# Fatigue of Material : Some introductory documents

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## Beyond Strength of Materials

Strength of Materials, or Mechanics of Materials, is an engineering subject discussing the ultimate strength, yield stress, and stiffness of a material and the structure it forms. The subject of Mechanics of Materials generally presumes that the initially flawless material is ideally connected and perfectly assembled into the designed shapes. The failure criteria depend solely on the ultimate strength or yield stress of the material. Unfortunately, in reality, many other factors may shorten the service life of a structure. Some of these factors are:

- *Material defects*: flaws, voids, dislocations.
- *Surface roughness and surface treatments*: scratches, pits, machining marks, electroplating, stamping.
- *Assembling induced imperfection*: press-fit,
- *Functionality requirements*: sharp corners, grooves, nicks.
- *Size*: everything being equal, the larger the size the more initial defects.
- *Loading types*, e.g., tri-axial, bi-axial, axial, bending, torsion, combined loadings.
- *Harsh environments*, e.g., thermal loadings (temperature changes), corrosion, UV light.
- *Damage in service*, e.g., repeated loading, large dead loads, vibrations, impacts, other unexpected loadings.
- *Poor maintenance and improper repair*, e.g., lack of lubrication, wear at joins or surfaces, not enough or unfit reinforcements in repair.

To overcome all these odds, safety factor is introduced in common industrial practices. Nevertheless, even with safety factors, it is estimated 80%+ structural failures occurs through a fatigue related mechanism.

Top of Page

## Definition of Fatigue

There are many harmful factors to the materials beyond the scope of strength of materials as discussed in the previous section. The accumulation of one or several of these factors eventually shorten the service life of materials. The combined effect of these factors is called "fatigue mechanism". Some common fatigue mechanisms include

- Time-varying Loading Fatigue
- Thermal Fatigue
- Corrosion Fatigue
- Surface/Contact Fatigue
- Combined Creep and Fatigue

According to ASTM International (originally American Society for Testing and Materials), fatigue is "the process of progressive localized permanent structural change occurring in a material subjected to conditions which produce fluctuating stresses and strains at some point or points and which may culminate in cracks or complete fracture after a sufficient number of fluctuations."

Top of Page

## Fatigue Failure upon Time-Varying Loading

**Time-varying Loading Fatigue** can be defined as a process caused by time-varying loads which never reach a high enough level to cause failure in a single application, and yet results in progressive localized permanent damages on the material. The damages, usually cracks, initiate and propagate in regions where the strain is most severe. When the local damages grow out of control, a sudden fracture/rupture ends the service life of the structure. Common categories and approaching methods include:

- **High-cycle fatigue**, associated with low loads and long life ($>10^3$ cycles), is commonly analyzed with a "stress-life" method (the *S*-*N* curve), which predicts the number of cycles sustained before failure, or with a "total-life" method (endurance limit), which puts a cap stress that allows the material to have infinite life ($>10^6$ cycles).
- **Low-cycle fatigue**, associated with higher loads (plastic deformation occurs) and shorter life ($<10^3$ cycles), is commonly used methods called "strain-life" to analyze or predict the fatigue life.
- Crack-growth approach uses **fracture mechanics** to examine

the propagation of a crack to predict its growth in length per cycle, and to determine the amount of loading that will result in true failure.

Top of Page

## Fatigue Reduction

The effort of fatigue prevention/reduction and service life extension can be summarized as

- Use stronger, more capable materials
- Reduce the margin of errors in assembly and manufacture
- Avoid, soften when inevitable, stress concentrations
- Keep residual stress at surface, if any, in compression
- Take service environment into account
- Schedule routine maintenance, firm and thorough

### High cycle fatigue

(src: http://www.efunda.com/formulae/solid_mechanics/fatigue/fatigue_highcycle.cfm)

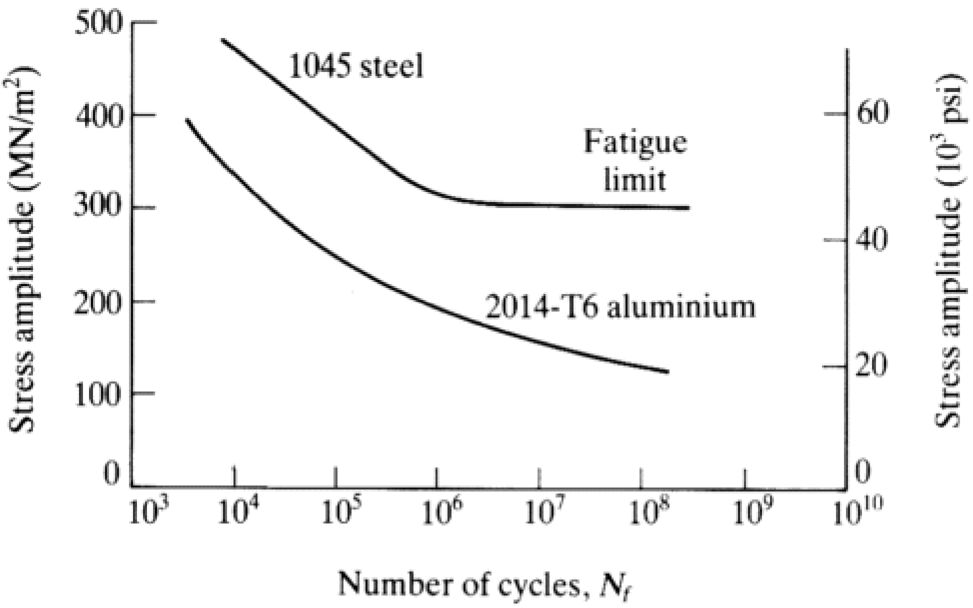
When the fatigue occurs above $10^3$ cycles (usually $10^4$ or more), it is usually called*High-cycle fatigue*. The material is subject to lower loads, usually less than 2/3 of the yield stress. The deformation is in elastic range. The fatigue life is "high-cycle" ($10^3 \sim 10^6$).

## The *S-N* Curve

The *S*-*N* curve, a.k.a., Stress Life Method, is the basic method presenting fatigue failure in high cycles ($N > 10^5$) which implies the stress level is relatively low and the deformation is in elastic range.

The *S*-*N* curve for a specific material is the curve of nominal stress *S* (*y* axis) against the number of cycles to failure *N* (*x* axis). A log scale is almost always used for *N*. The stress is usually nominal stress and is no adjustment for stress concentration. The curve is usually obtained one by reversed bending experiments with zero mean stress.

The *S*-*N* curve of 1045 steel and 2014-T6 aluminum alloy is enclosed below to represent two tipical *S-N* curves of metal materials.




The 1045 steel, as well as some other steels and titanium alloys, exhibit a *fatigue limit*. When the amplitude of repeat loading is below the fatigue limit, small stresses do not shorten the fatigue life of the material. On the other hand, the 2014-T6 aluminum alloy and most metal materials do not have the fatigue limit, i.e., small stresses will eventually cause failure.

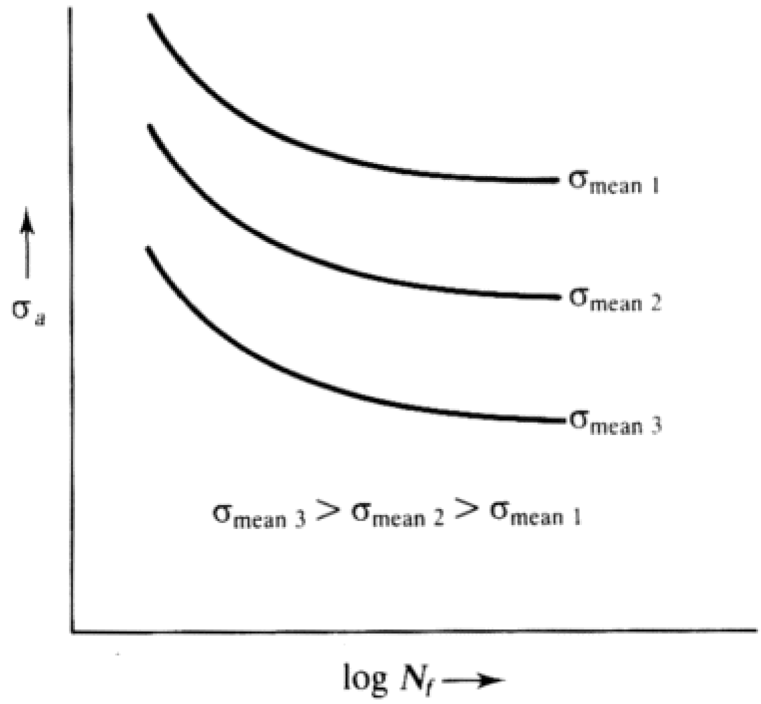
In short, the *S-N* curve is used to predicts the number of cycles sustained under certain stress before failure. The curve gives designers a quick reference of the allowable stress level for an intended service life.

Top of Page

## Mean Stress Effect on Fatigue

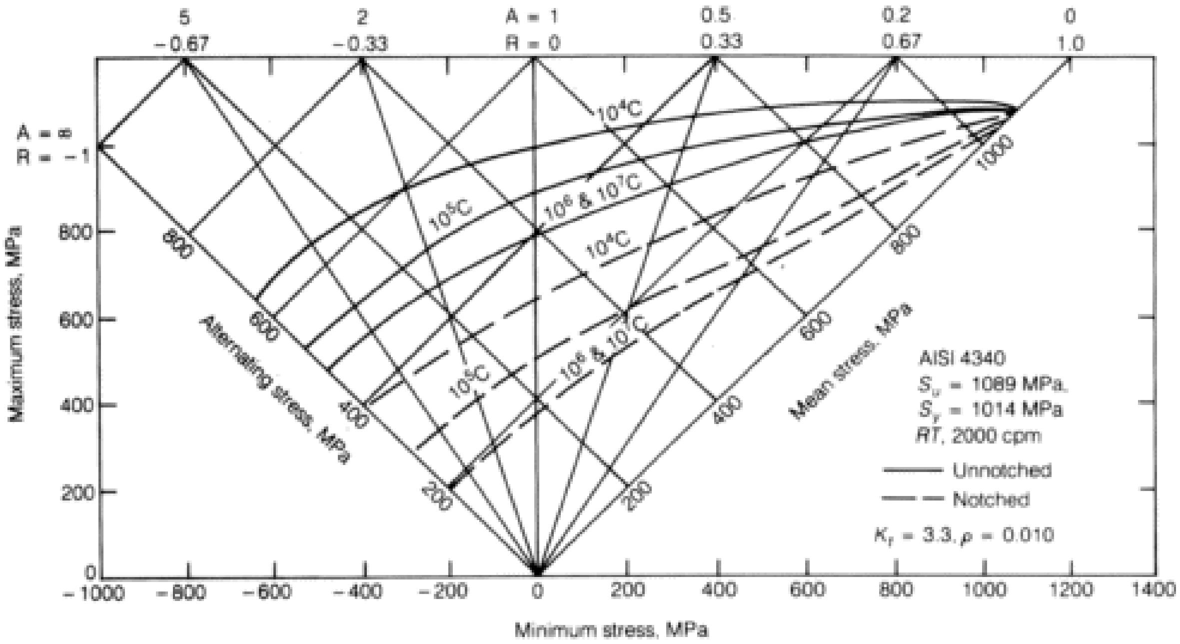
While *S-N* curve is clear and straight forward on addressing the service life under fatigue, its accuracy leaves some room to be improved. Partially because of the statistical nature of fatigue and Partially because of the difference between laboratory experiments and the real-life practice.

For example, most *S-N* curves are constructed based on zero mean stress. However, it is more often the time-varying stresses are oscillating near a non-zero mean stress. Multiple *S-N* curves are determined by a several sets of fatigue experiments. Each curve represents a specific mean stress of that particular material.




The non-zero mean stress *S-N* relation requires huge amount of experiments to obtain the required data and form the mesh over a wide range of mean stresses. There are two approaches to present the data. The first is to present it in a diagram format. The second is to resemble the data with a formula based on the zero-mean stress *S-N*curve.

A more popular diagram for design purposes is called *master diagram* which accumulates fatigue data under different mean stresses and presents each line as the fatigue life under the net of maximum and minimum stresses in addition to mean stress and alternating stress as the reference axises. An example of master diagram of AISI 4340 steel is enclosed for your reference.




Users may check the maximum and minimum stress directly. Define *R* is the ratio of minimum stress to the maximum stress. Alternatively, define *A* is the ratio of alternating stress to mean stress.

$$R = \frac{\sigma_{\min}}{\sigma_{\max}}$$

$$A = \frac{\sigma_a}{\sigma_m} = \frac{1-R}{1+R}$$

An approximation based on the zero-mean stress *S-N* curve proposed by Goodman and Gerber is written as

$$\sigma_a = \sigma_{f0}\left[1-\left(\frac{\sigma_m}{\sigma_u}\right)^r\right]$$

where $\sigma_a$ is the amplitude of allowable stress (alternating stress).

$\sigma_{f0}$ is the stress at fatigue fracture when the material under zero mean stress cycled loading.

$\sigma_m$ is the mean stress of the actual loading.

$\sigma_u$ is the tensile strength of the material.

*r* = 1 is called Goodman line which is close to the results of notched specimens.
*r* = 2 is the Gerber parabola which better represents ductile

metals.

Please note that the *S-N* curve and its elaborated master diagram require a lot of experiments to accumulate the necessary data. On the other hand, Goodman and Gerber's approximations, although simple, they might not properly represent the specific material. Finally, they are only good for uniform mean and alternating stresses.

Top of Page

## Combined Effect of a Sequence of Loads

With the complexity of a master diagram, not to mention the time and effort to create one, the fatigue prediction is still less that perfect. In practice, a mechanical component is exposed to a complex, often random, sequence of loads, large and small and different mean values.

The procedure to establish the combined effect of a sequence of loads may involve

1. Simplify and divide the complex loading to a series of simple cyclic loadings
2. Create a histogram of cyclic stress
3. For each stress level, calculate the degree of cumulative damage incurred from the *S-N* curve
4. Combine the individual contributions to the total effect

Palmgren (1924) and Miner (1945) suggested an algorithm to combine individual contributions, known as Palmgren-Miner's linear damage hypothesis or Miner's rule.

$$\sum_{i=1}^{k} \frac{n_i}{N_i} = c$$

where $k$ is the total number of different stress magnitudes in a spectrum

$S_i$ ($1 <= i <= k$) is the magnitudes of each different stress in a spectrum

$n_i(S_i)$ is the actual number of cycles under the specific stress $S_i$

$N_i(S_i)$ is the total number of cycles to failure under the specific stress $S_i$

0.7 < $c$ < 2.2 is an material dependent constant obtained by experiments. Set $c$ = 1, if there is no further information available.

Miner's rule assume the fatigue life is consumed by the linear combination of different portion of stress state, both cycles and magnitude. This approximation, which is simple and straight forward, does not take the squences of loading history into account. For example, a serial of high stress loading, which weaken the material, followed by a serial low stress loading may cause more damage than a serial of low stress loading followed by a serial of high stress loading. But Miner's rule can not catch this effect.

Finally, the probabilistic nature of fatigue makes Miner's rule look over simplified.