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Problem 1.

When a question has more than one correct answer, mark all statements that are true.

1) Suppose that Y is a robust control invariant set for the system $x^+ = Ax + Bu + w$ subject to the state and input constraints X and U and the noise $w \in W$. For which of the following scenarios is Y still a robust invariant set for the resulting system?

   - ◯ Disturbance set changes to $2W$
   - ◯ Disturbance set changes to $0.5W$
   - ◯ Input constraint set changes to $2U$
   - ◯ Input constraint set changes to $U + 1$
   - ◯ State constraint set changes to $-X$

2) Consider the system $x^+ = Ax + u + w$ subject to the constraints $\|x\| \leq 1$ and $\|u\| \leq 1$ and a bounded disturbance w, $\|w\| \leq 0.1$. If $A = \begin{bmatrix} 1 & 0 \\ 1 & 1.2 \end{bmatrix}$, what is the set of feasible states of an open-loop robust MPC problem as the horizon goes to infinity?

   - ◯ $\emptyset$
   - ◯ It won't converge
   - ◯ $\mathbb{R}^n$
   - ◯ A circle

3) Let $\pi(x) = K(x - z_0^\star(x)) + v_0^\star(x)$ be a tube-MPC control law, designed for the system $x^+ = Ax + Bu + w$ which is subject to the constraints $x \in X$, $u \in U$ and the disturbance $w \in W$. Let the set $\mathcal{E}$ be the minimum robust invariant set for the system $x^+ = (A + BK)x + w$ for $w \in W$. Suppose now that this controller is applied to the system but that the observed noise only lies within the set $\bar{W} = 0.5W$.

   What is the set to which the closed-loop system will converge to in the limit?

   - ◯ Not enough information
   - ◯ The system does not converge
   - ◯ $\mathcal{E}$
   - ◯ The terminal set of the MPC control law
   - ◯ $0.5\mathcal{E}$

4) Consider the system $x^+ = Ax+Bu$, which is subject to the constraints $x \in X$ and $u \in U$. Let $u^\star(x) = \text{argmin}\{\|u\| \mid Ax+Bu \in C_\infty,\ u \in U\}$, where C_∞ is the maximum control invariant set for this system. Let the state input sequence $\{x_i, u_i\}$ be generated by this system with $x_0 \in C_\infty$. Mark the correct statements.

   - ◯ $x_i \in X$ and $u_i \in U$ for all i
   - ◯ There may exist an $x_0 \in C_\infty$ such that $x_i \notin X$ for some i
   - ◯ The system is asymptotically stable
   - ◯ $\lim_{i\to\infty} \|x_i\|_\infty = 0$
   - ◯ $\lim_{i\to\infty} \|u_i\|_1 = 0$

5) Consider the following optimization problem, which has a local optimizer at $x = x^\star$

$$\begin{aligned} &\min \ c^T x \\ &\text{s.t. } x^T Q x + q^T x \geq 1 \end{aligned}$$

Which conditions guarantee that $x^\star$ is also a global minimum?

- ◯ Q positive definite
- ◯ Q positive semi-definite
- ◯ Q negative semi-definite
- ◯ $c > 0$
- ◯ $q > 0$

6) Consider the quadratic programming problem

$$\begin{aligned} p^\star = \min \ & \frac{1}{2} z^T H z + q^T z \\ \text{s.t.} \ & \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} z \leq d \\ & \begin{bmatrix} 1 & 10 \\ 2 & 20 \end{bmatrix} z = b \end{aligned}$$

i) Is it possible to choose d and b so that the feasible set is empty?

- ◯ Yes
- ◯ No
- ◯ Depends on H and q

ii) Is it possible to choose d and b so that the feasible set is nonconvex?

- ◯ Yes
- ◯ No
- ◯ Depends on whether H is symmetric

iii) Let $z \in \mathbb{R}^2$. Which of the following H matrices results in a nonconvex optimization problem?

- ◯ $\begin{bmatrix} 0.9 & 0 \\ 0 & 3 \end{bmatrix}$
- ◯ $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$
- ◯ $\begin{bmatrix} 0.8 & 0 \\ 0 & -0.8 \end{bmatrix}$
- ◯ $\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$
- ◯ None of the above

iv) Does there exist a bounded matrix H, a vector q and vectors d and b such that

$p^* = -\infty$ ◯ Yes ◯ No ◯ Not enough information

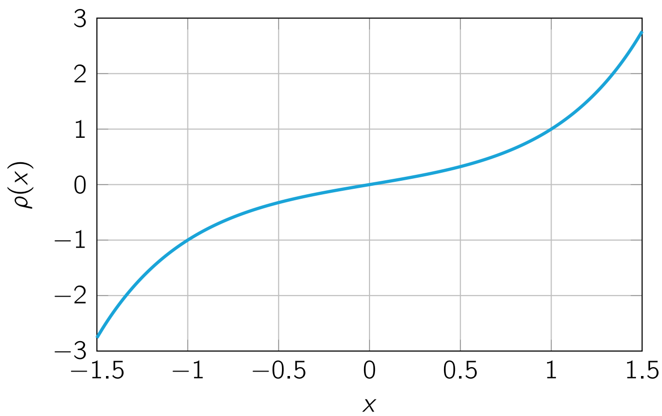
$p^* = \infty$ ◯ Yes ◯ No ◯ Not enough information

7) For which of the following cases is the optimization problem convex?

$$\min \; f(x) \quad \text{s.t. } g(x) \geq 5$$

- ◯ $f(x) = x^T x - \|x\|_1$ $\quad g(x) = x^T x$
- ◯ $f(x) = -x^T x + \|x\|_1$ $\quad g(x) = -x^T x$
- ◯ $f(x) = \begin{bmatrix} 1 & 2 \end{bmatrix} x$ $\quad g(x) = \|-4x\|_\infty$
- ◯ $f(x) = \begin{bmatrix} 1 & 2 \end{bmatrix} x$ $\quad g(x) = e^{-x_1} + e^{-x_2}$
- ◯ None of the above

8) What is the maximum invariant set of the scalar system $x^+ = \rho(x)$ contained in the set $X = \{x \,|\, \|x\|_\infty \leq 1.5\}$ where the function $\rho(x)$ is plotted below



- ◯ $\{x \,|\, -1 \leq x \leq 1\}$
- ◯ X
- ◯ $\{x \,|\, x \geq 0\}$
- ◯ $\emptyset$
- ◯ $\mathbb{R}$
- ◯ None of the above

9) Consider the system $x^+ = 4x + 5 + u$ subject to the constraints $|u| \leq 5$ and $|x| \leq 1$.

i) What is the pre-set of the set $\Omega = \{x \,|\, \alpha \leq x \leq \beta\}$?

◯ $[\alpha - 10, \beta]$ ◯ $[\alpha, \beta + 5]$ ◯ $\frac{1}{4}[\alpha - 10, \beta]$ ◯ $\frac{1}{2}[-\alpha, -\beta]$ ◯ $\mathbb{R}$

ii) What is the maximum control invariant set?

◯ $[-1, 1]$ ◯ $\frac{1}{4}[-1, 1]$ ◯ $\emptyset$ ◯ $[-1, 0]$ ◯ $\mathbb{R}$

10) Consider the system $x^+ = 2x + 1 + u$ subject to the constraints $u \leq 0$. Which of the following sets are control invariant?

- ◯ $\{0, 1, 2\}$
- ◯ $\{x \,|\, \|x\| \leq 1\}$
- ◯ $\{x \,|\, x \leq 0\}$
- ◯ $\{x \,|\, x \geq 0\}$
- ◯ $\mathbb{R}$

11) Let x_0 be a state in the maximum control invariant set $\mathcal{C}_\infty$ for the system $x^+ = f(x, u)$ under the constraints $(x, u) \in \mathbb{X} \times \mathbb{U}$. Is it possible that there exists a controller $\kappa(x)$ such that the 3-step sequence $\{x_0, x_1, x_2\}$ is in $\mathbb{X}$ and $\{\kappa(x_0), \kappa(x_1)\}$ is in $\mathbb{U}$, where $x_1 = f(x_0, \kappa(x_0))$ and $x_2 = f(x_1, \kappa(x_1))$?

◯ Yes ◯ No

12) Consider the following standard MPC problem which generates the control law $\pi_N(x)$

$$
\begin{aligned}
J_N^\star(x) = \min \ & \sum_{i=0}^{N-1} l(x_i, u_i) \\
\text{s.t.} \quad & x_{i+1} = Ax_i + Bu_i \\
& (x_i, u_i) \in X \times U \\
& x_N = 0
\end{aligned}
$$

i) Assume for all of the following cases that the state x is feasible and non-zero and mark the correct statements.

- ◯ $J_N^\star(x) < J_{N-1}^\star(x)$
- ◯ $J_{N-1}^\star(x) < J_N^\star(Ax + B\pi_N(x))$
- ◯ $J_{N-1}^\star(x) > J_N^\star(Ax + B\pi_{N-1}(x))$
- ◯ $J_N^\star(Ax + B\pi_{N-1}(x)) = J_{N-1}^\star(x)$
- ◯ $J_{N-1}^\star(Ax + B\pi_N(x)) = J_N^\star(x) - l(x, \pi_N(x))$
- ◯ $J_{N-1}^\star(Ax + B\pi_N(x)) < J_N^\star(x) - l(x, \pi_N(x))$

ii) Let $\{z_i^N(x)\}$ be the closed-loop sequence generated by the dynamic system $z^+ = Az + B\pi_N(z)$ starting at state x, and the function $\bar{J}_N(x) = \sum_{i=0}^{\infty} l(z_i^N(x), \pi_N(z_i^N(x)))$ be the resulting closed-loop cost. Mark all statements that are correct for all feasible, non-zero x

- ◯ $\bar{J}_N(x) \leq J_N^\star(x)$
- ◯ $\bar{J}_N(x) \leq \bar{J}_{N-1}(x)$
- ◯ $\bar{J}_N(x) \geq \bar{J}_{N-1}(x)$
- ◯ $\lim_{N\to\infty} \bar{J}_N(x) = \lim_{N\to\infty} J_{N-1}^\star(x)$
- ◯ $\lim_{N\to\infty} \bar{J}_N(x) > \lim_{N\to\infty} J_{N-1}^\star(x)$
- ◯ $\lim_{N\to\infty} \bar{J}_N(x) < \lim_{N\to\infty} J_{N-1}^\star(x)$

iii) If the state x is feasible for the MPC problem above and the controller $\pi_N(x)$ is applied, then the closed-loop system will arrive at the origin in exactly N steps.

◯ True ◯ False

13) A controller is called static if its output depends only on its input, and not on an internal state. Which of the controllers studied in class are static?

- ◯ Nominal MPC regulation
- ◯ Offset-free MPC
- ◯ Tube-MPC
- ◯ Open-loop robust MPC

14) The function V_f is a Lyapunov function for the system $x^+ = Ax$, the stage cost l is positive definite and the MPC controller $\pi(x)$ defined by the following optimization problem is recursively feasible and stabilizes the closed-loop system $x^+ = Ax + B\pi(x)$ for a prediction horizon of $N = N_0$. Mark all statements that are correct.

$$\begin{aligned} \min \ & \sum_{i=0}^{N-1} l(x_i, u_i) + V_f(x_N) \\ \text{s.t. } & x_{i+1} = Ax_i + Bu_i \quad \forall i = 0, \dots, N-1 \\ & u_i \in U \quad \forall i = 0, \dots, N-1 \end{aligned}$$

The closed-loop system...

- ◯ ... is stable for a prediction horizon $N > N_0$, but it may not be recursively feasible
- ◯ ... is stable and recursively feasible for any prediction horizon $N \geq N_0$
- ◯ ... may be stable for a prediction horizon $N < N_0$, but it may not be recursively feasible
- ◯ ... is stable and recursively feasible for any prediction horizon $N \leq N_0$
- ◯ ... is recursively feasible for a prediction horizon $N < N_0$, but may not be stable

15) Consider the following MPC problems

$$\begin{aligned} J^\star_{LQR}(x_0) = \min \ & \sum_{i=0}^{N-1} l(x_i, u_i) + V_f(x_N) \\ \text{s.t. } & x_{i+1} = Ax_i + Bu_i \\ & x_i \in X, \ u_i \in U \\ & x_N \in X_{LQR} \end{aligned}$$

$$\begin{aligned} J^\star_{C_\infty}(x_0) = \min \ & \sum_{i=0}^{N-1} l(x_i, u_i) + V_f(x_N) \\ \text{s.t. } & x_{i+1} = Ax_i + Bu_i \\ & x_i \in X, \ u_i \in U \\ & x_N \in C_\infty \end{aligned}$$

where $l(x, u) = x^T Qx + u^T Ru$ and $V_f(x)$ is the optimal value function of the corresponding LQR controller. X_{LQR} is the maximum invariant set for the system $x^+ = (A+BK)x$ for the LQR control law K subject to the constraints $x \in X$ and $Kx \in U$, and C_∞ is the maximum control invariant set for the system $x^+ = Ax + Bu$ subject to the constraints $x \in X$ and $u \in U$. If the problem is infeasible, we define the optimal value to be $+\infty$. Mark all the correct statements.

- ◯ $J^\star_{LQR}(x) \leq J^\star_{C_\infty}(x)$ for all x
- ◯ $J^\star_{LQR}(x) \geq J^\star_{C_\infty}(x)$ for all x
- ◯ $J^\star_{C_\infty}(x)$ is recursively feasible
- ◯ $J^\star_{C_\infty}(x)$ is a Lyapunov function for the system $x^+ = Ax + Bu^\star_{LQR}(x)$
- ◯ The domain of $J^\star_{LQR}$ contains the domain of $J^\star_{C_\infty}$
- ◯ The domain of $J^\star_{C_\infty}$ contains the domain of $J^\star_{LQR}$

16) Give an example of a system for which you cannot use soft state constraints. Explain.

17) Give an example of a system for which you cannot use soft input constraints. Explain.

18) Let $\pi(x)$ be the MPC control law defined by the following optimization problem

$$\begin{aligned} J^\star(x) = \min \ & \sum_{i=0}^{N-1} l(x_i, u_i) + V_f(x_N) \\ \text{s.t.} \quad & x_{i+1} = Ax_i + Bu_i \\ & (x_i, u_i) \in X \times U \\ & x_N \in X_f \\ & x_0 = x \end{aligned}$$

where X, X_f and U are polyhedral sets, and l and V_f are positive definite quadratic functions that are zero at zero.

i) The control law $\pi(x)$ ...

- ◯ ... is a convex function
- ◯ ... is a piecewise affine function
- ◯ ... is an affine function
- ◯ ... is a piecewise quadratic function
- ◯ ... is a quadratic function
- ◯ ... has a polyhedral domain

ii) The value function $J^\star(x)$ ...

- ◯ ... is a convex function
- ◯ ... is a piecewise affine function
- ◯ ... is an affine function
- ◯ ... is a piecewise quadratic function
- ◯ ... is a quadratic function
- ◯ ... has a polyhedral domain

19) Two infinite horizon LQR controllers are designed with weighting matrices Q_1, R_1 used for the first one and Q_2, R_2 used for the second one. It holds that $Q_1 = 10Q_2$ and $R_1 = 0.1R_2$. Which of the following statements is true?

- ◯ The first LQR controller will tend to use less input than the second one and will thus have slower convergence to the origin.
- ◯ The second LQR controller will tend to use less input than the first one and will thus have slower convergence to the origin.
- ◯ The speed of response depends only on the choice of the Q matrix.
- ◯ None of the above

20) Consider the following finite-horizon LQR controller $u_0^\star$ applied to the system $x^+ = Ax + Bu$ in a receding horizon fashion.

$$\begin{aligned} \min \sum_{i=0}^{N-1} & x_i^T x_i + u_i^T u_i \\ \text{s.t. } & x_{i+1} = Ax_i + Bu_i \end{aligned}$$

Mark the correct statements:

- ◯ If the system is stable for $N = 5$, then it will also be stable for $N = 6$
- ◯ The system will be stable if N is at least five times larger than the largest eigenvalue of A
- ◯ The performance of the system will be better for $N = 6$ than it will be for $N = 5$
- ◯ None of the above

21) $V(x) = x^T x$ is a Lyapunov function for which of the following systems?

- ◯ $x^+ = x + u$
- ◯ $x^+ = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} x$
- ◯ $x^+ = \begin{bmatrix} 1.4 & 0 \\ 1 & 0.1 \end{bmatrix} x$
- ◯ $x^+ = 0.6x$
- ◯ None of the above

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Problem 2.

Consider the finite-horizon optimal control problem

$$V_N^\star(x_0) = \min \sum_{i=0}^{N} l(x_i, u_i) \quad \text{s.t.} \quad x_{i+1} = 3x_i + u_i$$

where the stage cost is

$$l(x, u) = \begin{pmatrix} x \\ u \end{pmatrix}^T \begin{bmatrix} 1 & 1 \\ 1 & 2 \end{bmatrix} \begin{pmatrix} x \\ u \end{pmatrix}$$

and let $\kappa_N(x)$ be the resulting MPC controller.

1) If the cost-to-go is $V_{i+1}(x) = x^T H x$, give an expression for $\kappa_i(x)$ and $V_i(x)$ as functions of H.

$\kappa_i(x) =$ ______ $V_i(x) =$ ______

2) Compute the smallest horizon N° and the controller $\kappa_{N^\circ}(x)$ such that the closed-loop system $x^+ = Ax + B\kappa_{N^\circ}(x)$ is asymptotically stable.

$N^\circ =$ ______ $\kappa_{N^\circ}(x) =$ ______

3) Suppose now that the system is subject to input constraints $|u| \leq 1$ and consider the MPC controller below

$$
\begin{aligned}
\min \; & \sum_{i=0}^{10} l(x_i, u_i) + V_{N^\circ}(x_{10}) \\
\text{s.t. } & x_{i+1} = 3x_i + u_i \quad \forall i = 0, \ldots, 9 \\
& |u_i| \leq 1 \quad\quad\quad\;\; \forall i = 0, \ldots, 9
\end{aligned}
$$

What is the set of states $x_0 \in X_0$ for which this MPC controller has a solution?

$X_0 =$ [ ]

Will the closed-loop system $x^+ = 3x + u^\star(x)$ be asymptotically stable for all $x \in X_0$, where $u^\star(x)$ is the optimal answer of the above MPC problem?

◯ Yes ◯ No

If yes, prove it. If no, explain why.

[ ]

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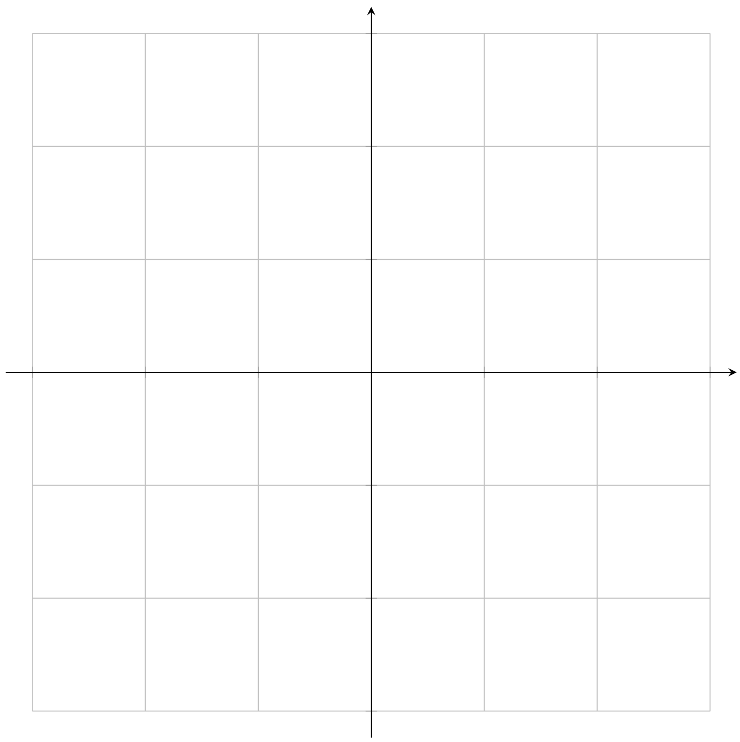
Problem 3.

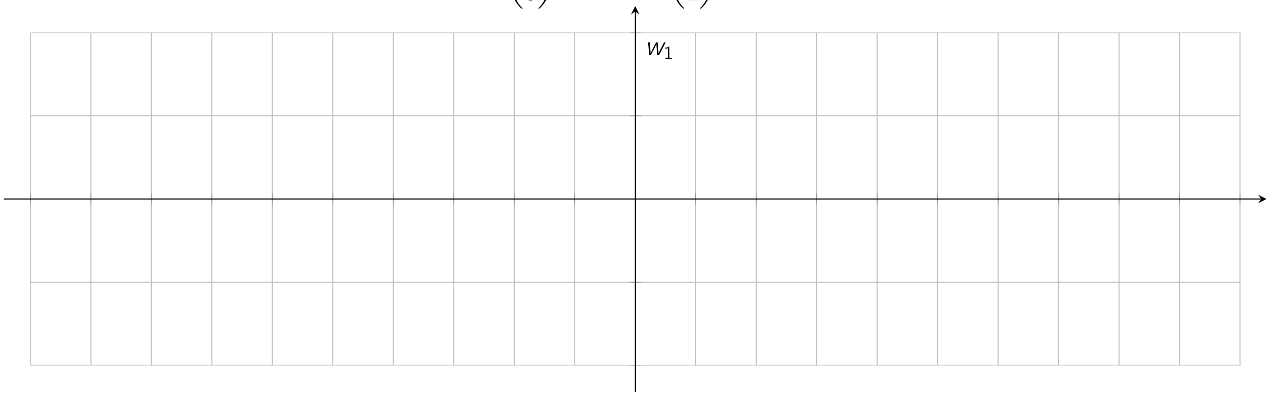
Consider the following parametric LCP

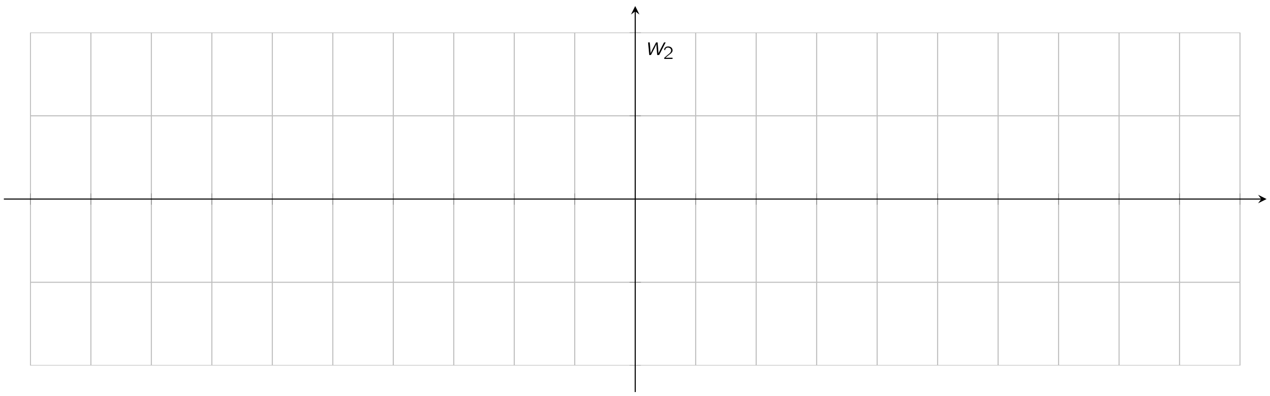
$$
\begin{aligned}
w - \begin{bmatrix} 1 & -1 \\ -1 & 2 \end{bmatrix} z &= Qx + q \\
w, z &\geq 0 \\
w^T z &= 0
\end{aligned}
$$

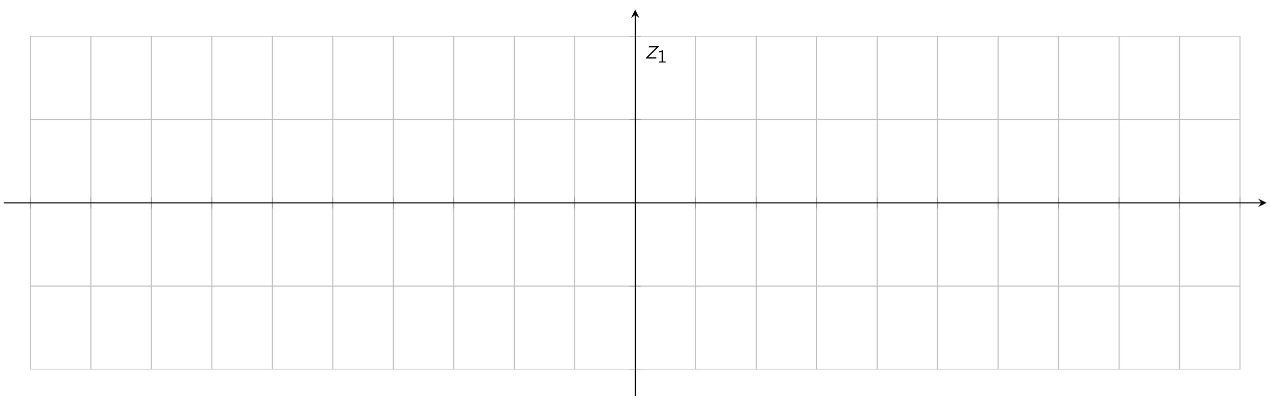
1) Give the complementarity cones for this problem and sketch them.

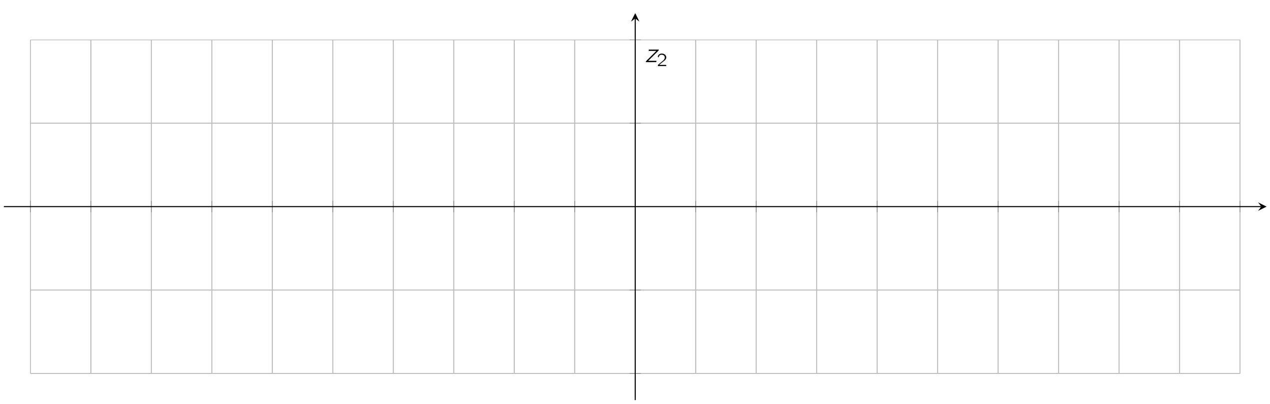


2) Sketch the solution $w(x)$ and $z(x)$ for $Q = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$ and $q = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$.









Problem 4.

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Recall the following definitions

$X \oplus Y := \{x + y \mid x \in X,\ y \in Y\}$	Minkowski sum of X and Y
$\alpha X := \{\alpha x \mid x \in X\}$	Scaling of a set
If X convex: $x_1, x_2 \in X \Rightarrow \alpha x_1 + (1-\alpha)x_2 \in X\ \forall \alpha \in [0,1]$	Convex set

1) Let X_1 and X_2 be convex invariant sets for the system $x^+ = Ax$. Show that $\alpha X_1 \oplus (1-\alpha)X_2$ is also an invariant set for any $\alpha \in [0,1]$.

2) Let $X_1 \subseteq \mathbb{X}$ and $X_2 \subseteq \mathbb{X}$, where X_1, X_2 and $\mathbb{X}$ are convex sets. Show that $\alpha X_1 \oplus (1-\alpha)X_2 \subseteq \mathbb{X}$ for any $\alpha \in [0,1]$.

3) Let $V_i(x) := x^T P_i x$ be a Lyapunov function for the system $x^+ = Ax$ for $i = 1, 2$, with a rate of decrease of $x^T \Gamma x$, i.e.:

$$V_i(x^+) - V_i(x) \leq -x^T \Gamma x \ .$$

Show that $V(x) = \alpha V_1(x) + (1-\alpha)V_2(x)$ is also a Lyapunov function with a rate of decrease of $x^T \Gamma x$ for any $\alpha \in [0,1]$.

4) Let K be a stabilizing controller for the system $x^+ = Ax + Bu$, and $X_i \subset \mathbb{X}$ be a convex invariant set for the system $x^+ = (A + BK)x$, with $KX_i \subset \mathbb{U}$ for each $i = 1, 2$. $V_i(x) = x^T P_i x$ are Lyapunov functions for the system $x^+ = (A + BK)x$ with a rate of decrease of $Q + K^T RK$, for some $Q = Q^T \succ 0$ and $R = R^T \succ 0$.

$$
\begin{aligned}
J^\star(x(t)) = \min \ & \sum_{i=0}^{N} x_i^T Q x_i + u_i^T R u_i + \alpha V_1(x) + (1-\alpha) V_2(x) \\
\text{s.t. } & x_{i+1} = Ax_i + Bu_i, \ i = 0, \dots, N \\
& x_i \in \mathbb{X}, \ i = 1, \dots, N \\
& u_i \in \mathbb{U}, \ i = 0, \dots, N-1 \\
& x_N \in \alpha X_1 \oplus (1-\alpha) X_2 \\
& x_0 = x(t)
\end{aligned}
$$

Prove that this MPC controller is stabilizing and recursively feasible for any $\alpha \in [0, 1]$ by

i) listing sufficient conditions for stability and

ii) proving them

Hint: You can use the results of the previous three questions, even if you couldn't answer them.

Problem 5. / 15

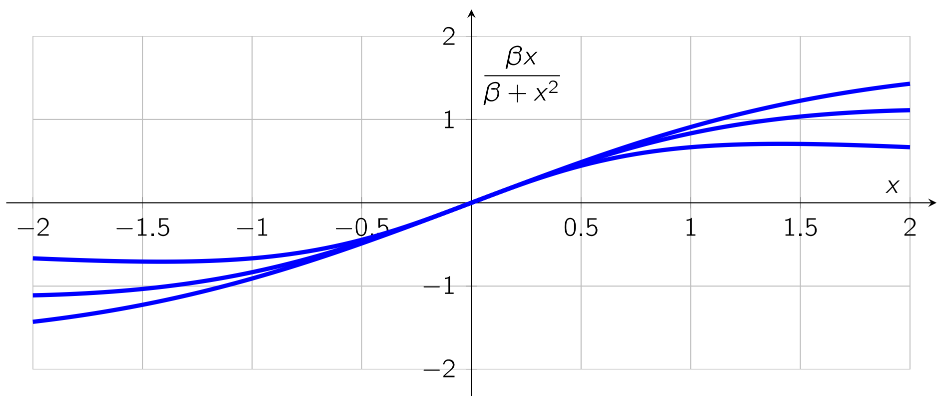
Consider the following nonlinear discrete-time dynamic system.

$$x^+ = \frac{\beta x}{\beta + x^2} + u := f(x,u)$$

subject to the state and input constraints $X = \{x \mid |x| \leq 1\}$ and $U = \{u \mid |u| \leq 0.2\}$. Treat β symbolically as a constant $\beta > 1$.

Your goal is to design a robust **linear** MPC controller for this nonlinear system to regulate it to the origin.

The function $\frac{\beta x}{\beta + x^2}$ is shown in the plot below for $\beta \in \{2, 5, 10\}$.



1) Find values a and b by linearizing the system around the origin and the smallest $\bar{w}$ such that the evolution of the following uncertain linear dynamic system contains that of the nonlinear system above for all $x \in X$ and all $u \in U$.

$$x^+ = ax + bu + w \qquad w \in W = \bar{w} \cdot [-1, 1] \tag{1}$$

$a =$ ______ $b =$ ______ $\bar{w} =$ ______

Hint: Ensure that $\forall (x,u) \in X \times U \; \exists w \in W$ such that $ax + bu + w = f(x,u)$

2) Compute the maximum robust invariant set X_f for the linear system in the previous part within the constraint set X for the control law $u = -0.5x$. Use the value $\bar{w} = 0.1$ for this question.

$X_f =$ ______

3) Fill in the blanks in the following set so that any controller $u(x) \in C(x)$ will ensure robust constraint satisfaction for system (1). Use the value $\bar{w} = 0.1$ for this question.

$$C(x) := \left\{ u_0 \;\middle|\; \exists u_1 \begin{pmatrix} \_\_ \\ \_\_ \\ \_\_ \\ \_\_ \end{pmatrix} \leq \begin{bmatrix} \_\_ & \_\_ & \_\_ \\ \_\_ & \_\_ & \_\_ \\ \_\_ & \_\_ & \_\_ \\ \_\_ & \_\_ & \_\_ \end{bmatrix} \begin{pmatrix} u_0 \\ u_1 \\ x \end{pmatrix} \leq \begin{pmatrix} \_\_ \\ \_\_ \\ \_\_ \\ \_\_ \end{pmatrix} \right\}$$

4) Define the controller $u^\star(x) = \text{argmin}_{u \in C(x)} u^T u$, where $C(x)$ is the function from the previous part. Will the dynamic system $x^+ = f(x, u^\star(x))$ satisfy the constraints $x \in X$ and $u \in U$? Explain your answer.

◯ Yes ◯ No ◯ Not enough information

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Problem 6.

Consider the following scalar system, which has the nonlinearity γ

$$\dot{x} = f(x, u) = x + u + \gamma(x) \tag{2}$$

subject to the constraints $|x| \leq 25$ and $-1 \leq u \leq 15$.

1) Discretize the system $x^+ = \phi(x, u)$ with a sample period of 1 second using Runge-Kutta 2. Assume that during the sample period, the input is constant, i.e., $u(t) = u_i$ for all $t \in [hi, h(i+1)]$ for $i \in \{0, 1, 2, \ldots\}$.

$x^+ = \phi(x, u) \approx$

2) Linearize the discrete-time system around the point $x_s = 0$ and $u_s = 0$ and for $\gamma(x) = \sin(x)$

$x^+ \approx$

3) Design a stabilizing and recursively feasible linear MPC controller for the linear discrete-time model computed in the last part.

4) If the input from your linear MPC controller is applied to the system (2), will the closed-loop system necessarily satisfy the constraints?

◯ Yes ◯ No

If yes, prove that this is the case.

If no, give suggestions for how you can modify your controller to ensure that it does.