Correction Series 2.

Exercice 1

(a) We have seen during the lecture that the surface intensity of a tungsten filament ($T = 2500\,\mathrm{K}$ and $\varepsilon = 0.35$) is

$$I_{\text{fila.}} \simeq 0.77\,\mathrm{W/mm^2}. \tag{1}$$

Since the emitting surface is the lateral surface of a cylinder: $S_{\text{fila.}} = 2\pi rh$, the numerical data give:

$$S_{\text{fila.}} \simeq 2 \times 3.14 \times 0.5 \times 20 \simeq 62.8\,\mathrm{mm^2},$$

and we conclude that the electro-magnetic power of the bulb is:

$$P_{\text{bulb}} = I_{\text{fila.}} S_{\text{fila.}} \simeq 0.77 \times 62.8 \simeq 48.4\,\mathrm{W}. \tag{2}$$

(b) We have seen during the lecture that the image intensity can be estimated by the formula:

$$I_{\text{image}} = \frac{1}{4} I_{\text{fila.}} \left(\frac{R_{\text{l}}}{f}\right)^2$$

where R_{l} is the radius of the lens and f its focus. With the numerical data for R_{l} and f and using the value (1) of the source intensity, we get that

$$I_{\text{image}} = \frac{1}{4} \times 0.77 \times \left(\frac{25}{150}\right)^2 \simeq 0.00534\,\mathrm{W/mm^2}. \tag{3}$$

(c) At a large distance R, the bulb emission can be approached by a spherical wave. The value of the intensity depends on the power through the formula:

$$I_0(R) = \frac{P_{\text{bulb}}}{4\pi R^2}.$$

Therefore we will have $I_0(R) = I_{\text{image}}$ if

$$R = \sqrt{\frac{P_{\text{bulb}}}{4\pi I_{\text{image}}}}.$$

Using the value (2) of the power and the value (3) of the image intensity, we get that

$$R \simeq \sqrt{\frac{48.4}{4 \times 3.14 \times 0.00534}} \simeq 26.85\,\mathrm{mm}.$$

Which means that focusing, in the case of a bulb, is not the only solution to increase intensity. Approaching the bulb might be more efficient.

Exercice 2

(a) We have seen during the lecture that the surface intensity on the sun is

$$I_{\text{sun}} \simeq 63\,\text{W/mm}^2. \tag{4}$$

Since the sun surface $S_{\text{sun}} = 4\pi R_{\text{sun}}^2 \simeq 6.084\times 10^{24}\,\text{mm}^2$, we conclude that its electro-magnetic power is:

$$P_{\text{sun}} = 63 \times 6.084 \times 10^{24} \simeq 3.833 \times 10^{26}\,\text{W}. \tag{5}$$

(b) We have seen during the lecture that the image intensity can be estimated by the formula:

$$I_{\text{image}} = \frac{1}{4} I_{\text{sun}} \left(\frac{R_{\text{l}}}{f}\right)^2$$

where R_{l} is the radius of the lens and f its focus. With the numerical data for R_{l} and f and using the value (4) of the surface intensity, we get that

$$I_{\text{image}} = \frac{1}{4} \times 63 \times \left(\frac{25}{150}\right)^2 \simeq 0.438\,\text{W/mm}^2. \tag{6}$$

(c) At a large distance R from its center: $R \gg R_{\text{sun}}$, the sun emission can be approached by a spherical wave. The value of the intensity depends on the power through the formula:

$$I_0(R) = \frac{P_{\text{sun}}}{4\pi R^2}.$$

Therefore we will have $I_0(R) = I_{\text{image}}$ if

$$R = \sqrt{\frac{P_{\text{sun}}}{4\pi I_{\text{image}}}}.$$

Using the value (5) of the power and the value (6) of the image intensity, we get that

$$R \simeq \sqrt{\frac{3.833 \times 10^{26}}{4 \times 3.14 \times 0.438}} \simeq 8.345 \times 10^{12}\,\text{mm} = 8.345 \times 10^{6}\,\text{km}.$$

Since the intensity collected at the image is probably able to burn paper, we expect the location R to be far away from earth. It is the case, because the distance earth to sun is of about 228×10^6 km.
In the case of the sun, focusing is certainly a good solution to increase intensity. Getting closer to the sun is also a possibility to increase intensity in the same proportion, but a spacecraft would be needed to travel millions of km.