Networked Control Systems (ME-427)- Exercise session 5

Prof. G. Ferrari Trecate

1. **Stabilizing state-feedback design.** Consider the NCS in Figure 1,

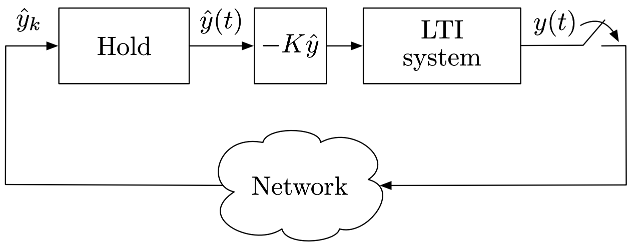


Figure 1: Networked control system

where the LTI system is the first-order model

$$\begin{cases} \dot{x} = -x - 5u \\ y = x \end{cases}.$$

Let $T = 0.5$ and $\tau = 0.2$ be the nominal sampling time and network induced delay, respectively. Find by hand all values of K stabilizing the NCS.

Hint: Recall the Jury's criterion: the roots of $\phi(\lambda) = \lambda^2 + \alpha\lambda + \beta$ verify $|\lambda| < 1$ if and only if

$$\begin{aligned} \beta &> -\alpha - 1 \\ \beta &> \alpha - 1 \\ \beta &< 1 \end{aligned}.$$

Solution: Given that $A = -1$, $B = -5$, $C = 1$, $T = 0.5$, and $\tau = 0.2$, one has

$$\begin{aligned} \psi(T,\tau) &= \begin{bmatrix} e^{AT} - BK\Gamma(T-\tau) & -BKe^{A(T-\tau)}\Gamma(\tau) \\ 1 & 0 \end{bmatrix} \\ &= \begin{bmatrix} e^{-0.5} + 5K\Gamma(0.3) & 5Ke^{-0.3}\Gamma(0.2) \\ 1 & 0 \end{bmatrix} \end{aligned}.$$

Approximately,

$$\begin{aligned} e^{-0.5} &= 0.6065 \\ e^{-0.3} &= 0.7408 \\ \Gamma(s) = 1 - e^{-s} \rightarrow \Gamma(0.3) &= 0.2592 \quad \Gamma(0.2) = 0.18 \end{aligned}.$$

Therefore,

$$\psi(0.5, 0.2) = \begin{bmatrix} 0.6065 + K \times 1.2939 & K \times 0.6716 \\ 1 & 0 \end{bmatrix}.$$

By computing the characteristic polynomial of ψ and using Jury's conditions, one has

$$-K \times 0.6716 > 0.6065 + K \times 1.2939 - 1 \quad \text{(1a)}$$

$$-K \times 0.6716 > -0.6065 - K \times 1.2939 - 1 \quad \text{(1b)}$$

$$-K \times 0.6716 < 1 \quad \text{(1c)}$$

- From (1a), $-K\underbrace{(0.6716+1.2939)}_{1.9655} > -0.3935 \rightarrow K < \frac{0.3935}{1.9655} = 0.1595$
- From (1b), $-K\underbrace{(0.6716-1.2939)}_{-0.6223} > -1.6065 \rightarrow K > \frac{-1.6065}{0.6223} = -2.5816$
- From (1c), $-K < \frac{1}{0.6716} \rightarrow K > -1.4890$

Conclusion: $K \in (-1.4890, 0.1595)$.

2. **Remote control with delay compensation.** Consider the cart-stick balancer system in Figure 2,

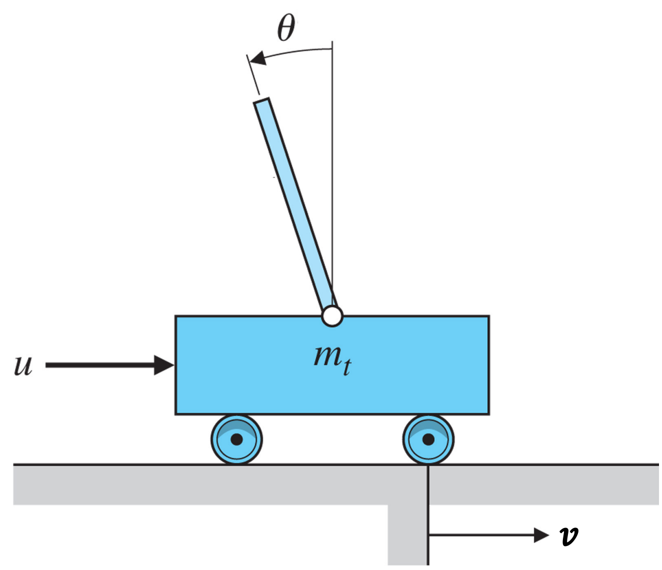


Figure 2: Cart-stick balancer.

with states x_1: stick angle $\theta/10$, x_2: stick angular velocity $\dot{\theta}$, x_3: cart velocity v. The input u is the voltage to the motor driving the wheels. The measured ouput $y = \theta$ is the stick angle. The control goal is to mantain the stick vertical by moving the cart through u. Consider the remote controller with delay compensation defined in Figure 3 with uniform sampling period $T = 0.1$ s.

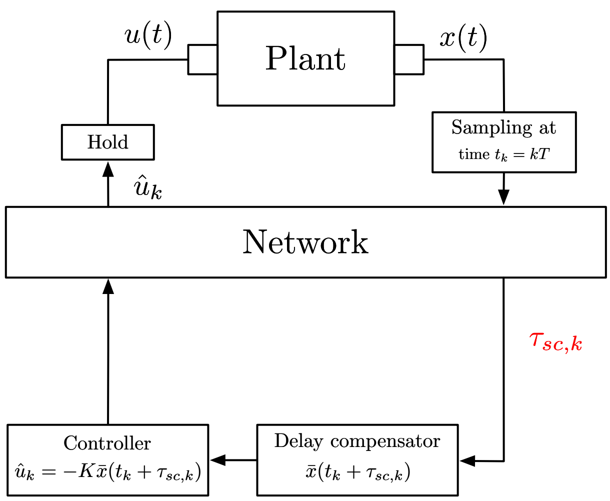


Figure 3: Remote controller with delay compensation.

Setting $x^T = \begin{bmatrix} x_1 & x_2 & x_3 \end{bmatrix}$ one has the linearized model

$$\dot{x} = \underbrace{\begin{bmatrix} 0 & 1 & 0 \\ 31.33 & 0 & 0.016 \\ -31.33 & 0 & -0.216 \end{bmatrix}}_{A} x + \underbrace{\begin{bmatrix} 0 \\ -0.649 \\ 8.649 \end{bmatrix}}_{B} u$$

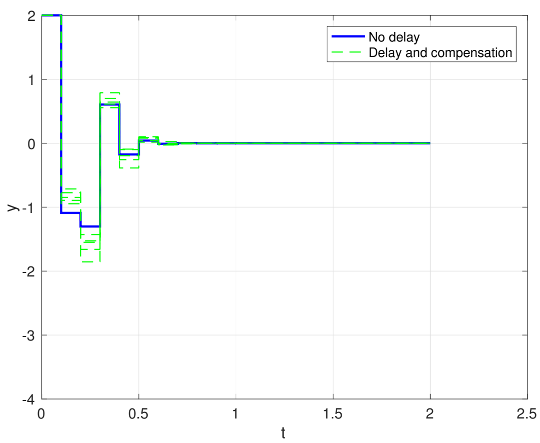
Recall from the lectures that, setting $\delta_k = T + \tau_{sc,k+1} - \tau_{sc,k}$, the closed-loop NCS model is given by the discrete-time system

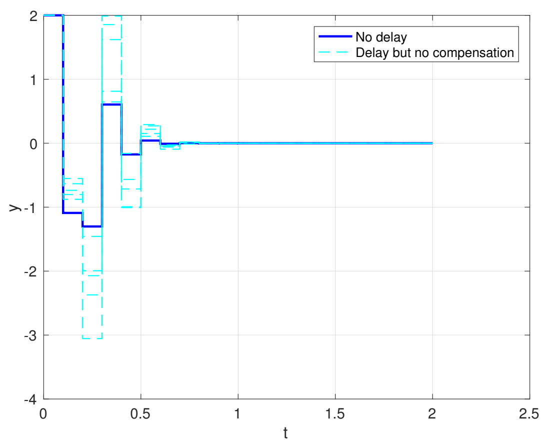
$$\begin{aligned} & x(t_{k+1} + \tau_{sc,k+1}) = \tilde{A}_k x(t_k + \tau_{sc,k}) \\ & \tilde{A}_k = e^{A\delta_k} - \Gamma(\delta_k) BK, \quad \Gamma(\delta_k) = \int_0^{\delta_k} e^{As} ds \end{aligned} \tag{2}$$

A simulator of the NCS is provided by the file `NCS_car_stick_balancer.m` available on moodle. The `.m` file is configured for running the *nominal experiment* defined by

$t_0 = 0$
$x(0) = \begin{bmatrix} 0.2 & 0.3 & -0.5 \end{bmatrix}^T$
$K = \begin{bmatrix} -556.1829 & -208.3171 & -12.9905 \end{bmatrix}$

and it produces two plots, similar to the ones below





Both figures display the angular position of the stick (in degrees) and

- the blue line is the ideal NCS with $\tau_{sc,k} = 0$, $k = 0, 1, \ldots$
- other lines are obtained extracting the values of $\tau_{sc,k}$ five times from the uniform distribution on $[\tau_{min}, \tau_{max}]$ (these parameters can be specified at the beginning of the file) and
  - using a delay-compensation controller, as seen in the lectures (green lines);
  - using an uncompensated controller (cyan lines).

(a) **Familiarize with the simulator.** Perform the following experiments.

i. The default gain K has been produced by nominal design (see the lecture slides for the precise meaning of "nominal design") for placing the closed-loop eigenvalues in -0.42, -0.49, and -0.56. Check stability in simulation by looking at the plots for
   - $\tau_{min} = \tau_{max} = 0$
   - $\tau_{min} = \tau_{max} = \tau < T$. In this case performances are different if using delay compensation or not. Why ?

Solution: As seen in the lectures, considering the sampling interval $[t_k, t_{k+1}]$, the compensated controller produces the best possible control actions in the sub-interval $[t_k + \tau_{sc,k}, t_{k+1}]$, while the uncompensated controller does not.

ii. Assume that performance of the NCS is acceptable if

$$|\theta(t_k)| < 15, \ \forall k = 0, 1, \ldots \tag{3}$$

By increasing $\tau = \tau_{min} = \tau_{max}$, find $\bar{\tau}$ such that (3) is verified by the delay-compensated controller, but not by the uncompensated controller.

Solution: $\bar{\tau} \simeq 0.25T$ produces the desired behavior.

iii. Run simulations with $\tau_{sc,k}$ generated randomly in $[0, \bar{\tau}]$. The system behavior gets worse (for instance, oscillations are less dampened). Can you guess why ?

Solution: Nominal design implies that the controller has been synthesized for $\tau_{sc,k}$ constant. This assumption, however, is not fulfilled in the simulation.

iv. Can the NCS become unstable by increasing $\bar{\tau}$ in the previous point ? Run simulations for answering.

Solution: Yes, for $\bar{\tau} \simeq 0.45T$ unstable behaviors start appearing. Note that the car-stick-balancer is open-loop unstable. This is why network delays can have a particularly detrimental effect.

(b) **Control design.** Network delays that can be tolerated for stability and performance depend on the eigenvalues of the nominal NCS. To see this,

- design a nominal gain K for placing the closed-loop eigenvalues in -0.12, -0.14, and -0.16 (so that NCS transients are shorter than before).

  Hint: Fill in the missing code in `NCS_car_stick_balancer.m` for computing K.

  Solution: See the MatLab code on moodle.

- run simulations with $\tau_{sc,k}$ extracted randomly in $[0, \bar{\tau}]$ and increase $\bar{\tau}$ until unstable behaviors start appearing. How does $\bar{\tau}$ compare with the result of point (2(a)iv) above?

  Solution: Instability starts appearing for $\bar{\tau} \simeq 0.85T$. Therefore, compared to point (2(a)iv), the new controller brings more robustness to delays.