mathrm{m}\right]$	Location
Z	$\left[\mathrm{m}\right]$	Altitude
gH	$\left[\mathrm{J\ kg^{-1}}\right]$	Specific energy
$NPSE$	$\left[\mathrm{J\ kg^{-1}}\right]$	Net positive suction specific energy
$NPSH$	$\left[\mathrm{m}\right]$	Net positive suction head

GREEK

Symbol	Unit [SI]	Description
α	$\left[\mathrm{rad}\right]$	Absolute flow angle
β	$\left[\mathrm{rad}\right]$	Relative flow angle
η	$\left[-\right]$	Efficiency
θ	$\left[\mathrm{rad}\right]$	Angle
λ	$\left[-\right]$	Regular specific energy losses local coefficient
μ	$\left[\mathrm{Pa\ s}\right]$	Dynamic viscosity
ν	$\left[\mathrm{m^2\ s^{-1}}\right]$	Kinematic viscosity
ρ	$\left[\mathrm{kg\ m^{-3}}\right]$	Density
τ	$\left[\mathrm{m\ s^{-2}}\right]$	Stress tensor

ω	$\left[\text{rad s}^{-1}\right]$	Rotational speed
Π	$\left[\text{W kg}^{-1}\right]$	Production of turbulence
Φ	$\left[\text{W kg}^{-1}\right]$	Viscous dissipation
∂	$\left[-\right]$	Partial derivative operator
∇	$\left[\text{m}^{-1}\right]$	Derivative operator

DIMENSIONLESS NUMBER

Symbol	**Definition**	**Description**
Re	$\frac{C \cdot D_{ref}}{\nu}$	Reynolds number
Fr	$\sqrt{\frac{E}{gD}}$	Froude number
Cp	$\frac{p - p_{\bar{1}}}{\rho E}$	Static pressure factor
n_{ED}	$\frac{nD}{\sqrt{E}}$	IEC speed factor
Q_{ED}	$\frac{Q}{D^2\sqrt{E}}$	IEC discharge factor
T_{ED}	$\frac{T}{\rho D^3 E}$	IEC torque factor
P_{ED}	$\frac{P}{\rho D^2 E^{1.5}}$	IEC power factor
ψ	$\frac{2E}{\omega^2 R^2}$	Specific energy coefficient
φ	$\frac{Q}{\pi\omega R^3}$	Discharge coefficient
ν	$\frac{\omega \cdot \sqrt{Q}}{\sqrt{\pi}\left(2E\right)^{\frac{3}{4}}} = \frac{\varphi^{\frac{1}{2}}}{\psi^{\frac{3}{4}}}$	Specific speed
n_{11}	$\frac{nD}{\sqrt{H}}$	Unit speed

Q_{11}	$\dfrac{Q}{D^2\sqrt{H}}$	Unit discharge
T_{11}	$\dfrac{T}{D^3\sqrt{H}}$	Unit torque
P_{11}	$\dfrac{P}{D^2 H^{3/2}}$	Unit power
n_q	$\dfrac{n\sqrt{Q}}{H^{3/4}}$	Metric specific speed
n_s	$\dfrac{n\sqrt{P}}{H^{5/4}}$	Imperial units specific speed
σ	$\dfrac{NPSE}{E}$	Thoma number
χ_E	$\dfrac{p_{\bar{1}} - p_v}{\rho E}$	Local cavitation factor

SUPERSCRIPTS

Symbol	**Description**
$\tilde{}$	Instantaneous quantity in Reynolds averaging
$'$	Fluctuating quantity in Reynolds averaging
$\bar{}$	Mean operator in Reynolds averaging
$\rightarrow$	Vector
$=$	Tensor
^{T}	Transposition operator

SUBSCRIPTS

Symbol	Description
e	Energetic
e	External
i	Component i of the Cartesian coordinate system
i	Internal
j	Component j of the Cartesian coordinate system
h	Hydraulic
m	Mechanical
opt	Optimal
q	Volumetric
r	Losses
r	Runaway
ref	Reference
s	Sand
t	Transformed
t	Turbulent
v	Singular losses
v	Vapor
A	Water intake reference section
B	Upper reservoir reference section
$\bar{B}$	Lower reservoir reference section
0	Starting value
1	Component 1 of the Cartesian coordinate system
1	Runner high pressure reference section
$\bar{1}$	Runner low pressure reference section
2	Component 2 of the Cartesian coordinate system

$_2$	Reference section 2
$_3$	Component 3 of the Cartesian coordinate system
$_3$	Reference section 3
$_I$	Power unit high pressure reference section
$_{\bar{I}}$	Power unit low pressure reference section
$_{II}$	Reference section II
$_{III}$	Reference section III

ABBREVIATIONS

Symbol	**Description**
EPFL	École Polytechnique Fédérale de Lausanne
LMH	Laboratory for Hydraulic Machine
RANS	Reynolds-Averaged Navier-Stokes

II Fluid Mechanics

1 MASS CONSERVATION

1.1 Continuity

The continuity equation for an incompressible fluid is given by

$$\vec{\nabla} \cdot \vec{C} = 0$$

where $\vec{C}$ is the absolute velocity vector.

Therefore, the mass balance for a volume V of the fluid is defined as

$$\int_{\partial V} \vec{C} \cdot \vec{n} dA = 0$$

where $\vec{n}$ is the normal vector on the area A.

1.2 Discharge Conservation

For a pipe flow, the mass balance is reduced to

$$\int_{A_1} \vec{C} \cdot \vec{n} dA + \int_{A_2} \vec{C} \cdot \vec{n} dA = 0$$

because Σ is a material surface.

The discharge Q is defined as

$$Q = \int_{A_2} \vec{C} \cdot \vec{n} dA \geq 0$$

and so, the discharge conservation is expressed as

$$\int_{A_2} \vec{C} \cdot \vec{n} dA = -\int_{A_1} \vec{C} \cdot \vec{n} dA = Q > 0$$

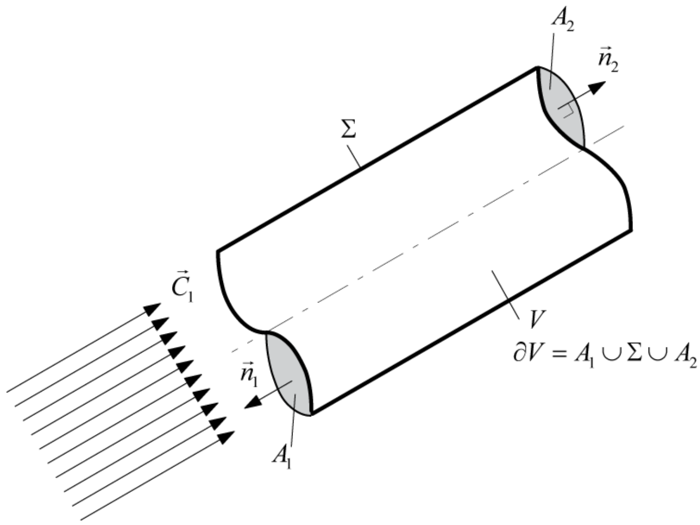

Figure 1 - Pipe section

1.3 Discharge Velocity

The discharge velocity for the given section 2 is defined as

$$C_2 = \frac{Q}{A_2} = \frac{1}{A_2} \int_{A_2} \vec{C} \cdot \vec{n} dA$$

Consequently, the discharge conservation between sections 1 and 2 is written

$$C_1 A_1 = C_2 A_2 = Q$$

In the case of a flow coming from a section 1, which is split into two sections 2 and 3 the conservation of discharge imposes

$$Q_1 = C_1 A_1 = C_2 A_2 + C_3 A_3 = Q_2 + Q_3$$

2 GOVERNING EQUATIONS

2.1 Specific Energy

The average specific energy gH over a section 1 is given by

$$gH_1 = \frac{p_1}{\rho} + gZ_1 + \frac{C_1^2}{2}$$

where ρ is the density, g the gravity and Z the altitude.

2.2 Navier-Stokes Equations

The fluid is assumed as incompressible water at constant temperature and viscosity. The instantaneous value of static pressure $\tilde{p}$ and absolute velocity $\vec{\tilde{C}}$ are governed by the continuity equation

$$\vec{\nabla} \cdot \vec{\tilde{C}} = 0$$

and the momentum equation

$$\frac{\partial \vec{\tilde{C}}}{\partial t} + \left(\vec{\tilde{C}} \cdot \vec{\nabla}\right)\vec{\tilde{C}} = -\vec{\nabla}\left(\frac{\tilde{p}}{\rho} + gZ\right) + \vec{\nabla} \cdot \left(2\nu \overline{\overline{\tilde{D}}}\right)$$

where t is the time, ν the kinematic viscosity and $\overline{\overline{\tilde{D}}}$ the instantaneous deformation tensor, which is given for a Newtonian fluid

$$\overline{\overline{\tilde{D}}} = \frac{1}{2}\left[\left(\vec{\nabla} \otimes \vec{\tilde{C}}\right) + \left(\vec{\nabla} \otimes \vec{\tilde{C}}\right)^T\right]$$

2.3 Reynolds-Averaged Navier-Stokes Equations

The Reynolds-Averaged Navier-Stokes (RANS) equations are derived from the Navier-Stokes equations according to the Reynolds-Averaged decomposition of the instantaneous quantities into mean values and fluctuating parts

$$\vec{\tilde{C}} = \vec{C} + \vec{c}' \quad \text{and} \quad \tilde{p} = p + p'$$

The mean operator in Reynolds averaging is defined as

$$\overline{\vec{\tilde{C}}} = \vec{C} \text{ and } \overline{\vec{c}'} = 0$$

$$\overline{\tilde{p}} = p \text{ and } \overline{p'} = 0$$

By applying the mean operator to the Navier-Stokes equations, the RANS equations are deduced

$$\vec{\nabla} \cdot \vec{c}' = 0$$

$$\frac{\partial \vec{C}}{\partial t} + \left(\vec{C} \cdot \vec{\nabla}\right)\vec{C} = -\vec{\nabla}\left(\frac{p}{\rho} + gZ\right) + \vec{\nabla} \cdot \left(2\nu \overline{\overline{D}} + \frac{\overline{\overline{\tau}}_t}{\rho}\right)$$

where $\overline{\overline{\tau}}_t$ is the symmetric turbulent stress tensor, which is defined as

$$\overline{\overline{\tau}}_t = -\rho\left(\overline{\vec{c}' \otimes \vec{c}'}\right)$$

2.4 Kinetic Specific Energy of the Mean Flow

In order to simplify the notation, the total stress tensor $\overline{\overline{P}}$ is introduced

$$\overline{\overline{P}} = (-\frac{p}{\rho} + gZ)\overline{\overline{I}} + 2\nu\overline{\overline{D}} + \frac{\overline{\overline{\tau}}_t}{\rho}$$

Using this total stress tensor and the material derivative operator, the RANS momentum equation can be reduced to

$$\frac{D\vec{C}}{Dt} = \vec{\nabla} \cdot \overline{\overline{P}}$$

Consequently, the conservation of kinetic specific energy is defined as

$$\vec{C} \cdot \frac{D\vec{C}}{Dt} = \vec{C} \cdot \left[\vec{\nabla} \cdot \frac{\overline{\overline{P}}}{\rho} \right] \qquad \left[\mathrm{W\,kg^{-1}}\right]$$

In a first time, the variation of kinetic specific energy is reduced to a sum of external and internal contributions

$$\underbrace{\frac{D}{Dt}\left(\frac{\vec{C}^2}{2}\right)}_{\text{Kinetic specific energy variation}} = \underbrace{\vec{\nabla} \cdot \left(\vec{C} \cdot \frac{\overline{\overline{P}}}{\rho} \right)}_{\text{External contribution}} - \underbrace{\left(\overline{\overline{D}} : \frac{\overline{\overline{P}}}{\rho} \right)}_{\text{Internal contribution}}$$

In a second time, the specific energy flux of the mean flow is expressed

$$\frac{\partial}{\partial t}\frac{\vec{C}^2}{2} + \vec{\nabla} \cdot \left[\underbrace{\left\{ \frac{p}{\rho} + gZ + \frac{\vec{C}^2}{2} \right\}}_{\text{Specific energy}} \vec{C} \right] = \vec{\nabla} \cdot \left[\left\{ 2\nu\overline{\overline{D}} + \frac{\overline{\overline{\tau}}_t}{\rho} \right\} \cdot \vec{C} \right] - \underbrace{2\nu\overline{\overline{D}} : \overline{\overline{D}}}_{\Phi\text{: viscous dissipation}} - \underbrace{\frac{1}{\rho}\overline{\overline{\tau}}_t : \overline{\overline{D}}}_{\Pi\text{: turbulence production}}$$

In order to simplify this expression, the viscous dissipation Φ and production of turbulence Π are defined

$$\Phi = 2\nu\overline{\overline{D}} : \overline{\overline{D}} \quad \text{and} \quad \Pi = \frac{1}{\rho}\overline{\overline{\tau}}_t : \overline{\overline{D}}$$

2.5 Power Balance of the Mean Flow

The variation of hydraulic specific energy flux through the border ∂V from a control volume V is given by

$$\int_V \frac{\partial}{\partial t}\frac{C^2}{2}\rho\, dV + \int_{\partial V} \left[\frac{p}{\rho} + gZ + \frac{\vec{C}^2}{2} \right] \rho\vec{C} \cdot \vec{n}\, dA = \int_{\partial V} \left(\left[2\nu\overline{\overline{D}} + \frac{\overline{\overline{\tau}}_t}{\rho} \right] \cdot \rho\vec{C} \right) \cdot \vec{n}\, dA - \int_V \Phi\,\rho\, dV - \int_V \Pi\,\rho\, dV \qquad [\mathrm{W}]$$

In a pipe flow case, the homogenous turbulence expresses that

$$\int_{A_1 \cup A_2} \left(\left[2\nu\overline{\overline{D}} + \frac{\overline{\overline{\tau}}_t}{\rho} \right] \cdot \rho\vec{C} \right) \cdot \vec{n}\, dA = 0$$

because the distributions of mean and fluctuating velocities are invariant per translation along the pipe axis. Therefore, the power balance in this case is reduced to

$$\int_{A_i \cup A_2} \left(\frac{p}{\rho} + gZ + \frac{\vec{C}^2}{2} \right) \rho \vec{C} \cdot \vec{n} \, dA = -\int_V \left(\Phi + \Pi \right) \rho \, dV$$

According to the definition of hydraulic power in each section

$$P_{h_1} = -\int_{A_1} \left(\frac{p}{\rho} + gZ + \frac{\vec{C}^2}{2} \right) \rho \vec{C} \cdot \vec{n} \, dA \quad \text{and} \quad P_{h2} = \int_{A_2} \left(\frac{p}{\rho} + gZ + \frac{\vec{C}^2}{2} \right) \rho \vec{C} \cdot \vec{n} \, dA$$

the power balance of the mean flow becomes

$$P_{h_1} = P_{h_2} + \int_V \left(\Phi + \Pi \right) \rho dV \qquad \left[\mathrm{W} \right]$$

The specific energy can be deduced from the power balance with the specific discharge ρQ

$$\frac{P_{h_1}}{\rho Q_1} = gH_1 = gH_2 + gH_{r1\div 2} \qquad \left[\mathrm{J\ kg^{-1}} \right]$$

Consequently, the expression of the pipe flow's energetic losses is given by the viscous dissipation and turbulence production.

$$gH_{r1\div 2} = \int_V \left(\Phi + \Pi \right) \frac{\rho}{\rho Q} dV = \frac{1}{Q} \int_V \left(2\nu \overline{\overline{D}}^2 + \overline{\overline{D}} : \frac{\overline{\overline{\tau_t}}}{\rho} \right) dV$$

III Specific Energy Losses

1 REGULAR SPECIFIC ENERGY LOSSES

1.1 Expression

The regular specific energy losses expression is given by

$$gH_{r1\div2} = \lambda \frac{L_{1\div2}}{D} \cdot \frac{C^2}{2} = K_r \cdot \frac{C^2}{2}$$

with

- λ the local coefficient of regular specific energy losses
- $L_{1\div2} = \int_{1\div2} dl$ the length of the pipe
- $K_r = \lambda \frac{L}{D}$ another notation in order to sum regular and singular losses

The local coefficient depends on the Reynolds number, Mach number (compressibility effect) and two different shape factors.

1.2 Local Coefficient

In the case of a constant diameter pipe, the local coefficient of regular specific energy losses depends only on the Reynolds number and the ratio between the sand rugosity k_s and the pipe diameter.

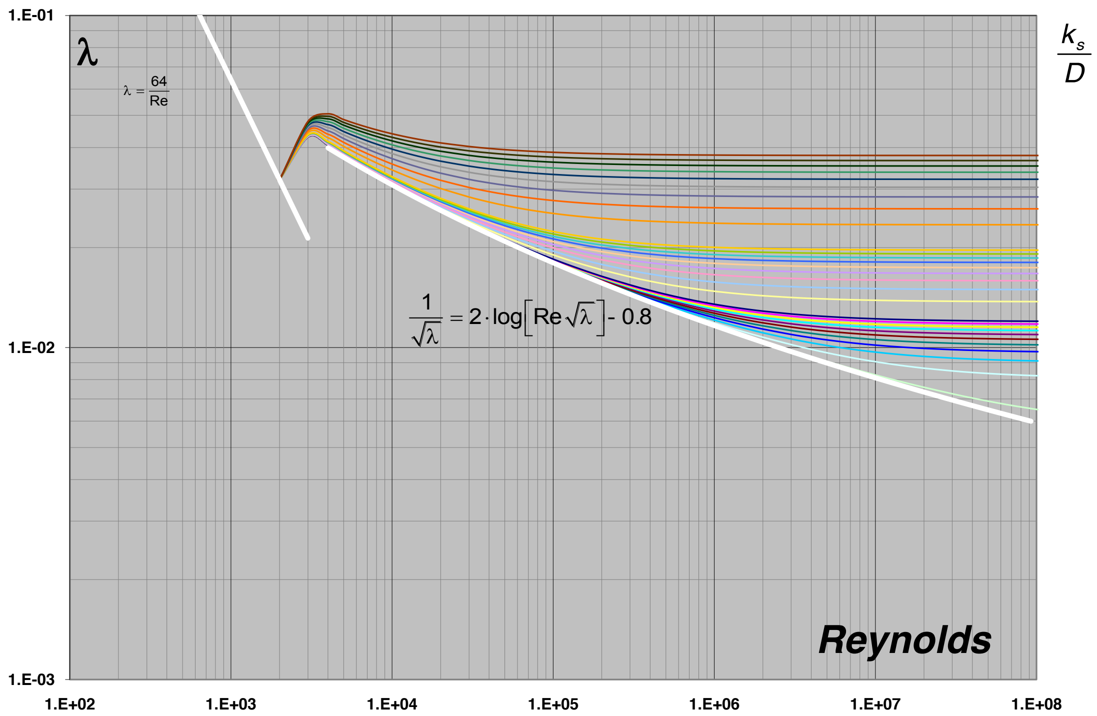

Figure 2 - Regular specific energy losses local coefficient for pipe flow

The Figure 2 provides the local coefficient evolution in function of the Reynolds number. On this figure, three different zones are highlighted: a laminar, transition and turbulent zone.

In order to compute the local coefficient, the analytical Churchill formula allows a good approximation in the three different zones.

$$\lambda = 8\left[\left(\frac{8}{\text{Re}}\right)^{12} + \frac{1}{\left(A+B\right)^{\frac{3}{2}}}\right]^{\frac{1}{12}}$$

with $A = \left[2.457 \cdot \ln \frac{1}{\left(\frac{7}{\text{Re}}\right)^{0.9} + 0.27 \cdot \frac{k_s}{D}}\right]^{16}$ and $B = \left[\frac{37'530}{\text{Re}}\right]^{16}$

1.3 Roughness

The equivalent sand roughness, which is needed in the Churchill formula, is given for different materials in Table 1.

Table 1 – Equivalent sand roughness for some industrial pipes (Comolet 1994)

Pipe material	Surface quality	Equivalent sand roughness k_s
Drawn glass, plastic, copper, brass, stainless steel		$< 10^{-6}$ m
Industrial PVC		from $10 \cdot 10^{-6}$ m to $20 \cdot 10^{-6}$ m
Industrial painted pipe		$20 \cdot 10^{-6}$ m
Industrial brass		$25 \cdot 10^{-6}$ m
Rolled steel	New	$50 \cdot 10^{-6}$ m
	Rusted	from $150 \cdot 10^{-6}$ m to $250 \cdot 10^{-6}$ m
	Incrusted	from $1.5 \cdot 10^{-3}$ m to $3 \cdot 10^{-3}$ m
Welded steel	New	from $30 \cdot 10^{-6}$ m to $100 \cdot 10^{-6}$ m
	Rusted	$400 \cdot 10^{-6}$ m
Cast iron	New	$250 \cdot 10^{-6}$ m
	Rusted	from $1 \cdot 10^{-3}$ m to $1.5 \cdot 10^{-3}$ m
Centrifuged concrete	Smooth	$300 \cdot 10^{-6}$ m
Concrete	Smooth	from $300 \cdot 10^{-6}$ m to $800 \cdot 10^{-6}$ m
	Rough	up to $3 \cdot 10^{-3}$ m
Riveted steel		from $900 \cdot 10^{-6}$ m to $9 \cdot 10^{-3}$ m
Tunnel		from $90 \cdot 10^{-3}$ m to $600 \cdot 10^{-3}$ m

2 SINGULAR SPECIFIC ENERGY LOSSES

2.1 Expression

The regular specific energy losses expression is given by

$$gH_{rv} = K_v \cdot \frac{C^2}{2}$$

with K_v the singular specific energy loss coefficient, which is given for a few examples below.

2.2 Examples

Sudden Enlargement:

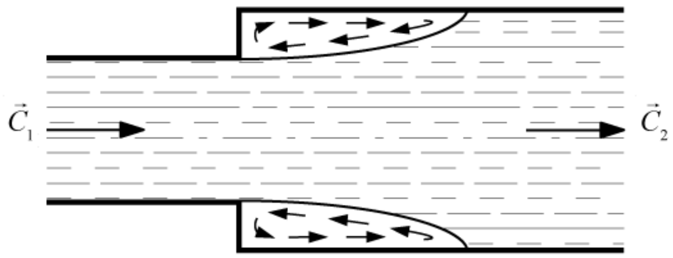

Figure 3 - Sudden enlargement flow pattern

$$gH_{rv} = K_v \cdot \frac{C_1^2}{2} \quad \text{with} \quad K_v = \left[1 - \frac{A_1}{A_2}\right]^2$$

Sudden Contraction:

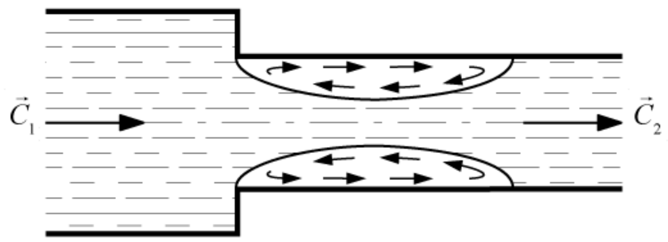

Figure 4 - Sudden contraction flow pattern

$$gH_{rv} = K_v \cdot \frac{C_1^2}{2} \quad \text{with} \quad K_v = \frac{1}{2}\left[1 - \frac{A_2}{A_1}\right]$$

Water Intake:

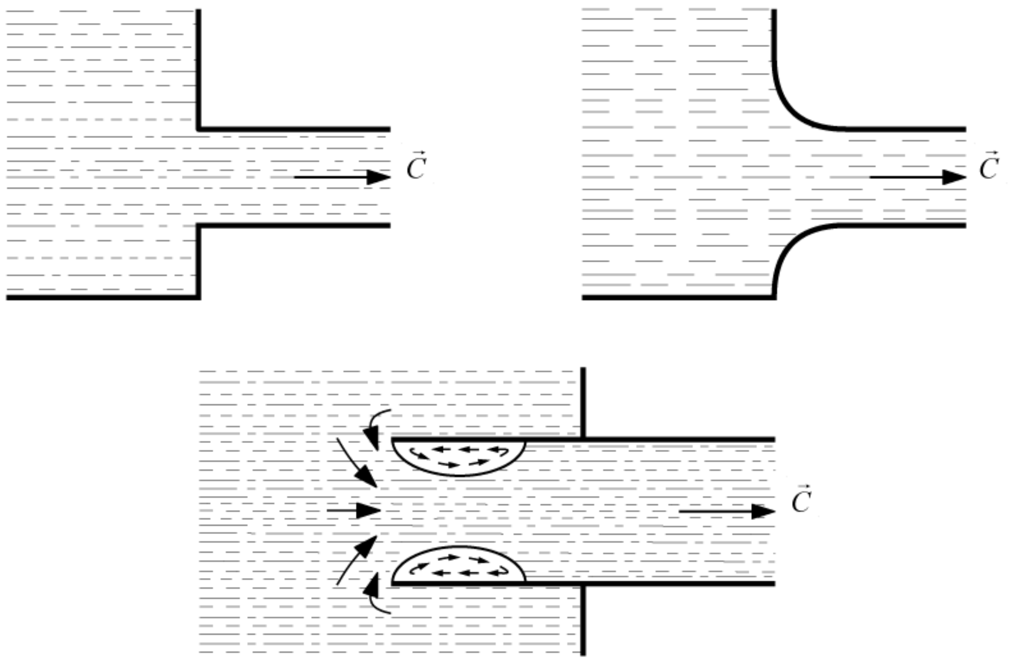

Figure 5 - Water intake with sharp (upper left), smooth (upper right) and re-entrant pipe (lower) connections

$$gH_{rv} = K_v \cdot \frac{C^2}{2} \quad \text{with} \quad \begin{cases} K_v = 0.5 & \text{for a sharp connection} \\ K_v = 0.05 & \text{for a smooth connection} \\ K_v = 1 & \text{for a re-entrant pipe connection} \end{cases}$$

Water Outflow:

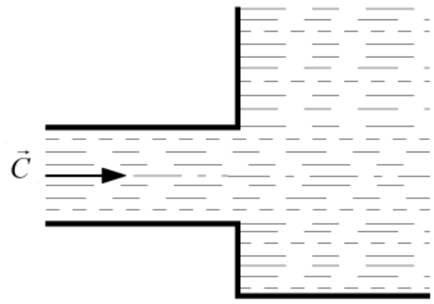

Figure 6 - Outflow with sharp connection

$$gH_{rv} = K_v \cdot \frac{C^2}{2} \quad \text{with} \quad K_v = 1$$

Elbow:

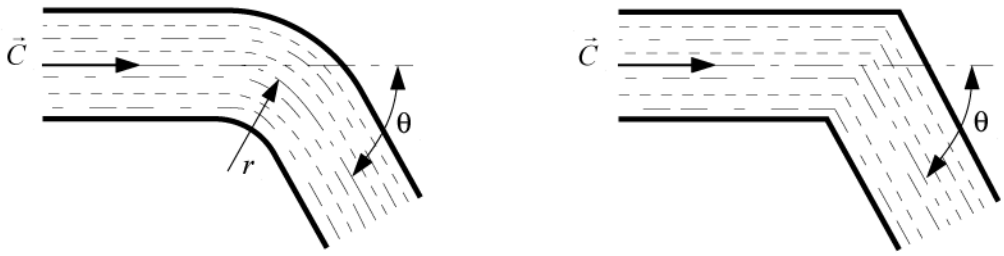

Figure 7 - Smooth (left) and sharp (right) elbows

$$gH_{rv} = K_v \cdot \frac{C^2}{2} \quad \text{with} \quad K_v = \left[0.131 + 1.847\left(\frac{D}{2r}\right)^{3.5}\right]\frac{\theta}{90} \quad \text{for a smooth elbow}$$

For a sharp elbow, the loss coefficient K_v is given in Table 2.

Table 2 – Loss coefficient for a sharp elbow

θ °	22.5	30	45	60	90
K_v	0.07	0.11	0.24	0.47	1.13

Valve:

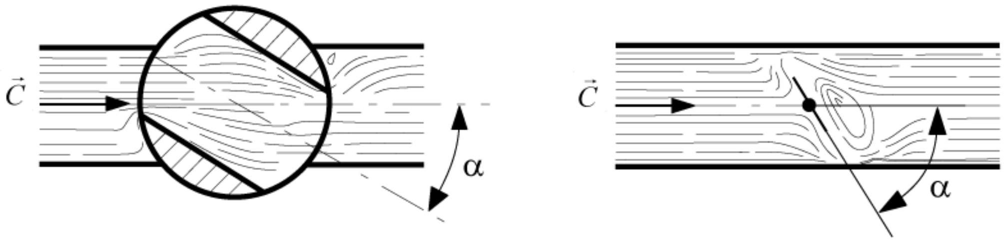

Figure 8 - Spherical (left) and butterfly (right) valves

$$gH_{rv} = K_v \cdot \frac{C^2}{2}$$ with the loss coefficient K_v given in the tables below.

Table 3 – Loss coefficient for a spherical valve

θ °	5	10	15	20	30	40	45	50	60	70
K_v	0.24	0.52	0.90	1.5	3.9	11	19	33	120	750

Table 4 – Loss coefficient for a butterfly valve

θ °	5	10	15	25	35	45	55	65
K_v	0.05	0.29	0.75	3.1	9.7	31	110	490

IV Hydraulic Power Plant

1 MAIN COMPONENTS

1.1 Power Plant

The hydraulic power plant uses a hydraulic circuit between an upstream reservoir and a downstream reservoir in order to produce (turbine mode) or store (pump mode) energy. The Figure 9 represents a common power plant outline with its main components and reference sections.

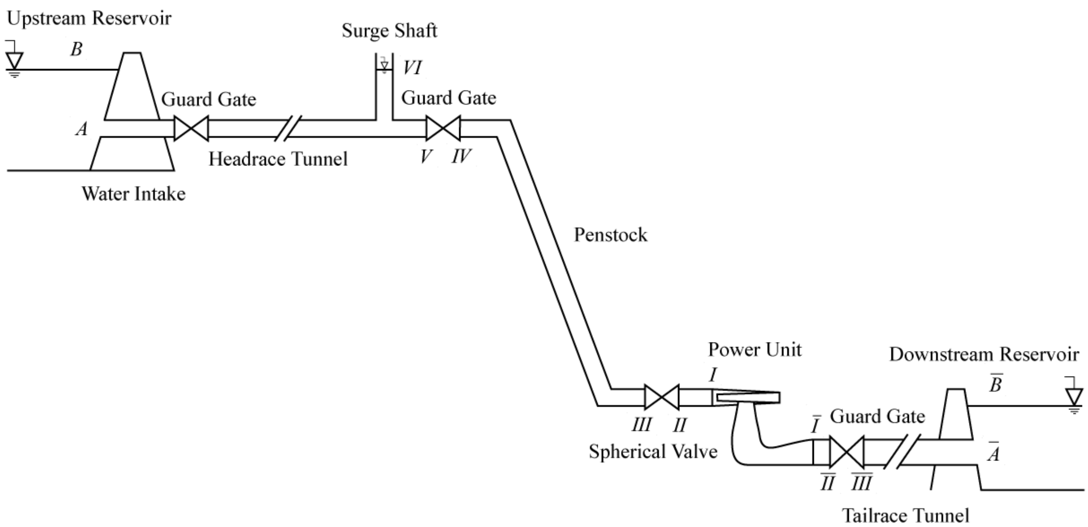

Figure 9 - Power plant outline

1.2 Power Unit

Between the sections I and $\overline{I}$ of the power plant, the power unit transfers the mechanical energy to the flow. The Figure 10 provides the main components and reference sections for a turbine.

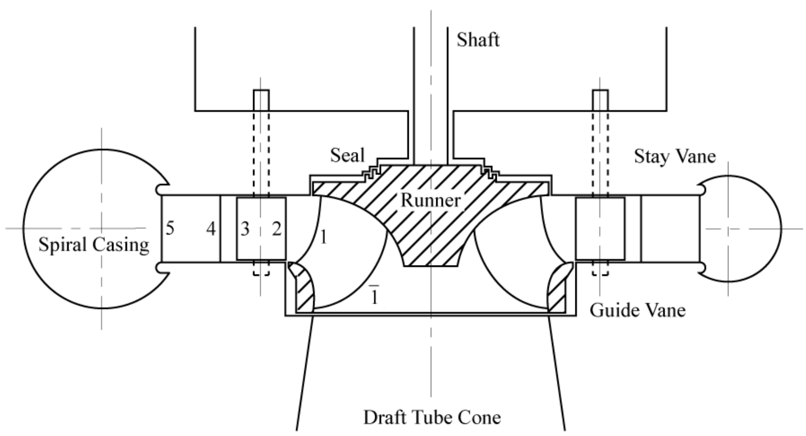

Figure 10 - Turbine cut view of a power unit

1.3 Spiral Casing

The spiral casing is used to distribute uniformly the flow around the turbine. The Figure 11 represents the two common designs for a spiral casing. The double curvature is an older design, which provides a complex distribution of stresses. The Piguet type is a recent design, which is appropriate for large machines.

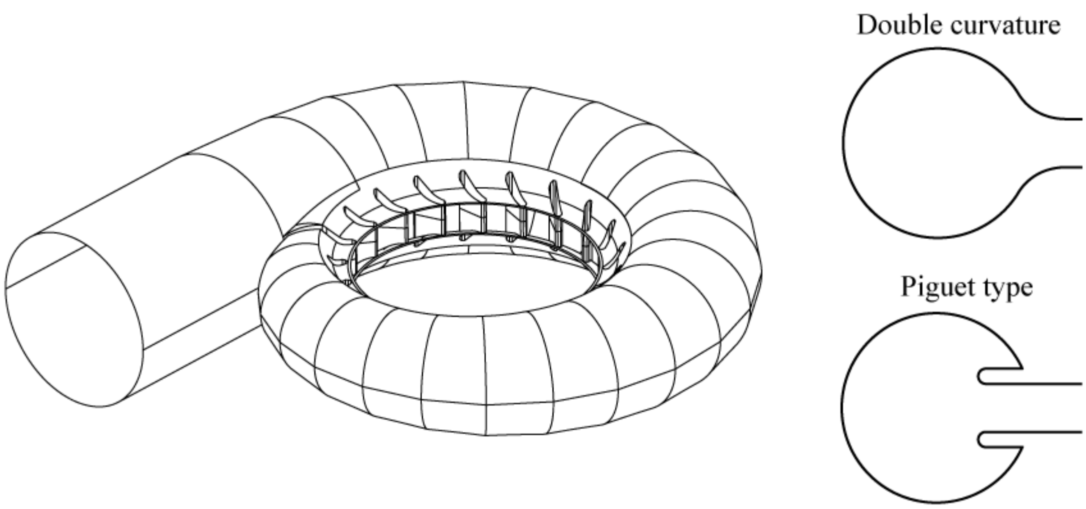

Figure 11 - Spiral casing types; water passages

1.4 Stay Vane and Guide Vane

The stay vanes guide the flow inside the turbine and preserve the static stability of the spiral casing, which is submitted to a high pressure load case. The guide vanes (or wicket gates) are used to control the discharge and the incidence of the flow given to the runner. The different relations related to the guide vanes position are given in Figure 12.

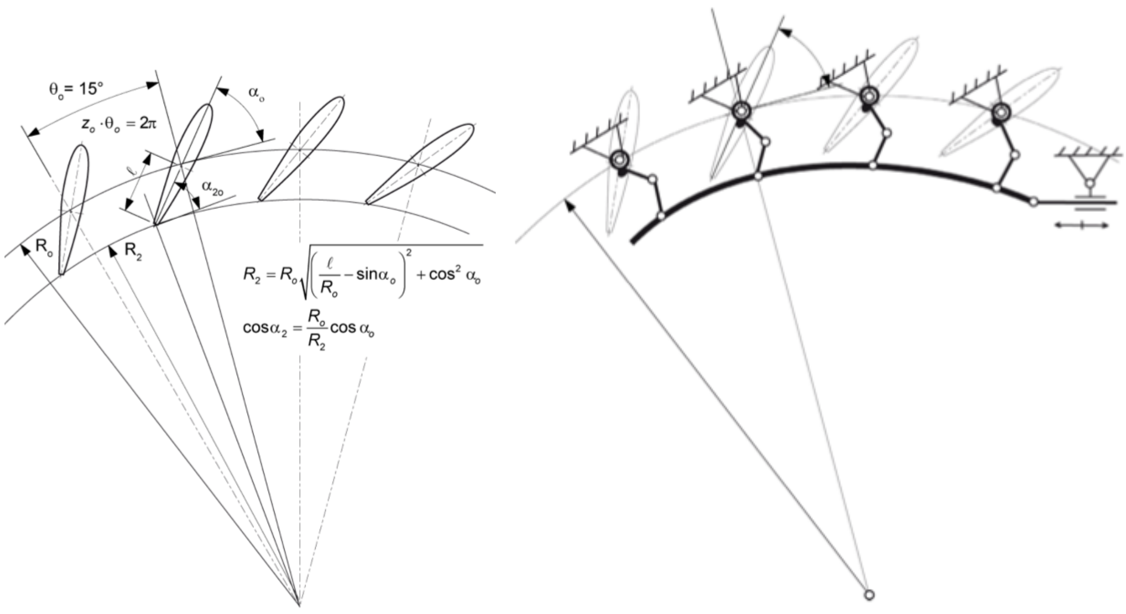

Figure 12 - Guide vane relations

1.5 Runner and Impeller

The runner converts the energy of the flow into mechanical power. Indeed, the flow passing through the runner generates a pressure difference between the pressure and suction sides of the runner blades. From this pressure difference, forces apply on the runner blades, which generates a torque on the runner. This torque is transmitted to the generator by the shaft.

There are four main runner types: Pelton, Francis, Kaplan and Bulb. The Figure 13 provides for each runner type, the operating range, which induces the best efficiency operating condition of the power unit. This figure shows the appropriate operating range of a runner type in function of the head H and specific speed ν, which is defined as

$$\nu = \frac{\omega \cdot \sqrt{Q}}{\sqrt{\pi}\left(2E\right)^{\frac{3}{4}}}$$

with

- ω the rotational speed of the runner
- $E = gH_I - gH_{\bar{I}}$ the available (turbine) or supplied (pump) specific energy

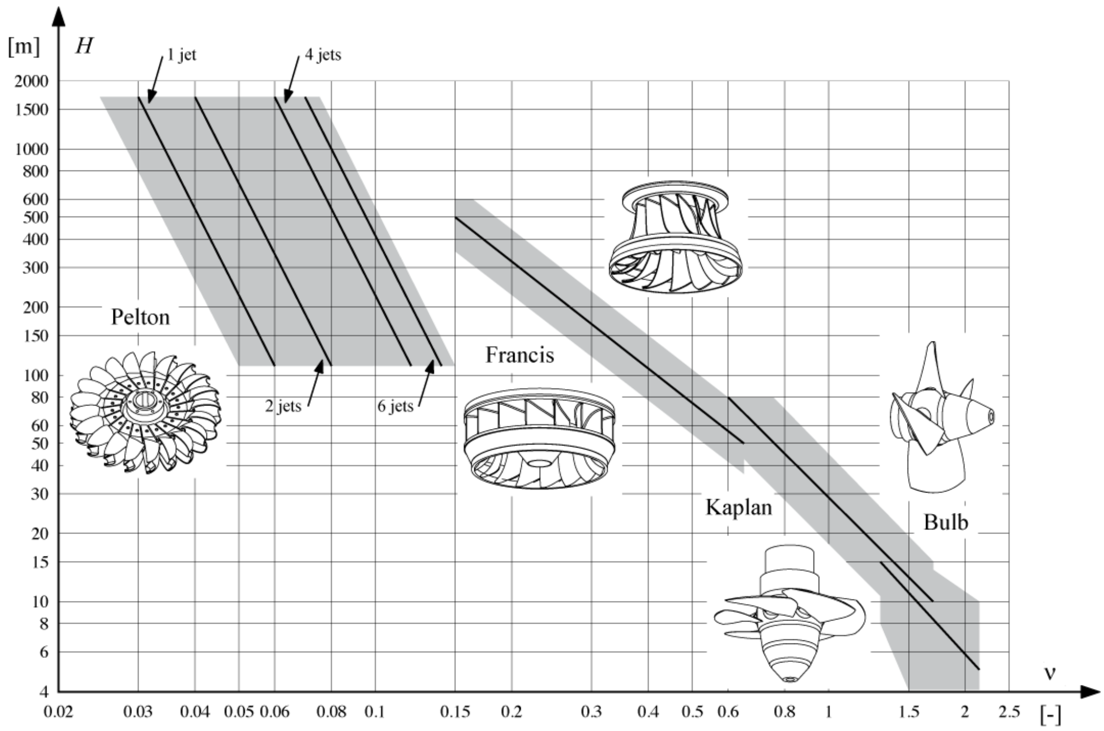

Figure 13 - Appropriate operating range for the different runner types

An impeller converts the mechanical power into a pressure increase. However, an impeller is also used as a turbine in the particular case of pump-turbine power unit.

The Figure 14 shows the appropriate operating range of a pump-turbine in function of the head H and specific speed ν.

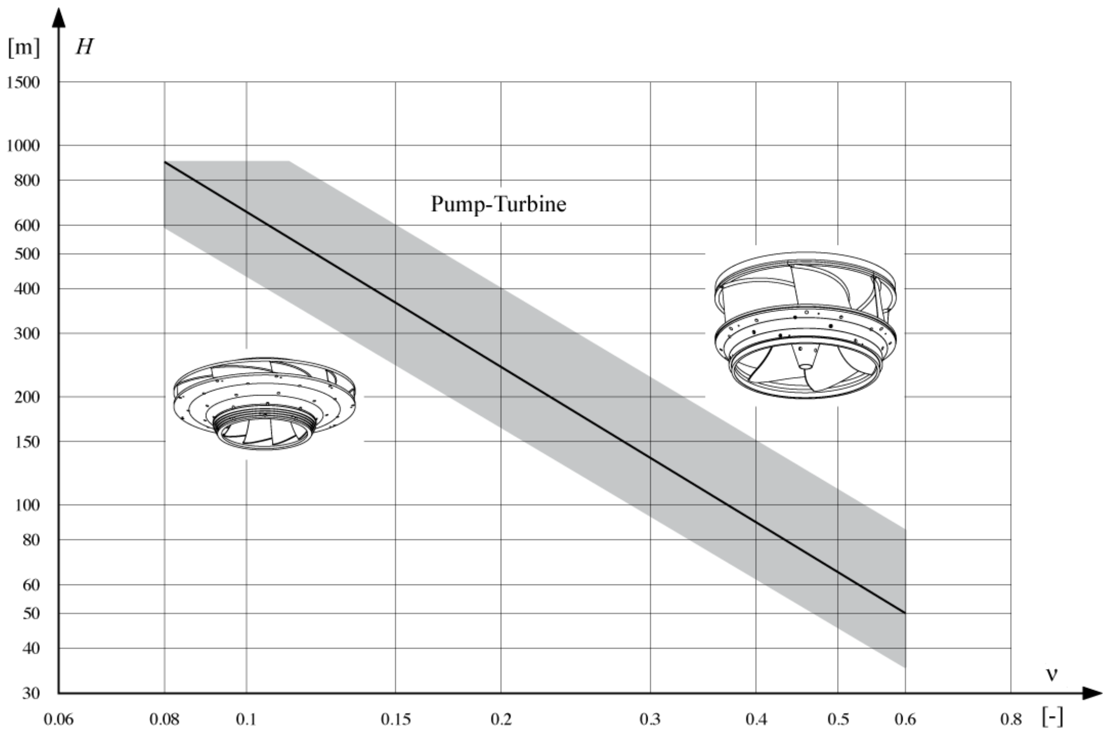

Figure 14 - Appropriate operating range of impeller

1.6 Draft Tube

As shown in Figure 15, the draft tube is composed of a cone, an elbow and a diffuser zone. The aim of the draft tube is to recover the static pressure at the turbine outlet.

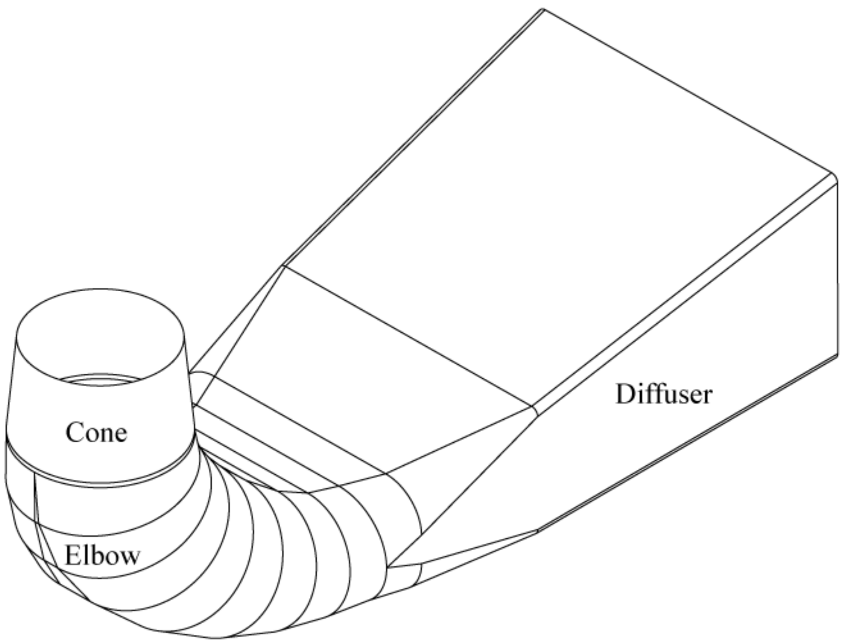

Figure 15 - Draft tube zones

2 OPERATING PRINCIPLES

2.1 Turbine Mode

In turbine mode, the specific energy available E is defined as

$$E = gH_I - gH_{\bar{I}} = \left(gH_B - gH_{\bar{B}}\right) - \sum gH_{rB \div \bar{B}}$$

and the transformed specific energy is expressed as

$$E_t = E - \sum E_{rI \div \bar{I}}$$

The Figure 16 represents the specific energy evolution in a turbine circuit.

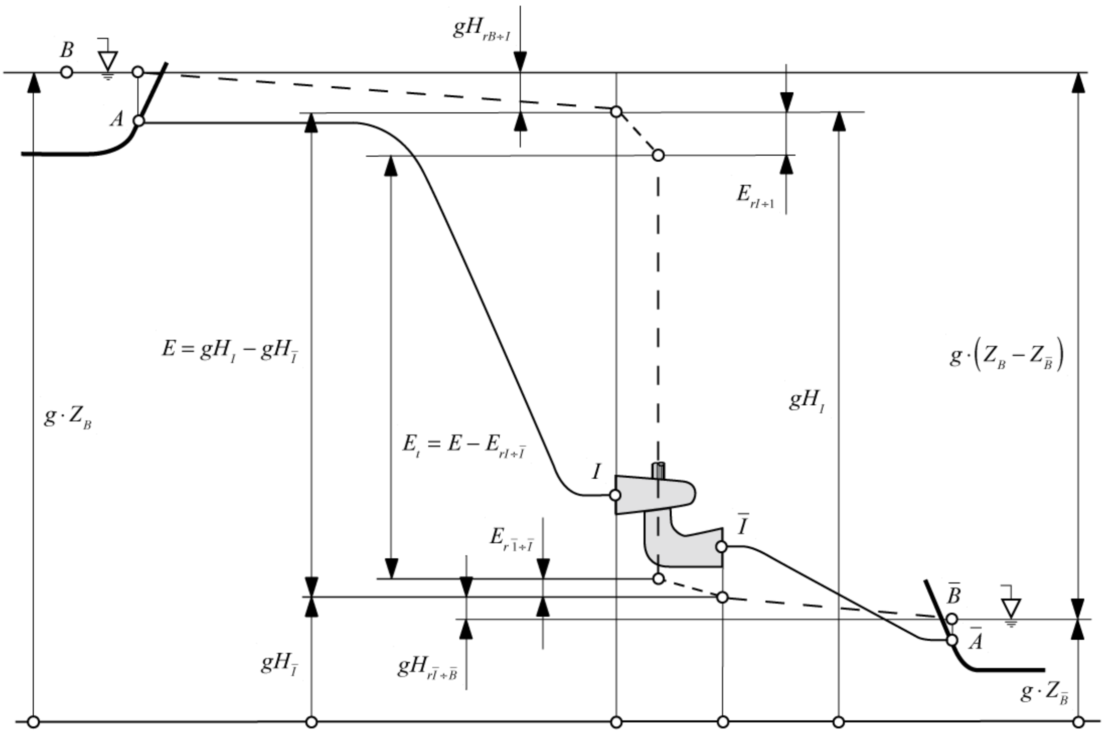

Figure 16 - Specific energy evolution in a turbine circuit

2.2 Pump Mode

In pump mode, the supplied specific energy E is defined as

$$E = gH_I - gH_{\bar{I}} = \left(gH_B - gH_{\bar{B}}\right) + \sum gH_{rB \div \bar{B}}$$

and the transformed specific energy is expressed as

$$E_t = E + \sum E_{rI \div \bar{I}}$$

The Figure 17 represents the specific energy evolution in a pump circuit.

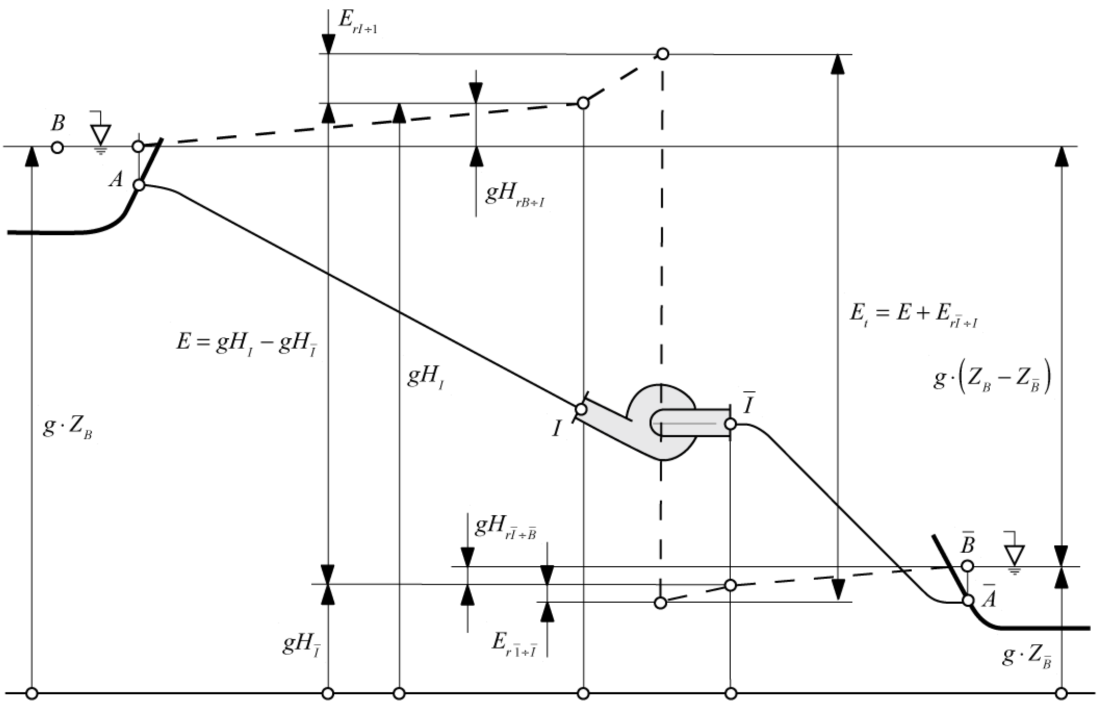

Figure 17 - Specific energy evolution in a pump circuit

2.3 Power and Efficiency

The hydraulic power of the power unit is defined as

$$P_h = \rho \cdot Q \cdot E \quad \text{with} \quad \begin{cases} P_h > 0 & \text{in turbine mode} \\ P_h < 0 & \text{in pump mode} \end{cases}$$

Therefore, the transformed power is

$$P_t = \rho \cdot Q_t \cdot E_t \quad \text{with} \quad \begin{cases} P_t > 0 & \text{in turbine mode} \\ P_t < 0 & \text{in pump mode} \end{cases}$$

with Q_t the part of the discharge, which goes through the runner. Finally, the available or supplied power is defined as

$$P = \omega \cdot T \quad \text{where } T \text{ is the torque at the shaft} \quad \text{with} \quad \begin{cases} P > 0 & \text{in turbine mode} \\ P < 0 & \text{in pump mode} \end{cases}$$

The negative power value in pump mode is due to the negative definition of the discharge and rotational speed in pump mode.

The global efficiency of the power unit is given by

$$\eta = \eta_h \cdot \eta_m = \begin{cases} \dfrac{P}{P_h} & \text{in turbine mode} \\ \dfrac{P_h}{P} & \text{in pump mode} \end{cases}$$

where η_m is efficiency of the bearing and η_h the hydraulic efficiency defined by

$$\eta_h = \eta_e \cdot \eta_q \cdot \eta_{rm}$$

with the efficiency of the disc friction η_{rm}, the volumetric efficiency η_q and the energetic efficiency η_e. The volumetric efficiency is given by

$$\eta_q = \begin{cases} \dfrac{Q_t}{Q} & \text{in turbine mode} \\ \dfrac{Q}{Q_t} & \text{in pump mode} \end{cases}$$

and the energetic efficiency by

$$\eta_e = \begin{cases} \dfrac{E_t}{E} & \text{in turbine mode} \\ \dfrac{E}{E_t} & \text{in pump mode} \end{cases}$$

In order to summarize the power and efficiency definitions, the Figure 18 provides an outline of the power evolution through a turbine and a pump.

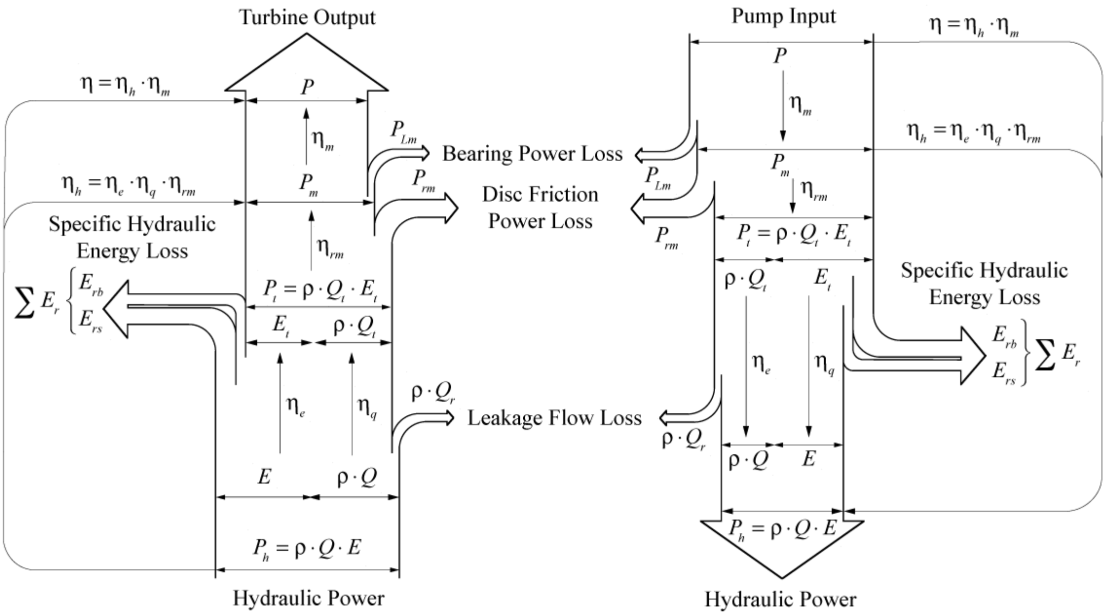

Figure 18 - Power evolution through a power unit

2.4 IEC Factors and Specific Speed

The IEC Factors are different dimensionless numbers, which provide information on the operating condition of a power unit. Therefore, for a given power unit with the following

- n rotational speed (rotation per second)
- D reference diameter ($D_{\bar{1}e}$ for turbine and D_{1e} for pump)
- $R = \dfrac{D}{2}$ reference radius
- E available or supplied specific energy
- Q discharge
- T torque

The different IEC factors are defined as

- $n_{ED} = \dfrac{nD}{\sqrt{E}}$ IEC Speed Factor
- $Q_{ED} = \dfrac{Q}{D^2\sqrt{E}}$ IEC Discharge Factor
- $T_{ED} = \dfrac{T}{\rho D^3 E}$ IEC Torque Factor
- $P_{ED} = \dfrac{P}{\rho D^2 E^{1.5}}$ IEC Power Factor

In the same way as the IEC Factors, the Specific Energy Coefficient ψ and Discharge Coefficient φ provide also information on the operating condition of a power unit.

- $\psi = \dfrac{2E}{\omega^2 R^2}$ Specific Energy Coefficient
- $\varphi = \dfrac{Q}{\pi\omega R^3}$ Discharge Coefficient

Consequently, the specific speed ν can also be expressed as

- $\nu = \dfrac{\varphi^{\frac{1}{2}}}{\psi^{\frac{3}{4}}}$ Specific Speed

2.5 Engineering Factors and Specific Speed

In order to provide information on the operating condition of a power unit, other engineering factors and specific speed definitions exist. However, these factors have the disadvantage to be non dimensionless.

- $n_{11} = \dfrac{nD}{\sqrt{H}}$ Unit Speed
- $Q_{11} = \dfrac{Q}{D^2\sqrt{H}}$ Unit Discharge
- $T_{11} = \dfrac{T}{D^3 H}$ Unit Torque
- $P_{11} = \dfrac{P}{D^2 H^{3/2}}$ Unit Power
- $n_q = \dfrac{n\sqrt{Q}}{H^{3/4}}$ Metric Specific Speed
- $n_s = \dfrac{n\sqrt{P}}{H^{5/4}}$ Imperial Units Specific Speed

3 CAVITATION

3.1 Description

The cavitation phenomena appear when the pressure inside the flow is lower than the vapor pressure p_v. The Thoma's cavitation factor σ is defined as

$$\sigma = \frac{p_{ref} - p_v}{\frac{1}{2}\rho C_{ref}^2}$$

The Figure 19 represents the cavitation onset for a hydraulic profile.

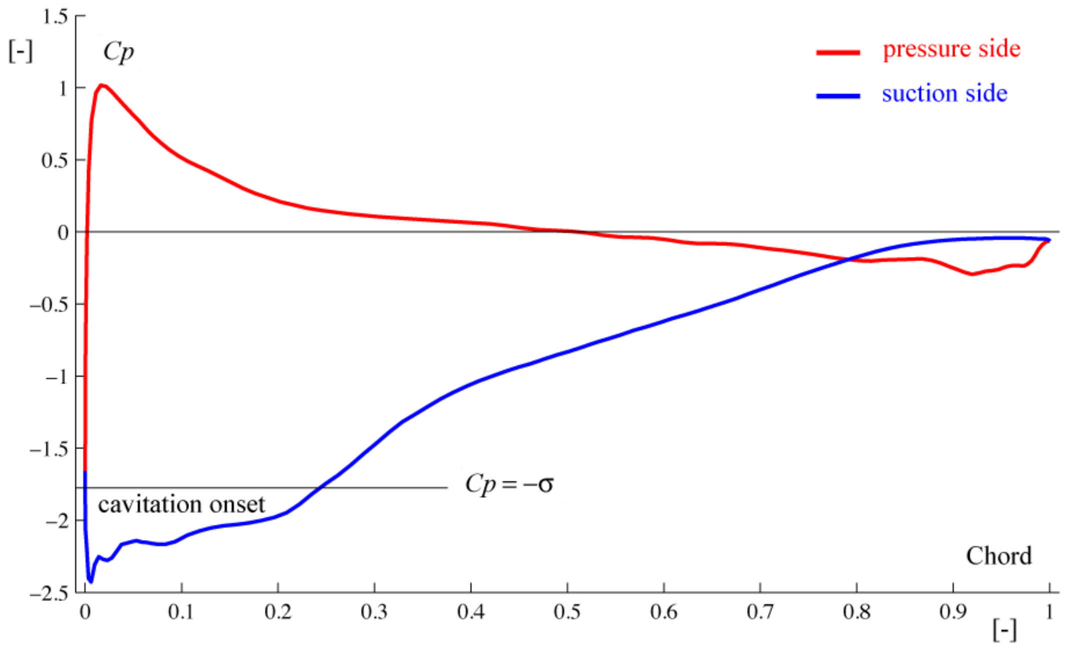

Figure 19 - Cavitation onset on a hydraulic profile

In the case of a Francis turbine, the different reference altitudes are given in Figure 20.

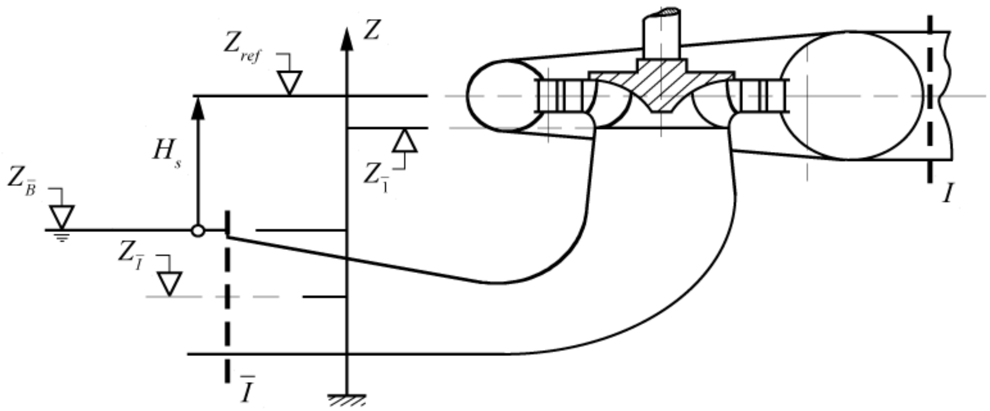

Figure 20 - Reference altitudes on a Francis turbine outline

Therefore, the Thoma number σ is now defined as

$$\sigma = \frac{NPSE}{E}$$

where $NPSE$ is the Net Positive Suction Specific Energy of the machine

$$NPSE = \frac{p_{\bar{B}}}{\rho} - \frac{p_v}{\rho} - gH_s + \frac{C_{\bar{I}}^2}{2}$$

The Net Positive Suction Head $NPSH$ is expressed as

$$NPSH = \frac{NPSE}{g}$$

In the case of the Francis turbine, the condition of cavity onset is given by

$$Cp < -\chi_E$$

with

- $Cp = \dfrac{p - p_{\bar{1}}}{\rho E}$ Static Pressure Factor
- $\chi_E = \dfrac{p_{\bar{1}} - p_v}{\rho E} = \sigma + \dfrac{1}{Fr^2}\left(\dfrac{Z_{ref} - Z_{\bar{1}}}{D_{\bar{1}e}}\right) - \dfrac{C_{\bar{1}}^2}{2E} + \dfrac{\sum gH_{r\bar{1}\div\bar{I}}}{E}$ Local Cavitation Factor
- $Fr = \sqrt{\dfrac{E}{gD}}$ Froude Number

V Power Balance

1 TURBOMACHINERY EQUATIONS

1.1 Rotating Frame

In order to describe the flow inside the rotating zone of the power unit, a multi reference frame is defined and represented in Figure 21. The Cartesian coordinate system is used to represent absolute velocity $\vec{C}$ and the cylindrical coordinate system represents the relative velocity $\vec{W}$.

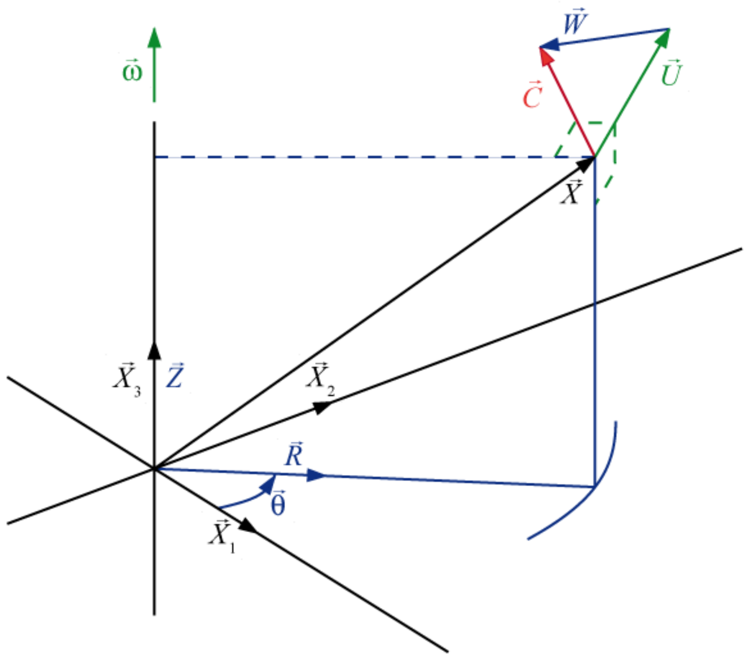

Figure 21 - Cartesian (black) and cylindrical (blue) reference frames

The rotating velocity $\vec{U}$ is defined as

$$\vec{U} = \vec{\omega} \times \vec{X} = \omega R \cdot \vec{\theta}$$

Consequently, the relation between the absolute and relative flow is given by

$$\vec{C} = \vec{U} + \vec{W} \quad \text{or} \quad \vec{W} = \vec{C} - \vec{U}$$

1.2 Rotalpy Conservation

The Reynolds equation in the relative reference frame is expressed as

$$\frac{D\vec{W}}{Dt} = -\vec{\nabla}\left(\frac{p}{\rho} + gZ - \frac{\vec{U}^2}{2}\right) + 2\vec{\omega} \times \vec{W} + \vec{\nabla} \cdot \left(2\nu \overline{\overline{D}} + \frac{\overline{\overline{\tau_t}}}{\rho}\right)$$

Multiplying this expression by $\vec{W}$, the relative specific kinetic energy variation is given by

$$\frac{D}{Dt}\left(\frac{\vec{W}^2}{2}\right) = \vec{W} \cdot -\vec{\nabla}\left(\frac{p}{\rho} + gZ - \frac{\vec{U}^2}{2}\right) + \underbrace{\vec{W} \cdot \left(2\vec{\omega} \times \vec{W}\right)}_{=0} + \vec{W} \cdot \vec{\nabla} \cdot \left(2\nu \overline{\overline{D}} + \frac{\overline{\overline{\tau_t}}}{\rho}\right)$$

As the relative flow is stationary and incompressible, the left term becomes

$$\frac{D}{Dt}\left(\frac{\vec{W}^2}{2}\right)=\frac{\partial}{\partial t}\left(\frac{\vec{W}^2}{2}\right)+\left(\vec{W}\cdot\vec{\nabla}\right)\frac{\vec{W}^2}{2}=\frac{\partial}{\partial t}\left(\frac{\vec{W}^2}{2}\right)+\vec{\nabla}\cdot\left[\frac{\vec{W}^2}{2}\vec{W}\right]-\frac{\vec{W}^2}{2}\underbrace{\vec{\nabla}\cdot\vec{W}}_{=0}$$

The right term of the relative specific kinetic energy variation can be reduced to

$$\frac{D}{Dt}\left(\frac{\vec{W}^2}{2}\right)=-\vec{\nabla}\cdot\left[\left\{\frac{p}{\rho}+gZ-\frac{\vec{U}^2}{2}\right\}\vec{W}\right]+\vec{\nabla}\cdot\left[\left\{2\nu\overline{\overline{D}}+\frac{\overline{\overline{\tau_t}}}{\rho}\right\}\cdot\vec{W}\right]-2\nu\overline{\overline{D}}:\overline{\overline{D}}-\frac{1}{\rho}\overline{\overline{\tau_t}}:\overline{\overline{D}}$$

The rotalpy conservation applied the runner's control volume is given by

$$\int_{\partial V}\left(\frac{p}{\rho}+gZ-\frac{\vec{U}^2}{2}+\frac{\vec{W}^2}{2}\right)\vec{W}\cdot\vec{n}\,dA=\int_{\partial V}\left[\left\{2\nu\overline{\overline{D}}+\frac{\overline{\overline{\tau_t}}}{\rho}\right\}\cdot\vec{W}\right]\cdot\vec{n}\,dA$$
$$-\int_V\left\{+2\nu\overline{\overline{D}}:\overline{\overline{D}}+\frac{1}{\rho}\overline{\overline{\tau_t}}:\overline{\overline{D}}\right\}dV$$

As $\vec{W}\cdot\vec{n}=\vec{C}\cdot\vec{n}$, the rotalpy conservation is expressed as

$$\int_{A_{\bar{1}}\cup A_{\bar{1}}}\left(\frac{p}{\rho}+gZ\right)\vec{C}\cdot\vec{n}\,dA=-\int_{A_{\bar{1}}\cup A_{\bar{1}}}\left(-\frac{\vec{U}^2}{2}+\frac{\vec{W}^2}{2}\right)\vec{C}\cdot\vec{n}\,dA+\int_{A_{\bar{1}}\cup A_{\bar{1}}}\left[\left\{2\nu\overline{\overline{D}}+\frac{\overline{\overline{\tau_t}}}{\rho}\right\}\cdot\vec{W}\right]\cdot\vec{n}\,dA$$
$$-\int_V\left\{+2\nu\overline{\overline{D}}:\overline{\overline{D}}+\frac{1}{\rho}\overline{\overline{\tau_t}}:\overline{\overline{D}}\right\}dV$$

The hydraulic power through the runner is defined

$$P_{hb}=\int_{A_{\bar{1}}\cup A_{\bar{1}}}\left(\frac{p}{\rho}+gZ+\frac{\vec{C}^2}{2}\right)\rho\vec{C}\cdot\vec{n}dA-\int_{A_{\bar{1}}\cup A_{\bar{1}}}\rho\left[\left\{2\nu\overline{\overline{D}}+\frac{\overline{\overline{\tau_t}}}{\rho}\right\}\cdot\vec{C}\right]\cdot\vec{n}\,dA$$

As $\vec{W}^2=\vec{C}^2-2\vec{C}\cdot\vec{U}+\vec{U}^2$ and by substituting the rotalpy conservation in the power expression, the hydraulic power through the runner becomes

$$P_{hb}=\int_{A_{\bar{1}}\cup A_{\bar{1}}}\vec{C}\cdot\vec{U}\;\rho\vec{C}\cdot\vec{n}dA-\int_{A_{\bar{1}}\cup A_{\bar{1}}}\rho\left[\left\{2\nu\overline{\overline{D}}+\frac{\overline{\overline{\tau_t}}}{\rho}\right\}\cdot\vec{U}\right]\cdot\vec{n}\,dA-\int_V\rho\left\{+2\nu\overline{\overline{D}}:\overline{\overline{D}}+\frac{1}{\rho}\overline{\overline{\tau_t}}:\overline{\overline{D}}\right\}dV$$

1.3 Resulting Torque

The resulting torque generated by the flow on the runner blade is defined

$$-\vec{T}_t=\int_{\Sigma_b}\vec{X}\times\left(-p-\rho gZ\right)\vec{n}\,dA+\int_{\Sigma_b}\vec{X}\times\left(\overline{\overline{\tau}}+\overline{\overline{\tau_t}}\right)\vec{n}\,dA$$

As it is difficult to know directly the pressure and the wall stresses, a momentum balance is performed on the runner control volume.

$$\frac{D}{Dt}\int_V \vec{X}\times\rho\vec{W}\,dV = -\int_V \vec{X}\times\left(2\vec{\omega}\times\rho\vec{W}\right)dV \;-\int_V \vec{X}\times\left(\vec{\omega}\times\left(\vec{\omega}\times\rho\vec{X}\right)\right)dV$$
$$-\int_{A_1\cup A_{\bar{1}}} \vec{X}\times\left(p+\rho gZ\right)\vec{n}\,dA \;+\int_{A_1\cup A_{\bar{1}}} \vec{X}\times\left(\overline{\overline{\tau}}+\overline{\overline{\tau}}_t\right)\vec{n}\,dA$$
$$\underbrace{-\int_{\Sigma_b} \vec{X}\times\left(p+\rho gZ\right)\vec{n}\,dA \;+\int_{\Sigma_b} \vec{X}\times\left(\overline{\overline{\tau}}+\overline{\overline{\tau}}_t\right)\vec{n}\,dA}_{-\vec{T}_t}$$

By assuming a stationary flow, the resulting torque expression is deduced

$$\vec{T}_t = -\int_{\partial V}\left(\vec{X}\times\rho\vec{W}\right)\vec{W}\cdot\vec{n}\,dA$$
$$-\int_V \vec{X}\times\left(2\vec{\omega}\times\rho\vec{W}\right)dV \;-\int_V \vec{X}\times\left(\vec{\omega}\times\left(\vec{\omega}\times\rho\vec{X}\right)\right)dV$$
$$-\int_{A_1\cup A_{\bar{1}}} \vec{X}\times\left(p+\rho gZ\right)\vec{n}\,dA \;+\int_{A_1\cup A_{\bar{1}}} \vec{X}\times\left(\overline{\overline{\tau}}+\overline{\overline{\tau}}_t\right)\vec{n}\,dA$$

According to the transformed power definition $P_t = \vec{T}_t\cdot\vec{\omega}$ and substituting the relation $\vec{U} = \vec{\omega}\times\vec{X} = -\vec{X}\times\vec{\omega}$, the transformed power is given by

$$P_t = -\int_{\partial V} -\rho\vec{W}\cdot\left(-\vec{U}\right)\vec{W}\cdot\vec{n}\,dA$$
$$-\int_V -\left(2\vec{\omega}\times\rho\vec{W}\right)\cdot\left(-\vec{U}\right)dV \;-\underbrace{\int_V -\rho\left(\vec{\omega}\times\left(-\vec{U}\right)\right)\cdot\left(-\vec{U}\right)dV}_{=0}$$
$$-\underbrace{\int_{A_1\cup A_{\bar{1}}} -\left(p+\rho gZ\right)\vec{n}\cdot\left(-\vec{U}\right)dA}_{=0} \;+\int_{A_1\cup A_{\bar{1}}} -\left(\overline{\overline{\tau}}+\overline{\overline{\tau}}_t\right)\vec{n}\cdot\left(-\vec{U}\right)dA$$

As $\vec{W} = \vec{C} - \vec{U}$, the first term can be expressed as

$$-\int_{\partial V} -\rho\vec{W}\cdot\left(-\vec{U}\right)\vec{W}\cdot\vec{n}\,dA = -\int_{\partial V}\rho\left(\vec{C}\cdot\vec{U}\right)\vec{W}\cdot\vec{n}\,dA + \int_V \vec{\nabla}\cdot\rho\left(\vec{U}^2\vec{W}\right)dV$$

and with the relation $\vec{\nabla}\vec{U}^2 = 2\vec{U}\times\vec{\omega}$, it becomes

$$\int_{\partial V} -\rho\vec{W}\cdot\left(-\vec{U}\right)\vec{W}\cdot\vec{n}\,dA = -\int_{\partial V}\rho\left(\vec{C}\cdot\vec{U}\right)\vec{W}\cdot\vec{n}\,dA + \int_V \rho\left(2\vec{\omega}\times\vec{W}\right)\cdot\vec{U}\,dV$$

Therefore, the transformed power is reduced to

$$P_t = -\int_{A_1\cup A_{\bar{1}}}\rho\left(\vec{C}\cdot\vec{U}\right)\vec{C}\cdot\vec{n}\,dA + \int_{A_1\cup A_{\bar{1}}}\left(\overline{\overline{\tau}}+\overline{\overline{\tau}}_t\right)\vec{n}\cdot\vec{U}\,dA$$

The power balance in the runner is expressed for both turbine and pump mode

$$P_{hb} = \begin{cases} P_t + P_{rb} & \text{for turbine mode} \\ P_t - P_{rb} & \text{for pump mode} \end{cases}$$

Consequently, the expression of the dissipated power in the runner by turbulence and viscosity is expressed as

$$\pm P_{rb} = \int_V \rho\left\{+2\nu\overline{\overline{D}}:\overline{\overline{D}} + \frac{1}{\rho}\overline{\overline{\tau}}_t:\overline{\overline{D}}\right\}dV$$

1.4 Global Euler Equation

From the transformed power expression expressed in the precedent section, the transformed specific energy is defined as

$$E_t = \frac{P_t}{\rho \cdot Q_t} = -\frac{\int\limits_{A_1 \cup A_{\bar{1}}} \left(\vec{C} \cdot \vec{U}\right) \vec{C} \cdot \vec{n}\, dA}{\rho \cdot Q_t} + \frac{\int\limits_{A_1 \cup A_{\bar{1}}} \left(\overline{\overline{\tau}} + \overline{\overline{\tau}}_t\right) \vec{n} \cdot \vec{U}\, dA}{\rho \cdot Q_t}$$

This expression is expressed in a local form by considering a particular streamline. For example, the transformed specific energy is defined using the external streamline between 1 and $\bar{1}$.

$$E_t = k_{Cu1e}\left(\vec{C}_{1e} \cdot \vec{U}_{1e}\right) - k_{Cu\bar{1}e}\left(\vec{C}_{\bar{1}e} \cdot \vec{U}_{\bar{1}e}\right) - E_{rb}$$

As the Euler equation is defined for the mean flow, the local form uses distribution coefficients

$$k_{Cu1e} = \frac{\int\limits_{A_1} \left(\vec{C} \cdot \vec{U}\right) \vec{C} \cdot \vec{n}\, dA}{\rho \cdot Q_t \left(\vec{C}_{1e} \cdot \vec{U}_{1e}\right)} \quad \text{and} \quad k_{Cu\bar{1}e} = \frac{\int\limits_{A_{\bar{1}}} \left(\vec{C} \cdot \vec{U}\right) \vec{C} \cdot \vec{n}\, dA}{\rho \cdot Q_t \left(\vec{C}_{\bar{1}e} \cdot \vec{U}_{\bar{1}e}\right)}$$

to represent le velocity distribution effect. Although the specific energy losses is often neglected, its expression is given by

$$E_{rb} = -\frac{\int\limits_{A_1 \cup A_{\bar{1}}} \left(\overline{\overline{\tau}} + \overline{\overline{\tau}}_t\right) \vec{n} \cdot \vec{U}\, dA}{\rho \cdot Q_t}$$

The distribution coefficient value for a uniform cylindrical velocity profile at inlet is computed

$$k_{Cu1e} = \frac{\int\limits_{A_1} \left(\vec{C} \cdot \vec{U}\right) \vec{C} \cdot \vec{n}\, dA}{Q_t \left(\vec{C}_{1e} \cdot \vec{U}_{1e}\right)} = \frac{\left(\vec{C}_{1e} \cdot \vec{U}_{1e}\right) \overbrace{\int\limits_{A_1} \vec{C} \cdot \vec{n}\, dA}^{Q_t}}{Q_t \left(\vec{C}_{1e} \cdot \vec{U}_{1e}\right)} = 1$$

In the case of a solid body rotation at outlet, the cylindrical velocity profile depends from the radius as

$$\frac{Cu}{Cu_{\bar{1}e}} = \frac{R}{R_{\bar{1}e}} \quad \text{and} \quad \frac{U}{U_{\bar{1}e}} = \frac{R}{R_{\bar{1}e}}$$

Therefore, the distribution coefficient is computed

$$\begin{aligned} k_{Cu\bar{1}e} &= \frac{\int\limits_{A_{\bar{1}}} \left(\vec{C} \cdot \vec{U}\right) \cdot \vec{C} \cdot \vec{n}\, dA}{Q_t \left(\vec{C}_{\bar{1}e} \cdot \vec{U}_{\bar{1}e}\right)} = \frac{Cm_{\bar{1}e}}{Q_t} \frac{1}{R_{\bar{1}e}^2} \int\limits_{A_{\bar{1}}} R^2\, dA = \frac{Cm_{\bar{1}e}}{Q_t} \frac{1}{R_{\bar{1}e}^2} \int\limits_0^{R_{\bar{1}e}} 2\pi R^3\, dR \\ &= 2\pi \frac{Cm_{\bar{1}e}}{Q_t} \frac{1}{R_{\bar{1}e}^2} \frac{R_{\bar{1}e}^4}{4} = \frac{\pi R_{\bar{1}e}^2 Cm_{\bar{1}e}}{Q_t} \cdot \frac{1}{2} = \frac{1}{2} \end{aligned}$$

2 VELOCITY TRIANGLES

2.1 Turbine Operating Conditions

In this section, an analysis of the specific energy evolution through the runner is performed by neglecting the losses between 1 and $\bar{1}$.

$$E_t = k_{Cu1e} \cdot Cu_{1e} \cdot U_{1e} - k_{Cu\bar{1}e} \cdot Cu_{\bar{1}e} \cdot U_{\bar{1}e}$$

The Figure 22 provides examples of velocity triangle for different specific speed and for the best efficiency operating condition (assumption $Cu_{\bar{1}e} = 0$).

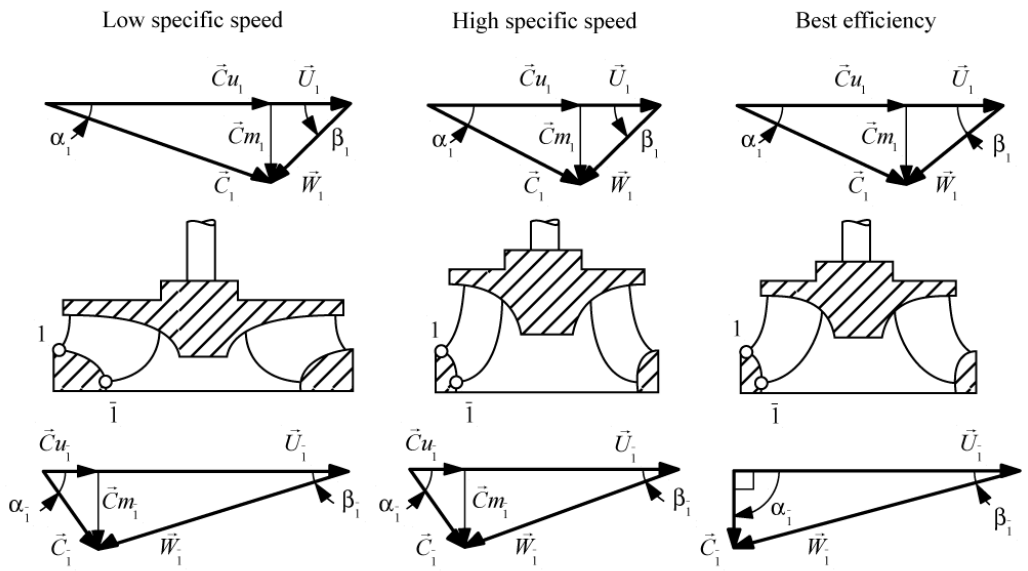

Figure 22 - Velocity triangle at turbine inlet and outlet, for different operating points

From the velocity triangle, the peripheral component of absolute velocity Cu can be expressed in function of the meridional component Cm, absolute flow angle α_1 and relative flow angle $\beta_{\bar{1}}$, which is related to the blade angle at runner outlet.

$$Cu_{1e} = \frac{Cm_{1e}}{\tan\alpha_{1e}} \quad \text{and} \quad Cu_{\bar{1}e} = U_{\bar{1}e} - \frac{Cm_{\bar{1}e}}{\tan\beta_{\bar{1}e}}$$

Moreover, the meridional component of absolute velocity can also be expressed in function of the runner discharge Q_t and distributions coefficient k_{Cm}.

$$Cm_{1e} = \frac{Q_t}{k_{Cm1e} \cdot A_1} \quad \text{and} \quad Cm_{\bar{1}e} = \frac{Q_t}{k_{Cm\bar{1}e} \cdot A_{\bar{1}}}$$

Therefore, the transformed specific energy in the runner depends only on the absolute flow angle, blade angle at runner outlet, discharge and rotational speed (because $U_{\bar{1}e} = \omega \cdot R_{\bar{1}e}$).

$$E_t = -k_{Cu\bar{1}e} \cdot U_{\bar{1}e}^2 + \left[\frac{k_{Cu1e}}{k_{Cm1e}} \frac{R_{1e}}{R_{\bar{1}e}} \frac{A_{\bar{1}}}{A_1} \frac{1}{\tan\alpha_{1e}} + \frac{k_{Cu\bar{1}e}}{k_{Cm\bar{1}e}} \frac{1}{\tan\beta_{\bar{1}e}} \right] \frac{Q_t \cdot U_{\bar{1}e}}{A_{\bar{1}}}$$

In order to simplify the following notation, a uniform distribution of Cu and Cm is assumed for the runner inlet and outlet.

$$E_t = -U_{\bar{1}}^2 + \left[\frac{R_1}{R_{\bar{1}}}\frac{A_{\bar{1}}}{A_1}\frac{1}{\tan\alpha_1} + \frac{1}{\tan\beta_{\bar{1}}}\right]\frac{Q_t \cdot U_{\bar{1}}}{A_{\bar{1}}}$$

The evolution of transformed specific energy in the runner is given in Figure 23 for a constant rotational speed.

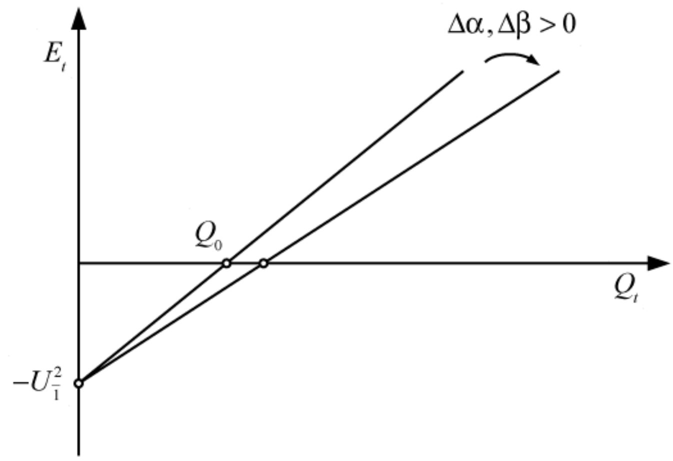

Figure 23 - Hydraulic characteristic of a turbine runner for a given rotational speed

The transformed power expression is deduced from the specific energy

$$P_t = \rho \cdot Q_t \cdot E_t = -\rho \cdot Q_t \cdot U_{\bar{1}}^2 + \left[\frac{R_1}{R_{\bar{1}}}\frac{A_{\bar{1}}}{A_1}\frac{1}{\tan\alpha_1} + \frac{1}{\tan\beta_{\bar{1}}}\right]\frac{\rho \cdot Q_t^2 \cdot U_{\bar{1}}}{A_{\bar{1}}}$$

Therefore, the torque on the shaft is given by

$$T_t = \frac{P_t}{\omega} = -\rho \cdot Q_t \cdot \omega \cdot R_{\bar{1}}^2 + \left[\frac{R_1}{R_{\bar{1}}}\frac{A_{\bar{1}}}{A_1}\frac{1}{\tan\alpha_1} + \frac{1}{\tan\beta_{\bar{1}}}\right]\frac{\rho \cdot Q_t^2 \cdot R_{\bar{1}}}{A_{\bar{1}}}$$

As the evolutions of power and torque depend on both rotational speed and discharge, these expressions are given on Figure 24 for a constant rotational speed and for a constant discharge.

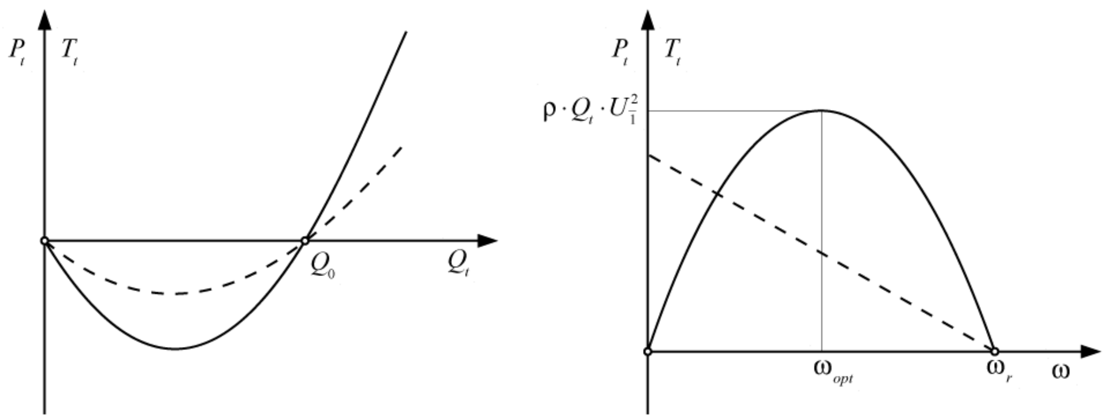

Figure 24 - Evolutions of transformed power (filled) and torque (dashed) in a turbine runner for a given rotational speed (left) and discharge (right)

For a given rotational speed the machine needs a discharge greater than Q_0 in order to produce energy.

For a given discharge, the machine produces a maximum power for an optimal rotational speed

$$\omega_{opt} = \frac{1}{2}\left(\frac{R_{1e}}{R_{\bar{1}e}} \frac{A_{\bar{1}}}{A_1} \frac{1}{\tan\alpha_{1e}} + \frac{1}{\tan\beta_{\bar{1}e}} \right) \frac{Q_t}{A_{\bar{1}} \cdot R_{\bar{1}}}$$

Consequently, the value of maximum power and energy are given by

$$P_{t,\max} = \rho \cdot Q_t \cdot U_{\bar{1}}^2 \quad \text{and so} \quad E_{t,\max} = U_{\bar{1}}^2$$

From the torque expression, the runaway speed is deduced as

$$\omega_r = \left(\frac{R_1}{R_{\bar{1}}} \frac{A_{\bar{1}}}{A_1} \frac{1}{\tan\alpha_1} + \frac{1}{\tan\beta_{\bar{1}}} \right) \frac{Q_t}{A_{\bar{1}} \cdot R_{\bar{1}}}$$

2.2 Pump Operating Conditions

In this section a uniform distribution of velocities at pump outlet and an axial flow at the pump inlet are assumed

$$k_{Cu} = 0, \quad k_{Cm} = 0, \quad \alpha_{\bar{1}} = \frac{\pi}{2} \quad \text{and} \quad Cu_{\bar{1}} = 0 \quad \text{so} \quad E_t = Cu_1 \cdot U_1$$

The Figure 25 provides examples of velocity triangle for different specific speed and for the maximum power operating condition.

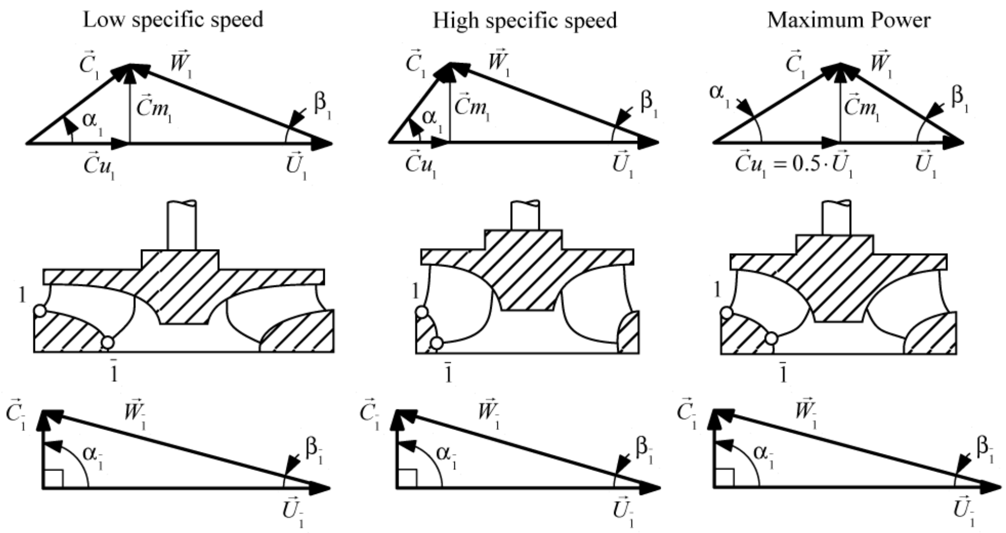

Figure 25 - Velocity triangle at pump inlet and outlet, for different operating points

From the velocity triangle, the peripheral component of absolute velocity Cu can be expressed in function of the meridional component Cm and relative flow angle β_1, which is related to the blade angle at runner outlet.

$$Cu_1 = U_1 - \frac{Cm_1}{\tan\beta_1}$$

Moreover, the meridional component of absolute velocity can also be expressed in function of the impeller discharge Q_t, which is defined negative in pump mode.

$$Cm_1 = -\frac{Q_t}{A_1}$$

Therefore, the transformed specific energy in the impeller depends only on the blade angle at outlet, discharge and rotational speed (because $U_1 = -\omega \cdot R_1$ with $\omega < 0$ in pump mode).

$$E_t = U_1^2 + \frac{1}{\tan\beta_1}\frac{Q_t \cdot U_1}{A_1}$$

The evolution of transformed specific energy in the impeller is given in Figure 26 for a constant rotational speed.

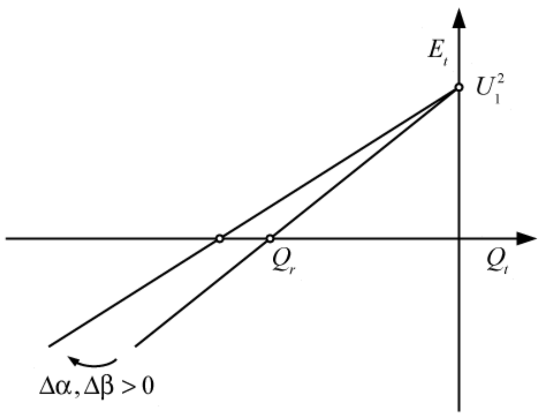

Figure 26 - Hydraulic characteristic of a pump impeller for a given rotational speed

The transformed power expression is deduced from the specific energy

$$P_t = \rho \cdot Q_t \cdot E_t = \rho \cdot Q_t \cdot U_1^2 + \frac{1}{\tan\beta_1}\frac{\rho \cdot Q_t^2 \cdot U_1}{A_1}$$

Therefore, the torque on the shaft is given by

$$T_t = \frac{P_t}{\omega} = \rho \cdot Q_t \cdot \omega \cdot R_1^2 + \frac{1}{\tan\beta_1}\frac{\rho \cdot Q_t^2 \cdot R_1}{A_1}$$

As the evolutions of power and torque depend on both rotational speed and discharge, these expressions are given on Figure 27 for a constant rotational speed and for a constant discharge.

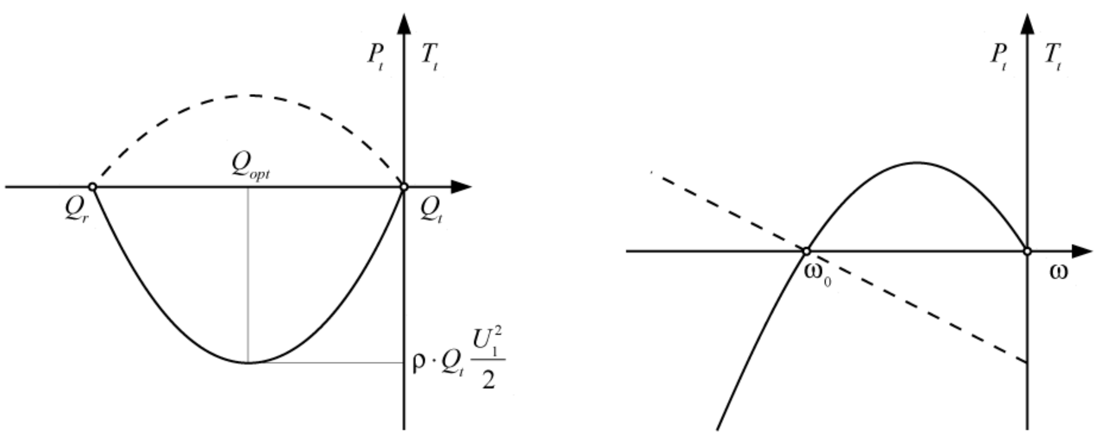

Figure 27 - Evolutions of transformed power (filled) and torque (dashed) in a pump impeller for a given rotational speed (left) and discharge (right)

For a given discharge, the machine needs a rotational speed greater than ω_0 in order to store energy.

For a given rotational speed, the machine store a maximum power for an optimal discharge

$$Q_{opt} = \frac{1}{2} \cdot U_1 \cdot A_1 \cdot \tan\beta_1$$

Consequently, the value of maximum power and energy are given by

$$P_{t,\max} = \rho \cdot Q_t \cdot \frac{U_1^2}{2} \quad \text{and so} \quad E_{t,\max} = \frac{U_1^2}{2}$$

From the torque expression, the runaway discharge is deduced as

$$Q_r = U_1 \cdot A_1 \cdot \tan\beta_1$$

VI References

COMOLET, R., *Mécanique expérimentale des fluides* : Tome 2, Dynamique des fluides réels, turbomachines, Paris : Masson, 1994 cote LMH FLUI 097/V2